Unit 1: Algebraic Tools for Operating with Functions: Rational Expressions

1. Simplify. State any restrictions on the variables.

a) $(4x^2-7x-7)-(8x^2-5x-9)$

b) $2(x-3)^2-(2x+1)(3x+2)$

c) $\dfrac{3x-3y}{5x-5y}$

d) $\dfrac{x^2-16}{x^2-x-12}$

e) $\dfrac{x^2+2x-3}{x^2+6x+8}\times\dfrac{x^2+2x-8}{x^2+x-6}$

f) $\dfrac{2x^2-x-1}{3x^2+x-2}\div\dfrac{2x^2-3x-2}{3x^2-11x+6}$

g) $\dfrac{x+2}{3}+\dfrac{2x-1}{4}-\dfrac{3x+1}{2}$

h) $\dfrac{4}{2x-3}-\dfrac{1}{3-2x}$

i) $\dfrac{2}{x^2+5x+4}-\dfrac{3}{x^2-3x-4}$

j) $\dfrac{x+1}{3x^2+4x+1}+\dfrac{2x-1}{3x^2-5x-2}$

Unit 2: Radical Mathematics and Quadratic Functions

2. Simplify.

a) $\sqrt{50}$

b) $\sqrt{44}$

c) $2\sqrt{3}\times\sqrt{6}$

d) $\dfrac{\sqrt{72}}{\sqrt{6}}$

e) $5\sqrt{10}\times3\sqrt{2}$

f) $\left(2\sqrt{5}\right)^2$

g) $\dfrac{8-\sqrt{40}}{2}$

h) $\dfrac{15\sqrt{48}}{5\sqrt{3}}$

i) $\sqrt{48}-\sqrt{27}+\sqrt{12}$

j) $\sqrt{6}(3\sqrt{2}+2\sqrt{8})$

k) $(2-\sqrt{3})(1+3\sqrt{3})$

l) $\dfrac{2}{\sqrt{7}}$

m) $\dfrac{3}{\sqrt{3}-4}$

n) $\dfrac{5}{2\sqrt{6}+\sqrt{3}}$

3. Solve by factoring.

a) $2x^2-7x=4$

b) $3x^2=6-7x$

4. Solve using the quadratic formula.

a) $x^2-5x=13$

b) $3x^2=-3x+7$

5. Complete the square. State the maximum or minimum value of each function and the value of x when it occurs.

a) $y=x^2-7x+2$

b) $y=-4x^2-8x+5$

c) $y=-2x^2+5x+5$

d) $y=\frac{1}{2}x^2-4x+6$

6. A picture that measures 10 cm by 8cm is to be surrounded by a mat before being framed. The width of the mat is to be the same on all sides of the picture. The area of the mat is to equal the area of the picture. What is the width of the mat to the nearest tenth of a centimeter?

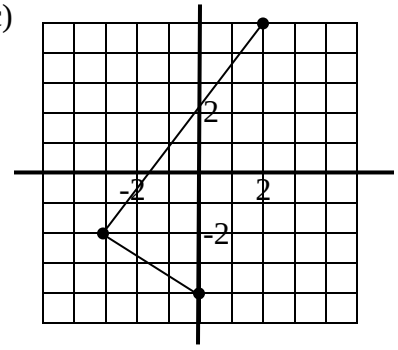
7. Two whole numbers differ by 3. The sum of their squares is 89. What are the numbers?

8. The function $h = -5t^2 + 20t + 2$ gives the approximate height, *h* metres of a thrown football as a function of the time, *t* seconds since it was thrown. The ball hit the ground before a receiver could get near it.
   a) How long was the ball in the air, to the nearest tenth of second?
   b) For how many seconds was the height of the ball at least 17 m?
   c) What is the maximum height of the ball?

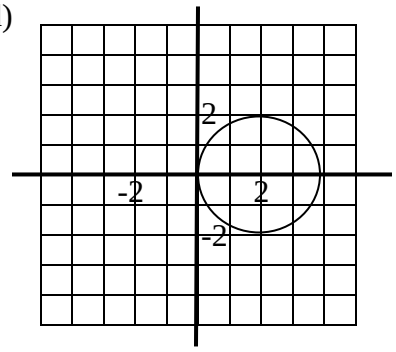
9. The difference between the length of the hypotenuse and the length of the next longest side of a right triangle is 3 cm. The difference between the lengths of the two perpendicular sides is 3 cm. Find the three side lengths.

Unit 3: Transformations of Functions

10. For each of the following, state the domain, range and whether or not it is a function.
   a) $\{ (2,4),\ (3,5),\ (7,9)\ (2,-5)\ ,(3,-7) \}$
   b) $\{ (-1,6),\ (0,-6),\ (1,-6)\ (2,-6) \}$
   c)



   d)



11. If $f(x) = 3 - 2x^2$, find:
   a) $f(5)$
   b) $f\left(-\frac{1}{2}\right)$

12. Describe the transformations of the following functions from the graph of $f(x)$.
   a) $y = f(x-2) - 3$
   b) $y = -f(x+5) - 1$
   c) $y = \frac{1}{3} f(-3x) + 5$
   d) $y = -2f(2(x+3)) + 6$

13. Describe the transformations on $f(x) = x^2$ required to graph $y = -\frac{1}{4}\left[\frac{1}{2}(x-8)\right]^2$.

14. Find the inverse of each function. Is the inverse a function? Explain.
   a) $y = 3x - 5$
   b) $y = x^2 - 7$
   c) $y = (x+2)^2$
   d) $y = \sqrt{x-3}$

15. i) Use transformations to sketch the graphs of each of the following pairs of functions on the same set of axes. The first function is the Parent/Base Function.
   a) $y = \sqrt{x}$ and $y = \sqrt{x} - 4$
   b) $y = x^2$ and $y = -\frac{1}{2}(x+1)^2 - 3$

c) $y = 2^x$ and $y = -2^x$

d) $y = \frac{1}{x}$ and $y = \frac{3}{x+2}$

ii) State the domain and range of each function.

16. The graph of $y = x^2$ is stretched vertically by a factor of 2, translated 3 units to the left and translated 4 units upward. Write the equation of the transformed function and state its domain and range.

17. Given $f(x) = x^2 + 6x$

a) Write equations for $-f(x)$ and $f(-x)$.

b) Sketch the three graphs on the same set of axes.

c) Determine any points that are invariant for each reflection.

18. Copy and complete the chart below.

Relation	Rough Sketch	Domain	Range	Function? Yes or No
a) $y = 3x$				
b) $y = 2(x-1)^2 - 4$				
c) $y = -\sqrt{x} + 2$				
d) $y = \frac{1}{x}$				
e) $y = 3^x$				
f) $x^2 + y^2 = 25$				

Unit 4: Exponential Functions

19. Simplify. Express each answer with positive exponents.

a) $x^{-1} \cdot x^{-3} \cdot x^2$

b) $(x^{-1}y^2)^{-2}$

c) $5x^4 \cdot 3x^2$

d) $(6x^{-1}y^2)(-x^{-3}y^{-4})$

e) $\frac{3xy^3 \times 10x^4y^2}{15x^2y^6}$

f) $\left(\frac{4x^{-3}y^4}{8x^2y^{-2}}\right)^{-2}$

20. Use exponent laws to evaluate the following. NO DECIMALS!!

a) 5^{-2}

b) 6^0

c) $(-3)^{-4}$

d) $\frac{x^0 + 3^2}{2^4 - y^0}$

e) $25^{\frac{1}{2}}$

f) $\left(\frac{1}{27}\right)^{\frac{1}{3}}$

g) $(-32)^{\frac{4}{5}}$

h) $\left(\frac{81}{16}\right)^{\frac{5}{4}}$

i) $\left(\frac{27}{125}\right)^{-\frac{2}{3}}$

21. Express using exponents. Simplify where necessary.

a) $\sqrt[3]{-x}$ b) $\sqrt{\sqrt[3]{x^2}}$ c) $\left(\sqrt{x^3}\right)\left(\sqrt{x}\right)$

22. An insect colony, with an initial population of 50, triples every day.

(a) Which function models this exponential growth:

A: $p(n) = 50 \times 2^n$ B: $p(n) = 150 \times 3n$ C: $p(n) = 50 \times 3^n$

(b) For the correct model, explain what each part of the equation means.

23. Shylo is very excited about her brand new car! Although she paid $20,000 for the car, its resale value will depreciate (decrease) by 30% of its current value every year. The equation relating the car's depreciated value, *v*, in dollars, to the time, *t*, in years since her purchase is $v(t) = 20000(0.7)^t$.

(a) Explain the significance of each part of this equation.

(b) How much will Shylo's car be worth in

(i) 1 year? (ii) 2 years?

(c) How long will it take for Shylo's car to depreciate to 10% of its original price?

24. (a) Is an exponential function either always increasing or always decreasing? Explain.

(b) Is it possible for an exponential function of the form $y = ab^x$ to have an x-intercept? If yes, give an example. If no, explain why not.

25. Match each transformation with the corresponding equation, using the function $y = 10^x$ as the base. Give reasons for your answers. Not all transformations will match an equation.

Transformation

(a) horizontal stretch by a factor of 3
(b) shift 3 units up
(c) shift 3 units left
(d) vertical compression by a factor of $\frac{1}{3}$
(e) vertical stretch by a factor of 3
(f) shift 3 units right
(g) reflect in the *x*-axis

Equation

A $y = 10^x + 3$
B $y = 10^{x+3}$
C $y = -10^x$
D $y = 10^x - 3$
E $y = 10^{3x}$
F $y = 10^{-x}$
G $y = \left(\frac{1}{3}\right)10^x$

26. (a) Describe the transformations that must be applied to the graph of $y = 3^x$ to obtain the graph of : i) $y = 5(3)^{2x} - 1$ ii) $y = -\left(\frac{1}{3}\right)^{12-3x} + 2$.

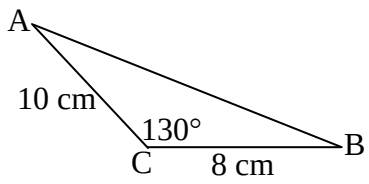
(b) Graph each function from part a).

(c) Identify the following properties of the transformed function.

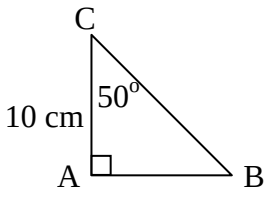
(i) domain
(ii) range
(iii) equation of the asymptote
(iv) intercept(s), if they exist

27. Determine the value of c to one decimal place.

a)



b)



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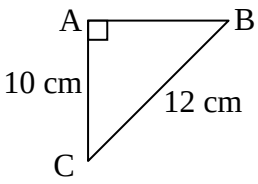
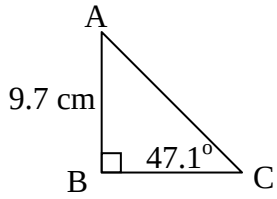


28. Solve each triangle. Round each side length and angle to the nearest tenth.

a)



b) In ΔKLM, $\angle K = 90^\circ$, $m = 12.4\ cm$ and $l = 8.8\ cm$.

29. The Toronto Stock Exchange is housed in the Exchange Tower. From the top of the building, the angle of depression to a point on the ground 100 m from the foot of the building is 55.6°. Determine the height of the building, to the nearest metre.

30. The point (20, 21) is on the terminal arm of an angle θ in standard position. Find $\sin\theta$ and $\cos\theta$.

31. Find $\angle A$ to the nearest tenth of a degree, if $0^\circ \le A \le 180^\circ$.
a) $\sin A = 0.6157$ b) $\cos A = 0.2756$ c) $\cos A = -0.8988$

32. Solve each triangle. Round each side length and angle to the nearest tenth.
a) In ΔABC, $\angle A = 52.5^\circ$, $\angle B = 73.4^\circ$ and $b = 36.6\ cm$
b) In ΔRST, $r = 12.6\ m$, $s = 11.5\ m$ and $t = 13.2\ m$
c) In ΔEFG, $\angle F = 67.8^\circ$, $f = 12.6\ m$ and $e = 9.8\ m$

33. An isosceles triangle has two 5.5 cm sides and two 32.4° angles. Find:
a) the perimeter of the triangle to the nearest tenth of a centimetre.
b) the area of the triangle, to the nearest tenth of a square centimetre.

34. Airport X is 150 *km* east of airport Y. An aircraft is 240 km from airport Y and 23° north of due west from airport Y. How far is the aircraft from airport X to the nearest kilometer?

35. Two ships left Port Hope on Lake Ontario at the same time. One travelled at 12 km/h on a course of 235°. The other travelled at 15 km/h on a course of 105°. How far apart were the ships after four hours to the nearest kilometer?

36. Determine the number of triangles that could be drawn with the given measures. Then, find the measures of the other angles and the other side in each possible triangle.
a) In ΔGHI, $\angle G = 20^\circ$, $g = 2\ cm$ and $h = 5\ cm$
b) In ΔXYZ, $\angle X = 43^\circ$, $x = 2\ m$ and $y = 4\ m$
c) In ΔABC, $\angle B = 104.5^\circ$, $c = 1.4\ m$ and $b = 3.9\ m$

37. The coordinates of a point P on the terminal arm of an angle, θ, in standard position, where $0 \le \theta \le 360^o$. Determine the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.
a) P(4, 5) b) P(7, -4)

38. Find the exact value of each trigonometric ratio:
a) $\tan 225^\circ$ b) $\cos 150^o$

39. If $0^\circ \le \theta \le 360^\circ$, find the possible measures of $\angle A$:
a) $\cos A = \frac{1}{\sqrt{2}}$ b) $\tan A = -\sqrt{3}$

40. Sketch one cycle of the graph of each of the following. State the domain, range, amplitude, period, vertical translation (when necessary!) and phase shift (when necessary!).
a) $y = \sin x$ b) $y = -2\sin 2x + 2$ c) $y = \frac{1}{2}\sin\left(x + 45^o\right)$

d) $y = \cos x$ e) $y = 3\cos\frac{1}{3}x$ f) $y = 2\cos\frac{1}{2}\left(x - 180^o\right) + 1$

41. Prove each identity.
a) $\frac{1-\sin^2 x}{\cos x} = \cos x$ b) $1 + \tan^2 x = \frac{1}{\cos^2 x}$ c) $\frac{1}{\sin x} - \sin x = \frac{\cos x}{\tan x}$

d) $\frac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x$ e) $(1-\cos^2 x)(1+\tan^2 x) = \tan^2 x$ f) $(\sin x - \cos x)^2 = 1 - 2\sin x\cos x$

g) $(1+\cot^2 x)\tan^2 x = \sec^2 x$ h) $\sin x \sec x = \tan x$ i) $\tan x(1+\cot x) = 1 + \tan x$

42. Solve each equation for $0 \le x \le 360^o$.
a) $\sin x = \frac{-\sqrt{3}}{2}$ b) $\sqrt{2}\cos x + 1 = 0$ c) $2\sin x - 1 = 0$

d) $\tan x = \sqrt{3}$ e) $(\sqrt{2}\cos x + 1)(\sin x - 1) = 0$ f) $2\cos^2 x + 3\cos x = -1$

g) $\cos x + 1 = 2\sin^2 x$ h) $\cos^2 x - 1 = \sin^2 x$ i) $15\sin^2 x + \sin x = 2$

Unit 7: Sequences and Series

$t_n = a + (n-1)d$	$t_n = ar^{n-1}$	$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_n = \frac{n}{2}[a + t_n]$	$S_n = \frac{a(r^n - 1)}{r - 1}$	

43. Find the formula for the *n*th term and find the indicated term for each arithmetic sequence.
a) $3,\ 5,\ 7,\ \ldots\ ; t_{30}$ b) $-4,\ 3,\ 10,\ \ldots\ ; t_{18}$

44. Find the number of terms in each arithmetic sequence.
a) $4,\ 9,\ 14,\ \ldots\ ,\ 169$ b) $19,\ 11,\ 3,\ \ldots\ , -229$

45. The Women's World Cup of Soccer tournament was first held in 1991. The next two tournaments were held in 1995 and 1999.
a) Write a formula for finding the year in which the nth tournament will be held.
b) Predict he year of the 35^{th} tournament.

46. Find the formula for the *n*th term and find the indicated term for each geometric sequence.
a) $27,\ 9,\ 3,\ \ldots\ ; t_6$ b) $1,\ -3,\ 9,\ \ldots\ ; t_7$

47. Find $a,\ r,\ and\ t_n$ for each geometric sequence.
a) $t_4 = 24\ and\ t_6 = 96$ b) $t_2 = -6\ and\ t_5 = -162$

48. Use the recursion formula to write the first 5 terms of each sequence.
a) $t_1 = 3;\ t_2 = 3;\ t_n = t_{n-1} + t_{n-2}$ b) $f(1) = 8;\ f(n) = 0.5 f(n-1)$

49. Identify whether the series is Arithmetic or Geometric. Then, find *n*.
a) $1 + 2 + 4 + \ldots + 1024$ b) $-5 - 2 + 1 + 4 \ldots + 133$ c) $16384 + 4096 + \ldots + 1$

50. Find the indicated sum for each arithmetic series.
a) S_{25} for $-20 - 18 - 16$ b) $1 + \frac{5}{4} + \frac{3}{2} + \ldots + 20$

51. The side lengths in a quadrilateral from an arithmetic sequence. The perimeter is 38 cm and the shortest side measures 5 cm. What are the other side lengths?

52. Find the indicated sum for each geometric series.
a) S_{12} for $4 - 8 + 16 - 32 + \ldots$ b) $3645 - 1215 + 405 - \ldots + 5$

53. A ball is kicked from the ground 6.4 m into the air. The ball falls, rebounds to 60% of its previous height and falls again. If the ball continues to rebound and fall in this manner, find the total distance the ball travels until it hits the ground for the fifth time (assume the ball bounces vertically with no curvature in its path).

ANSWER

UNIT 1

1a $-4x^2-2x+2$ 1b $-4x^2-19x+16$ 1c $\frac{3}{5},\ x \neq y$ 1d $\frac{x+4}{x+3},\ x \neq -3,\ 4$ 1e $\frac{x-1}{x+2},\ x \neq -4,\ -3,\ -2,\ 2$ 1f $\frac{(x-1)(x-3)}{(x+1)(x-2)},\ x \neq -1,\ -\frac{1}{2},\ \frac{2}{3},\ 2,\ 3$

1g $\frac{-8x-1}{12},$ 1h $\frac{5}{2x-3},\ x \neq \frac{3}{2}$ 1i $\frac{-x-20}{(x+1)(x+4)(x-4)},\ x \neq -4,\ -1,\ 4$ 1j $\frac{3x-3}{(3x+1)(x-2)},\ x \neq -1,\ -\frac{1}{3},\ 2$

UNIT 2

2a $5\sqrt{2}$ 2b $2\sqrt{11}$ 2c $6\sqrt{2}$ 2d $2\sqrt{3}$ 2e $30\sqrt{5}$ 2f 20

2g $4-\sqrt{10}$ 2h 12 2i $3\sqrt{3}$ 2j $14\sqrt{3}$ 2k $5\sqrt{3}-7$ 2l $\frac{2\sqrt{7}}{7}$

2m $-\frac{3(\sqrt{3}+4)}{13}$ 2n $\frac{5\left(2\sqrt{6}-\sqrt{3}\right)}{21}$ 3a $x=-\frac{1}{2},\ 4$ 3b $x=-3,\ \frac{2}{3}$ 4a $x=\frac{5\pm\sqrt{77}}{2}$ 4b $x=\frac{-3\pm\sqrt{93}}{6}$

5a $\min=\frac{-41}{4},\ x=\frac{7}{2}$ 5b $\max=9,\ x=-1$ 5c $\max=\frac{65}{8},\ x=\frac{5}{4}$ 5d $\min=-2,\ x=4$ 6 1.8 *cm* 7 5 and 8 or -5 and -8

8a 4.1 *s* 8b 2 *s* 8c 22 *m* 9 9, 12, 15 *cm*

UNIT 3

10a D: {2 ,3, 7}
R: {–7, –5, 4, 5, 9}
Not a function

10b D: {–1 ,0, 1, 2}
R: {–6, 6}
Function

10c D: $\{-3 \le x \le 2,\ x \in R\}$
R: $\{-4 \le y \le 5\ \ y \in R\}$
Not a function

10d D: $\{0 \le x \le 4,\ x \in R\}$
R: $\{-2 \le y \le 2\ \ y \in R\}$
Not a function

11a -47

11b $\frac{5}{2}$

12a Translated 2 units right
Translated 3 units down

12b Reflected in x-axis
Translated 5 units left
Translated 1 unit down

12c Reflected in y-axis
Vertical compression by factor of 3
Horizontal compression by factor of 1/3
Translated 5 units up

12d Reflected in x-axis
Vertical stretch by a factor of 2
Horizontal compression by factor of 2
Translated 3 units left
Translated 6 units up

13 Reflected in x-axis
Vertical compression by factor of ¼
Horizontal stretch by factor of 2
Translated 8 units right

14a $f^{-1}(x)=\frac{x}{3}+\frac{5}{3}$
Function

14b $f^{-1}(x)=\pm\sqrt{x+7}$
Not a function

14c $f^{-1}(x)=\pm\sqrt{x}-2$
Not a function

14d $f^{-1}(x)=x^2+3$
Function

15a Translated 4 units down

15b Reflected in x-axis
Vertical compression by factor of 1/2
Translated 1 unit left
Translated 3 units down

15c Reflected in x-axis

15d Vertical stretch by factor of 3
Translated 2 units left

16 $f(x)=2(x+3)^2+4$
D: $\{x \in R\}$
R: $\{y \ge 4,\ \ y \in R\}$

17a $-f(x)=-x^2-6x$
$f(-x)=x^2-6x$

17c $-f(x)$: (0, 0), (–6, 0)
$f(-x)$: (0, 0)

18a D: $\{x \in R\}$
R: $\{y \in R\}$
Function

18b D: $\{x \in R\}$
R: $\{y \geq -4,\ y \in R\}$
Function

18c D: $\{x \geq 0,\ x \in R\}$
R: $\{y \leq 2,\ y \in R\}$
Function

18d D: $\{x \neq 0,\ x \in R\}$
R: $\{y \neq 0,\ y \in R\}$
Function

18e D: $\{x \in R\}$
R: $\{y > 0,\ y \in R\}$
Function

18f D: $\{-5 \leq x \leq 5,\ x \in R\}$
R: $\{-5 \leq y \leq 5\ y \in R\}$
Not a function

UNIT 4

19a $\frac{1}{x^2}$

19b $\frac{x^2}{y^4}$

19c $15x^6$

19d $\frac{-6}{x^4y^2}$

19e $\frac{2x^3}{y}$

19f $\frac{4x^{10}}{y^{12}}$

20a $\frac{1}{25}$

20b 1

20c $\frac{1}{81}$

20d $\frac{2}{3}$

20e 5

20f $\frac{1}{3}$

20g 16

20h $\frac{243}{32}$

20i $\frac{25}{9}$

21a $(-x)^{\frac{1}{3}}$

21b $x^{\frac{1}{3}}$

21c x^2

22a C

22b $p(n) = 50 \times (3)^n$

50: Initial population
3: rate of increase
n: number of days

23a $v(t) = 20000 \times (0.7)^t$

20000: Initial value of the car
0.7: percent of value carried to the next year
t: number of years

23bi \$14 000

23bii \$ 9800

23c 6.5 years

24a Yes

24b No

25a $y = 10^{\frac{1}{3}x}$: not listed

25b A

25c B

25d G

25e $y = 3(10^x)$: not listed

25f $y = 10^{x-3}$: not listed

25g C

26ai Vertical stretch of factor 5
Horizontal compression of factor 1/2
Shift down 1 unit

26ci Domain:
$\{x \in R\}$
Range:
$\{y > -1, y \in R\}$

Asymptote:
$y = -1$
x-intercept:
$x \cong 0.73$

26aii Reflection over x-axis
Horizontal compression of factor 1/3
Shift 4 units right
Shift 2 units up

26cii Domain:
$\{x \in R\}$
Range:
$\{y < 2, y \in R\}$
Asymptote:
$y = 2$
x-intercept:
$x \cong 4.21$

UNIT 5

27a 16.3 cm

27b 11.9 cm

27c 6.6 cm

28a $A = 42.9^\circ$
$a = 9.0\ cm$
$b = 13.2\ cm$

28b $L = 35.4^\circ$
$M = 54.6^\circ$
$k = 15.2\ cm$

29 146 *m*

30 $\sin\theta = \frac{21}{29}$
$\cos\theta = \frac{20}{29}$

31a $A = 38.0^\circ$ *or*
$A = 142.0^\circ$

31b $A = 74.0^\circ$

31c $A = 154.0^\circ$

32a $C = 54.1^\circ$
$a = 30.3\ cm$
$c = 30.9\ cm$

32b $S = 52.9^\circ$
$R = 60.9^\circ$
$T = 66.2^\circ$

32c $G = 66.1^\circ$, $E = 46.1^\circ$, $g = 12.4\ m$

33a 20.3 *cm*

33b $13.5\ cm^2$

34 383 *km*

35 98 *km*

36a 2 Triangles
$H = 58.8^\circ$, $I = 101.2^\circ$, $i = 5.7cm$ *or* $H = 121.2^\circ$, $I = 38.8^\circ$, $i = 3.7cm$

36b 0 Triangles

36c 1 Triangle,
$C = 20.3^\circ$, $A = 55.2^\circ$, $a = 3.3m$

UNIT 6

37a $\sin\theta = \frac{5}{\sqrt{41}},\ \cos\theta = \frac{4}{\sqrt{41}}, \tan\theta = \frac{5}{4}$

37b $\sin\theta = \frac{-4}{\sqrt{65}},\ \cos\theta = \frac{7}{\sqrt{65}}, \tan\theta = -\frac{4}{7}$

38a 1

38b $-\frac{\sqrt{3}}{2}$

39a $A = 45^\circ,\ 315^\circ$

39b $A = 120^\circ,\ 300^\circ$

40a $D:\ \{0^\circ \le x \le 360^\circ,\ x \in R\}$
$R:\ \{-1 \le y \le 1\ \ y \in R\}$
Amplitude $= 1$
Period $= 360^\circ$
Phase Shift $=$ *none*

40b $D:\ \{0^\circ \le x \le 180^\circ,\ x \in R\}$
$R:\ \{0 \le y \le 4\ \ y \in R\}$
Amplitude $= 2$
Period $= 180^\circ$
Phase Shift $=$ *none*
Up 2 *units*

40c $D:\ \{-45^\circ \le x \le 315^\circ,\ x \in R\}$
$R:\ \{-\frac{1}{2} \le y \le \frac{1}{2}\ \ y \in R\}$
Amplitude $= \frac{1}{2}$
Period $= 360^\circ$
Phase Shift $=$ *left* 45°

40d $D:\ \{0^\circ \le x \le 360^\circ,\ x \in R\}$
$R:\ \{-1 \le y \le 1\ \ y \in R\}$
Amplitude $= 1$
Period $= 360^\circ$
Phase Shift $=$ *none*

40e $D:\ \{0^\circ \le x \le 1080^\circ,\ x \in R\}$
$R:\ \{-3 \le y \le 3\ \ y \in R\}$
Amplitude $= 3$
Period $= 1080^\circ$
Phase Shift $=$ *none*

40f $D:\ \{180^\circ \le x \le 900^\circ,\ x \in R\}$
$R:\ \{-1 \le y \le 3\ \ y \in R\}$
Amplitude $= 2$
Period $= 720^\circ$
Phase Shift $=$ *right* 180°
Up 1 *unit*

42a $x = 240^\circ,\ 300^\circ$

42b $x = 135^\circ,\ 225^\circ$

42c $x = 30^\circ,\ 150^\circ$

42d $x = 60^\circ,\ 240^\circ$

42e $x = 90^\circ,\ 135,^\circ\ 225^\circ$

42f $x = 120^\circ,\ 180,^\circ\ 240^\circ$

42g $x = 60^\circ,\ 180,^\circ\ 300^\circ$

42h $x = 0^\circ, 180^\circ, 360^\circ$

42i $x = 19.5^\circ,\ 160.5,^\circ\ 203.6^\circ,\ 336.4^\circ$

UNIT 7

43a $t_n = 2n + 1$
$t_{30} = 61$

43b $t_n = 7n - 11$
$t_{18} = 115$

44a 34

44b 32

45a $t_n = 4n + 1987$

45b 2127

46a $t_n = 27(3)^{1-n}$
$t_6 = \frac{1}{9}$

46b $t_n = (-3)^{n-1}$
$t_7 = 729$

47a $a = 3,\ r = 2\ , t_n = 3(2)^{n-1}, or$
$a = -3,\ r = -2, t_n = -3(-2)^{n-1}$

47b $a = -2,\ r = 3,$
$t_n = -2(3)^{n-1}$

48a 3, 3, 6, 9, 15

48b 8, 4, 2, 1, 0.5

49a Geometric
$n = 11$

49b Arithmetic
$n = 47$

49c Geometric
$n = 8$

50a 100

50b $\frac{1617}{2}$

51 5, 8, 11, 14 cm

52a -5460

52b 2735

53 29.5 *m*