Date:____________

UNIT 2: QUADRATIC EQUATIONS & APPLICATIONS

2.1 Solving Quadratic Equations: Two More Methods

1. Solve using ***inverse operations*** to isolate the variable when it appears only once, $x \in C$.

a) $x^2 + 36 = 0$ **b)** $-2x^2 + 24 = 0$ **c)** $36(x+2)^2 - 4 = 0$

2. Solve using the ***quadratic formula*** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $ax^2 + bx + c = 0$ and $x \in C$.

a) $x - 1 = \dfrac{2x+3}{x-1}$ **b)** $\dfrac{x^2}{2} + \dfrac{x}{3} = -1$

3. Write a ***quadratic equation*** in expanded form with integer coefficients having the given roots.

a) $-3, \ \dfrac{3}{4}$ **b)** $-1 - 2\sqrt{3}\,i, \ -1 + 2\sqrt{3}\,i$

HW. Exercise 2.1

Date:____________ **2.2 Solving Cubic and Quartic Equations**

Note: *A polynomial of the nth degree has n roots.*

Ex. Solve for x in each of the following, $x \in C$.

a) $-4x^3 - 18x^2 + 10x = 0$

b) $x^4 - 24x^2 - 25 = 0$

c) $3x^3 + x^2 + 24x + 8 = 0$

d) $(x^2 - 5x)^2 - 2(x^2 - 5x) - 24 = 0$

e) $\left(x + \frac{1}{x}\right)^2 - 7\left(x + \frac{1}{x}\right) + 12 = 0$

HW. Exercise 2.2

Date:______________

2.3 Solving Linear and Quadratic Inequalities

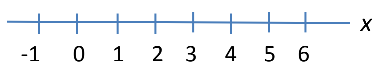
Complete the table for the ***inequality* 9 > 6**:

Operation:	Add 3	Subtract 3	Multiply by 3	Divide by 3	Multiply by -3	Divide by -3
Resulting Inequality:						

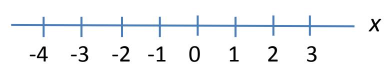
Rule:
*When **multiplying** or **dividing** an inequality by a **negative**, **change the direction** of the inequality sign.*

1. Solve each of the following ***linear inequalities*** and ***graph*** the solution on a number line.

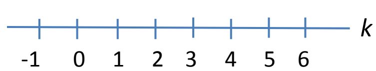
a) $4x-1<11$



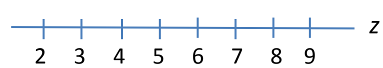
b) $3(2-x)+1\leq 13$



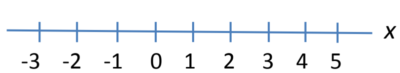
c) $\frac{3}{4}k+\frac{1}{2}k>5$



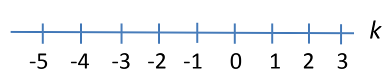
d) $\frac{z-1}{5}\geq\frac{z+2}{4}-1$



e) $-3\leq 2x-1<5$



f) $2-k\geq 5\ or\ 2-3k<-1$



2. Solve each of the following ***quadratic inequalities*** by following the steps below and ***graph*** the solution on a number line.

- *Rearrange to compare* $ax^2 + bx + c$ *to* 0, with $a > 0$.
- *Find the zeros of the corresponding quadratic equation and graph accordingly.*
- *If* $ax^2 + bx + c < 0$, *the solution is between the zeros.*
- *If* $ax^2 + bx + c > 0$, *the solution is outside the zeros.*

a) $x^2 - 3x - 4 \le 0$

b) $k^2 + 5k > 0$

c) $-2k^2 + 18 > 0$

d) $-4x^2 - 8x \le -5$

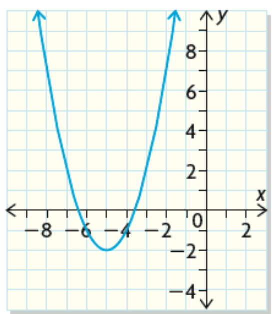
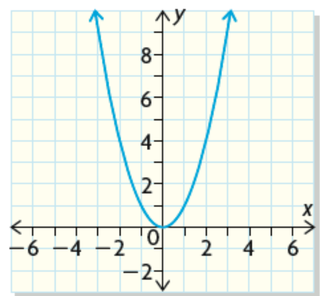
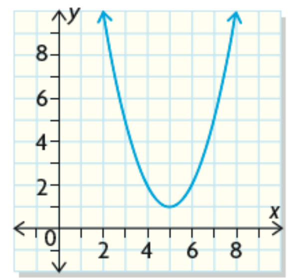
HW. Exercise 2.3

Date:______________

2.4 The Discriminant

Recall: *If* $ax^2 + bx + c = 0$, *and* $a \neq 0$, *then* $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ⟵ ***Quadratic Formula***

Definition: In the quadratic formula, the ***discriminant*** is the quantity $b^2 - 4ac$ under the radical sign. The ***discriminant***, $b^2 - 4ac$ can determine the nature of the roots to the corresponding quadratic equation and the number of x-intercepts to the corresponding quadratic relation. The ***nature*** of the two roots can be one of three types based on the value of the ***discriminant***, $b^2 - 4ac$.

two distinct (different) real roots	**two equal (same) real roots**	**two non-real (imaginary) roots**
$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
two *x*-intercepts	**one *x*-intercept**	**no *x*-intercepts**
		

Note: *If the* **discriminant**, $b^2 - 4ac$, *is a positive perfect square, the quadratic equation can be solved by factoring.*

1. Use the ***discriminant*** to determine the nature of the roots for each of the following.

a) $16x^2 - 16x + 4 = 0$ **b)** $1 - 2x - 3x^2 = 0$ **c)** $3 = 2\sqrt{2}x - x^2$

2. Determine the value(s) of *k* that will give the indicated types of roots.

a) $kx^2 - 2x + 1 = 0$; *distinct real roots*

b) $(1-3k)x^2 + 4x - 4 = 0$; *equal real roots*

c) $2x^2 - kx + 8 = 0$; *non-real roots*

d) $(2k-1)x^2 + (3k+2)x + (k-1) = 0$; *real roots*

HW. Exercise 2.4

Date:______________

2.5 Applications: Quadratic Equations

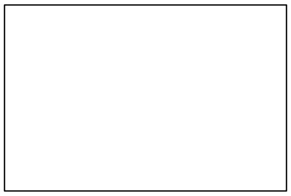
1. The height, *h*, in metres of an object thrown off the top of a cliff after *t* seconds is given by the following equation: $h = -5t^2 + 20t + 80$

a) Use the ***discriminant*** to determine if the object ever reaches a height of 110 m.

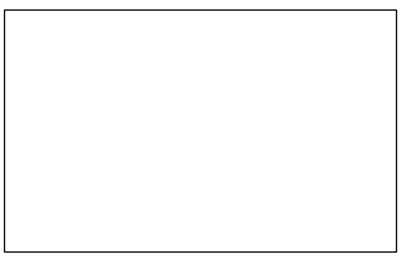
b) Exactly when does the object hit the ground?

2. The base of a triangle is 2 cm more than the height. If the area is 5 cm^2, find the exact length of the base.

3. A dining room measures 5 m by 4 m. A strip of uniform width is added to two adjacent sides to increase the area by 5 m^2. Find the width of the strip to 1 decimal place.



4. A uniform-width boardwalk is built around the inside edge of a rectangular park that measures 15 m by 10 m. If the boardwalk takes up 20% of the lot, how wide is the boardwalk to the nearest centimetre?



HW. Exercise 2.5

Date:______________ **2.6 Graphing Quadratic Relations Given Any Form**

A. Vertex Form of a Quadratic Relation: $y = a(x-h)^2 + k$

- *The vertex is* (h, k).
- *The parabola opens up if* $a > 0$ *and the vertex is a minimum.*
- *The parabola opens down if* $a < 0$ *and the vertex is a maximum.*
- *The parabola is congruent to* $y = |a|x^2$.

1. For each of the following quadratic relations, state the vertex of the parabola, the direction of opening, the equation of the parabola it's congruent to and the maximum or minimum value of the relation and when it occurs. Graph each relation on the grids below.

a) $y = (x+2)^2 - 4$

i) *vertex:* _________________

ii) *opens:* _________________

iii) *congruent to:* _________________

iv) *The* _____________________ *value of* y *is* ________ *when* $x =$ ______.

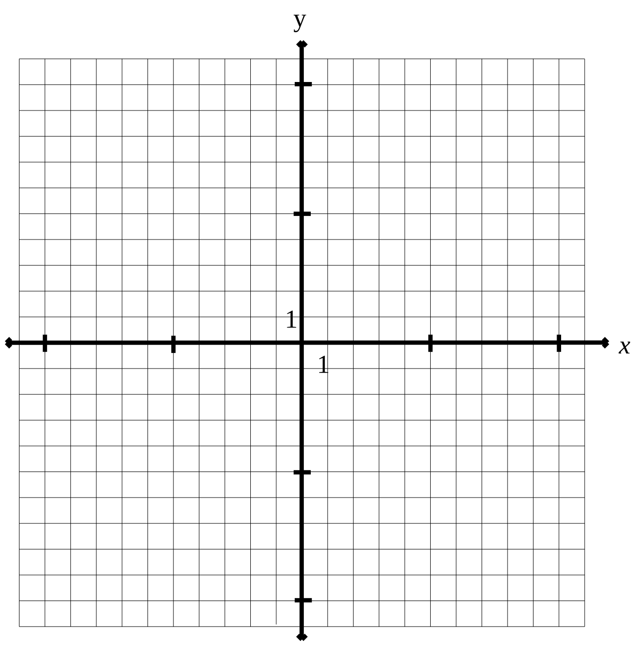
b) $y = -3(x-5)^2 + 1$

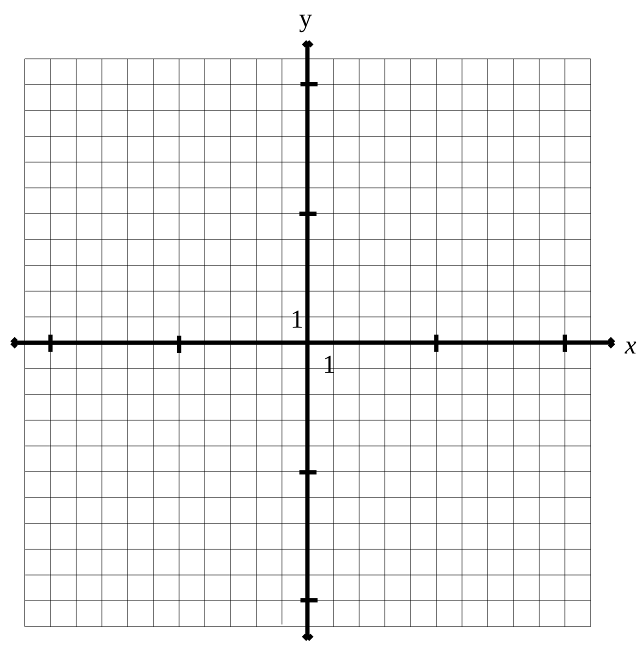
i) *vertex:* _________________

ii) *opens:* _________________

iii) *congruent to:* _________________

iv) *The* _____________________ *value of* y *is* ________ *when* $x =$ ______.





B. Factored Form of a Quadratic Relation: $y = a(x-r)(x-s)$

- *The x-intercepts are r and s.*
- *At the vertex,* (x, y), $x = \dfrac{r+s}{2}$.
- *The parabola opens up if* $a > 0$ *and the vertex is a minimum.*
- *The parabola opens down if* $a < 0$ *and the vertex is a maximum.*
- *The parabola is congruent to* $y = |a|x^2$.

2. For each of the following quadratic relations, state the *x*-intercepts and vertex of the parabola, the direction of opening, the equation of the parabola it's congruent to and the maximum or minimum value of the relation and when it occurs. Graph each relation on the grids below.

a) $y = 2(x-3)(x-7)$

i) *x-intercepts:* ________________

ii) *vertex:* ________________

iii) *opens:* ________________

iv) *congruent to:* ________________

v) *The* ____________________ *value of y is* ________ *when* $x =$ ______ .

b) $y = -\dfrac{1}{2}x(x+4)$

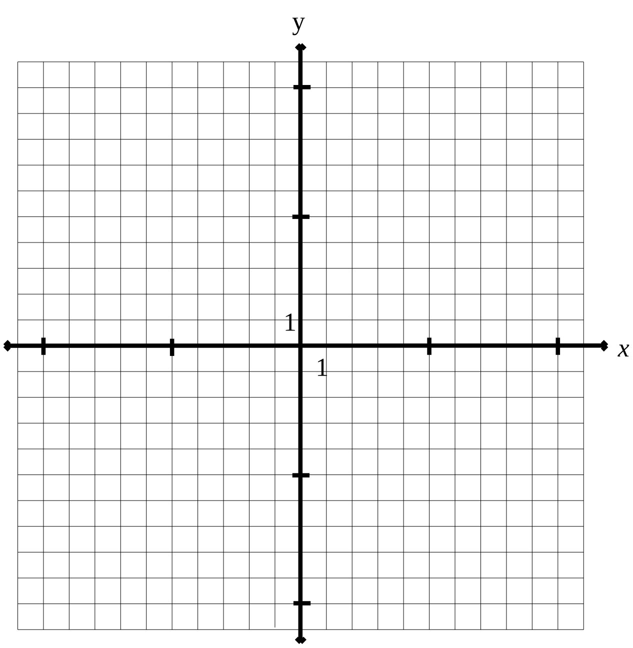
i) *x-intercepts:* ________________

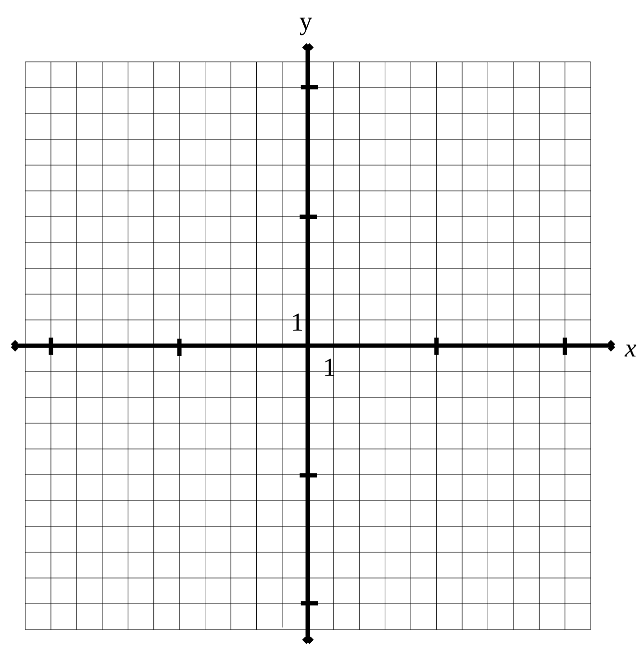
ii) *vertex:* ________________

iii) *opens:* ________________

iv) *congruent to:* ________________

v) *The* ____________________ *value of y is* ________ *when* $x =$ ______ .





C. Standard Form of a Quadratic Relation: $y = ax^2 + bx + c$

- *Complete the square to express the relation in* ***Vertex Form*** *.*
- *The parabola opens up if* $a > 0$ *and the vertex is a minimum.*
- *The parabola opens down if* $a < 0$ *and the vertex is a maximum.*
- *The parabola is congruent to* $y = |a| x^2$.

3. For each of the following quadratic relations, express the relation in vertex form by completing the square, state the vertex of the parabola, the direction of opening, the equation of the parabola it's congruent to and the maximum or minimum value of the relation and when it occurs. Graph each relation on the grids below.

Vertex Form is $y = a(x-h)^2 + k$

a) $y = -x^2 + 4x - 7$

b) $y = \frac{1}{3}x^2 + 2x + 3$

a)

i) *vertex:* ________________

ii) *opens:* ________________

iii) *congruent to:* ________________

iv) *The* ____________________ *value of* y *is* ________ *when* $x =$ ______ .

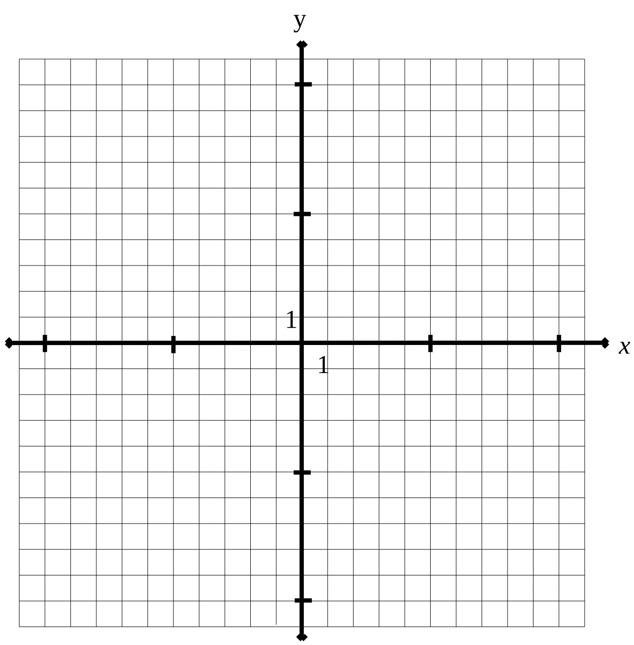
b)

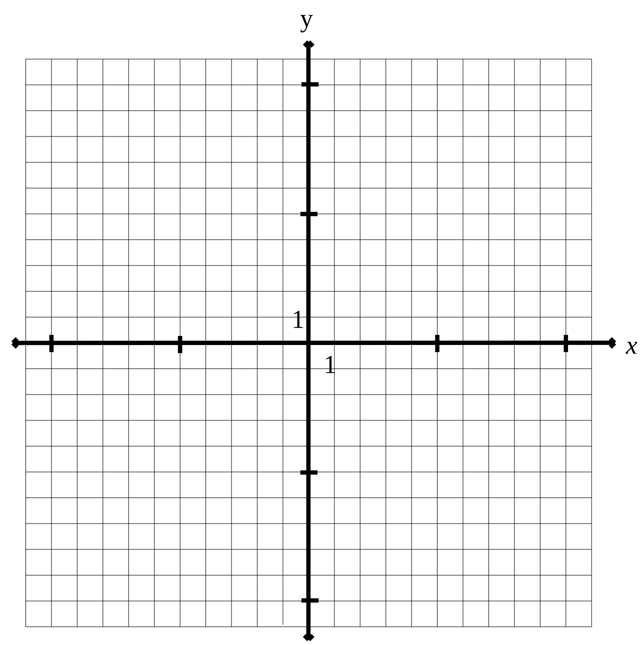
i) *vertex:* ________________

ii) *opens:* ________________

iii) *congruent to:* ________________

iv) *The* ____________________ *value of* y *is* ________ *when* $x =$ ______ .





HW. Exercise 2.6

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2.7 Applications: Maximum and Minimum Values

1. Determine the maximum or minimum value of each relation and when it occurs.

a) $y = 0.3x^2 + 1.2x + 4.5$

b) $A = -3w^2 + 5w - 1$

2. A rocket is fired down a practice range. The height in metres after t seconds is given by $h = -\frac{1}{4}t^2 + 3t + 45$. Find the the maximum height attained by the rocket and when it occurs.

3. The concentration of bacteria in a pool, t days after treatment is $C = 30t^2 - 240t + 500$, where C is the concentration of bacteria per cm^3.

Find the lowest concentration of bacteria and the day on which it occurs.

4. Find two positive quantities whose sum is 18, if the sum of their squares is a minimum.

5. A magazine producer can sell 600 of her magazines at \$6.00 each. A marketing survey shows her that for every \$0.50 she increases the price, she will lose 30 sales. What price should she set to obtain the greatest revenue?

HW. Exercise 2.7

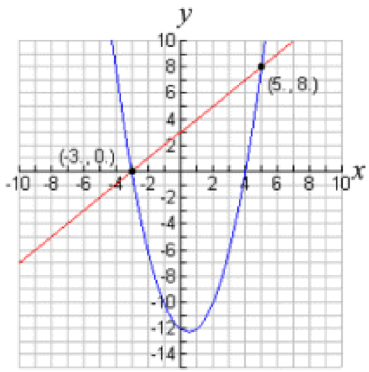
Date:_____________

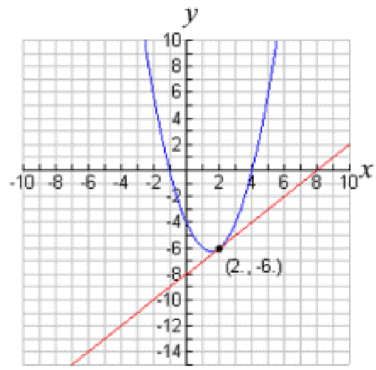
2.8 Solving Systems of Equations

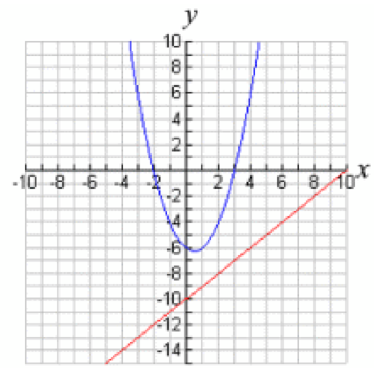
A. Linear-Quadratic Systems

Graphing a line and a parabola on the same set of axes yields one of three possible scenarios.

i) *Two points of intersection* **ii)** *One point of intersection* **iii)** *No points of intersection*



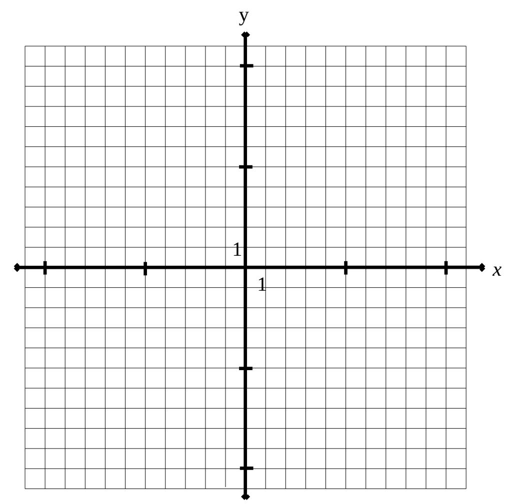




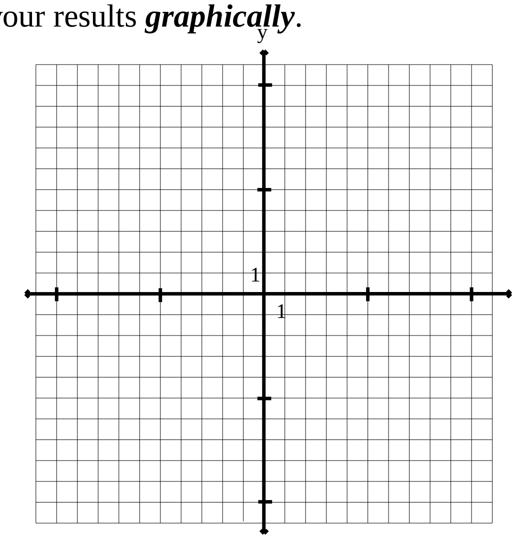
1. Solve the following linear-quadratic systems ***algebraically*** and illustrate ***graphically***.

$y=-(x+3)^2+2$

$y=\frac{3}{2}x+4$



2. *Without solving* determine the number of points of intersection of the quadratic and linear relations $y=(x-1)^2$ and $3=x-y$ by using the ***discriminant*** . Illustrate your results ***graphically***.

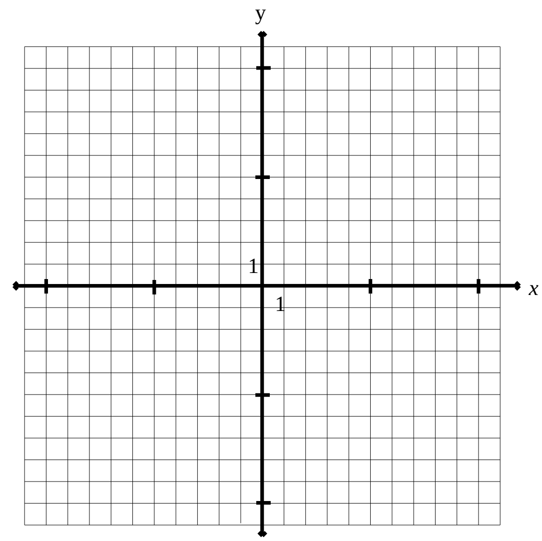


B. Other Systems of Equations

3. Solve the following systems ***algebraically*** and illustrate ***graphically***.

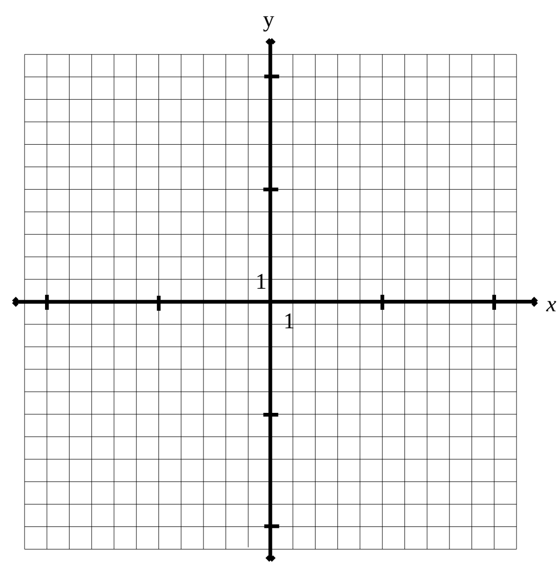
a) $x^2 + y^2 = 25$

$3x - 4y = 0$



b) $y = -\frac{1}{2}x^2 - x + \frac{3}{2}$

$y = x^2 - 4x + 3$



HW. Exercise 2.8

For Unit 2 Test: do Unit 2 Review Exercise