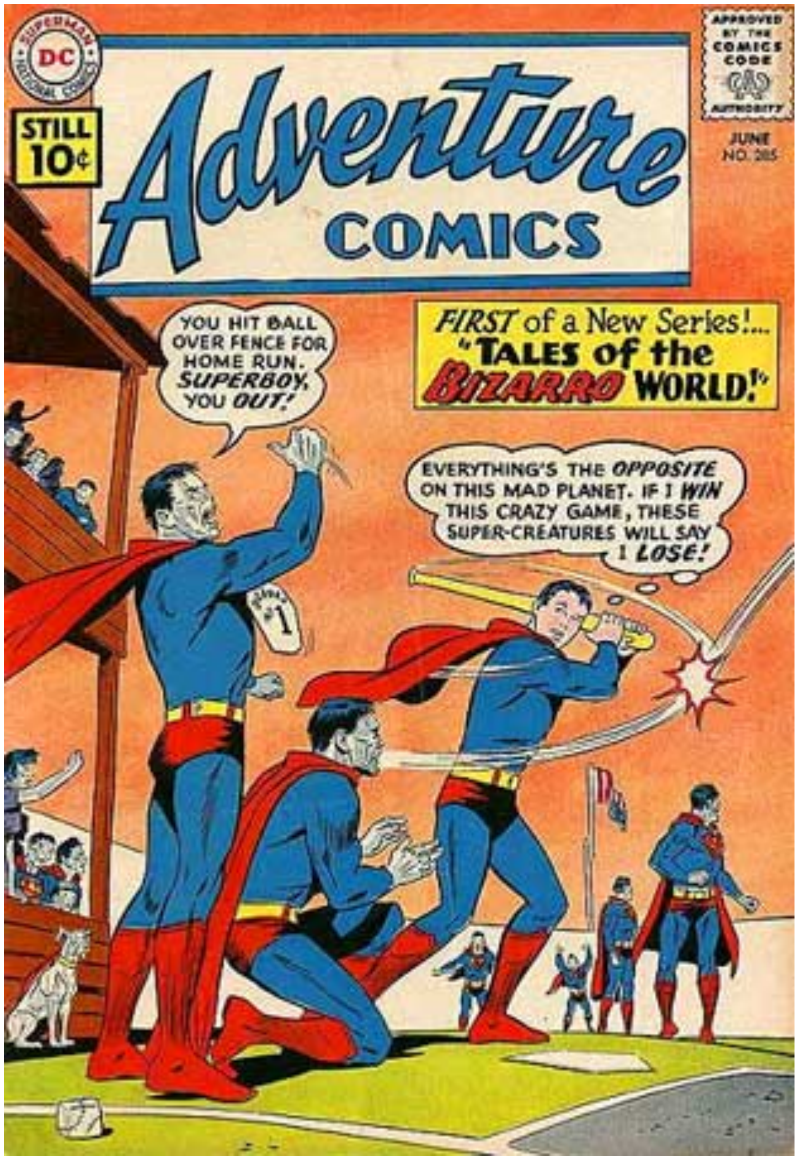


MCR3UI

Unit 1: Functions and Transformations

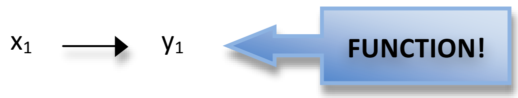
Functions and Function Notation

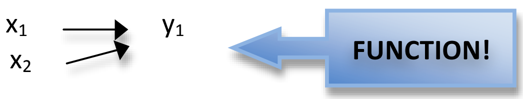
A. Functions and Relations

A **relation** is defined as an identified *pattern* or *relationship* between two variables. Relations can be represented in various ways: as a set of **ordered pairs**, a **table of values**, a **graph**, or an **equation**.

A **function** is a type of relation between two variables, an *input* variable and an *output* variable, in which each value of the *input* variable corresponds to no more than **ONE** value of the *output* variable.

∴ A relation is **NOT** a function if *one x value has 2 or more different y-values associated* with it.





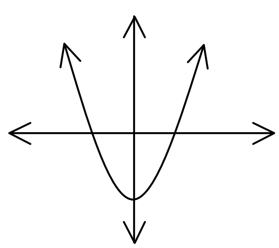
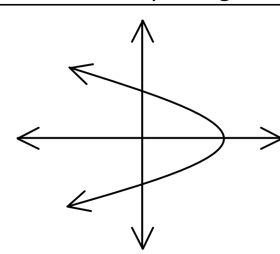
B. The Vertical Line Test (or VLT!)

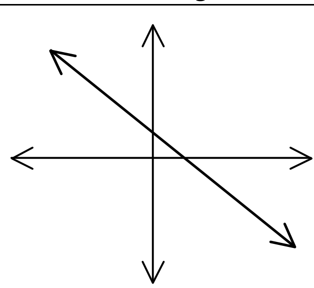
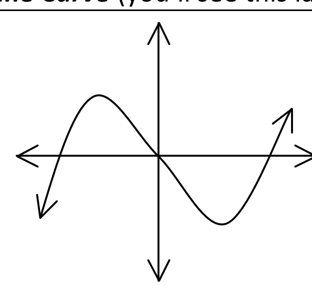
An easy way of determining whether or not a relation is a function is to use its graph and the Vertical Line Test.

The **Vertical Line Test** states that a relation is a function if you can draw a vertical line that passes through **ZERO** points or **ONE** point on the graph of the relation.

∴ A relation ***fails*** the vertical line test if the line intersects the graph at **more than one** point!

Ex. 1: Let's look at some examples below…

Parabola Opening Up	***Parabola*** Opening to the Left
	
Does this pass the Vertical Line Test? YES / NO Therefore, is this relation a function? YES / NO	Does this pass the Vertical Line Test? YES / NO Therefore, is this relation a function? YES / NO

Straight Line	***Sine Curve*** (you'll see this later)
	
Is this relation a function? YES / NO	Is this relation a function? YES / NO

C. What is Function Notation?

In an equation such as $y = 2x + 3$, ______ depends on ______ and is said to be a *function* of *x*. Since we are dealing with functions, and not all relations represent functions, we're going to use a special type of notation when writing equations that represent functions. It's called **FUNCTION NOTATION**!

Equation	Function Notation
$y = 3x + 1$	$f(x) = 3x + 1$
$d = 3t^2 - 2t + 1$	
$A = \pi r^2$	
$v = t^2 - 21$	

"*f(x)*" is read "*f of x*" or "*f at x*". It represents the height of the function at a given independent (x) value.

Using **function notation** is similar to using equations involving ***x*** and ***y*** values. To find a ***y***-value given an ***x***-value simply requires ***substitution***. Thus, we can write ordered pairs (*x, f(x)*) which are the same as (*x, y*) since ***y = f(x).***

D. Performing Operations Using Function Notation

i) **Given an *x*-value**, we can use substitution to **find the height** of the function at that particular *x*-value (i.e. we can find the corresponding *y*-coordinate on the Cartesian plane).

Ex. 2: If $f(x) = -3x + 2$, find:

a) $f(-1)$

b) $f(0)$

Ex. 3: If $g(x) = -2x^2 - 5x + 2$, find:

a) $g(-1)$

b) $2[g(2)]$

ii) **Given the height** of the function, we can use substitution to **solve the equation for x** at that particular height (i.e. we can find the corresponding *x*-coordinate on the Cartesian plane).

Ex. 4: If $f(x) = 5x - 1$ and $g(x) = x^2 + 3x$, find the value(s) of *x* when

a) $f(x) = 24$ **b)** $g(x) = 18$ **c)** $g(x) = 0$ **d)** $f(x) = g(x)$

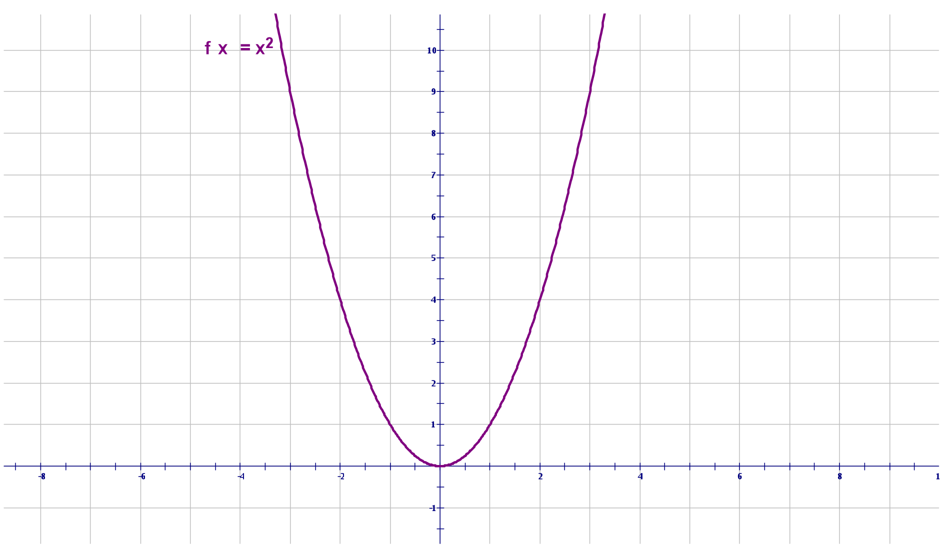
iii) **Given an expression that replaces the x-value** of the function, we can use substitution to **write and simplify** a new expression representing a new function!

Ex. 5: Given that $f(x) = 2x^2 - x + 1$ and $g(x) = \sqrt{4x+1}$:

a) write and simplify $f(x+3)$ **b)** write and simplify $g(a^2 - a)$

iv) **Given the graph** of the function, we can **interpret** the height at varying *x*-values, and vice versa.

Ex. 6: Find the required values using the graph of $f(x) = x^2$ below.



a) $f(2) =$ ________

b) $f(-1) =$ ________

c) $f(0) =$ ________

d) $f(x) = 9,\ x =$ ________________

e) $f(x) = 1,\ x =$ ________________

HW: p. 178-181 #1, 7-10(odds), 12, 26, 32

Domain and Range

A. Describing Relations and Functions Using Number Sets

In order to understand how relations behave, we need to understand the sets of numbers that they can inhabit:

N: denotes the set of ***natural*** numbers, which include all positive counting numbers but NOT zero;
N = {1, 2, 3, . . .}

W: denotes all ***whole*** numbers, including positive counting numbers and zero;
W = {0, 1, 2, 3, . . .}

I: denotes the set of ***integers***, which includes all positive and negative whole numbers, as well as zero;
I = {. . . , -3, -2, -1, 0, 1, 2, 3, . . .}

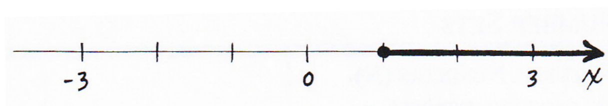
R: denotes the set of all ***real*** numbers, including all integers; fractions/rational numbers [Q] (terminating decimals and repeating decimals); and irrational numbers [$\overline{Q}$](non-terminating, non-repeating decimals)

B. Domain

The domain of a relation is the set of all _______________________ (___) elements of a relation. The domain describes the values that are acceptable for the **independent** variable.

Domains can be communicated in ***words***, using a ***number line***, or as a ***set*** (giving a *list* of numbers or using *inequality* statements). To create a set, you need to know proper set notation.

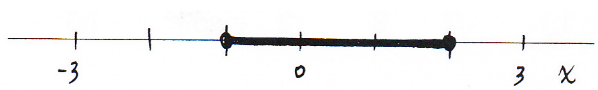
Ex. 1: Given the number line …



Words: *"x is any real number greater than or equal to 1"*

Set Notation: D = $\{ x \mid x \in R, x \geq 1 \}$

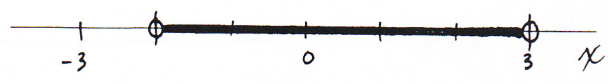
Ex. 2: Given the number line …



Words: *"x is any real number greater than or equal to –1 and less than or equal to 2"*

Set Notation: ____

Ex. 3: Given the number line …



Words: ___________ *"x is any real number greater than –2 and less than 3"* ___________________

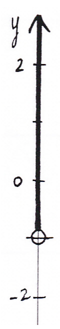
Set Notation: ____

C. Range

The range of a relation is the set of all _______________________ (___) elements of a relation. The range describes the values that are acceptable for the **dependent** variable.

Ranges can also be communicated in ***words***, using a ***number line***, or as a ***set***.

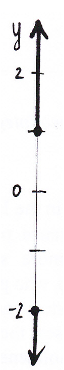
Ex. 4: Given the number line …



Words: ______________ *"y is any real number greater than –1"* ____________________________

Set Notation: ____

Ex. 5: Given the number line …



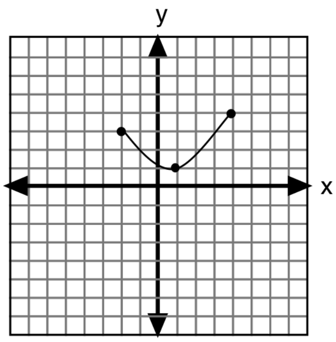
Words: ___ *"y is any real number greater than or equal to 1 or less than or equal to –2"* ________

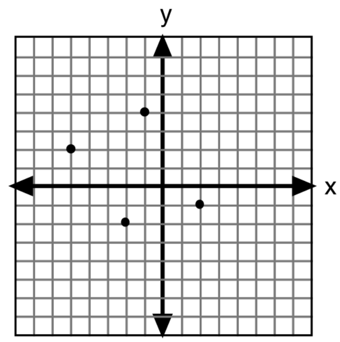
Set Notation: ____

D. Bringing Domain and Range Together

Now that we've worked with domain and range separately, let's bring them together!

Ex. 6: Use *set notation* to describe the domain and range of the following relations:





x	*y*
– 4	7
– 3	2
– 8	5
– 4	– 1

Function? ___________________________

D = _________________________________

R = _________________________________

Function? _______________________

D = _____________________________

R = _____________________________

Function? _______________________

D = _____________________________

R = _____________________________

Ex. 7: Write the domain and range for the following relations:

a) $0 = 2x - y$ *Function?* ___________

D = _____________________________________

R = _____________________________________

b) $16 = x^2 + y^2$ *Function?* ___________

D = _____________________________________

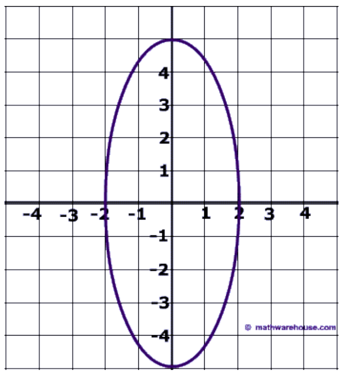
R = _____________________________________

c) $0 = 2x^2 - y$ *Function?* ___________

D = _____________________________________

R = _____________________________________

Ex. 8: Describe the domain and range of the following relations in the manner indicated:



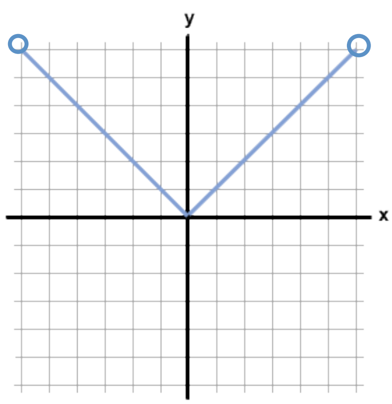
Domain – words:

x is any real number greater than or equal to –2 and less than or equal to 2

Domain – set notation:

Range – set notation:

Function (with reason)?



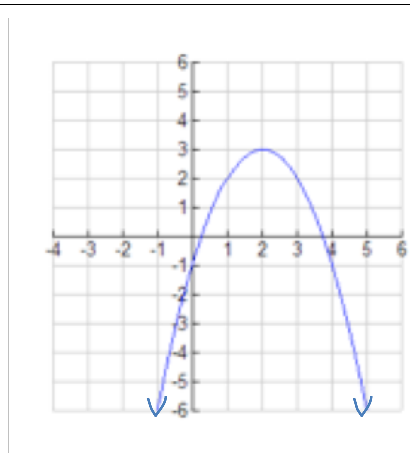
Domain – words:

x is any real number greater than ________ and less than ________

Domain – set notation:

Range – set notation:

Function (with reason)?



Domain – words:

Domain – set notation:

Range – set notation:

Function (with reason)?

HW: p. 178-181 #2-5, 17, 18, 21, 25

Workshop: Graphing Quadratic Functions Given Any Form

A. Characteristics of the Quadratic Function $f(x) = x^2$

The graph of the quadratic function is a *parabola*. Basic properties of the quadratic function $f(x) = x^2$ are as follows:

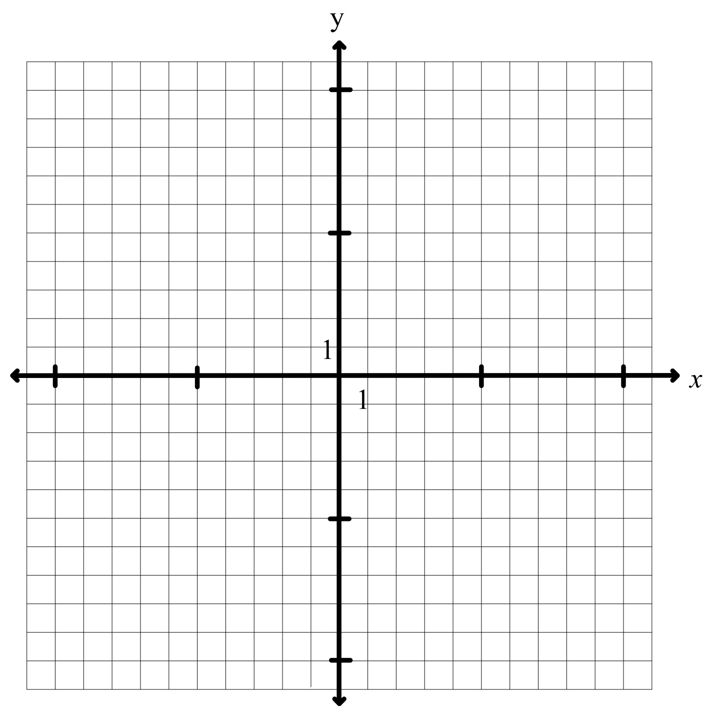
- **standard form** is written as $y = ax^2 + bx + c$, where (0, c) is the *y-intercept.*
- **factored form** is written as $y = a(x - r)(x - s)$, where (r, 0) and (s, 0) are the *x-intercepts, zeros* or *roots*.
- **vertex form** is written as $y = a(x - h)^2 + k$, where (h, k) is the *vertex*.
- the **direction of opening** can be determined by inspecting the '**a**' value:
  - when $a > 0$, the parabola opens up and the vertex is a minimum.
  - when $a < 0$, the parabola opens down and the vertex is a maximum.
- the **axis of symmetry** is a vertical line drawn through the vertex across which the parabola is symmetrical; it has the equation $x = h$.
- the **domain** is $D = \{x \mid x \in R\}$
- the **range** is $R = \{y \mid y \in R, y \geq k$ **or** $y \leq k\}$

Ex. 1: Graph the function $f(x) = x^2$ on the grid below. State the domain and range. When graphing quadratic functions, include the following on the graph:

- the equation of the function
- the vertex and any intercepts
- the axis of symmetry (sketched and labeled)
- key points (clearly marked with a dot)

$f(x) = x^2$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



D = ____

R = ____

B. Vertex Form of a Quadratic Function

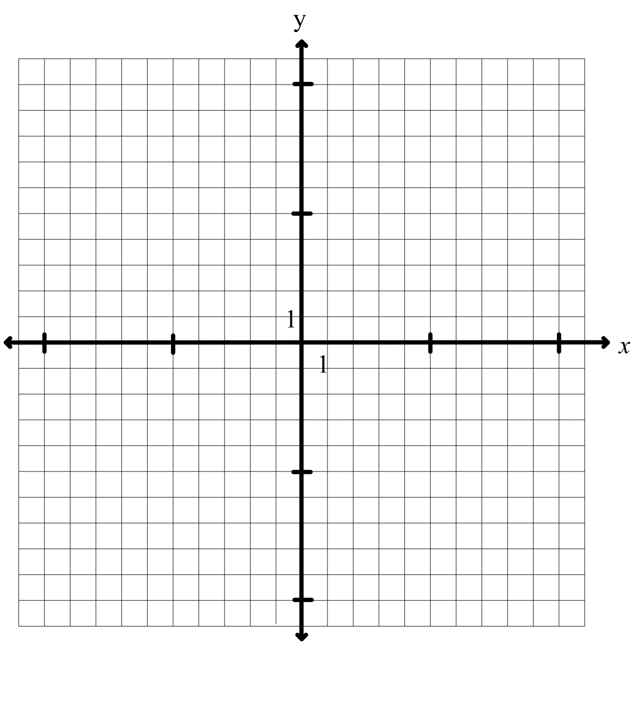
	Characteristics	**What To Do . . .**
Vertex Form	$f(x) = a(x-h)^2 + k$ ***or*** $y = a(x-h)^2 + k$ • In this form, the only <u>point</u> evident is the <u>vertex</u>, (h, k) • the a value determines the direction of opening and the step pattern	• Plot and label the vertex, (h, k) • From the vertex, move horizontally (⇔) 1 unit, and vertically (⇕) $1^2(a)$ units (on either side); then, move horizontally 2 units, and vertically $2^2(a)$ units; [repeat with 3 units and $3^2(a)$ units, if necessary, to get accurate end behaviour] • Sketch and label the axis of symmetry, $x = h$

Ex. 2: Sketch the following quadratic functions on the grids below. State the vertex (h, k), equation of the axis of symmetry (x = h), domain, and range.

a) $f(x) = (x + 2)^2 - 4$

Vertex: ________________________________

Axis of symmetry: ________________________



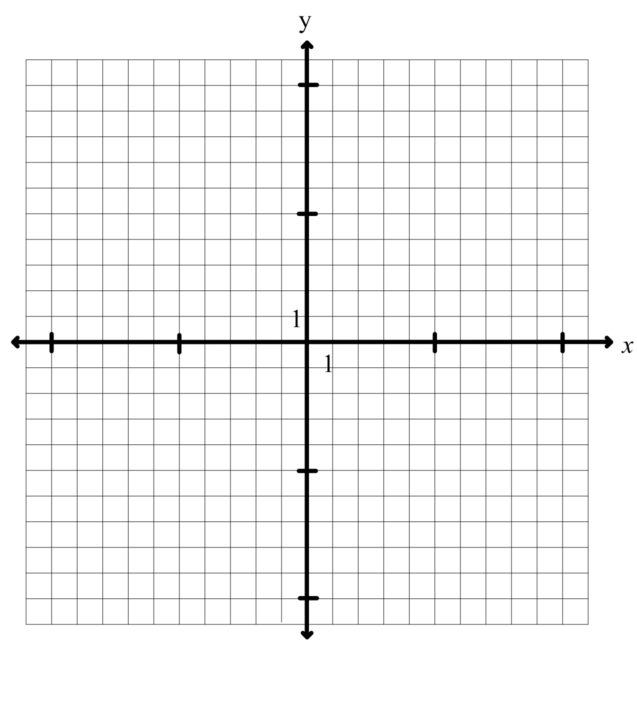
D = ____________________________________

R = ____________________________________

b) $f(x) = -\frac{1}{2}(x - 5)^2 + 1$

Vertex: ________________________________

Axis of symmetry: ________________________



D = ____________________________________

R = ____________________________________

C. Factored Form of a Quadratic Function

	Characteristics	What To Do . . .

Factored Form	$f(x) = a(x - r)(x - s)$ ***or*** $y = a(x - r)(x - s)$
	
	• In this form, the only <u>points</u> evident are the <u>x-intercepts</u>, $(r, 0)$ and $(s, 0)$
	• the x-value of the vertex ("h") can be calculated by finding the value of $\frac{r+s}{2}$
	• the y-value of the vertex ("k") can be calculated by finding the value of $f(h)$
	• the a value determines the direction of opening and the step pattern
		• Plot and label the x-intercepts, $(r, 0)$ and $(s, 0)$
		• Find and plot the vertex, (h, k), using:
		  ○ $h = \frac{r+s}{2}$
		  ○ $k = f(h)$
		• From the vertex, move horizontally (⇔) 1 unit, and vertically (⇳) $1^2(a)$ units (on either side); then, move horizontally 2 units, and vertically $2^2(a)$ units; [repeat with 3 units and $3^2(a)$ units, if necessary, to get accurate end behaviour]
		• Sketch and label the axis of symmetry, $x = h$

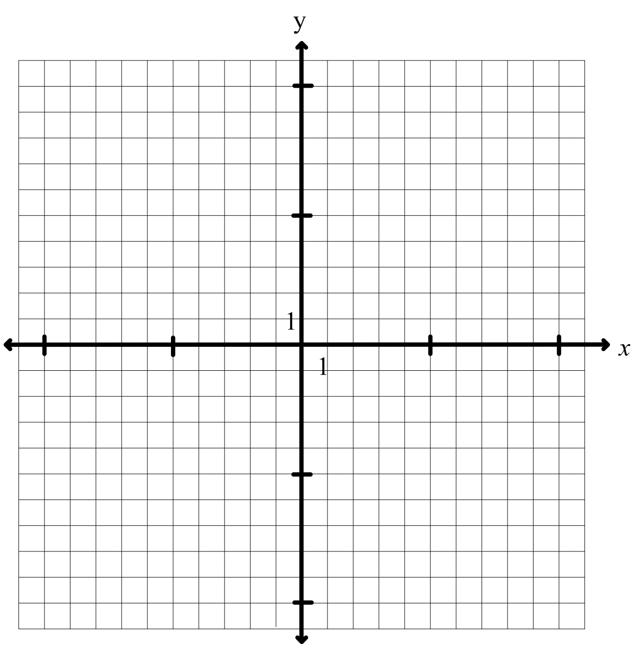
Ex. 3: Sketch the following quadratic functions on the grids below. State the *x*-intercepts, vertex (*h*, *k*), equation of the axis of symmetry (*x* = h), and range.

a) $f(x) = 2(x - 3)(x - 7)$

x-int(s): ________________________________

Vertex: ________________________________

Axis of symmetry: ________________________



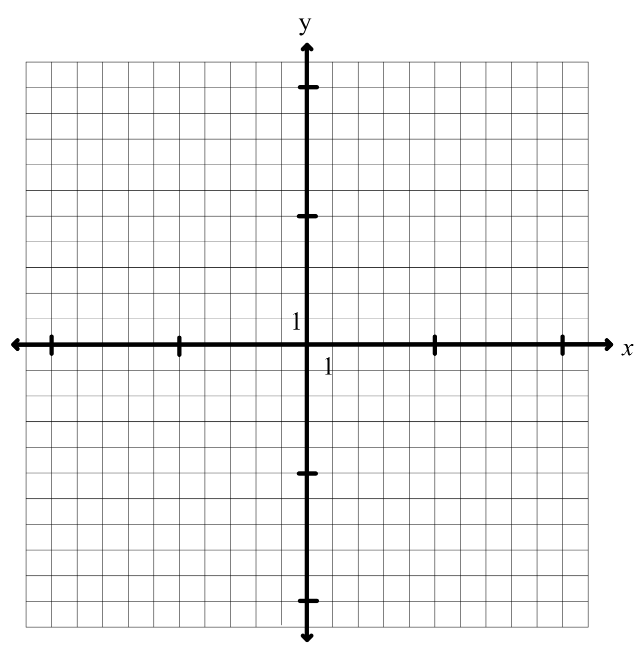
R = ____________________________________

b) $f(x) = -\frac{1}{3}x(x - 4)$

x-int(s): ________________________________

Vertex: ________________________________

Axis of symmetry: ________________________



R = ____________________________________

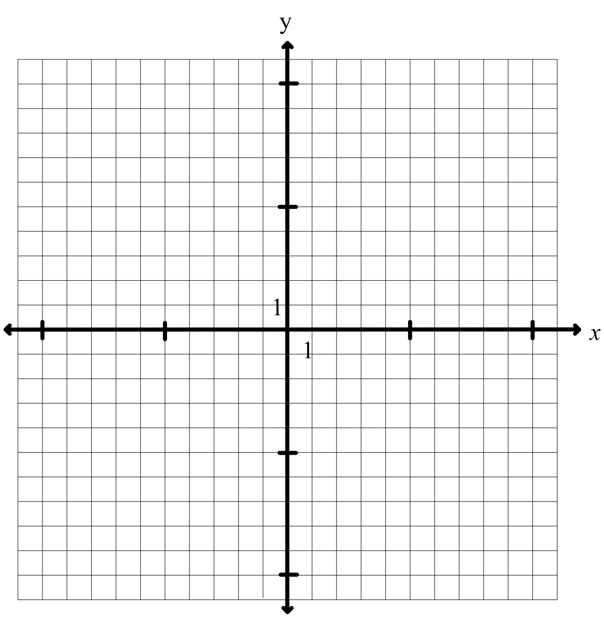
D. Standard Form of a Quadratic Function

	Characteristics	What To Do . . .
Standard Form	$f(x) = ax^2 + bx + c$ ***or*** $y = ax^2 + bx + c$ • In this form, the only <u>point</u> evident is the <u>y-intercept</u>, (0, *c*) • the vertex (*h, k*) can be determined by <u>completing the square</u> • the *a* value determines the direction of opening and the step pattern	• Plot and label the y-intercept, (0, *c*) • Find, plot, and label the vertex, (*h, k*), by first <u>completing the square</u> • From the vertex, move horizontally (⇔) 1 unit, and vertically (⇳) $1^2(a)$ units (on either side); then, move horizontally 2 units, and vertically $2^2(a)$ units; [repeat with 3 units and $3^2(a)$ units, if necessary, to get accurate end behaviour] • Sketch and label the axis of symmetry, $x = h$

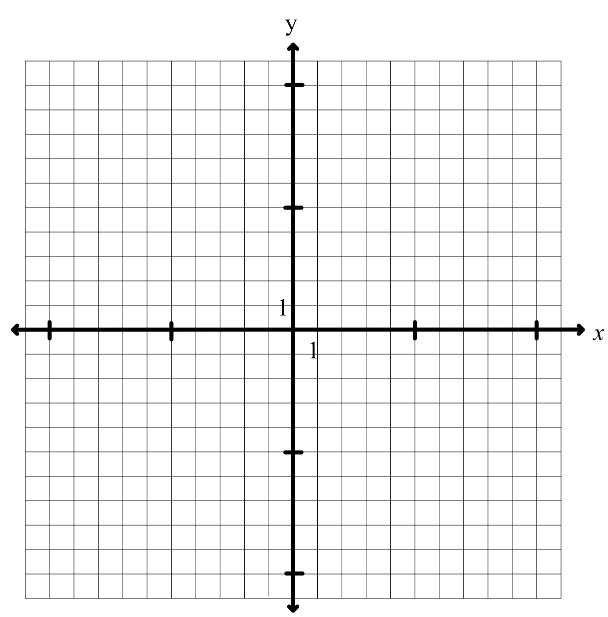
Ex. 4: Complete the square on the following quadratic functions to find the vertex; then, graph them on the grids below. Label the *y*-intercept, vertex (*h*, *k*), and equation of the axis of symmetry (*x* = h) on the graph. State the range.

a) $f(x) = x^2 - 4x + 3$

b) $f(x) = -3x^2 + 12x - 5$



R = ____



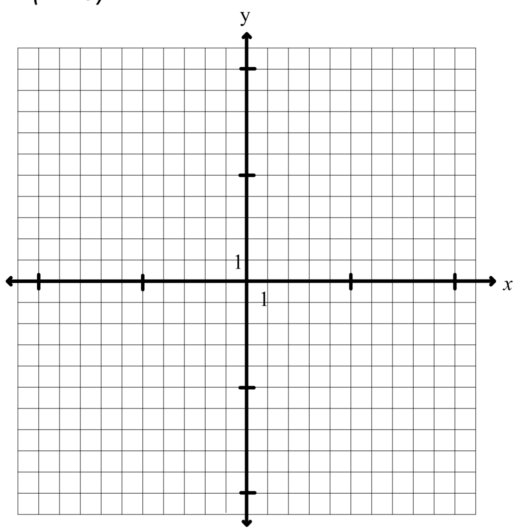
R = ____

HW: WORKSHEET: "Graphing Quadratic Functions Given Any Form"

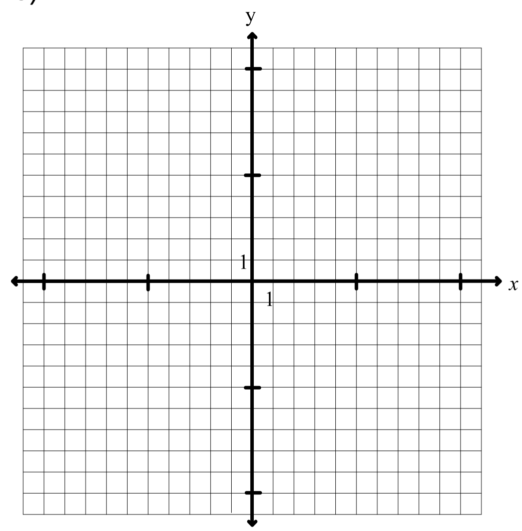
WORKSHEET: Graphing Quadratic Functions Given Any Form

Graph each of the following quadratic functions on the grids provided. Label each graph appropriately!

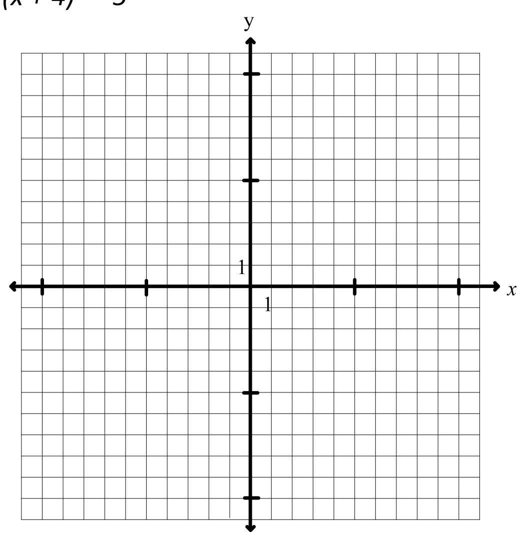
$f(x) = -x(x-6)$



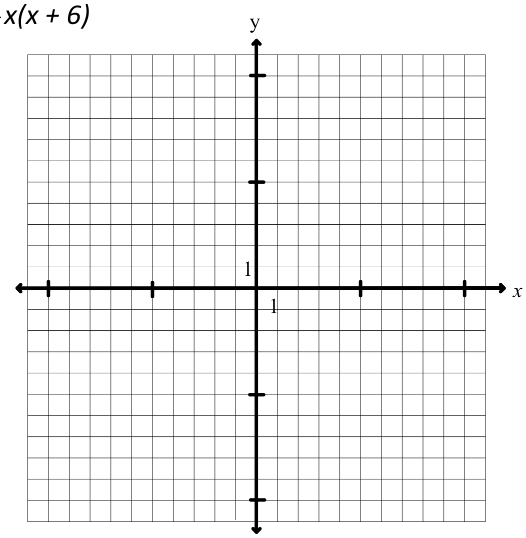
$f(x) = (x-3)^2 + 2$



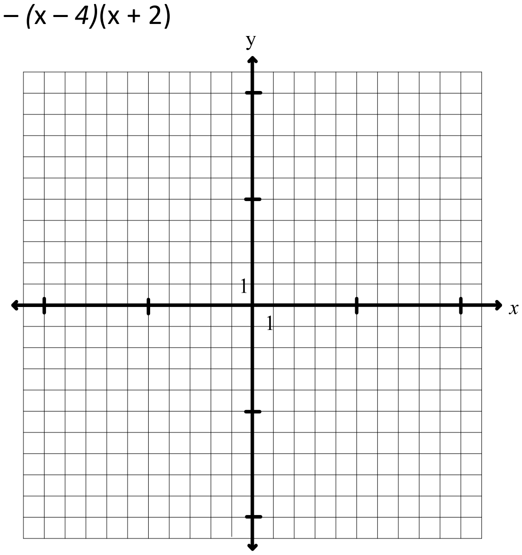
$f(x) = (x+4)^2 - 5$



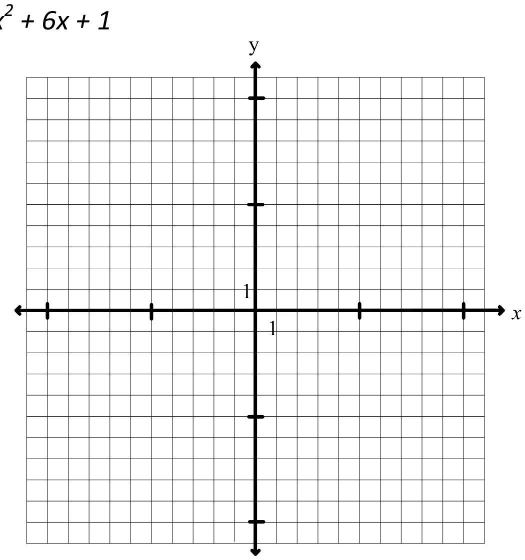
$f(x) = \frac{1}{2}x(x+6)$



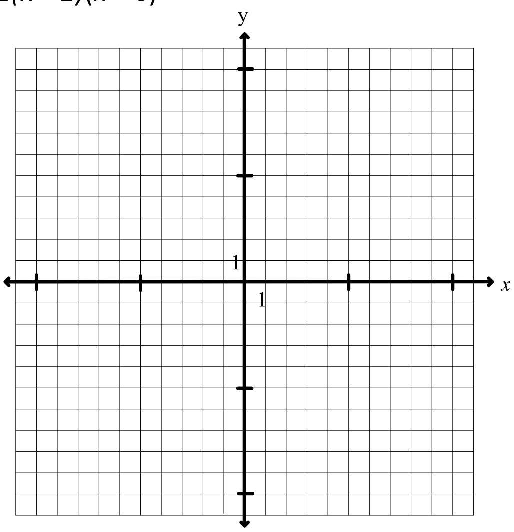
$f(x) = -(x-4)(x+2)$



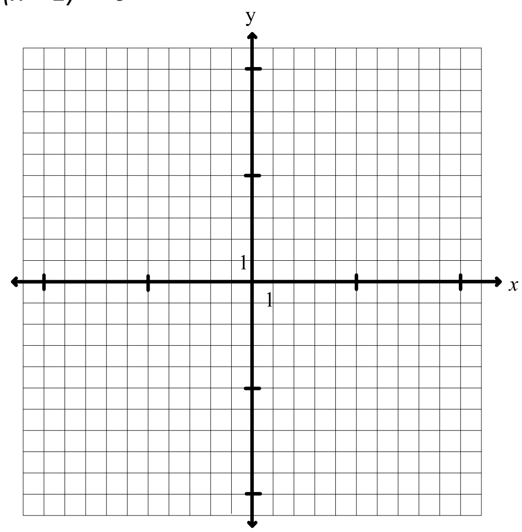
$f(x) = 3x^2 + 6x + 1$



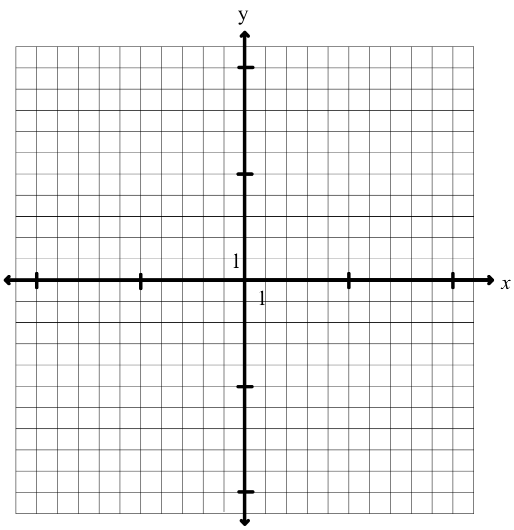
$f(x) = 2(x - 2)(x - 6)$



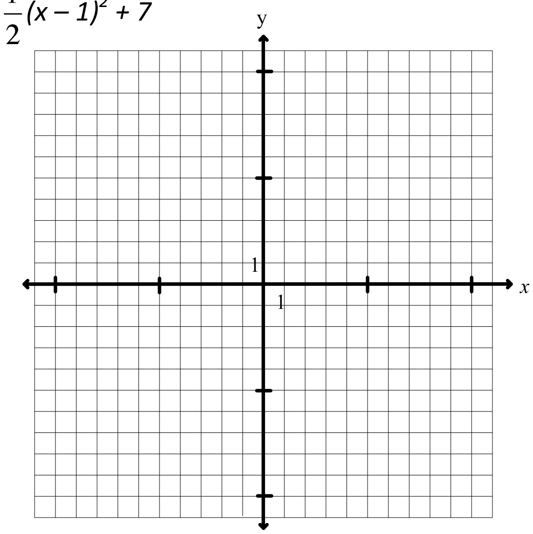
$f(x) = -3(x + 1)^2 - 6$



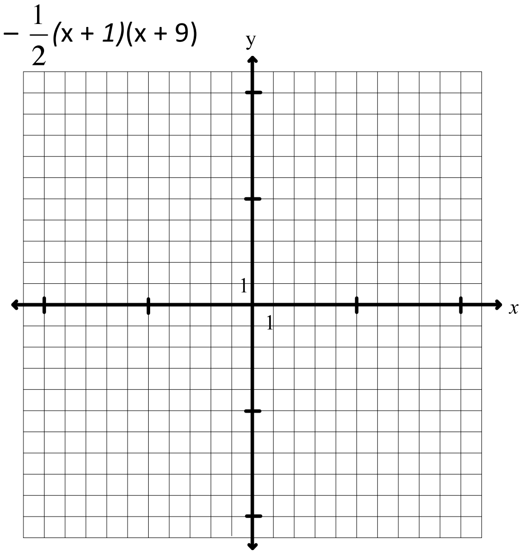
$f(x) = x^2 - 2x + 5$



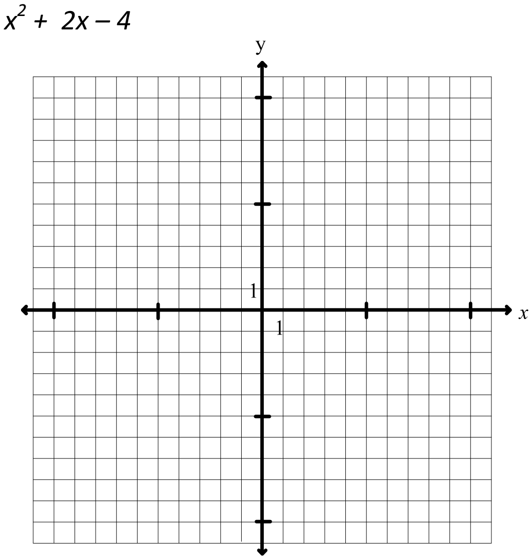
$f(x) = -\frac{1}{2}(x - 1)^2 + 7$



$f(x) = -\frac{1}{2}(x + 1)(x + 9)$



$f(x) = -x^2 + 2x - 4$



Transformations of Functions

A **Transformation** is an operation that moves (or maps) a figure from an original position to a new position. Transformations that we will consider are reflections, stretches, and translations on any base function ***y* = *f*(*x*)**:

$$y = af[k(x-d)] + c$$

Base Function	$y = x^2$	$y = \sqrt{x}$	$y = \frac{1}{x}$	$y = \sin x$	$y = (b)^x$
Transformed Function	$y = a(k(x-d))^2 + c$	$y = a\sqrt{k(x-d)} + c$	$y = a\left(\frac{1}{k(x-d)}\right) + c$	$y = a\sin k(x-d) + c$	$y = a(b)^{k(x-d)} + c$

*Perform combinations of transformations in the following **order**:*

1. Reflections on the Function *y* = *f*(x):

Reflection	Form	Effect
Vertical	*y* = – *f(x)*	Compared to y = *f*(x), the graph of y = – *f*(x) is a vertical reflection across the x- axis. The point (x, y) on *y* = *f*(x) becomes the point (x, – y) on *y* = – *f*(x).
Horizontal	*y* = *f*(– *x*)	Compared to y = *f*(x), the graph of y = *f*(– x) is a horizontal reflection across the y- axis. The point (x, y) on *y* = *f*(x) becomes the point (–x, y) on *y* = *f*(– x).

2. Stretches on the Function *y* = *f*(x):

Stretch	Form	Effect
Vertical	*y* = a *f(x)*	If **a > 1**, the graph is **vertically expanded** by a factor of **a**. If **0 < a < 1**, the graph is **vertically compressed** by a factor of **a**. The point (x, y) on *y* = *f*(x) becomes the point (x, ay) on *y* = a*f*(x).
Horizontal	*y* = *f*(k*x*)	If **k > 1**, the graph is **horizontally compressed** by a factor of $\frac{1}{k}$. If **0 < k < 1**, the graph is **horizontally expanded** by a factor of $\frac{1}{k}$ The point (x, y) on *y* = *f*(x) becomes the point ($\frac{1}{k}$x, y) on *y* = *f*(k*x*).

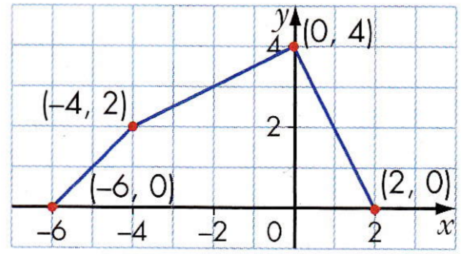
3. Translations on the Function *y* = *f*(x):

Translation	Form	Effect
Vertical	*y* = *f(x)* + c	Compared to the graph of *y* = *f*(x), the graph of *y* = *f*(x) + c is a vertical translation of c units. When **c > 0** the graph is **vertically translated up c units**. When **c < 0** the graph is **vertically translated down c units**. The point (x, y) on *y* = *f*(x) becomes the point (x, *y* + c) on *y* = *f*(x) + c.
Horizontal*	*y* = *f*(*x* – d)	Compared to the graph of *y* = *f*(x), the graph of *y* = *f*(*x* – d) is a horizontal translation of d units. When **d > 0** the graph is **horizontally translated to the RIGHT d units**. When **d < 0** the graph is **horizontally translated to the LEFT d units**. The point (x, y) on *y* = *f*(x) becomes the point (x + d, *y*) on *y* = *f*(*x* – d).

If necessary, **factor the coefficient of the x-term to identify the horizontal translation more easily.*

A. Translations on *y = f(x)*

1. The graph of *y = f(x)* is shown to the right. Sketch the following transformed functions on the grids below and state their domain and range:



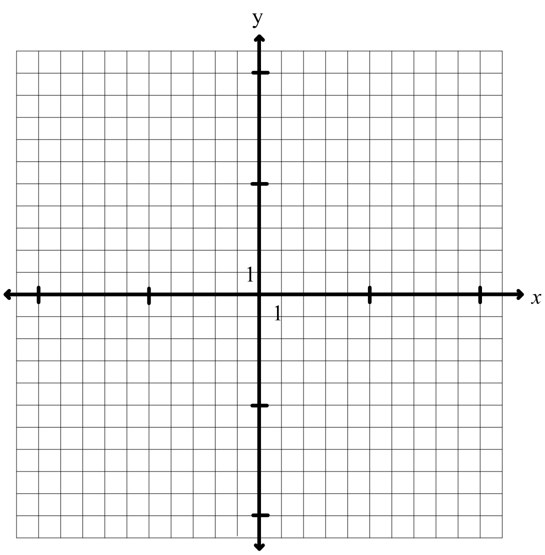
a) y = f(x) + 6

Description of transformations on y = f(x):

____

____

(x, y) → (___________, ___________)



D = ______________________________________

R = ______________________________________

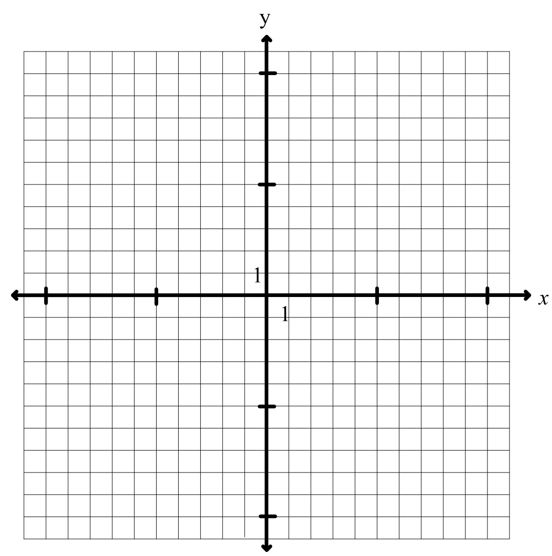
b) y = f(x + 2)

Description of transformations on y = f(x):

____

____

(x, y) → (___________, ___________)

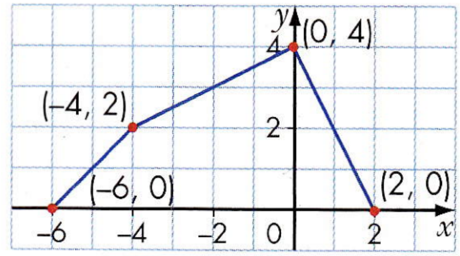


D = ______________________________________

R = ______________________________________

B. Stretches and Reflections on *y = f(x)*

1. The graph of *y = f(x)* is shown to the right. Sketch the following transformed functions on the grids below and state their domain and range:



a) y = −3f(x)

Description of transformations on y = f(x):

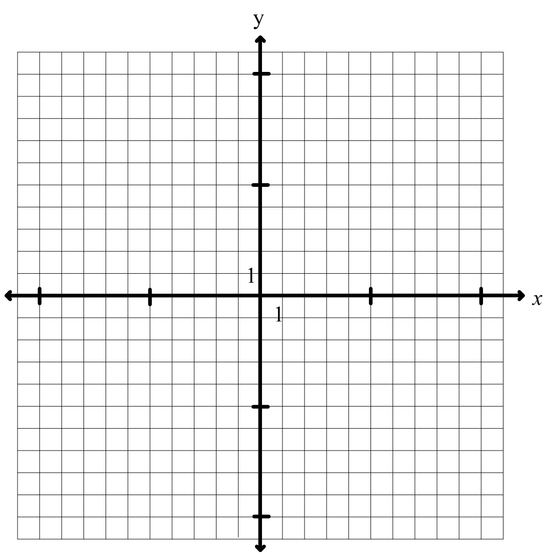
____

____

____

____

(x, y) → (____________, ____________)



D = ______________________________________

R = ______________________________________

b) y = f(−2x)

Description of transformations on y = f(x):

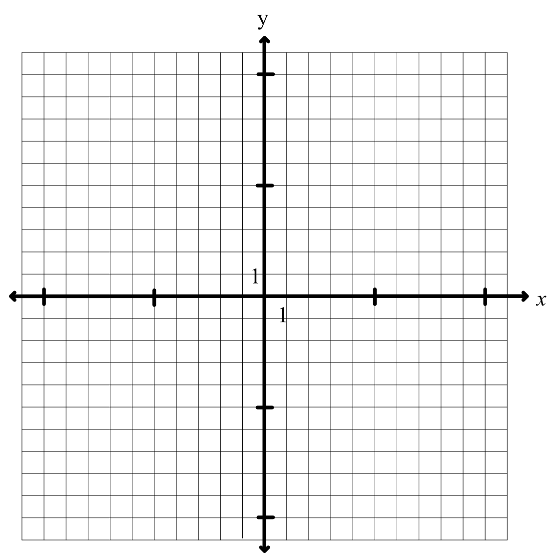
____

____

____

____

(x, y) → (____________, ____________)



D = ______________________________________

R = ______________________________________

C. Combinations of Transformations

When the function $y = f(x)$ has been transformed to $y = af\left[k(x-d)\right]+c$, each point (x,y) on the base function becomes point $\left(\frac{1}{k}x - d,\ ay + c\right)$ on the transformed function.

1. Describe how the graphs of the following functions can be obtained from the graph of the function *y = f(x)* and write the point-by-point transformation.

   a. $y = f(3x) + 8$

   Description of transformations on y = f(x):

   ____

   ____

   $(x, y) \rightarrow ($ ___________, ___________ $)$

   b. $y = -f[\frac{1}{2}(x-4)]$

   Description of transformations on y = f(x):

   ____

   ____

   ____

   $(x, y) \rightarrow ($ ___________, ___________ $)$

   c. $y = 4f(-x-2) - 5$

   Description of transformations on y = f(x):

   ____

   ____

   ____

   ____

   $(x, y) \rightarrow ($ ___________, ___________ $)$

   d. $y = 2f(6-3x) - 12$

   Description of transformations on y = f(x):

   ____

   ____

   ____

   ____

   $(x, y) \rightarrow ($ ___________, ___________ $)$

2. The function ***y = f(x)*** has been transformed to ***y* = a*f*[k(*x* – d)] + c.** Determine the values of a, k, d, and c for each of the following transformations:

   a. *a vertical compression by a factor of ½ and a horizontal translation 7 units left:*

   y = ___________________________________

   b. *a reflection across the x-axis, a vertical expansion by a factor of 3, a horizontal compression by a factor of ¼, and a vertical translation 10 units up:*

   y = ___________________________________

HW: p. 231 #10; p. 240-242 #3, 5, 7, 17

Transformations of Quadratic Functions

A. Transforming Quadratic Functions

Quadratic functions of the form $y = x^2$ can also be transformed according to: $y = a[k(x - d)]^2 + c$.

Ex. 1: Sketch the following transformed functions on the grids below. List the transformations in order on the base function $y = x^2$. State the vertex (d, c), equation of the axis of symmetry (x = d), domain, and range.

a) $y = (x+3)^2 - 2$

Description:

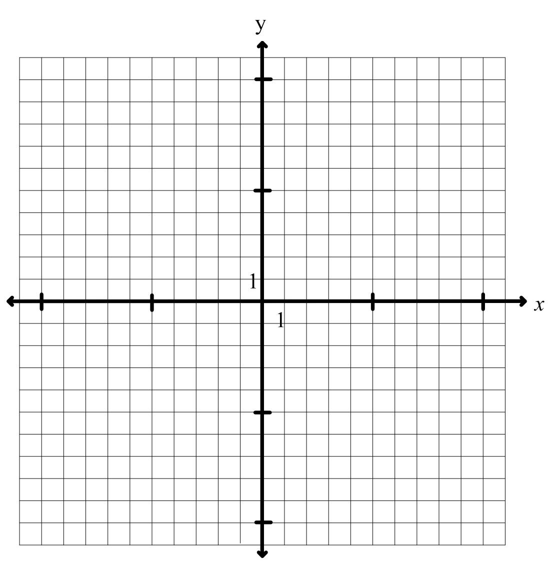
____

____

(x, y) → (_______________, _______________)

Vertex: ___________________________________

Axis of symmetry: _________________________



D = _______________________________________

R = _______________________________________

b) $y - 5 = -2(x-4)^2$

Description:

____

____

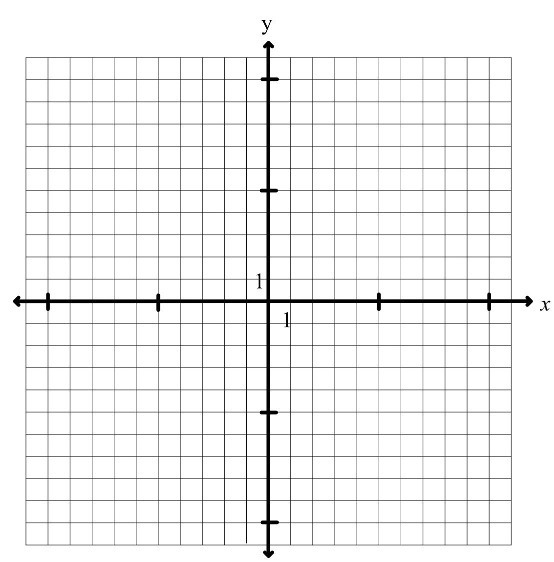
____

____

(x, y) → (_______________, _______________)

Vertex: ___________________________________

Axis of symmetry: _________________________



D = _______________________________________

R = _______________________________________

Ex. 2: Sketch the following transformed functions on the grids below. List the transformations in order on the base function $y = x^2$. State the vertex (d, c), equation of the axis of symmetry (x = d), domain, and range.

a) $y = \left(\frac{1}{3}x + 1\right)^2$

Description:

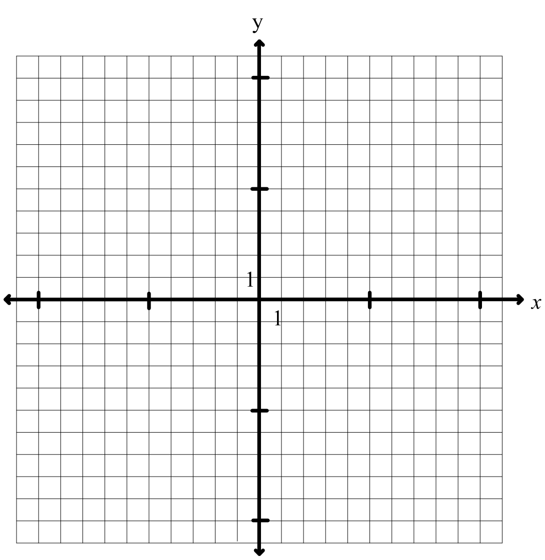
____

____

(x, y) → (_______________, _______________)

Vertex: ___________________________________

Axis of symmetry: _________________________



D = _______________________________________

R = _______________________________________

b) $y = -(2x - 10)^2 + 6$

Description:

____

____

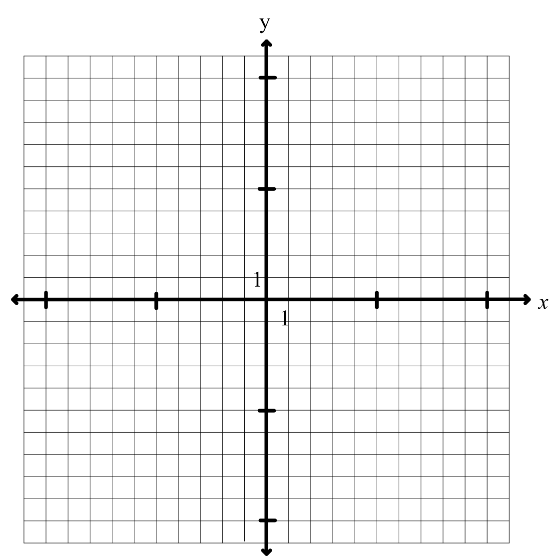
____

____

(x, y) → (_______________, _______________)

Vertex: ___________________________________

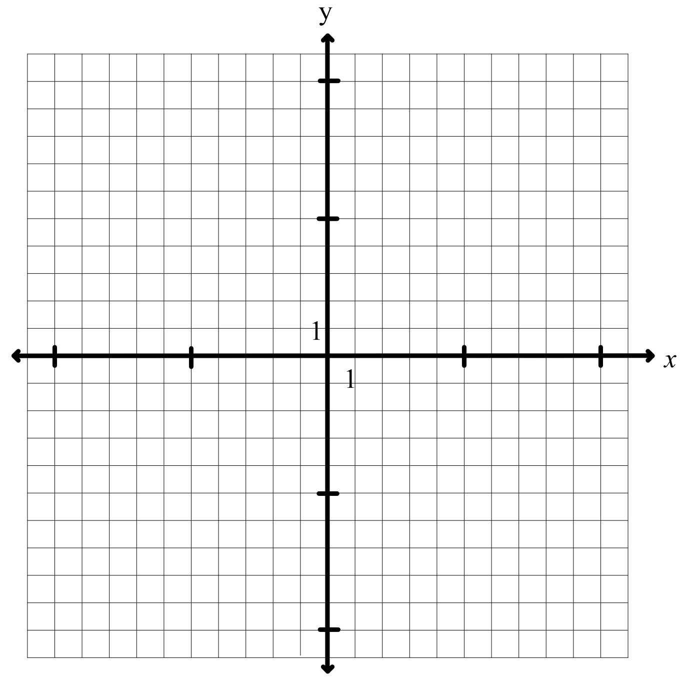
Axis of symmetry: _________________________



D = _______________________________________

R = _______________________________________

Ex. 3: i) Graph $f(x) = -x^2 + 6x$ on the grid provided.



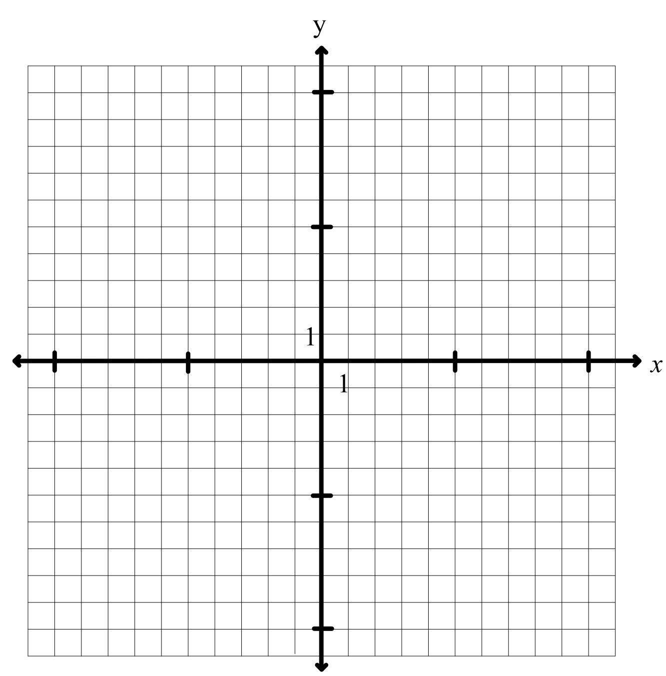
ii) Write equations for a) $y = -f(x)$ and b) $y = f(-x)$.
Graph these functions on the grid provided.

a) $y = -f(x)$

b) $y = f(-x)$

Ex. 4: Given $f(x) = x^2$, a) sketch a graph of $y = -f(2x - 10) + 6$ and b) write an equation for this transformed parabola.

Transformations on y = f(x) are:



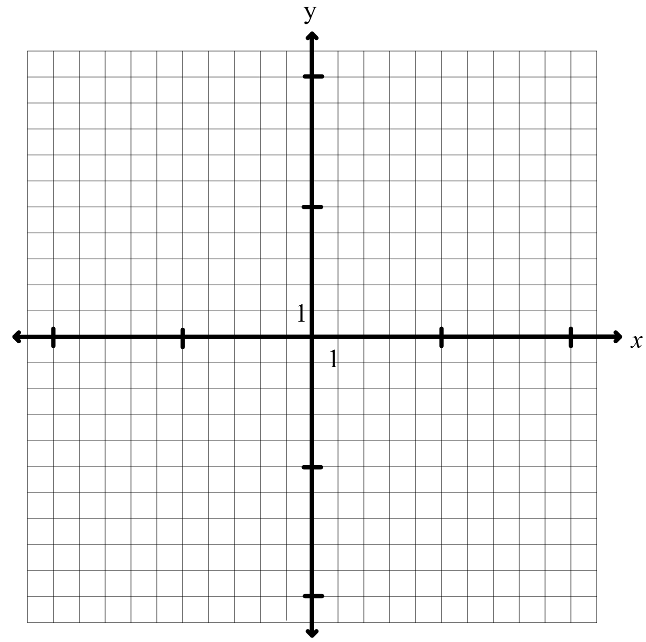
HW: p. 190 #8bd; p. 99 #2bd (include descriptions); p. 204 #7ab, 8a, 9a; p.241 #8bef

Transformations of Square Root Functions

A. Characteristics of the Square Root Function $f(x) = \sqrt{x}$

1. Graph the function $f(x) = \sqrt{x}$ on the grid below. State the domain and range. When graphing square root functions, label the following on the graph:

- the equation of the function
- the "vertex" of the half-parabola
- clearly mark each key point with a dot
- label any intercepts
- draw an arrow showing the direction of opening



$f(x) = \sqrt{x}$

x	y
0	
1	
4	
9	

D = ____

R = ____

The graph of the square root function is *half a parabola* opening to the side. Basic properties of the square root function $f(x) = \sqrt{x}$ are as follows:

- the expression under the radical sign is the **radicand** and takes on all *non-negative* real values
- the **domain** is D = $\{x \mid x \in R, x \geq 0\}$
- the **range** is R = $\{y \mid y \in R, y \geq 0\}$

2. Find the domain of the square root function by inspecting the radicand. It should only take on *non-negative* values.

a) $y = \sqrt{x+3}$

b) $y = \sqrt{3x+4}$

B. Transforming Square Root Functions

Square root functions of the form $y = \sqrt{x}$ can also be transformed according to: $y = a\sqrt{k(x-d)} + c$.

1. Sketch the following transformed functions on the grids below. List the transformations in order on the base function $y = \sqrt{x}$. State the domain and range.

a) $y + 3 = \sqrt{2 - 2x}$

Description:

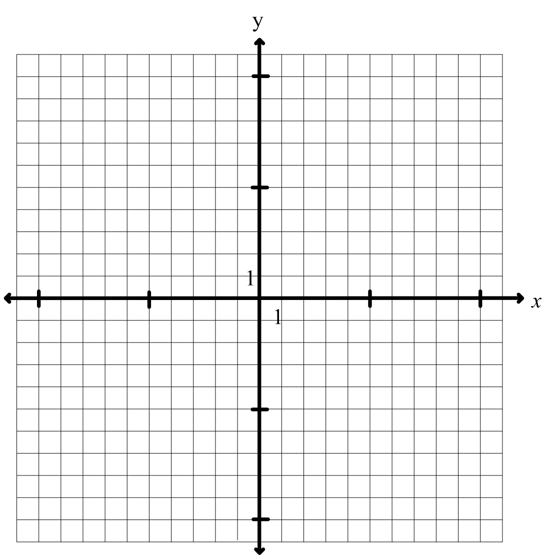
____

____

____

____

(x, y) → (______________ , ______________)



D = ___________________________________

R = ___________________________________

b) $y = -3\sqrt{\frac{1}{2}x + 1} + 4$

Description:

____

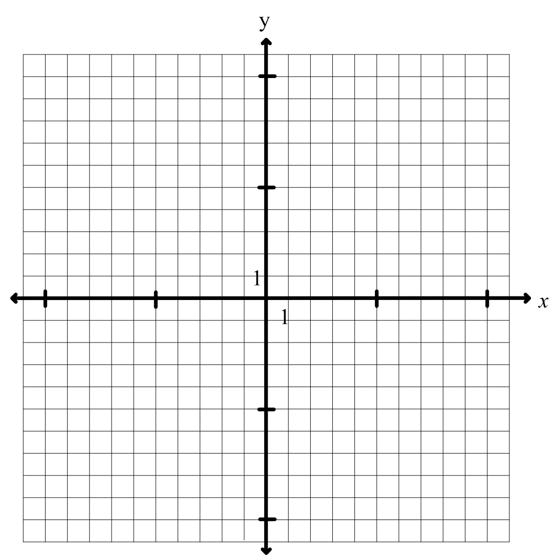
____

____

____

____

(x, y) → (______________ , ______________)



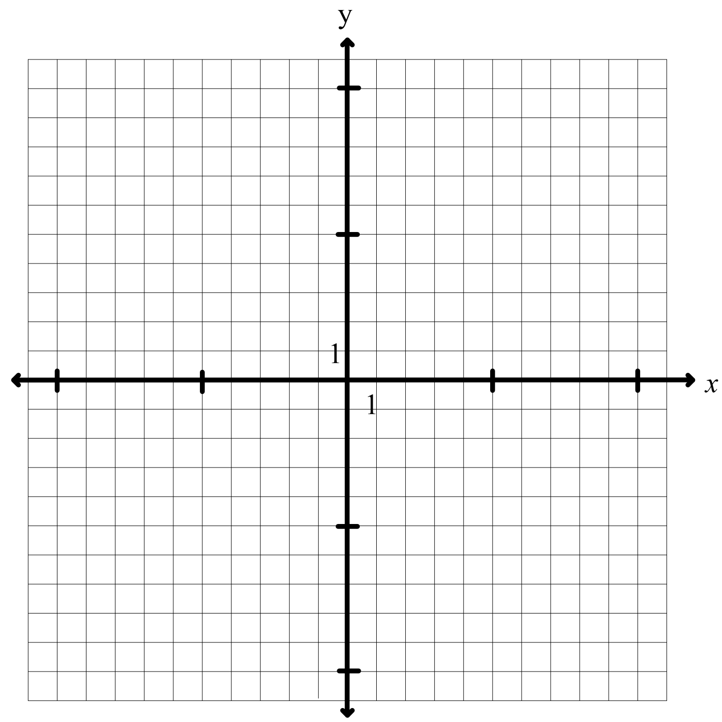
D = ___________________________________

R = ___________________________________

2. Given $f(x)=\sqrt{x+5}-4$, write equations for a) *y = – f(x)* and b) *y = f(–x)*. Graph all three functions.

a) *y = – f(x)*

b) *y = f(–x)*



3. Write the resulting equation when the base function $y=\sqrt{x}$ is vertically reflected across the x-axis, horizontally compressed by a factor of $\frac{1}{2}$, horizontally translated 5 units right, and vertically translated 7 units up.

f (*x*) = ____

4. Given the function $f(x)=\sqrt{x+9}-2$ determine the following:

a) Domain = ____________________________________ Range = ______________________________________

b) Find *x*, if $f(x)=4$

c) Simplify *f(4a² – 9)*

HW: p. 190 #5, 7bdf; p. 205-206 #11, 18; p.241-243 #9bdf, 19

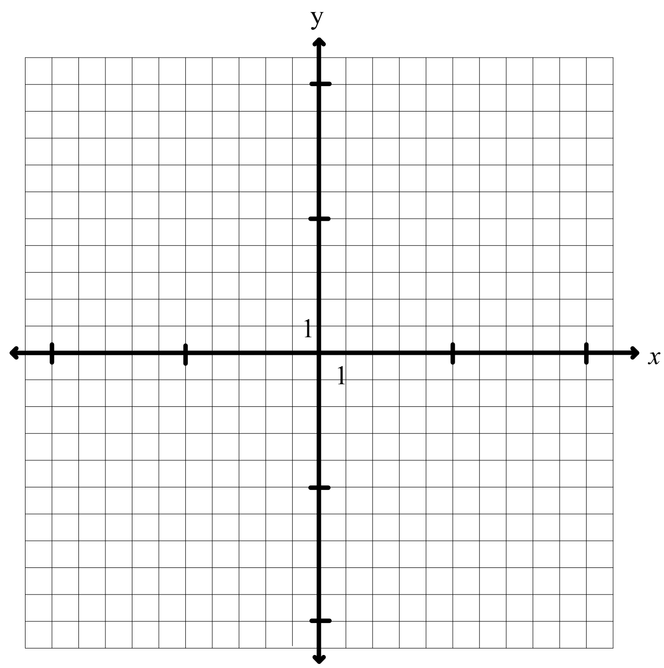
Transformations of Reciprocal Functions

A. Characteristics of the Reciprocal Function *f(x) = 1/x*

1. Graph the function $f(x) = \frac{1}{x}$ on the grid below. State the domain and range.

f(x) = 1/x

x	y
-2	
-1	
-1/2	
0	
1/2	
1	
2	



D = ________________________________

R = ________________________________

The graph of the reciprocal function is *a hyperbola* found in quadrant I and quadrant III of the Cartesian plane.
Basic properties of the reciprocal function ***f(x) = 1/x*** are as follows:

- the value 1/0 is **undefined**, so the expression in the *denominator cannot equal zero*
- any value of *x* that would yield 1/0 is a restriction on the function, and is visualized on the graph as a **vertical asymptote** (note**:** *in this course,* an ***asymptote*** is a line representing a value that the function will approach but *never reach*)
- if the numerator is a constant, no input value of *x* will result in an output value of zero, and is visualized on the graph as a **horizontal asymptote**
- the **domain** is any real number *except* the singularity: D = {x| x ∈ R, x ≠ 0} → v.a.: x = 0
- the **range** is any real number *except* zero: R = {y| y ∈ R, y ≠ 0} → h.a.: y = 0

2. When graphing reciprocal functions of the form $y = \frac{a}{k(x-d)} + c$, label the following on the graph:

- the equation of the function
- the equation of the vertical asymptote, *x = d*
- the equation of the horizontal asymptote, *y = c*
- clearly mark each key point with a dot
- label any intercepts

B. Transforming Reciprocal Functions

Reciprocal functions of the form $y=\frac{1}{x}$ can also be transformed according to: $y=\frac{a}{k(x-d)}+c$.

1. Sketch the following transformed functions on the grids below. List the transformations in order on the base function $y=\frac{1}{x}$. State the domain and range.

a) $y+5=-\frac{1}{x+2}$

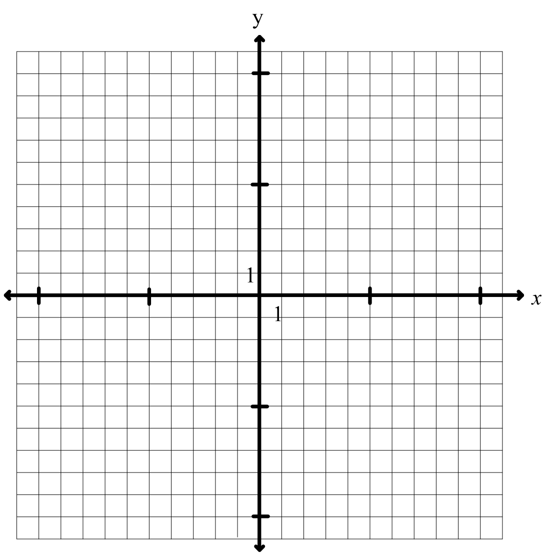
Description:

____

____

____

(x, y) $\rightarrow$ *(_______________, _______________)*



D = ____

R = ____

b) $y=\frac{2}{-x+3}+4$

Description:

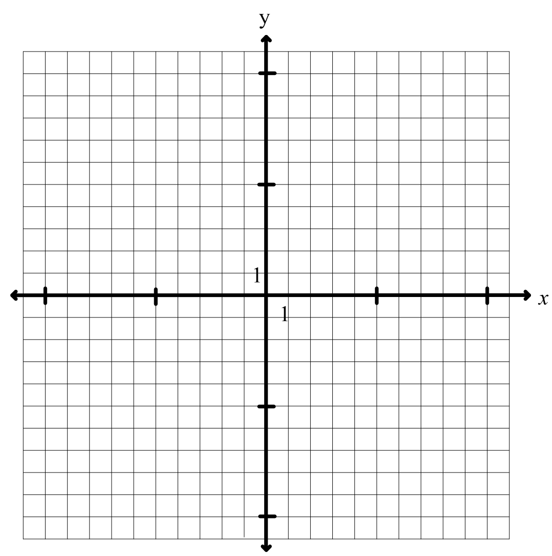
____

____

____

____

(x, y) $\rightarrow$ *(_______________, _______________)*



D = ____

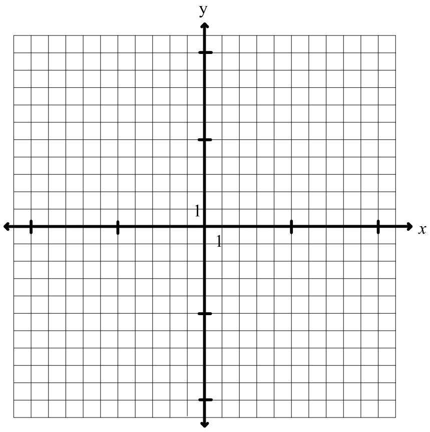
R = ____

HW: "WORKSHEET: Transformations of Reciprocal Functions"

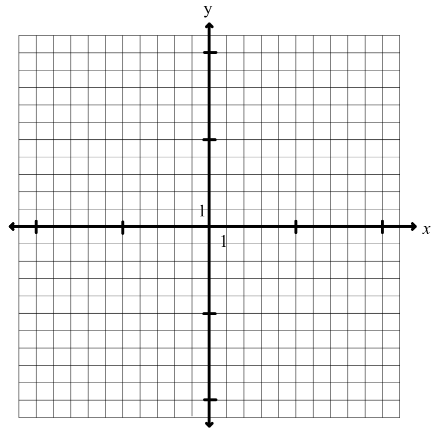
WORKSHEET: Transformations of Reciprocal Functions

Sketch a **graph** for each function below and state the **domain**, **range**, **vertical asymptote**, and **horizontal asymptote**. ***Show your work on a separate sheet of paper.***

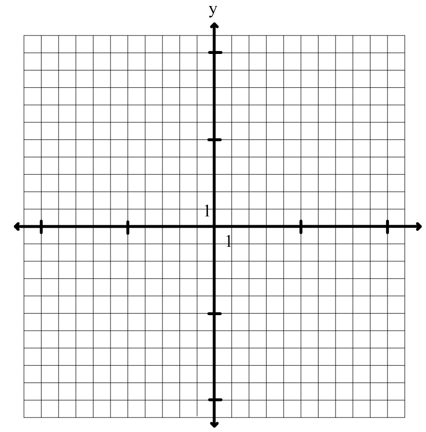
a. $y=\dfrac{1}{x+3}$



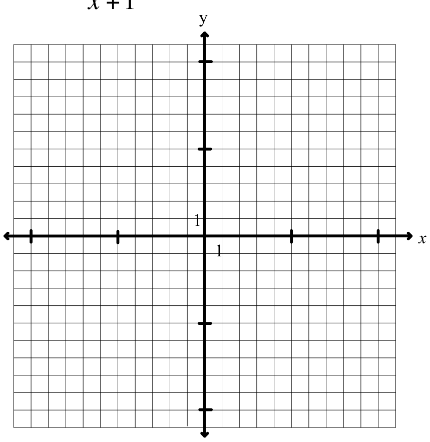
b. $y=\dfrac{1}{x}-3$



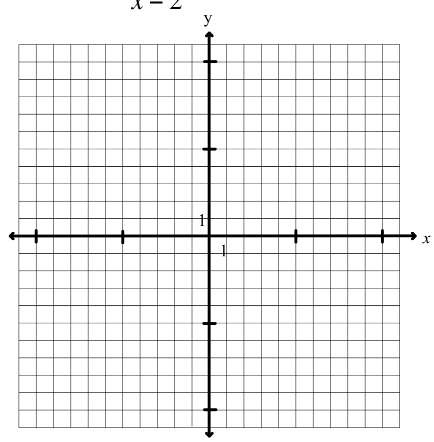
c. $y=\dfrac{2}{x-4}$



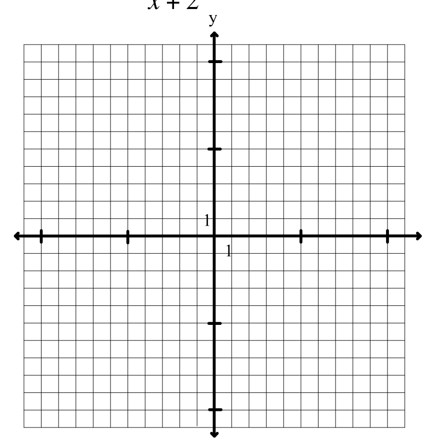
d. $y=\dfrac{3}{x+1}+1$



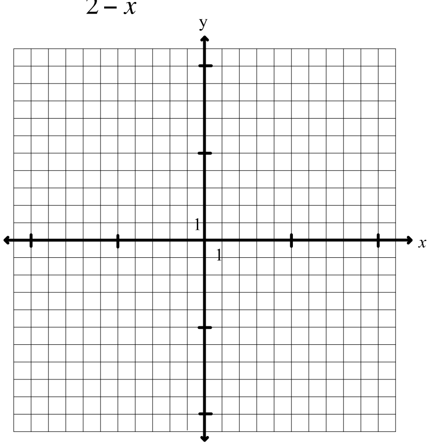
e. $y=\dfrac{-4}{x-2}$



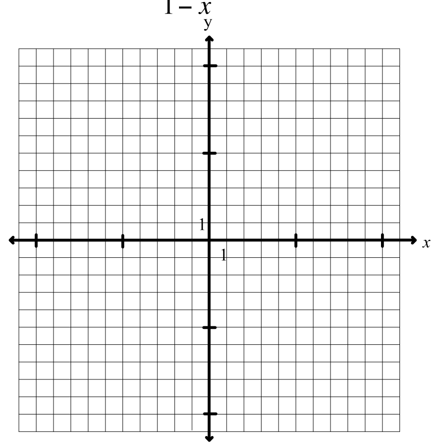
f. $y=3+\dfrac{1}{x+2}$



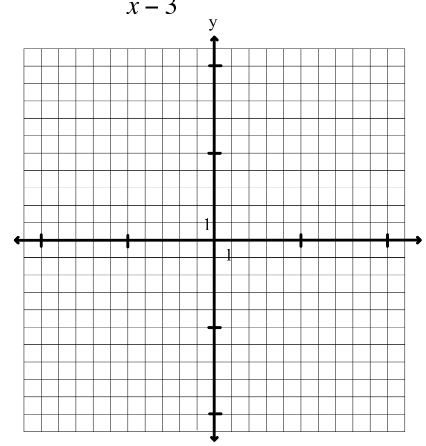
g. $y=\dfrac{2}{2-x}$



h. $y=1-\dfrac{1}{1-x}$



i. $y=-\dfrac{2}{x-3}+2$



Answers:

1. ii)	Domain $x \in R$	Range $y \in R$	Equation of the Asymptotes
a.	$x \neq -3$	$y \neq 0$	$x = -3, y = 0$
b.	$x \neq 0$	$y \neq -3$	$x = 0, y = -3$
c.	$x \neq 4$	$y \neq 0$	$x = 4, y = 0$
d.	$x \neq -1$	$y \neq 1$	$x = -1, y = 1$

e.	$x \neq 2$	$y \neq 0$	$x = 2, y = 0$
f.	$x \neq -2$	$y \neq 3$	$x = -2, y = 3$
g.	$x \neq 2$	$y \neq 0$	$x = 2, y = 0$
h.	$x \neq 1$	$y \neq 1$	$x = 1, y = 1$
i.	$x \neq 3$	$y \neq 2$	$x = 3, y = 2$

Functions and Their Inverses

A. Introduction

We have seen reflections across the x-axis (i.e. across the line y = 0) and the y-axis (i.e. across the line x = 0), but the **inverse of a function** can be visualized as a reflection across the line *y = x.* To achieve this reflection, the coordinates for each point in the function (___, ___) are inverted to create corresponding points (___, ____).

The inverse of a function, __________, is denoted by ______________.

If we look at a table of values for a function and its inverse function, we see a pattern:

f(x) = x + 3	
x	***f(x)***
2	
3	
4	
5	

$f^{-1}(x) = x - 3$	
x	$f^{-1}(x)$
5	
6	
7	
8	

Notice that the ___________________ values for the first function become the ___________________ values for the second function. The inverse function "undoes" what the original function did! The **domain** for the original function becomes the _____________________ of the inverse, and the **range** of the original function becomes the ______________________ of the inverse.

B. Finding the Inverse

There are 3 ways to find the inverse of a function: *1) interchanging* the coordinates of each point*; 2) graphing* the function and its inverse; and *3)* finding the inverse *algebraically*.

I. Interchanging Coordinates

1. Given that *g* = {(-2, -8), (0, -2), (3, 4), (4, 7)}

a) g^{-1}= ____

b) State the domain and range of *g* and g^{-1}.

D_g = ___________________________________

R_g = ___________________________________

$D_{g^{-1}}$ = ________________________________

$R_{g^{-1}}$ = ________________________________

c) Are *g* and g^{-1} both functions? Explain.

____

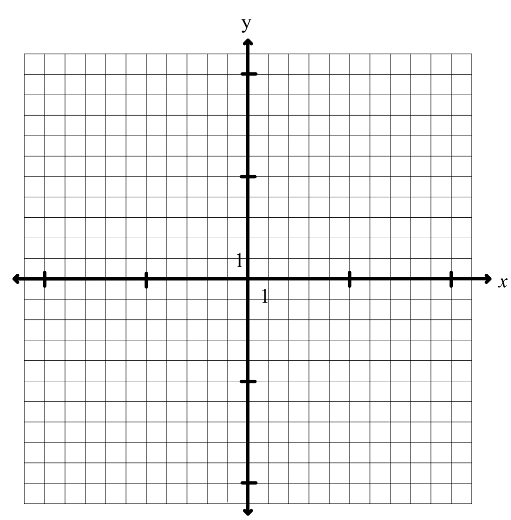
____

____

II. Graphing the Inverse of a Function

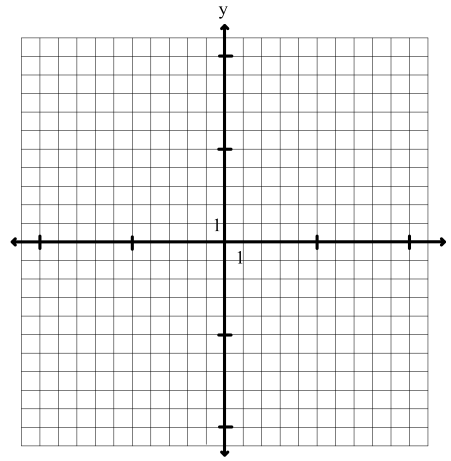
- Draw the line ____________
- Determine key points on *f(x)*
- Interchange *(x, y)* for *(y, x)*
- Plot the corresponding points as the inverse

1. Graph $f(x) = \sqrt{-(x+2)}$, and sketch its invers*e.*

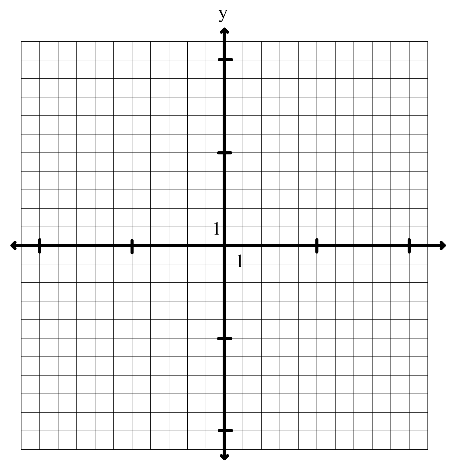


III. Finding the Inverse of a Function Algebraically

- Replace *f(x)* with *y*
- Interchange *x* and *y* in the equation
- Rearrange to isolate the 'new' *y*
- State whether or not the inverse is a function



1. Find the inverse of $f(x) = 4x + 3$. Graph both relations.



2. a) Find the inverse of $g(x) = x^2 + 2$. Graph both relations.

b) Is the inverse a function? ____________

c) State the domain and range of g^{-1}.

$D_{g^{-1}}$ = ______________________________

$R_{g^{-1}}$ = ______________________________

d) Restrict the domain of *g(x)* so that its inverse is a function.

____

____

3. Find the inverse of the following functions algebraically. For a) and b) use the range of the function to restrict the domain of the inverse, if necessary.

a) $f(x) = \sqrt{x+4}$

b) $g(x) = \dfrac{1}{x-3}$

c) $h(x) = \dfrac{2x+3}{x-1}$

HW: A) p. 215-219 #2, 3bdg, 10(iv)abc & (v)abc, 11, 15ac, 16(iii)abc & (viii)abc, 30; B) Find the inverse of: $y = \dfrac{x+5}{x-2},\ x \neq 2$

Unit 1 Review

1. Complete the ***point-by-point transformations*** from the base function onto the following transformed functions:

a) $y = -3(2x-6)^2 - 1$ (*x*, *y*) → (_______________, _______________)

b) $y = \frac{3}{2x-10} + 4$ (*x*, *y*) → (_______________, _______________)

c) $y = 2\sqrt{-\frac{1}{3}x + 1} - 8$ (*x*, *y*) → (_______________, _______________)

2. Find the inverse of the following functions. For b) and c) use the range of the function to find the domain of the inverse.

a) $f(x) = 3(x+2)^2 - 1$ b) $f(x) = \sqrt{2x} + 1$ c) $f(x) = \frac{2}{x-5} + 4$

3. Redo questions d., f., and i. from "WORKSHEET: Transformations of Reciprocal Functions".

4. Complete the following textbook questions:
Review: p. 246-253 #2, 4d, 6c, 23ab, 24a, 26a, 28, 29bdef, 32, 37, 41f, 42d, 43b, 44
Chapter Test: p. 254-256 #1ab, 2ab, 3ch, 6cd, 8b, 10, 11, 12a

Answers:

1. a) $(x, y) \rightarrow (x + 3,\ -12y - 1)$

b) $(x, y) \rightarrow (\frac{1}{2}x + 5,\ 3y + 4)$

c) $(x, y) \rightarrow (-3x + 3,\ 2y - 8)$

2. a) $f^{-1}(x) = \pm\sqrt{\frac{1}{3}(x+1)} - 2$; not a function

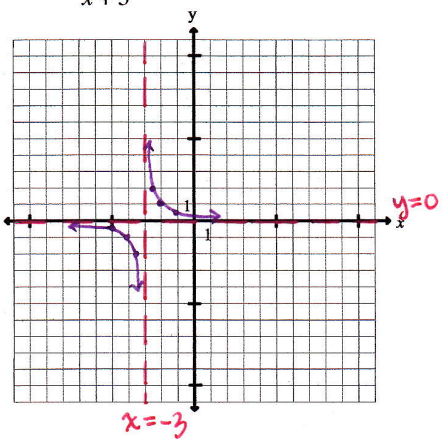
b) $f^{-1}(x) = \frac{1}{2}(x-1)^2$; $D_{f^{-1}} = \{x | x \in R,\ x \geq 1\}$

c) $f^{-1}(x) = \frac{2}{x-4} + 5$; $D_{f^{-1}} = \{x | x \in R,\ x \neq 4\}$

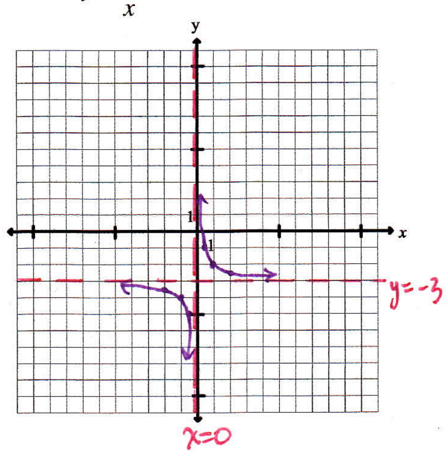
WORKSHEET: Transformations of Reciprocal Functions

Sketch a **graph** for each function below and state the **domain**, **range**, **vertical asymptote**, and **horizontal asymptote**.
Show your work on a separate sheet of paper.

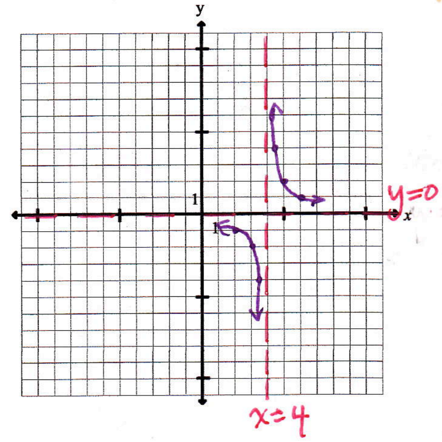
a. $y = \frac{1}{x+3}$



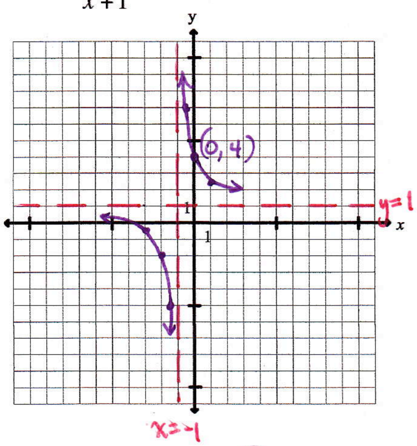
b. $y = \frac{1}{x} - 3$



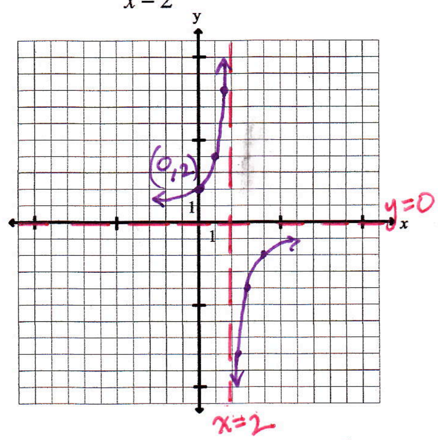
c. $y = \frac{2}{x-4}$



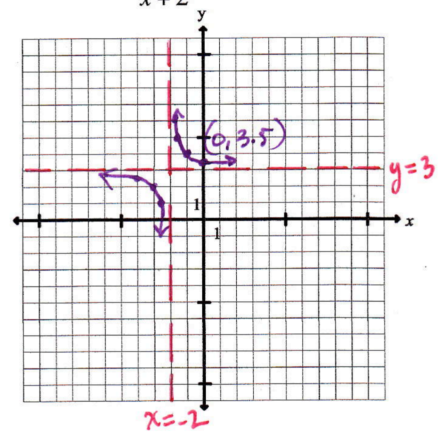
d. $y = \frac{3}{x+1} + 1$



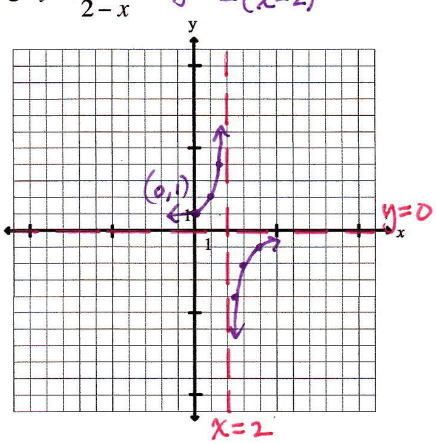
e. $y = \frac{-4}{x-2}$



f. $y = 3 + \frac{1}{x+2}$

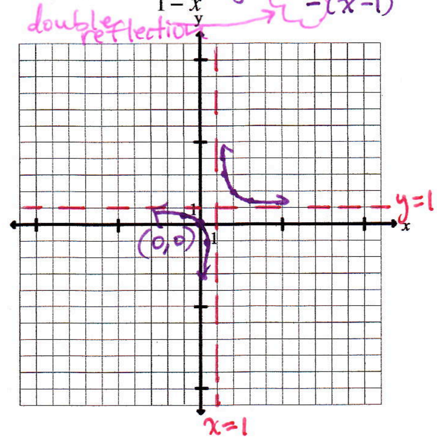


g. $y = \frac{2}{2-x}$ $y = \frac{2}{-(x-2)}$

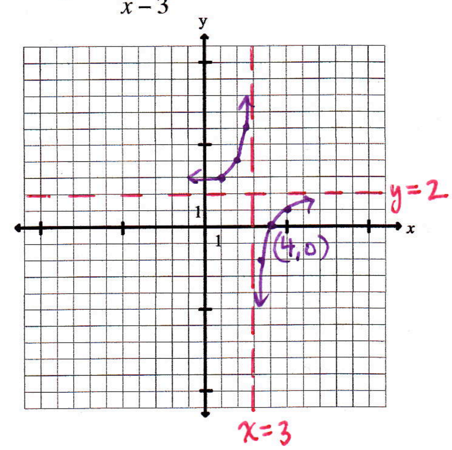


h. $y = 1 - \frac{1}{1-x}$ $y = - \frac{1}{-(x-1)} + 1$

double reflection



i. $y = -\frac{2}{x-3} + 2$



Answers:

1. ii)	Domain $x \in R$	Range $y \in R$	Equation of the Asymptotes
a.	$x \neq -3$	$y \neq 0$	$x = -3, y = 0$
b.	$x \neq 0$	$y \neq -3$	$x = 0, y = -3$
c.	$x \neq 4$	$y \neq 0$	$x = 4, y = 0$
d.	$x \neq -1$	$y \neq 1$	$x = -1, y = 1$
e.	$x \neq 2$	$y \neq 0$	$x = 2, y = 0$
f.	$x \neq -2$	$y \neq 3$	$x = -2, y = 3$
g.	$x \neq 2$	$y \neq 0$	$x = 2, y = 0$
h.	$x \neq 1$	$y \neq 1$	$x = 1, y = 1$
i.	$x \neq 3$	$y \neq 2$	$x = 3, y = 2$