Polynomials Review

- **Polynomials** are made up of a finite number of terms separated by **addition** or **subtraction.** A **monomial** is made up of <u>one</u> term, a **binomial** is made up of <u>two</u> terms, and a **trinomial** is made up of <u>three</u> terms. After that, we use the prefix *poly* meaning "**many**". Generally speaking, when we add or subtract monomials, we get a polynomial. Each monomial within the polynomial is a term.

- The **terms** of a polynomial have ***variables*** that are raised to whole-number exponents (0, 1, 2, 3, . . .) and ***constants*** (or coefficients) which form products with the variables.

$$3x^2 - 5x + 4$$

*<u>note</u>: terms of higher *degree* are usually written first, with the remaining terms written in *descending* order (degree = the sum of the exponents on all variables in that term)

- Terms that have the same variable factors (i.e. same letters with same exponents) are called **like terms**. To simplify a polynomial expression containing <u>like terms</u>, find the sum of their coefficients.

1. Simplify the following:

a) $2x^2 + 3x + (-4) - x^2 - (-x) + 9$

b) $10x^3 - 14x^2 - (-3x) - 4x^3 + 4x - 6$

c) $-6x^3 - (5x^3 - 11x^3)$

d) $(-2x^2)(5x^4) - (3x)(-3x^3)(x^2)$

- To **add** polynomial expressions, remove the brackets and collect like terms. To **subtract** a polynomial, remove the brackets by multiplying each term by (– 1). Then, collect like terms.

2. Simplify the following and evaluate for $x = 4$, $y = -1$, $z = 5$:

$(9x + 2y - 3z) - (7x + 4y) + (y - 4x)$

- To multiply any two polynomials, multiply each term of one polynomial by each term of the other polynomial.

3. Expand

Expand using the distributive property.

a) $3p(2p^2 - p + 4)$

b) $(x^2 - 2xy - y^2)(4x^2y)$

4. Expand and Simplify

Expand using the distributive property. Then, collect like terms. (When more than one set of brackets is used, simplify to remove the innermost brackets first.)

a) $3x(x^2 - 2x + 1) - (x^2 - 3x + 5)$

b) $(x - 2y)^2$

c) $(x - 3)(x^2 + 3x + 9)$

d) $3x(2x + 3)^2 - (x - 5)^2$

e) $(2y - 5)(y - 4)(y + 4)$

f) $(x^2 - 3x - 1)(2x^2 + x - 2)$

HW: WORKSHEET "Operations with Polynomials" #1abcdfgi, 2abdghij, 3efi, 4ab, 5abdgij, 6defgh

WORKSHEET: Operations with Polynomials

1. Simplify each of the following:

a. $4x^3 - 5x^3 - 6x^3$ **b.** $(4x^3)(-5x^3)(-6x^3)$ **c.** $x^2y - 2x^2y + 3x^2y$

d. $(x^2y)(-2x^2y)(3x^2y)$ **e.** $\frac{8x^4}{4x^3}$ **f.** $1 - x + 2 - x$

g. $2x^2 - 3x - x^2 + 3$ **h.** $\frac{24x^4y^3}{8xy}$ **i.** $2x^2y - xy^2 + 3xy - xy^2$

2. Simplify each of the following:

a. $4x^3(5x^3 - 6x^3)$ **b.** $4x^3 - (5x^3 - 6x^3)$ **c.** $\frac{4x^3}{5x^3 - 6x^3}$

d. $3x(-2x)^3$ **e.** $-2x^2(-2x)^2$ **f.** $\frac{(-3x)^4}{3x}$

g. $2x(3x + 4)$ **h.** $x^2(2x - 3)$ **i.** $3xy(x^2 - 2xy + 3y^2)$

j. $x^2 - (x^2 - 2x)$ **k.** $(4x - 8) \div 4$ **l.** $\frac{8x^2 - 2x}{2x}$

3. Simplify each of the following:

a. $(x + 1)(x + 2)$ **b.** $(2x - 3)(x - 4)$ **c.** $(3x - 2)(x + 6)$

d. $(x - 3)(x + 3)$ **e.** $(x + 3)^2$ **f.** $(4 - x)^2$

g. $(5 - 2x)(5 + 2x)$ **h.** $(x - y)(x + y)$ **i.** $(x + 3y)^2$

4. Simplify each of the following and evaluate for $x = 6$.

a. $7x^2 - 11x^2 + 3x^2$ **b.** $4x^2 - 6x - 3x^2 + 5x$ **c.** $\frac{15x^6}{5x^5}$

5. Simplify each of the following:

a. $(x^2 - 2x + 3) - (x^2 - 3x + 2)$ **b.** $(1 - 3x + x^3) + (x^2 + 2x - 2)$

c. $3x(2x - 1) + 3(x - 2)$ **d.** $x(x + 1) - 2x(x - 2) + 3x(x + 3)$

e. $(4x - 5)(6x - 7)$ **f.** $(2x - 1)(x^2 - 1)$

g. $(x - 2)(x^2 + 2x + 4)$ **h.** $(3x^2 - 4)(3x^2 + 4)$

i. $(5x - 6)^2$ **j.** $4x(2x - 1)(3x + 1)$

6. Perform the indicated operations in each of the following:

a. $(2x - 1)(x + 2) + (x - 2)(x + 3)$ **b.** $2x(3x - 1) - (x - 2)(x + 1)$

c. $(2x - 3)^2 - 2x(x - 3)$ **d.** $(x - 4)^2 + (x - 4)(x + 4)$

e. $(2x - 1)(2x - 3)(2x - 5)$ **f.** $(2x - 1)^3$

g. $(x^2 - 2x + 1)^2$ **h.** $4(5x - 1)^2 - 5(4x - 1)^2$

Answers:

1. a. $-7x^3$ **b.** $120x^9$ **c.** $2x^2y$ **d.** $-6x^6y^3$ **e.** $2x$ **f.** $3 - 2x$
g. $x^2 - 3x + 3$ **h.** $3x^3y^2$ **i.** $2x^2y - 2xy^2 + 3xy$ **2. a.** $-4x^6$ **b.** $5x^3$
c. -4 **d.** $-24x^4$ **e.** $-8x^4$ **f.** $27x^3$ **g.** $6x^2 + 8x$ **h.** $2x^3 - 3x^2$
i. $3x^3y - 6x^2y^2 + 9xy^3$ **j.** $2x$ **k.** $x - 2$ **l.** $4x - 1$ **3. a.** $x^2 + 3x + 2$
b. $2x^2 - 11x + 12$ **c.** $3x^2 + 16x - 12$ **d.** $x^2 - 9$ **e.** $x^2 + 6x + 9$
f. $16 - 8x + x^2$ **g.** $25 - 4x^2$ **h.** $x^2 - y^2$ **i.** $x^2 + 6xy + 9y^2$

4. a. $-x^2$, -36 **b.** $x^2 - x$, 30 **c.** $3x$, 18 **5. a.** $x + 1$
b. $x^3 + x^2 - x - 1$ **c.** $6x^2 - 6$ **d.** $2x^2 + 14x$ **e.** $24x^2 - 58x + 35$
f. $2x^3 - x^2 - 2x + 1$ **g.** $x^3 - 8$ **h.** $9x^4 - 16$ **i.** $25x^2 - 60x + 36$
j. $24x^3 - 4x^2 - 4x$ **6. a.** $3x^2 + 4x - 8$ **b.** $5x^2 - x + 2$
c. $2x^2 - 6x + 9$ **d.** $2x^2 - 8x$ **e.** $8x^3 - 36x^2 + 46x - 15$
f. $8x^3 - 12x^2 + 6x - 1$ **g.** $x^4 - 4x^3 + 6x^2 - 4x + 1$
h. $20x^2 - 1$

Factoring Part I: Common Factoring, Difference of Squares, and Simple Trinomial Factoring

Types of factoring:

1. Common Factoring
2. Difference of Squares ($\Delta^2 - \blacksquare^2$)
3. "Simple" Trinomial ($ax^2 + bx + c$, $a = 1$)
4. Grouping
5. "Tricky" Trinomial ($ax^2 + bx + c$, $a \neq 1$)

Algorithm for factoring:

START

Common Factoring

# of terms?

2 terms

3 terms

4^+ terms

Difference of Squares
$\Delta^2 - \blacksquare^2 = (\Delta - \blacksquare)(\Delta + \blacksquare)$

$ax^2 + bx + c$

Grouping
Group the terms into sets of binomials or trinomials, and factor these groups using other techniques. Then, look to factor the factors further!

$a = 1$

$a \neq 1$

"Simple" Trinomial
Find 2 integers that multiply to "c" and add to "b"

"Tricky" Trinomial
Use a systematic trial and error method involving the factors "a" and "c".

Can we factor any of the factors again?
If "yes", RESTART!

I. Factor *fully* using common factoring, difference of squares, and/or simple trinomial factoring:

a) $5x^2y - 45y$

b) $wxy + xyz$

c) $x^2y^2 - 9$

d) $3x^2 - 108$

e) $3x^4 - 243y^4$

f) $x^2 + x - 90$

g) $3x^2 - 24xy + 48y^2$

h) $2mnx^2 + 12mnx + 18mn$

II. Solve the following quadratic equations by factoring:

***Recall** : A quadratic equation is of the form* $ax^2 + bx + c = 0$ *and has 2 solutions.*

a) $9x^2 - 25 = 0$

b) $y^2 = 5y$

c) $x^2 - 4x = 12$

HW: WORKSHEET "Factoring Polynomials" Part A #1-3, 4abcd, 5cdgi, 6a-f; Part B #1

WORKSHEET: Factoring Polynomials

Part A:

1. Factor each of the following:

a. $x^2 + x$ **b.** $2x - 8$ **c.** $ab - bc$ **d.** $wxy + xyz$ **e.** $m^6 + m^2$

2. Factor each of the following:

a. $y^2 - 1$ **b.** $m^2 - 16$ **c.** $4 - a^2$ **d.** $x^2y^2 - 9$ **e.** $25 - t^4$

3. Factor each of the following:

a. $x^2 - 3x + 2$ **b.** $y^2 + 6y + 5$ **c.** $x^2 - 5x - 6$ **d.** $t^2 + 7t + 12$

4. Factor completely each of the following:

a. $9a^3 - 12a$ **b.** $5a^2b + ab^2$ **c.** $4a^3b^4 - 6a^2b^2 + 2ab$

d. $\pi r^2 + \pi rh$ **e.** $x(2x - 1) + 2(2x - 1)$ **f.** $(x + 3)^2 - 3(x + 3)$

5. Factor completely each of the following:

a. $x^2 - 49$ **b.** $100 - y^2$ **c.** $81x^2 - 4y^2$

d. $25 - x^2y^2$ **e.** $(x + 3)^2 - 16$ **f.** $(2x + 3)^2 - (x - 2)^2$

g. $x^3 - x$ **h.** $\pi R^2 - \pi r^2$ **i.** $x^4 - y^4$

6. Factor completely each of the following:

a. $x^2 - 11x + 18$ **b.** $x^2 + 11x - 42$ **c.** $x^2 + 15x + 54$

d. $x^2 - 21x + 54$ **e.** $x^2 - 16x + 64$ **f.** $x^2 - 16x - 80$

g. $x^4 + 15x^2 + 50$ **h.** $x^4 - 18x^2 + 81$ **i.** $x^2 + 6xy - 7y^2$

7. Factor completely each of the following:

a. $6x^2 - 23x - 18$ **b.** $12x^2 - 5x - 2$ **c.** $4x^2 + 17x + 4$

d. $9x^2 - 30x + 25$ **e.** $4x^4 - 3x^2 - 1$ **f.** $6x^2 - 12x - 18$

g. $2x^3 - 3x^2 + x$ **h.** $4x^4 - 13x^2 + 9$ **i.** $8x^4 - 31x^2 - 4$

j. $15x^2 - 29x - 14$ **k.** $21x^2 - 29x + 10$ **l.** $22x^2 + 43x - 2$

8. Factor completely each of the following:

a. $x^3 - 4x^2 + 3x - 12$ **b.** $2x^3 - 6x^2 - 3x + 9$

c. $4x^3 + 8x^2 - x - 2$ **d.** $2x^3 - 6x^2 + 10x - 30$

e. $3x^5 - 12x^3 - x^2 + 4$ **f.** $2x^4 - 4x^3 - 8x^2 + 16x$

9. Factor completely each of the following:

a. $x^2 - 13x + 22$ **b.** $3x^2 - 9x$ **c.** $6x^2 + 11x - 7$

d. $10x^3 - 21x^2 + 8x$ **e.** $x^3 - 3x^2 + 4x - 12$ **f.** $25x^2 - 49y^2$

g. $4x^2 + 20x + 25$ **h.** $5x^2 - 30x + 45$ **i.** $4x^2 + 19x - 5$

j. $100a^2 - 36b^2$ **k.** $4x^4 + 28x^2 + 49$ **l.** $12x^2 + 8x + 28$

10. Use difference of squares factoring to evaluate each of the following:

a. $51^2 - 49^2$ **b.** $27^2 - 23^2$ **c.** $121^2 - 111^2$ **d.** $10\,000^2 - 9\,999^2$

11. Factor completely each of the following:

a. $(x + 2)^2 - 3(x + 2) + 2$ **b.** $(x - 1)^4 - 1$ **c.** $x^2 - 8x + 16 - 4y^2$

Part A Answers:

1. a. $x(x + 1)$ **b.** $2(x - 4)$ **c.** $b(a - c)$ **d.** $xy(w + z)$
e. $m^2(m^4 + 1)$ **2. a.** $(y - 1)(y + 1)$ **b.** $(m - 4)(m + 4)$
c. $(2 - a)(2 + a)$ **d.** $(xy - 3)(xy + 3)$ **e.** $(5 - t^2)(5 + t^2)$
3. a. $(x - 2)(x - 1)$ **b.** $(y + 5)(y + 1)$ **c.** $(x - 6)(x + 1)$
d. $(t + 4)(t + 3)$ **4. a.** $3a(3a^2 - 4)$ **b.** $ab(5a + b)$
c. $2ab(2a^2b^3 - 3ab + 1)$ **d.** $\pi r(r + h)$ **e.** $(2x - 1)(x + 2)$
f. $x(x + 3)$ **5. a.** $(x + 7)(x - 7)$ **b.** $(10 - y)(10 + y)$
c. $(9x - 2y)(9x + 2y)$ **d.** $(5 - xy)(5 + xy)$ **e.** $(x - 1)(x + 7)$
f. $(x + 5)(3x + 1)$ **g.** $x(x - 1)(x + 1)$ **h.** $\pi(R - r)(R + r)$
i. $(x^2 + y^2)(x + y)(x - y)$ **6. a.** $(x - 2)(x - 9)$
b. $(x + 14)(x - 3)$ **c.** $(x + 6)(x + 9)$ **d.** $(x - 3)(x - 18)$
e. $(x - 8)(x - 8)$ **f.** $(x - 20)(x + 4)$ **g.** $(x^2 + 10)(x^2 + 5)$
h. $(x - 3)^2(x + 3)^2$ **i.** $(x + 7y)(x - y)$ **7. a.** $(3x + 2)(2x - 9)$
b. $(3x - 2)(4x + 1)$ **c.** $(4x + 1)(x + 4)$ **d.** $(3x - 5)^2$
e. $(4x^2 + 1)(x + 1)(x - 1)$ **f.** $6(x - 3)(x + 1)$
g. $x(2x - 1)(x - 1)$ **h.** $(2x - 3)(2x + 3)(x + 1)(x - 1)$
i. $(8x^2 + 1)(x + 2)(x - 2)$ **j.** $(5x + 2)(3x - 7)$
k. $(7x - 5)(3x - 2)$ **l.** $(22x - 1)(x + 2)$ **8. a.** $(x - 4)(x^2 + 3)$
b. $(x - 3)(2x^2 - 3)$ **c.** $(x + 2)(2x + 1)(2x - 1)$
d. $2(x - 3)(x^2 + 5)$ **e.** $(x + 2)(x - 2)(3x^2 - 1)$
f. $2x(x + 2)(x - 2)^2$ **9. a.** $(x - 2)(x - 11)$ **b.** $3x(x - 3)$
c. $(3x + 7)(2x - 1)$ **d.** $x(5x - 8)(2x - 1)$ **e.** $(x - 3)(x^2 + 4)$
f. $(5x - 7y)(5x + 7y)$ **g.** $(2x + 5)(2x + 5)$ **h.** $5(x - 3)^2$
i. $(4x - 1)(x + 5)$ **j.** $4(5a - 3b)(5a + 3b)$ **k.** $(2x^2 + 7)^2$
l. $4(3x^2 + 2x + 7)$ **10. a.** 200 **b.** 200 **c.** 2320 **d.** 19 999
11. a. $x(x + 1)$ **b.** $x(x - 2)(x^2 - 2x + 2)$
c. $(x - 4 - 2y)(x - 4 + 2y)$

Part B:

1. Solve each of the following quadratic equations by factoring:

a. $x^2 - x - 6 = 0$ **b.** $x^2 + 5x + 4 = 0$ **c.** $x^2 - 4 = 0$ **d.** $x^2 - 6x = 0$

e. $x^2 + x = 12$ **f.** $10x - x^2 = 25$ **g.** $8y + 15 + y^2 = 0$ **h.** $4t + 32 = t^2$

i. $2x^2 = 3x$ **j.** $16 = 9x^2$ **k.** $8 + 6x + x^2 = 0$ **l.** $y^2 - 15 = 2y$

2. Solve each of the following quadratic equations by factoring:

a. $2z^2 - 5z - 3 = 0$ **b.** $9s^2 + 6s + 1 = 0$ **c.** $6w^2 - w = 12$ **d.** $2x^2 - 12 = 5x$

e. $0 = 5m^2 + 8m + 3$ **f.** $15x + 4 = 4x^2$ **g.** $4x^2 = 3 + x$ **h.** $2r^2 - r = 3$

i. $5x + 6x^2 = 6$ **j.** $20 - 6x^2 = 7x$ **k.** $12x - 6x^2 = 0$ **l.** $100 - 36x^2 = 0$

Part B Answers:

1. a. $-2,\ 3$ **b.** $-4,\ -1$ **c.** $-2,\ 2$ **d.** $0,\ 6$
e. $-4,\ 3$ **f.** $5,\ 5$ **g.** $-5,\ -3$ **h.** $-4,\ 8$
i. $0,\ \frac{3}{2}$ **j.** $-\frac{4}{3},\ \frac{4}{3}$ **k.** $-4,\ -2$ **l.** $-3,\ 5$

2. a. $-\frac{1}{2},\ 3$ **b.** $-\frac{1}{3},\ -\frac{1}{3}$ **c.** $-\frac{4}{3},\ \frac{3}{2}$ **d.** $-\frac{3}{2},\ 4$
e. $-1,\ -\frac{3}{5}$ **f.** $-\frac{1}{4},\ 4$ **g.** $-\frac{3}{4},\ 1$ **h.** $-1,\ \frac{3}{2}$
i. $-\frac{3}{2},\ \frac{2}{3}$ **j.** $-\frac{5}{2},\ \frac{4}{3}$ **k.** $0,\ 2$ **l.** $-\frac{5}{3},\ \frac{5}{3}$

Factoring Part II: Tricky Trinomials

"Tricky Trinomials" are trinomials in the form $ax^2 + bx + c$, **where $a \neq 1$.** Several different methods can be used to factor these types of trinomials, but this lesson will focus on a method of **systematic trial & error.**

Part I: Factor the following trinomials completely, where *a* is *prime*.

a) $2x^2 + 11x + 5$

b) $2x^2 + 5x + 2$

c) $3x^2 + 2x - 5$

d) $3x^2 - x - 4$

e) $5x^2 - 22x + 8$

Part II: Factor the following trinomials completely, where *a* is *NOT prime*.

a) $6x^2 - 11x + 4$

b) $4x^2 + 4x + 1$

c) $9x^2 + 3x - 2$

d) $4x^2 - 8x - 5$

Part III: Factor the following trinomials completely, where the expression must be *common factored* first.

a) $12x^2 - 10x - 8$

b) $-2x^2 + 7x - 6$

c) $16x^2 + 32x + 12$

d) $-6x^2 - 9x - 3$

Part IV: Solve the following quadratic equations by factoring.

a) $2x^2 - 7x = 15$

b) $-9a^2 = 16 - 24a$

c) $18z - 18 + 8z^2 = 0$

HW: WORKSHEET "Factoring Polynomials" Part A #7bcd, 9cdghi; Part B #2

Factoring Part III: Grouping and Substitution

Factor completely each of the following:

1. $4x^4 - 3x^2 - 1$

2. $3x^5 - 12x^3 - x^2 + 4$

3. $3ax - 3ay - 6bx + 6by$

4. $x^2 + 10x + 25 - 9y^2$

5. $(x - 3)^4 - 16$

6. $(3x + 1)^2 - (x - 5)^2$

7. $(x + 2)^2 - 6(x + 2) + 8$

HW: WORKSHEET "Factoring Polynomials" #4ef, 5ef, 6ghi, 7hi, 8, 9ek, 11

Rearranging Equations and Formulas

Formulas are *equations* that express a *relationship* between more than one letter or variable. A formula is also called a **literal equation** when it involves several letters or variables. Sometimes algebra is needed to change the formula to a more useful equivalent equation, which is solved for a particular letter or variable.

Solve for the indicated variable in each of the following:

1. $P = 2(l + w)$, for l

2. $A = \pi r^2$, for r, $r \geq 0$

3. $A = \frac{bh}{2}$, for b

4. $A = \frac{1}{2}ah + \frac{1}{2}bh$, for h

5. $y = mx + b$, for x

6. $x^2 + y^2 = r^2$, for x

7. $\tan\theta = \frac{O}{A}$, for A

8. $\frac{a}{\sin A} = \frac{b}{\sin B}$, for $\sin A$

9. $a^2 = b^2 + c^2 - 2bc\cos A$, for $\angle A$

10. $y = ax^2 + k$, for x

11. $y = \sqrt{x - d} + c$, for x

12. $x = \frac{y - 3}{y + 2}$, for y

HW: "WORKSHEET: Rearranging Equations and Formulas"

WORKSHEET: Rearranging Equations and Formulas

Solve for the indicated variable in each of the following:

1. $d = vt$, for t
2. $c = 2\pi r$, for r
3. $P = 2l + 2w$, for w
4. $SA = \pi r^2 + 2\pi rh$, for h
5. $180° = \angle A + \angle B + \angle C$, for $\angle B$
6. $V = \pi r^2 h$, for r, $r \geq 0$
7. $V = \frac{4}{3}\pi r^3$, for r
8. $A = \frac{1}{2}h(a+b)$, for h
9. $C = \frac{5}{9}(F-32)$, for F
10. $F = 0.35T_1 + 0.35T_2 + 0.3E$, for E
11. $Ax + By + C = 0$, for y
12. $SA = \pi r^2 + 2\pi rh$, for π
13. $SA = 2lw + 2lh + 2wh$, for h
14. $\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$, for A
15. $a^2 + b^2 = c^2$, for b, $b \geq 0$
16. $x^2 = r^2 - y^2$, for y
17. $\cos\theta = \frac{A}{H}$, for H
18. $P = \frac{2\pi}{k}$, for k
19. $\frac{b}{\sin B} = \frac{c}{\sin C}$, for c
20. $\frac{\sin D}{d} = \frac{\sin E}{e}$, for $\angle D$
21. $r^2 = p^2 + q^2 - 2pq\cos R$, for $\cos R$
22. $x = \frac{2}{3}y + \frac{1}{4}$, for y
23. $x = y^2 + 2$, for y
24. $y = (x+2)^2 - 4$, for x
25. $y = ax^2 + k$, for x
26. $y = 2\sqrt{x-1}$, for x
27. $(y-3)(x+2) = 3$, for y
28. $y(3x-1) = x$, for y
29. $xy - 2y = 3x + 5$, for y
30. $x = \frac{y+4}{2y-1}$, for y
31. $\cos F = \frac{e^2 + g^2 - f^2}{2eg}$, for f, $f \geq 0$
32. $y^2 = x^2 + x^2 - 2(x)(x)\cos Y$, for $\angle Y$

Answers:

1. $t = \frac{d}{v}$
2. $r = \frac{c}{2\pi}$
3. $w = \frac{P-2l}{2}$
4. $h = \frac{SA - \pi r^2}{2\pi r}$
5. $\angle B = 180° - \angle A - \angle C$
6. $r = \sqrt{\frac{V}{\pi h}}$
7. $r = \sqrt[3]{\frac{3V}{4\pi}}$
8. $h = \frac{2A}{a+b}$
9. $F = \frac{9}{5}C + 32$
10. $E = \frac{F - 0.35T_1 - 0.35T_2}{0.3}$
11. $y = \frac{-Ax - C}{B}$
12. $\pi = \frac{SA}{r^2 + 2rh}$
13. $h = \frac{SA - 2lw}{2l + 2w}$
14. $A = \frac{r^2\theta}{2}$
15. $b = \sqrt{c^2 - a^2}$
16. $y = \pm\sqrt{r^2 - x^2}$
17. $H = \frac{A}{\cos\theta}$
18. $k = \frac{2\pi}{P}$
19. $c = \frac{b\sin C}{\sin B}$
20. $\angle D = \sin^{-1}\left(\frac{d\sin E}{e}\right)$
21. $\frac{r^2 - p^2 - q^2}{-2pq} = \cos R$ *or* $\cos R = \frac{p^2 + q^2 - r^2}{2pq}$
22. $y = \frac{12x - 3}{8}$
23. $y = \pm\sqrt{x-2}$
24. $x = -2 \pm \sqrt{y+4}$
25. $x = \pm\sqrt{\frac{y-k}{a}}$
26. $x = \frac{y^2}{4} + 1$
27. $y = \frac{3}{x+2} + 3$
28. $y = \frac{x}{(3x-1)}$
29. $y = \frac{3x+5}{x-2}$
30. $y = \frac{x+4}{2x-1}$
31. $f = \sqrt{e^2 + g^2 - 2eg\cos F}$
32. $\angle Y = \cos^{-1}\left(\frac{2x^2 - y^2}{2x^2}\right)$

Algebraic Tools Review Assignment

I. *Solve each of the following quadratic equations by factoring:*

a) $6w^2 - 7w - 3 = 0$ *b)* $12m^2 + 22m = -10$ *c)* $y + 2 = 10y^2$ *d)* $8x^2 - 18x = 0$

II. *Factor each of the following completely:*

a) $y^4 - 16y^2 + 63$ *b)* $2x^4 - 3x^2 - 20$ *c)* $n^8 + 5n^4 - 6$

III. *Factor each of the following completely:*

a) $x^3 - 2x^2 - x + 2$ *b)* $mx + my - nx - ny$ *c)* $4y^2 + 8xy + 3y + 6x$

IV. *Factor each of the following completely. DO NOT EXPAND before factoring!*

a) $(x-2)^2 - 9$ *b)* $49 - (x+9)^2$

c) $x^2 - 4x + 4 - y^2$ *d)* $a^2 - \left(b^2 - 10b + 25\right)$

e) $(t+3)^2 - 2(t+3)$

f) $(x+2)(x-2) + 3(x+2)$

g) $x^2(x-5) - 4(x-5)$

h) $3x(2x-4y) - 6(2x-4y)$

i) $(a+2)^2 - 12(a+2) + 32$

j) $4(x+3)^2 - 4(x+3) - 15$

V. *Expand and simplify each of the following:*

a) $3x(-2x^3) - 4x^2(2x)(-x)$

b) $3(2x+3)^2 - (x-5)^2 - (3x-4)(x-5)$

VI. *Solve for the indicated variable in each of the following:*

a) $x = \sqrt{y+2} - 3$ *for* y, $y \geq -2$

b) $y = \dfrac{x+5}{x-2}$ *for* x, $x \neq 2$