Polynomials Review

- **Polynomials** are made up of a finite number of terms separated by **addition** or **subtraction.** A **monomial** is made up of <u>one</u> term, a **binomial** is made up of <u>two</u> terms, and a **trinomial** is made up of <u>three</u> terms. After that, we use the prefix *poly* meaning "**many**". Generally speaking, when we add or subtract monomials, we get a polynomial. Each monomial within the polynomial is a term.

- The **terms** of a polynomial have ***variables*** that are raised to whole-number exponents (0, 1, 2, 3, . . .) and ***constants*** (or coefficients) which form products with the variables.

trinomial

leading coefficient

$$\underbrace{3x^2}_{\text{leading term}} - \underbrace{5x}_{\text{linear term } (x^1)} + \underbrace{4}_{\text{constant term } (x^0)}$$

[quadratic] (x^2)

*<u>note</u>: terms of higher *degree* are usually written first, with the remaining terms written in *descending* order (degree = the sum of the exponents on all variables in that term)

- Terms that have the same variable factors (i.e. same letters with same exponents) are called **like terms**. To simplify a polynomial expression containing <u>like terms</u>, find the sum of their coefficients.

1. Simplify the following:

a) $2x^2 + 3x + (-4) - x^2 - (-x) + 9$

$= 2x^2 + 3x - 4 - x^2 + x + 9$

$= \underbrace{2x^2 - x^2}\ \underbrace{+3x + x}\ \underbrace{-4 + 9}$

$= x^2 + 4x + 5$

b) $10x^3 - 14x^2 - (-3x) - 4x^3 + 4x - 6$

$= 10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6$

$= \underbrace{10x^3 - 4x^3} - 14x^2 \underbrace{+3x + 4x} - 6$

$= 6x^3 - 14x^2 + 7x - 6$

c) $-6x^3 - (5x^3 - 11x^3)$

$= -6x^3 - 5x^3 + 11x^3$

$= 0x^3$

$= 0$

d) $(-2x^2)(5x^4) - (3x)(-3x^3)(x^2)$

$= -10x^6 - (-9x^6)$

$= -10x^6 + 9x^6$

$= -x^6$

- To **add** polynomial expressions, remove the brackets and collect like terms. To **subtract** a polynomial, remove the brackets by multiplying each term by (– 1). Then, collect like terms.

2. Simplify the following and evaluate for *x* = 4, y = – 1, z = 5:

i) $(9x + 2y - 3z) - (7x + 4y) + (y - 4x)$

$= 9x + 2y - 3z - 7x - 4y + y - 4x$

$= -2x - y - 3z$

ii) Substitute $x = 4,\ y = -1,\ z = 5$:

$-2(4) - (-1) - 3(5)$

$= -8 + 1 - 15$

$= -22$

- To multiply any two polynomials, multiply each term of one polynomial by each term of the other polynomial.

3. Expand

Expand using the distributive property.

a) $3p(2p^2 - p + 4)$

$= 6p^3 - 3p^2 + 12p$

b) $(x^2 - 2xy - y^2)(4x^2y)$

$= 4x^4y - 8x^3y^2 - 4x^2y^3$

4. Expand and Simplify

Expand using the distributive property. Then, collect like terms. (When more than one set of brackets is used, simplify to remove the innermost brackets first.)

a) $3x(x^2 - 2x + 1) - (x^2 - 3x + 5)$

$= 3x^3 - 6x^2 + 3x - x^2 + 3x - 5$

$= 3x^3 - 7x^2 + 6x - 5$

b) $(x - 2y)^2$

$= (x-2y)(x-2y)$

$= x^2 - 2xy - 2xy + 4y^2$

$= x^2 - 4xy + 4y^2$

c) $(x - 3)(x^2 + 3x + 9)$

$= x^3 + 3x^2 + 9x - 3x^2 - 9x - 27$

$= x^3 - 27$

d) $3x(2x + 3)^2 - (x - 5)^2$ BEDMAS

$= 3x[(2x+3)(2x+3)] - [(x-5)(x-5)]$

$= 3x[4x^2 + 12x + 9] - [x^2 - 10x + 25]$

$= 12x^3 + 36x^2 + 27x - x^2 + 10x - 25$

$= 12x^3 + 35x^2 + 37x - 25$

e) $[(2y - 5)(y - 4)](y + 4)$ BEDMAS L→R

$= [2y^2 - 8y - 5y + 20](y+4)$

$= [2y^2 - 13y + 20](y+4)$

$= 2y^3 + 8y^2 - 13y^2 - 52y + 20y + 80$

$= 2y^3 - 5y^2 - 32y + 80$

f) $(x^2 - 3x - 1)(2x^2 + x - 2)$

$= 2x^4 + x^3 - 2x^2 - 6x^3 - 3x^2 + 6x - 2x^2 - x + 2$

$= 2x^4 + \underbrace{x^3 - 6x^3}\underbrace{- 2x^2 - 3x^2 - 2x^2}\underbrace{+ 6x - x} + 2$

$= 2x^4 - 5x^3 - 7x^2 + 5x + 2$

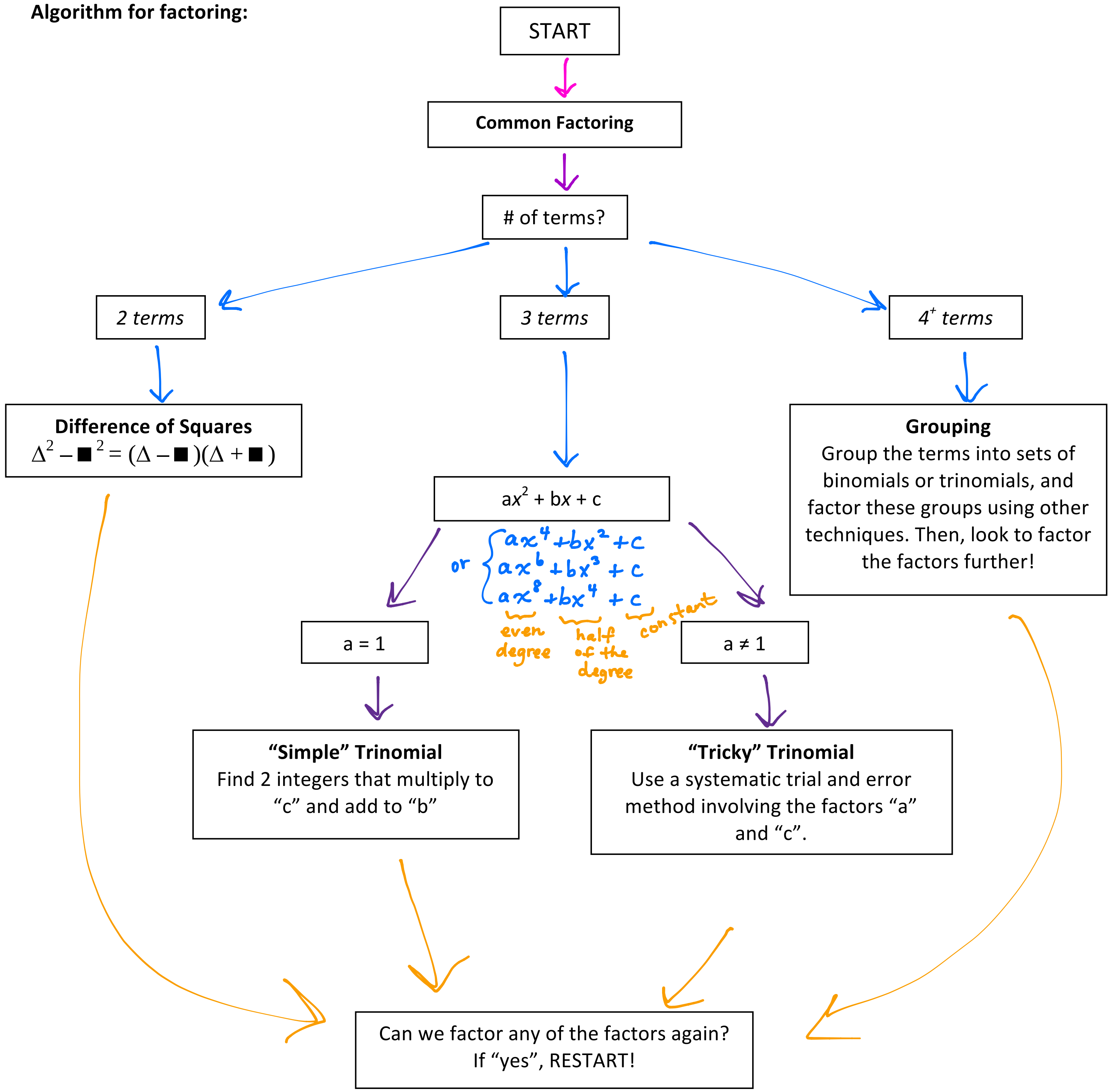
HW: WORKSHEET "Operations with Polynomials" #1abcdfgi, 2abdghij, 3efi, 4ab, 5abdgij, 6defgh

Factoring Part I: Common Factoring, Difference of Squares, and Simple Trinomial Factoring

Types of factoring:

1. Common Factoring
2. Difference of Squares ($\Delta^2 - \blacksquare^2$)
3. "Simple" Trinomial ($ax^2 + bx + c$, $a = 1$)
4. Grouping
5. "Tricky" Trinomial ($ax^2 + bx + c$, $a \neq 1$)

Algorithm for factoring:



I. Factor *fully* using common factoring, difference of squares, and/or simple trinomial factoring:

a) $5x^2y - 45y$

$= 5y(x^2 - 9)$ ① common

$= 5y(x+3)(x-3)$ ② diff. of squares

b) $wxy + xyz$

$= xy(w + z)$ ① common

c) $x^2y^2 - 9$

$= (xy+3)(xy-3)$ ① diff. of squares

d) $3x^2 - 108$

$= 3(x^2 - 36)$ ① common

$= 3(x+6)(x-6)$ ② diff. of squares

e) $3x^4 - 243y^4$

$= 3(x^4 - 81y^4)$ ① common

$= 3(x^2 + 9y^2)(x^2 - 9y^2)$ ② Diff. Sq.

$= 3(x^2 + 9y^2)(x+3y)(x-3y)$ ③ Diff. Sq.

f) $x^2 + x - 90$

$= (x+10)(x-9)$ ① simple trinomial

$1, -9$; $1, 10$; $-9 + 10 = +1$

check:

$(x+10)(x-9)$

$= x^2 - 9x + 10x - 90$

$= x^2 + x - 90$

g) $3x^2 - 24xy + 48y^2$

$= 3(f^2 - 8f + 16)$ ① common

$= 3(f-4)(f-4)$ ② simple trinomial

$1, -4$; $1, -4$; $-4 - 4 = -8$

$= 3(f-4)^2$

h) $2mnx^2 + 12mnx + 18mn$

$= 2mn(x^2 + 6x + 9)$ ① common

$= 2mn(x+3)(x+3)$ ② simple trinomial

$1, 3$; $1, 3$; $+3 + 3 = +6$

$1, 9$; $1, 1$; $9 \quad 1$

$= 2mn(x+3)^2$

II. Solve the following quadratic equations by factoring:

Recall *: A quadratic equation is of the form $ax^2 + bx + c = 0$ and has 2 solutions.*

To solve by factoring, isolate zero on one side of the equation!

a) $9x^2 - 25 = 0$

$(3x+5)(3x-5) = 0$

$3x+5=0$ or $3x-5=0$

$3x = -5$ $\quad$ $3x = 5$

$x = -\frac{5}{3}$ $\quad$ $x = \frac{5}{3}$

$\boxed{\therefore x = \pm \frac{5}{3}}$

b) $y^2 = 5y$

$y^2 - 5y = 0$

$y(y-5) = 0$

$y = 0$ or $y - 5 = 0$

$y = 5$

$\boxed{\therefore y = 0, 5}$

c) $x^2 - 4x = 12$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x - 6 = 0$ or $x + 2 = 0$

$x = 6$ $\quad$ $x = -2$

$\boxed{\therefore x = -2, 6}$

HW: WORKSHEET "Factoring Polynomials" Part A #1-3, 4abcd, 5cdgi, 6a-f; Part B #1

Factoring Part II: Tricky Trinomials

"Tricky Trinomials" are trinomials in the form $ax^2 + bx + c$, **where** $a \neq 1$. Several different methods can be used to factor these types of trinomials, but this lesson will focus on a method of **systematic trial & error.**

Part I: Factor the following trinomials completely, where *a* is *prime*.

a) $\boxed{2x^2} + \underbrace{11x}_{} + \boxed{5}$

"F" "O+I" "L"

2 X 5
1 X 1

$+5 + 2 \neq 11$

(2 X 1)
(1 X 5)

$+1 + 10 = +11$ ✓

$= (2x+1)(x+5)$

Check:

$= 2x^2 + 10x + x + 5$

$= 2x^2 + 11x + 5$ ✓

b) $2x^2 + 5x + 2$

2 X 2
1 X 1

$2 + 2 \neq 5$

2 X 1
1 X 2

$1 + 4 = 5$ ✓

$= (2x+1)(x+2)$

Check:

$= 2x^2 + 4x + x + 2$

$= 2x^2 + 5x + 2$ ✓

c) $3x^2 + 2x - 5$

3 X 1
1 X 5

1 15

3 X 5
1 X −1

$+5 - 3 = +2$ ✓

$= (3x+5)(x-1)$

check:

$= 3x^2 - 3x + 5x - 5$

$= 3x^2 + 2x - 5$ ✓

d) $3x^2 - x - 4$

3 X 2
1 X 2

2 6

3 X −4
1 X 1

$-4 + 3 = -1$ ✓

$= (3x-4)(x+1)$

check:

$= 3x^2 + 3x - 4x - 4$

$= 3x^2 - x - 4$ ✓

e) $5x^2 - 22x + 8$

5 X −2
1 X −4

$-2 - 20 = -22$ ✓

$= (5x-2)(x-4)$

check:

$= 5x^2 - 20x - 2x + 8$

$= 5x^2 - 22x + 8$ ✓

Part II: Factor the following trinomials completely, where *a* is *NOT prime*.

a) $6x^2 - 11x + 4$

$= (2x-1)(3x-4)$

$-3-8 = -11$ ✓

b) $4x^2 + 4x + 1$

$= (2x+1)(2x+1)$

$= (2x+1)^2$

$+2+2 = +4$ ✓

c) $9x^2 + 3x - 2$

$= (3x+2)(3x-1)$

$+6-3 = +3$ ✓

d) $4x^2 - 8x - 5$

$= (2x+1)(2x-5)$

$+2-10 = -8$ ✓

Part III: Factor the following trinomials completely, where the expression must be *common factored* first.

a) $12x^2 - 10x - 8$

$= 2(6x^2 - 5x - 4)$

$= 2(2x+1)(3x-4)$

$+3-8 = -5$ ✓

b) $-2x^2 + 7x - 6$

$= -(2x^2 - 7x + 6)$

$= -(2x-3)(x-2)$

$-3-4 = -7$ ✓

c) $16x^2 + 32x + 12$

$= 4(4x^2 + 8x + 3)$

$= 4(2x+3)(2x+1)$

$+6+2 = +8$ ✓

d) $-6x^2 - 9x - 3$

$= -3(2x^2 + 3x + 1)$

$= -3(2x+1)(x+1)$

$+1+2 = +3$ ✓

Part IV: Solve the following quadratic equations by factoring. Two solutions!

a) $2x^2 - 7x = 15$

$2x^2 - 7x - 15 = 0$

$(2x+3)(x-5) = 0$

$2x+3 = 0$ or $x-5 = 0$

$2x = -3$ $\quad x = 5$

$x = -\frac{3}{2}$

$\boxed{\therefore x = -\frac{3}{2}, 5}$

b) $-9a^2 = 16 - 24a$

$0 = 9a^2 - 24a + 16$

$0 = (3a-4)(3a-4)$

$3a-4 = 0$ or $3a-4 = 0$

$3a = 4$ $\quad a = \frac{4}{3}$

$a = \frac{4}{3}$

$\boxed{\therefore a = \frac{4}{3}, \frac{4}{3}}$

c) $21z - 18 + 8z^2 = 0$ 18z oops!

$8z^2 + 18z - 18 = 0$

$2(4z^2 + 9z - 9) = 0$

$2(4z-3)(z+3) = 0$

$4z - 3 = 0$ or $z + 3 = 0$

$-3 + 12$

$\boxed{\therefore z = -3, \frac{3}{4}}$

HW: WORKSHEET "Factoring Polynomials" Part A #7bcd, 9cdghi; Part B #2

Factoring Part III: Grouping and Substitution

- 4 or more terms
- 2 & 2, 1 & 3, 3 & 1

- Polynomial factors
- can use "dummy variable"

Factor completely each of the following:

1. $4x^4 - 3x^2 - 1$

$$\begin{array}{cc} 4 & 1 \\ 1 & -1 \\ +1 & -4 \end{array} \quad = -3 \checkmark \qquad \begin{array}{cc} 2 & 1 \\ 2 & 1 \\ 2 & 2 \end{array}$$

$= (4x^2+1)\underbrace{(x^2-1)}_{\text{Diff. Sq.}}$

$= (4x^2+1)(x+1)(x-1)$

2. $3x^5 - 12x^3 - x^2 + 4$

$= 3x^3(x^2-4) - 1(x^2-4)$

$= \underbrace{(x^2-4)}_{\text{Diff. Sq.}}(3x^3-1)$

$= (x+2)(x-2)(3x^3-1)$

3. $3ax - 3ay - 6bx + 6by$

$= 3a(x-y) - 6b(x-y)$

$= (x-y)(3a-6b)$

common

$= (x-y)(3)(a-2b)$

$= 3(x-y)(a-2b)$

4. $x^2 + 10x + 25 - 9y^2$

$= \underbrace{(x+5)^2 - 9y^2}_{\text{Diff. Sq.}}$ Let $a = (x+5)$

"dummy variable"

$= a^2 - 9y^2$

$= (a+3y)(a-3y)$

w/o dummy variable ⇨ $= [(x+5)+3y][(x+5)-3y]$

$= (x+3y+5)(x-3y+5)$

5. $(x-3)^4 - 16$ Let $a = (x-3)$

$= a^4 - 16$

$= (a^2+4)(a^2-4)$

$= (a^2+4)(a+2)(a-2)$

⇨ $= [(x-3)^2+4][(x-3)+2][(x-3)-2]$

$= [x^2-6x+9+4](x-1)(x-5)$

$= (x^2-6x+13)(x-1)(x-5)$

6. $(3x+1)^2 - (x-5)^2$ Let $a = (3x+1)$ Let $b = (x-5)$

$= a^2 - b^2$

$= (a+b)(a-b)$

⇨ $= [(3x+1)+(x-5)][(3x+1)-(x-5)]$

$= [3x+1+x-5][3x+1-x+5]$

$= (4x-4)(2x+6)$

common

$= 4(x-1)(2)(x+3)$

$= 8(x-1)(x+3)$

7. $(x+2)^2 - 6(x+2) + 8$ Let $a = (x+2)$

$= a^2 - 6a + 8$

$= (a-4)(a-2)$

⇨ $= [(x+2)-4][(x+2)-2]$

$= (x+2-4)(x+2-2)$

$= (x-2)(x)$

$= x(x-2)$

HW: WORKSHEET "Factoring Polynomials" #4ef, 5ef, 6ghi, 7hi, 8, 9ek, 11

Rearranging Equations and Formulas

Formulas are *equations* that express a *relationship* between more than one letter or variable. A formula is also called a **literal equation** when it involves several letters or variables. Sometimes algebra is needed to change the formula to a more useful equivalent equation, which is solved for a particular letter or variable.

Solve for the indicated variable in each of the following:

1. $P = 2(l+w)$, for l

$$\frac{P}{2} = \frac{\cancel{2}(l+w)}{\cancel{2}}$$

$$\frac{P}{2} = l + w$$

$$\frac{P}{2} - w = l$$

$$\boxed{\therefore l = \frac{P}{2} - w}$$

2. $A = \pi r^2$, for r, $r \ge 0$ → no negative values for r

$$\frac{A}{\pi} = \frac{\cancel{\pi} r^2}{\cancel{\pi}}$$

$$\frac{A}{\pi} = r^2$$

$$+\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$

$$\sqrt{\frac{A}{\pi}} = r$$

$$\boxed{\therefore r = \sqrt{\frac{A}{\pi}}}$$

3. $A = \frac{bh}{2}$, for b

$$2 \cdot A = \frac{bh}{\cancel{2}} \cdot \cancel{2}$$

$$\frac{2A}{h} = \frac{b\cancel{h}}{\cancel{h}}$$

$$\frac{2A}{h} = b$$

$$\boxed{\therefore b = \frac{2A}{h}}$$

4. $A = \frac{1}{2}ah + \frac{1}{2}bh$, for h

$$A = \frac{1}{2}h(a+b)$$

$$2(A) = \cancel{2}\left(\frac{1}{\cancel{2}}h(a+b)\right)$$

$$\frac{2A}{(a+b)} = \frac{h\cancel{(a+b)}}{\cancel{(a+b)}}$$

$$\frac{2A}{(a+b)} = h$$

$$\boxed{\therefore h = \frac{2A}{(a+b)}}$$

5. $y = mx + b$, for x

$$\frac{y-b}{m} = \frac{\cancel{m}x}{\cancel{m}}$$

$$\frac{y-b}{m} = x$$

$$\boxed{\therefore x = \frac{y-b}{m}}$$

6. $x^2 + y^2 = r^2$, for x → no restrictions on x

$$x^2 = r^2 - y^2$$

$$\sqrt{x^2} = \pm\sqrt{r^2 - y^2}$$

$$\boxed{\therefore x = \pm\sqrt{r^2 - y^2}}$$

cross-multiply ☺

7. $\tan\theta = \frac{O}{A}$, for A

$$A\cdot\tan\theta = \frac{O}{\cancel{A}}\cdot\cancel{A}$$

$$\frac{A\cancel{\tan\theta}}{\cancel{\tan\theta}} = \frac{O}{\tan\theta}$$

$$\boxed{\therefore\ A = \frac{O}{\tan\theta}}$$

8. $\frac{a}{\sin A} = \frac{b}{\sin B}$, for $\sin A$

$$\frac{\cancel{b}\sin A}{\cancel{b}} = \frac{a\sin B}{b}$$

$$\boxed{\therefore\ \sin A = \frac{a\sin B}{b}}$$

9. $a^2 = b^2 + c^2 - 2bc\cos A$, for $\angle A$

$$\frac{\cancel{2bc}\cos A}{\cancel{2bc}} = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos^{-1}(\cos A) = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

$$\boxed{\therefore\ \angle A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)}$$

10. $y = ax^2 + k$, for x → no restrictions on x

$$\frac{y-k}{a} = \frac{\cancel{a}x^2}{\cancel{a}}$$

$$\frac{y-k}{a} = x^2$$

$$\pm\sqrt{\frac{y-k}{a}} = \sqrt{x^2}$$

$$\pm\sqrt{\frac{y-k}{a}} = x$$

$$\boxed{\therefore\ x = \pm\sqrt{\frac{y-k}{a}}}$$

11. $y = \sqrt{x-d} + c$, for x, $x \geq d$

$$(y-c)^2 = \left(\sqrt{x-d}\right)^2$$

$$(y-c)^2 = x-d$$

$$(y-c)^2 + d = x$$

$$\boxed{\therefore\ x = (y-c)^2 + d}$$

12. $\frac{x}{1} = \frac{y-3}{y+2}$, for y, $y \neq -2$ cross-multiply

$$x(y+2) = 1(y-3) \qquad \text{expand}$$

$$xy + 2x = y - 3$$

$$xy - y = -2x - 3 \qquad \text{rearrange}$$

$$\frac{y\cancel{(x-1)}}{\cancel{(x-1)}} = \frac{-2x-3}{(x-1)} \qquad \text{factor \& divide out}$$

$$\boxed{\therefore\ y = \frac{-2x-3}{x-1},\ x \neq 1}$$

HW: "WORKSHEET: Rearranging Equations and Formulas"

 Name: Solutions !

Algebraic Tools Review Assignment

/60

I. *Solve each of the following quadratic equations by factoring:*

13

a) $6w^2 - 7w - 3 = 0$

$(2w-3)(3w+1)=0$

$2w-3=0$ or $3w+1=0$

$\boxed{\therefore w = -\frac{1}{3}, \frac{3}{2}}$

b) $12m^2 + 22m = -10$

$12m^2+22m+10=0$

$2(6m^2+11m+5)=0$

$2(6m+5)(m+1)=0$

$6m+5=0$ or $m+1=0$

$\boxed{\therefore m = -\frac{5}{6}, -1}$

c) $y + 2 = 10y^2$

$0 = 10y^2 - y - 2$

$0=(2y-1)(5y+2)$

$2y-1=0$ or $5y+2=0$

$\boxed{\therefore y = -\frac{2}{5}, \frac{1}{2}}$

d) $8x^2 - 18x = 0$

$2x(4x-9)=0$

$x=0$ or $4x-9=0$

$\boxed{\therefore x = 0, \frac{9}{4}}$

II. *Factor each of the following completely:*

7

a) $y^4 - 16y^2 + 63$

$=(y^2-7)(y^2-9)$

$=(y^2-7)(y-3)(y+3)$

b) $2x^4 - 3x^2 - 20$

$=(2x^2+5)(x^2-4)$

$=(2x^2+5)(x-2)(x+2)$

c) $n^8 + 5n^4 - 6$

$=(n^4+6)(n^4-1)$

$=(n^4+6)(n^2+1)(n^2-1)$

$=(n^4+6)(n^2+1)(n-1)(n+1)$

III. *Factor each of the following completely:*

4

a) $x^3 - 2x^2 - x + 2$

$=x^2(x-2)-1(x-2)$

$=(x-2)(x^2-1)$

$=(x-2)(x-1)(x+1)$

b) $mx + my - nx - ny$

$=m(x+y)-n(x+y)$

$=(x+y)(m-n)$

c) $4y^2 + 8xy + 3y + 6x$

$=4y(y+2x)+3(y+2x)$

$=(y+2x)(4y+3)$

IV. *Factor each of the following completely. DO NOT EXPAND before factoring!*

9

a) $(x-2)^2 - 9$ Let $a=(x-2)$

$=a^2-9$

$=(a-3)(a+3)$

$=[(x-2)-3][(x-2)+3]$

$=(x-5)(x+1)$

b) $49 - (x+9)^2$ Let $b=(x+9)$

$=49-b^2$

$=(7-b)(7+b)$

$=[7-(x+9)][7+(x+9)]$

$=(7-x-9)(7+x+9)$

$=(-x-2)(x+16)$

$=-(x+2)(x+16)$

c) $x^2 - 4x + 4 - y^2$ Let $a=(x-2)$

$=(x-2)^2-y^2$

$=a^2-y^2$

$=(a+y)(a-y)$

$=[(x-2)+y][(x-2)-y]$

$=(x+y-2)(x-y-2)$

d) $a^2 - (b^2 - 10b + 25)$ Let $y=(b-5)$

$=a^2-(b-5)^2$

$=a^2-y^2$

$=(a+y)(a-y)$

$=[a+(b-5)][a-(b-5)]$

$=(a+b-5)(a-b+5)$

e) $(t+3)^2-2(t+3)$ Let $c=(t+3)$

$= c^2-2c$

$= c(c-2)$

$= (t+3)\left[(t+3)-2\right]$

$= (t+3)(t+1)$

f) $(x+2)(x-2)+3(x+2)$ Let $d=(x+2)$

$= d(x-2)+3d$

$= d\left[(x-2)+3\right]$

$= d(x+1)$

$= (x+2)(x+1)$

g) $x^2(x-5)-4(x-5)$

$= (x-5)(x^2-4)$

$= (x-5)(x-2)(x+2)$

h) $3x(2x-4y)-6(2x-4y)$

$= \underbrace{(2x-4y)}_{\text{common}}\underbrace{(3x-6)}_{\text{common}}$

$= 2(x-2y)(3)(x-2)$

$= 6(x-2y)(x-2)$

13

i) $(a+2)^2-12(a+2)+32$ Let $b=(a+2)$

$= b^2-12b+32$

$= (b-4)(b-8)$

$= \left[(a+2)-4\right]\left[(a+2)-8\right]$

$= (a-2)(a-6)$

j) $4(x+3)^2-4(x+3)-15$ Let $y=(x+3)$

$= 4y^2-4y-15$

$= (2y+3)(2y-5)$

$= \left[2(x+3)+3\right]\left[2(x+3)-5\right]$

$= \left[2x+6+3\right]\left[2x+6-5\right]$

$= (2x+9)(2x+1)$

V. *Expand and simplify each of the following:*

a) $3x(-2x^3)-4x^2(2x)(-x)$

$= -6x^4-4x^2(-2x^2)$

$= -6x^4+8x^4$

$= 2x^4$

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b) $3(2x+3)^2-(x-5)^2-(3x-4)(x-5)$

$= 3\left[(2x+3)(2x+3)\right]-\left[(x-5)(x-5)\right]-\left[(3x-4)(x-5)\right]$

$= 3\left[4x^2+12x+9\right]-\left[x^2-10x+25\right]-\left[3x^2-19x+20\right]$

$= 12x^2+36x+27-x^2+10x-25-3x^2+19x-20$

$= 8x^2+65x-18$

VI. *Solve for the indicated variable in each of the following:*

a) $x=\sqrt{y+2}-3$ for y, $y\ge -2$

$(x+3)^2 = \left(\sqrt{-(y+2)}\right)^2$

$(-1)\times(x+3)^2 = -(y+2)\times(-1)$

$-(x+3)^2 = y+2$

$-(x+3)^2-2 = y$

$\therefore y = -(x+3)^2-2$

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b) $\frac{y}{1} = \frac{(x+5)}{(x-2)}$ for x, $x\ne 2$

$y(x-2)=1(x+5)$

$xy-2y = x+5$

$xy-x = 2y+5$

$\frac{x\cancel{(y-1)}}{\cancel{(y-1)}} = \frac{2y+5}{(y-1)}$

$\therefore x = \frac{2y+5}{y-1},\ y\ne 1$