Simplifying Rational Expressions

A *rational* <u>*number*</u> is a fraction written in the form $\frac{m}{n}$ where $n \neq 0$

A *rational* <u>*expression*</u> is also a fraction written in the form $\frac{f(x)}{g(x)}$ where $g(x) \neq 0$.

A. Restrictions on Rational Expressions

When the denominator has a value of 0, the question is impossible to evaluate. When this occurs, the rational expression is considered to be **UNDEFINED**.

Therefore, the value that makes the denominator equal to 0 is called the *non-permissive value* or the **restriction** of the expression.

<u>Practice</u>: Determine the restrictions for each of the following rational expressions:

a) $\frac{2x-7}{x+3}$

$x+3 \neq 0$

$\therefore x \neq -3$

b) $\frac{3y+5}{xy^2}$

$x \neq 0$

$y \neq 0$

c) $\frac{4x-1}{3}$

$3 \neq 0$ ✓

$\therefore$ no restrictions

d) $\frac{2x}{y^2-4}$

$y^2-4 \neq 0$

$\sqrt{y^2} \neq \pm\sqrt{4}$

$\therefore y \neq \pm 2$

e) $\frac{5y}{b^2+9}$

$b^2+9 \neq 0$

$\sqrt{b^2} \neq \pm\sqrt{-9}$

$b \neq \pm\sqrt{-9}$

$\therefore$ no restrictions

f) $\frac{5x}{10x+15y}$

$10x+15y \neq 0$

$10x \neq -15y$

$x \neq -\frac{15y}{10}$

$\therefore x \neq -\frac{3}{2}y$

g) $\frac{3y}{x^2+9x+18}$

$(x+6)(x+3) \neq 0$

$\therefore x \neq -6, -3$

h) $\frac{2b+4}{c^2-2c-24}$

$(c-6)(c+4) \neq 0$

$\therefore c \neq -4, 6$

B. Simplifying Rational Expressions

In order to simplify rational numbers, we factor the numerator and the denominator.

Ex. 1: $\frac{24}{30} = \frac{\cancel{6}(4)}{\cancel{6}(5)}$

$= \frac{4}{5}$

We simplify rational expressions in the same manner: the *constants* are reduced using the greatest common factor; the *variables* are reduced using the exponent laws.

Ex. 2: $\frac{8x^2}{12x} = \frac{\cancel{4}(2)\,\cancel{x}\cdot x}{\cancel{4}(3)\,\cancel{x}}$

$= \frac{2x}{3}, \; x \neq 0$

Ex. 3: $\frac{25xy^2}{30xy} = \frac{\cancel{5}(5)\,\cancel{x}\cdot\cancel{y}\cdot y}{\cancel{5}(6)\,\cancel{x}\cdot\cancel{y}}$

$= \frac{5y}{6}, \; x \neq 0, \; y \neq 0$

When we encounter multiple terms in either the numerator or denominator, we will place brackets around them. These brackets will ensure that the expression is dealt with as a whole unit (meaning that we cannot eliminate one part of the expression and leave the rest). Once the brackets are in place, we have <u>two options</u>:

1) remove a common factor from **every term** in the expression ***and/or***
2) in the case of a quadratic, factor the expression into two binomials

Ex. 4: $\frac{x^2 - 5x - 6}{x^2 - 36}$

$= \frac{\cancel{(x-6)}(x+1)}{\cancel{(x-6)}(x+6)}$

$= \frac{x+1}{x+6}, \; x \neq -6, +6$

Ex. 5 : $\frac{2xy - 12y^2}{18y^2 - 3xy}$

$= \frac{2y(x-6y)}{3y(6y-x)}$

$= \frac{2y(x-6y)}{3y(-x+6y)}$

$= \frac{2\cancel{y}\cancel{(x-6y)}}{3\cancel{y}(-1)\cancel{(x-6y)}}$ *

$= -\frac{2}{3}, \; y \neq 0, \; x \neq 6y$

*At this point, the binomial in the numerator and the binomial in the denominator look very similar; however, they are **not** the same. The only way that we can make them identical is to **rearrange** one of the binomials and then **factor out a value of (- 1)**. This will effectively "change the signs" inside the brackets, making the binomial factors identical so that we can divide and reduce them!

C. Simplify each of the following. State any restrictions on the variables.

a) $\dfrac{20x^3+5x^2-10x}{5x}$

$$= \frac{\cancel{5x}(4x^2+x-2)}{\cancel{5x}}$$

$$= 4x^2+x-2, \quad x \neq 0$$

b) $\dfrac{3x}{4x^2-8x}$

$$= \frac{3\cancel{x}}{4\cancel{x}(x-2)}$$

$$= \frac{3}{4(x-2)}, \quad x \neq 0, 2$$

c) $\dfrac{3y-2x}{4x-6y}$

$$= \frac{(3y-2x)}{2(2x-3y)}$$

$$= \frac{1\cancel{(3y-2x)}}{-2\cancel{(3y-2x)}}$$

$$= -\frac{1}{2}, \quad x \neq \frac{3y}{2}$$

d) $\dfrac{x^2+3x-10}{x^2-3x-40}$

$$= \frac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}(x-8)}$$

$$= \frac{(x-2)}{(x-8)}, \quad x \neq -5, 8$$

e) $\dfrac{x^2-2xy-8y^2}{16y^2-x^2}$

$$= \frac{(x-4y)(x+2y)}{(4y-x)(4y+x)}$$

$$= \frac{\cancel{(x-4y)}(x+2y)}{-\cancel{(x-4y)}(4y+x)}$$

$$= -\frac{x+2y}{x+4y}, \quad x \neq 4y, -4y$$

HW: p. 40-42 #2cegi, 3cegi, 4cegij, 5ace, 6egikl, 10

Multiplying and Dividing Rational Expressions

To multiply and divide rational *expressions*, we employ the same basic principles as with rational *numbers* (i.e. fractions ☺)!

When **multiplying** fractions: $\frac{A}{B} \times \frac{C}{D} = \frac{AC}{BD}, \ \textit{where} \ \ B \neq 0 \ \ \textit{and} \ \ D \neq 0$

When **dividing** fractions, **keep** the first fraction, **change** division to multiplication, and **invert** ("flip") the second fraction:

$$\frac{E}{F} \div \frac{G}{H} = \frac{E}{F} \times \frac{H}{G} = \frac{EH}{FG}, \ \textit{where} \ \ F \neq 0, \ H \neq 0, \ \textit{and} \ \ G \neq 0$$

Steps for multiplying or dividing rational expressions:

i) *factor the numerators and denominators*
(if you can't factor, place brackets around the expressions)

ii) *if dividing, invert the second rational and change to multiplication*

iii) *state any restrictions on the variables*

iv) *multiply numerators and denominators*

v) *simplify the rational expression, if possible*

Ex. 1: Simplify each of the following and state any restrictions on the variables.

a) $\frac{\cancel{12}^{\cancel{4}\,1} x^3}{\cancel{3}_1 y^4} \times \frac{\cancel{14}^{7} y^2}{\cancel{8}_{\cancel{2}\,1} x^2}$

$= \frac{7x^3y^2}{x^2y^4}$

$= \frac{7x}{y^2}, \quad x \neq 0, \ y \neq 0$

b) $\frac{63a^3bc^2}{40ab^2} \div \frac{27a^2bc^4}{-15ab^3c}$

$= \frac{\cancel{63}^{7} a^3bc^2}{{}_8\cancel{40}ab^2} \times \frac{\cancel{-15}^{\cancel{-3}\,-1} ab^3c}{\cancel{27}_{\cancel{3}\,1} a^2bc^4}$

$= \frac{-7a^4b^4c^3}{8a^3b^3c^4}$

$= -\frac{7ab}{8c}, \quad a \neq 0, \ b \neq 0, \ c \neq 0$

c) $\frac{\cancel{6}^{2}\cancel{(x-5y)}^{1}}{xy^2}\times\frac{y(x+3y)}{\cancel{9}_{3}\cancel{(x-5y)}_{1}}$

$= \frac{2y(x+3y)}{3xy^2}$

$= \frac{2(x+3y)}{3xy}$, $x \neq 0$
$y \neq 0$
$x \neq 5y$

d) $\frac{6x-3x^2}{4x+20}\div\frac{9x^2-18x}{3x+15}$

$= \frac{3x(2-x)}{4(x+5)} \div \frac{9x(x-2)}{3(x+5)}$

$= \frac{\cancel{-3}^{-1}\cancel{x}^{1}\cancel{(x-2)}^{1}}{4\cancel{(x+5)}_{1}} \times \frac{\cancel{3}^{1}\cancel{(x+5)}^{1}}{\cancel{9}_{\cancel{3}_{1}}\cancel{x}_{1}\cancel{(x-2)}_{1}}$

$= -\frac{1}{4}$, $x \neq -5, 0, 2$

e) $\frac{2x+6}{x^2+7x+10}\times\frac{x^2+3x-10}{x^2-4}$

$= \frac{2(x+3)}{(x+2)\cancel{(x+5)}_{1}} \times \frac{\cancel{(x+5)}^{1}\cancel{(x-2)}^{1}}{\cancel{(x-2)}_{1}(x+2)}$

$= \frac{2(x+3)}{(x+2)^2}$, $x \neq -5, -2, 2$

f) $\frac{a^2-4a-21}{2a^2+a}\div\frac{a^3-9a}{2a^2-5a-3}$

2 1
1 X 3
1 -6

$= \frac{(a-7)(a+3)}{a(2a+1)} \div \frac{a(a^2-9)}{(2a+1)(a-3)}$

$= \frac{(a-7)\cancel{(a+3)}^{1}}{a\cancel{(2a+1)}_{1}} \times \frac{\cancel{(2a+1)}^{1}\cancel{(a-3)}^{1}}{a\cancel{(a-3)}_{1}\cancel{(a+3)}_{1}}$

$= \frac{a-7}{a^2}$, $a \neq -3, -\frac{1}{2}, 0, 3$

HW: p. 50-52 #5 to 8 odd parts, 10, 15 *(Recall: the area of a triangle is A =* $\frac{1}{2}bh$*)*

Adding and Subtracting Rational Expressions, I

To add or subtract rational expressions:

i) *factor the numerators and denominators*
ii) *state any restrictions on the variables*
iii) *find the lowest common denominator, LCD*
iv) *add or subtract the numerators, as required*
v) *simplify the rational expression, if possible*

Simplify each of the following; state any restrictions on the variables.

a) $\frac{5}{x^2} + \frac{3y}{x^2} - \frac{2}{x^2}$ LCD $= x^2$

$= \frac{5+3y-2}{x^2}$

$= \frac{3y+3}{x^2}$, $x \neq 0$

b) $\frac{4x-1}{x+5} - \frac{(3x+2)}{x+5}$ LCD $= x+5$

$= \frac{4x-1-(3x+2)}{x+5}$

$= \frac{4x-1-3x-2}{x+5}$

$= \frac{x-3}{x+5}$, $x \neq -5$

c) $\frac{6(2x-5)}{6(2)} + \frac{3(x-3)}{3(4)} - \frac{2(3x-5)}{2(6)}$ LCD $= 12$

$= \frac{12x-30+3x-9-6x+10}{12}$

$= \frac{9x-29}{12}$

d) $\frac{x+4}{x-7} + \frac{x-5}{7-x}$ LCD $= x-7$

$= \frac{x+4}{x-7} + \frac{(x-5)}{-(x-7)}$

$= \frac{x+4}{(x-7)} - \frac{(x-5)}{(x-7)}$

$= \frac{x+4-x+5}{x-7}$

$= \frac{9}{x-7}$, $x \neq 7$

e) $\frac{a(2)}{a(ab^2)} + \frac{b(4)}{b(a^2b)}$ LCD $= a^2b^2$

$= \frac{2a}{a^2b^2} + \frac{4b}{a^2b^2}$

$= \frac{2a+4b}{a^2b^2}$, $a \neq 0$
$b \neq 0$

f) $\frac{xy(4)}{xy(1)} - \frac{3}{xy} + \frac{x(x)}{x(y)}$ LCD $= xy$

$= \frac{4xy}{xy} - \frac{3}{xy} + \frac{x^2}{xy}$

$= \frac{4xy-3+x^2}{xy}$

$= \frac{x^2+4xy-3}{xy}$, $x \neq 0$
$y \neq 0$

HW: p. 58-60 #4, 5aceg, 6ace, 11 *(Recall: the area of a circle is $A = \pi r^2$)*

Adding and Subtracting Rational Expressions, II

Recall: ***To add or subtract rational expressions:***

i) *factor the numerators and denominators*
ii) *state any restrictions on the variables*
iii) *find the lowest common denominator, LCD*
iv) *add or subtract the numerators, as required*
v) *simplify the rational expression, if possible*

Simplify each of the following; state any restrictions on the variables.

a) $\frac{2a^2(4)}{2a^2(5a)}-\frac{5a(3)}{5a(2a^2)}+\frac{10(1)}{10(a^3)}$ LCD $= 10a^3$

$= \frac{8a^2}{10a^3}-\frac{15a}{10a^3}+\frac{10}{10a^3}$

$= \frac{8a^2-15a+10}{10a^3}$, $a \neq 0$

b) $\frac{5}{3x-12}-\frac{7}{4x-16}$ LCD $= 12(x-4)$

$= \frac{4(5)}{4(3(x-4))}-\frac{3(7)}{3(4(x-4))}$

$= \frac{20}{12(x-4)}-\frac{21}{12(x-4)}$

$= -\frac{1}{12(x-4)}$, $x \neq 4$

c) $\frac{7m}{2m-4}+\frac{2m}{4m+4}$ LCD $= 4(m-2)(m+1)$

$= \frac{2(m+1)(7m)}{2(m+1)(2(m-2))}+\frac{(m-2)(2m)}{(m-2)(4(m+1))}$

$= \frac{14m(m+1)}{4(m-2)(m+1)}+\frac{2m(m-2)}{4(m-2)(m+1)}$

$= \frac{14m^2+14m+2m^2-4m}{4(m-2)(m+1)}$

$= \frac{16m^2+10m}{4(m-2)(m+1)}$

$= \frac{\cancel{2}^{1}m(8m+5)}{\cancel{4}_{2}(m-2)(m+1)}$

$= \frac{m(8m+5)}{2(m-2)(m+1)}$, $m \neq 2, -1$

d) $\frac{7x}{x^2-x-12}-\frac{4x}{x^2+2x-3}$ LCD $= (x-1)(x-4)(x+3)$

$= \frac{(x-1)7x}{(x-1)(x-4)(x+3)}-\frac{4x(x-4)}{(x-1)(x+3)(x-4)}$

$= \frac{7x^2-7x-4x^2+16x}{(x-1)(x-4)(x+3)}$

$= \frac{3x^2+9x}{(x-1)(x-4)(x+3)}$

$= \frac{3x\cancel{(x+3)}^{1}}{(x-1)(x-4)\cancel{(x+3)}^{1}}$

$= \frac{3x}{(x-1)(x-4)}$, $x \neq -3, 1, 4$

e) $\dfrac{2x^2-2x}{x^2+4x-5}-\dfrac{12x}{4x+20}$

$$= \frac{2x\cancel{(x-1)}^{1}}{(x+5)\cancel{(x-1)}_{1}} - \frac{{}^{3}\cancel{12}x}{{}_{1}\cancel{4}(x+5)}$$

$$= \frac{2x}{(x+5)} - \frac{3x}{(x+5)} \qquad LCD = (x+5)$$

$$= -\frac{x}{x+5}, \quad x \neq -5, 1$$

f) $\dfrac{x+1}{x^2+2x-3}-\dfrac{x+2}{x^2+4x-5}$

$$= \frac{(x+5)(x+1)}{(x+5)(x+3)(x-1)} - \frac{(x+2)(x+3)}{(x+5)(x-1)(x+3)} \qquad LCD = (x+3)(x+5)(x-1)$$

$$= \frac{(x+5)(x+1) - \left[(x+2)(x+3)\right]}{(x+3)(x+5)(x-1)}$$

$$= \frac{x^2+6x+5-(x^2+5x+6)}{(x+3)(x+5)(x-1)}$$

$$= \frac{x^2+6x+5-x^2-5x-6}{(x+3)(x+5)(x-1)}$$

$$= \frac{\cancel{(x-1)}^{1}}{(x+3)(x+5)\cancel{(x-1)}_{1}}$$

$$= \frac{1}{(x+3)(x+5)}, \quad x \neq -5, -3, 1$$

HW: p. 67-69 #3fg, 5def, 8ef, 9ef, 10gi, 14df

Working with Radicals - I

Rules: 1) *When **multiplying** radicals,* $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ 2) *When **dividing** radicals,* $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$

A. Changing Entire Radicals to Mixed Radicals

Ex. Simplify.

a) $\sqrt{50}$
$=\sqrt{25}\sqrt{2}$
$=5\sqrt{2}$

b) $\sqrt{20}$
$=\sqrt{4}\sqrt{5}$
$=2\sqrt{5}$

c) $\sqrt{32}$
$=\sqrt{16}\sqrt{2}$
$=4\sqrt{2}$

d) $\sqrt{48}$
$=\sqrt{16}\sqrt{3}$
$=4\sqrt{3}$

e) $\sqrt{72}$
$=\sqrt{36}\sqrt{2}$
$=6\sqrt{2}$

f) $\sqrt{98}$
$=\sqrt{49}\sqrt{2}$
$=7\sqrt{2}$

g) $\sqrt{162}$
$=\sqrt{81}\sqrt{2}$
$=9\sqrt{2}$

h) $\sqrt{200}$
$=\sqrt{100}\sqrt{2}$
$=10\sqrt{2}$

B. Dividing Radicals

Ex. Simplify.

a) $\frac{\sqrt{48}}{\sqrt{6}}$
$=\sqrt{\frac{48}{6}}$
$=\sqrt{8}$
$=\sqrt{4}\sqrt{2}$
$=2\sqrt{2}$

b) $\frac{\sqrt{225}}{\sqrt{3}}$
$=\sqrt{\frac{225}{3}}$
$=\sqrt{75}$
$=\sqrt{25}\sqrt{3}$
$=5\sqrt{3}$

c) $\frac{2\sqrt{5}}{\sqrt{180}}$
$=\frac{2\cancel{\sqrt{5}}^{1}}{\sqrt{36}\cancel{\sqrt{5}}^{1}}$
$=\frac{2}{6}$
$=\frac{1}{3}$

d) $\frac{-10\sqrt{384}}{2\sqrt{8}}$
$=-5\sqrt{48}$
$=-5\sqrt{16}\sqrt{3}$
$=-5(4)\sqrt{3}$
$=-20\sqrt{3}$

C. Multiplying Radicals

Ex. Simplify.

a) $2\sqrt{3}\times5\sqrt{6}$
$=10\sqrt{18}$
$=10\sqrt{9}\sqrt{2}$
$=10(3)\sqrt{2}$
$=30\sqrt{2}$

b) $2\sqrt{5}\times3\sqrt{10}$
$=6\sqrt{50}$
$=6\sqrt{25}\sqrt{2}$
$=6(5)\sqrt{2}$
$=30\sqrt{2}$

c) $9\sqrt{2}\times4\sqrt{14}$
$=36\sqrt{28}$
$=36\sqrt{4}\sqrt{7}$
$=36(2)\sqrt{7}$
$=72\sqrt{7}$

d) $3\sqrt{6}\times5\sqrt{6}$
$=15\sqrt{36}$
$=15(6)$
$=90$

D. Simplifying Radical Expressions

Ex. Simplify.

a) $\frac{21-14\sqrt{6}}{7}$
$=\frac{\cancel{7}^{1}(3-2\sqrt{6})}{\cancel{7}^{1}}$
$=3-2\sqrt{6}$

b) $\frac{12-\sqrt{48}}{4}$ $=\sqrt{16}\sqrt{3}$ $=4\sqrt{3}$
$=\frac{12-4\sqrt{3}}{4}$
$=\frac{\cancel{4}^{1}(3-\sqrt{3})}{\cancel{4}^{1}}$
$=3-\sqrt{3}$

c) $\frac{10+\sqrt{60}}{2}$ $=\sqrt{4}\sqrt{15}$ $=2\sqrt{15}$
$=\frac{10+2\sqrt{15}}{2}$
$=\frac{\cancel{2}^{1}(5+\sqrt{15})}{\cancel{2}^{1}}$
$=5+\sqrt{15}$

d) $\frac{-15+2\sqrt{125}}{5}$ $=\sqrt{25}\sqrt{5}$ $=5\sqrt{5}$
$=\frac{-15+2(5)\sqrt{5}}{5}$
$=\frac{-15+10\sqrt{5}}{5}$
$=\frac{\cancel{5}^{1}(-3+2\sqrt{5})}{\cancel{5}^{1}}$
$=-3+2\sqrt{5}$

HW: p. 106-109 #1 to 4 odd parts, 12, 13, 19

Working with Radicals - II

A. Adding and Subtracting Radicals

Ex. 1. Simplify (ie. add and subtract *like radicals*).

a) $\sqrt{24}-\sqrt{54}+\sqrt{150}$

$=\sqrt{4}\sqrt{6}-\sqrt{9}\sqrt{6}+\sqrt{25}\sqrt{6}$

$=2\sqrt{6}-3\sqrt{6}+5\sqrt{6}$

$=4\sqrt{6}$

b) $3\sqrt{48}-4\sqrt{8}+4\sqrt{27}-2\sqrt{72}$

$=3\sqrt{16}\sqrt{3}-4\sqrt{4}\sqrt{2}+4\sqrt{9}\sqrt{3}-2\sqrt{36}\sqrt{2}$

$=3(4)\sqrt{3}-4(2)\sqrt{2}+4(3)\sqrt{3}-2(6)\sqrt{2}$

$=12\sqrt{3}-8\sqrt{2}+12\sqrt{3}-12\sqrt{2}$

$=24\sqrt{3}-20\sqrt{2}$

B. Multiplying Radicals and Binomials

Ex. 2. Simplify.

a) $2\sqrt{2}\left(\sqrt{10}-3\sqrt{14}\right)$

$=2\sqrt{20}-6\sqrt{28}$

$=2\sqrt{4}\sqrt{5}-6\sqrt{4}\sqrt{7}$

$=2(2)\sqrt{5}-6(2)\sqrt{7}$

$=4\sqrt{5}-12\sqrt{7}$

b) $\left(2\sqrt{3}+\sqrt{5}\right)\left(\sqrt{3}-4\sqrt{5}\right)$

$=2\sqrt{9}-8\sqrt{15}+\sqrt{15}-4\sqrt{25}$

$=2(3)-7\sqrt{15}-4(5)$

$=6-7\sqrt{15}-20$

$=-14-7\sqrt{15}$

C. Fractions with Radicals in the Denominator

Fractions that contain a radical in the denominator are <u>not</u> yet reduced to simplest terms. To fully simplify a fraction with a radical in the denominator we must RATIONALIZE the denominator as follows:

Ex. 3. Simplify (ie. *rationalize* the denominator).

a) & b) monomials in the denominator (multiply by the radical)

a) $\dfrac{5}{\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}=\dfrac{5\sqrt{6}}{\sqrt{36}}$

$=\dfrac{5\sqrt{6}}{6}$

b) $\dfrac{\sqrt{5}}{\sqrt{40}}=\dfrac{\cancel{\sqrt{5}}^{1}}{\sqrt{8}\cancel{\sqrt{5}}^{1}}$

$=\dfrac{1}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$

$=\dfrac{\sqrt{2}}{4}$

c) & d) binomials in the denominator (multiply by the conjugate)

c) $\dfrac{4}{\left(2\sqrt{5}-\sqrt{3}\right)}\cdot\dfrac{\left(2\sqrt{5}+\sqrt{3}\right)}{\left(2\sqrt{5}+\sqrt{3}\right)}$

conjugates

$=\dfrac{8\sqrt{5}+4\sqrt{3}}{4\sqrt{25}+\underbrace{2\sqrt{15}-2\sqrt{15}}_{\emptyset}-\sqrt{9}}$

$=\dfrac{8\sqrt{5}+4\sqrt{3}}{4(5)-(3)}$

$=\dfrac{8\sqrt{5}+4\sqrt{3}}{17}$

d) $\dfrac{\left(2\sqrt{7}-\sqrt{3}\right)}{\left(3\sqrt{7}+2\sqrt{3}\right)}\cdot\dfrac{\left(3\sqrt{7}-2\sqrt{3}\right)}{\left(3\sqrt{7}-2\sqrt{3}\right)}$

conjugates

$=\dfrac{6\sqrt{49}-4\sqrt{21}-3\sqrt{21}+2\sqrt{9}}{9\sqrt{49}\underbrace{-6\sqrt{21}+6\sqrt{21}}_{\emptyset}-4\sqrt{9}}$

$=\dfrac{6(7)-7\sqrt{21}+2(3)}{9(7)-4(3)}$

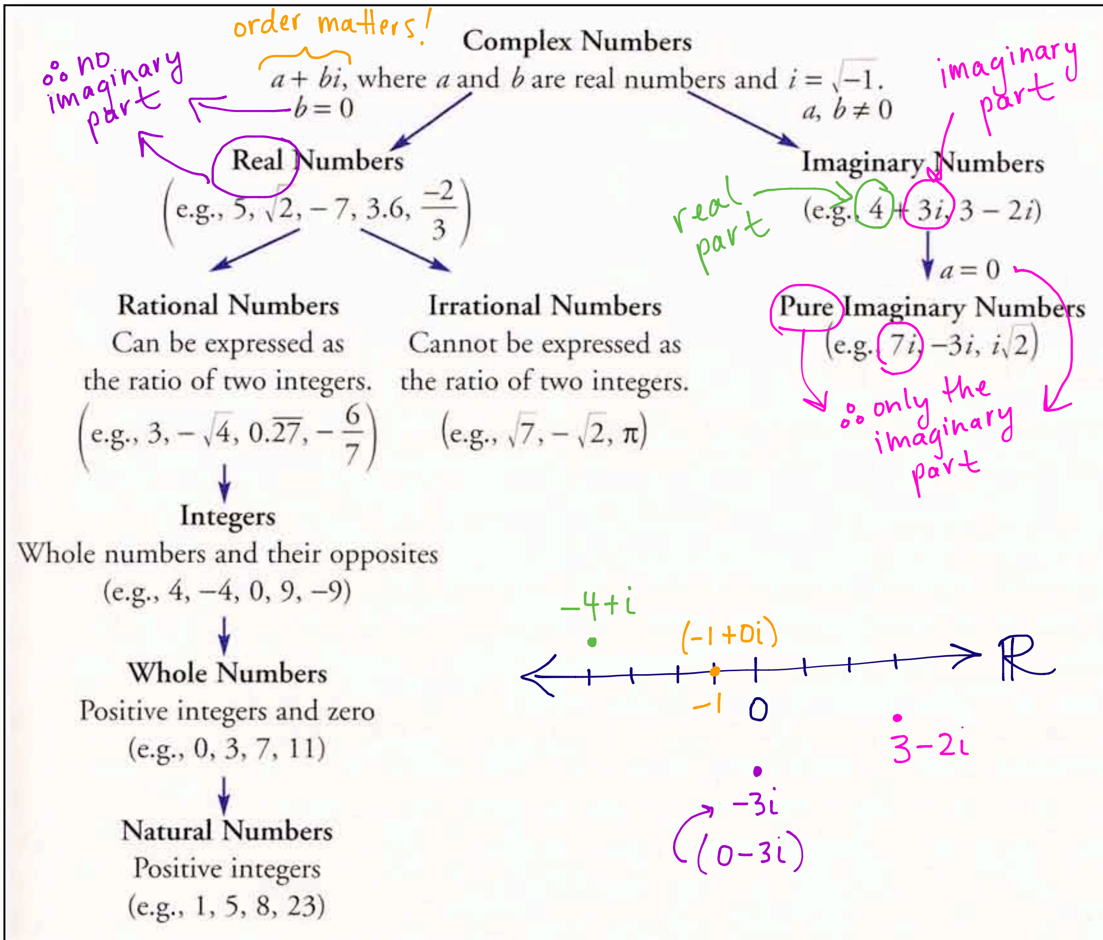
$=\dfrac{42-7\sqrt{21}+6}{63-12}$

$=\dfrac{48-7\sqrt{21}}{51}$

HW: p. 139-141 #1 to 7 even parts, 12, 13

Complex Numbers - I

Definition: ***Complex Numbers, C, are of the form a + bi**, where **a** and **b** are real numbers and **i**, the imaginary unit, is equal to* $\sqrt{-1}$, *i.e.* $\boxed{i=\sqrt{-1}}$ *and* $\boxed{i^2=-1}$.



Simplifying Complex Numbers:

a) $\sqrt{-25}$

$=\sqrt{-1}\sqrt{25}$

$=5i$

b) $\sqrt{-12}$

$=\sqrt{-1}\sqrt{4}\sqrt{3}$

$=2i\sqrt{3}$

c) $\sqrt{-48}$

$=\sqrt{-1}\sqrt{16}\sqrt{3}$

$=4i\sqrt{3}$

d) $\sqrt{-125}$

$=\sqrt{-1}\sqrt{25}\sqrt{5}$

$=5i\sqrt{5}$

e) $\sqrt{-4}$

$=\sqrt{-1}\sqrt{4}$

$=2i$

f) $3i\times 6i$

$=18i^2$

$=18(-1)$

$=-18$

g) $(2i)(-5i)(3i)$

$=-30i^3$

$=-30i^2i$

$=-30(-1)i$

$=30i$

h) $(3i\sqrt{2})^2$

$=(3i\sqrt{2})(3i\sqrt{2})$

$=9i^2\sqrt{4}$

$=9(-1)(2)$

$=-18$

i) $3-\sqrt{-28}$

$=3-\sqrt{-1}\sqrt{4}\sqrt{7}$

$=3-2i\sqrt{7}$

j) $\dfrac{-12+\sqrt{-32}}{2}$

$=\dfrac{-12+\sqrt{-1}\sqrt{16}\sqrt{2}}{2}$

$=\dfrac{-12+4i\sqrt{2}}{2}$

$=\dfrac{\cancel{2}^{1}(-6+2i\sqrt{2})}{\cancel{2}_{1}}$

$=-6+2i\sqrt{2}$

HW: p. 107-108 #5 to 9 odd parts, 10, 11

Complex Numbers - II

Adding and Subtracting Complex Numbers

a) $(6-4i)+(-2+3i)$

$= 6-4i-2+3i$

$= \underbrace{4}_{\text{real}}\underbrace{-i}_{\text{imaginary}}$

b) $(-4-5i)-(3-2i)$

$= -4-5i-3+2i$

$= \underbrace{-7}_{\text{real}}\underbrace{-3i}_{\text{imaginary}}$

Multiplying Complex Numbers

a) $3i(2+4i)$

$= 6i+12i^2 \longrightarrow i^2=-1$

$= 6i+12(-1)$

$= -12+6i$

b) $(1-4i)(3+2i)$

$= 3+2i-12i-8i^2$

$= 3-10i-8(-1)$

$= 11-10i$

c) $(1-5i)^3 = [(1-5i)(1-5i)](1-5i)$

$= (1-10i+25i^2)(1-5i)$

$= (1-10i-25)(1-5i)$

$= (-24-10i)(1-5i)$

$= -24+120i-10i+50i^2$

$= -74+110i$

Dividing Complex Numbers

Simplify by ***rationalizing*** the denominators. For binomial denominators, find the complex ***conjugate***.

a) $\frac{5}{2i}\cdot\frac{i}{i}$

$= \frac{5i}{2i^2}$

$= \frac{5i}{2(-1)}$

$= -\frac{5i}{2}$

b) $\frac{3i}{(-3+i)}\cdot\frac{(-3-i)}{(-3-i)}$

$= \frac{-9i-3i^2}{9\underbrace{+3i-3i}_{0}-i^2}$

$= \frac{-9i-3(-1)}{9-(-1)}$

$= \frac{3-9i}{10}$

c) $\frac{(2+3i)}{(1-2i)}\cdot\frac{(1+2i)}{(1+2i)}$

$= \frac{2+4i+3i+6i^2}{1\underbrace{+2i-2i}_{0}-4i^2}$

$= \frac{2+7i+6(-1)}{1-4(-1)}$

$= \frac{-4+7i}{5}$

Working with Equations and Complex Numbers

Solve using the quadratic formula:

$x^2-4x+6=0$ $a=1$ $b=-4$ $c=6$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(6)}}{2(1)}$

$x = \frac{4\pm\sqrt{-8}}{2}$

$x = \frac{4\pm\sqrt{-1}\sqrt{4}\sqrt{2}}{2}$

$x = \frac{4\pm 2i\sqrt{2}}{2}$

$x = \frac{\cancel{2}(2\pm i\sqrt{2})}{\cancel{2}}$

$$\boxed{\therefore x = 2+i\sqrt{2} \text{ or } x = 2-i\sqrt{2}}$$

Formal Check:

Check $x = 2+i\sqrt{2}$:

LS $= x^2-4x+6$ RS $=0$

$= (2+i\sqrt{2})^2-4(2+i\sqrt{2})+6$

$= (2+i\sqrt{2})(2+i\sqrt{2})-8-4i\sqrt{2}+6$

$= 4+4i\sqrt{2}+i^2\sqrt{4}-8-4i\sqrt{2}+6$

$= 4+(-1)(2)-8+6$

$= 4-2-8+6$

$= 0$

$\boxed{\therefore LS = RS}$

Check $x = 2-i\sqrt{2}$ on a sheet of lined paper!

HW: p. 150-151 #1hij, 2gjl, 3df, 4de, 5, 6fgh, 7e, 12ab

Unit 2 Review

Sample questions for additional practice:

a) $\dfrac{25-10x}{2x-5}$

$= \dfrac{5(\cancel{5-2x})}{-(\cancel{5-2x})}$

$= \dfrac{5}{-1}$

$= -5, \quad x \neq 5/2$

b) $\dfrac{m^2-5m+4}{4m+12} \times \dfrac{m^2-9}{m^2-7m+12}$

$= \dfrac{\cancel{(m-4)}(m-1)}{4\cancel{(m+3)}} \times \dfrac{\cancel{(m+3)}\cancel{(m-3)}}{\cancel{(m-4)}\cancel{(m-3)}}$

$= \dfrac{m-1}{4}, \quad m \neq -3, 3, 4$

c) $\dfrac{x^2+4x+4}{x-2} \div \dfrac{3x+6}{x^2-5x+6}$

$= \dfrac{(x+2)(x+2)}{(x-2)} \div \dfrac{3(x+2)}{(x-3)(x-2)}$

$= \dfrac{(x+2)\cancel{(x+2)}}{\cancel{(x-2)}} \times \dfrac{(x-3)\cancel{(x-2)}}{3\cancel{(x+2)}}$

$= \dfrac{(x+2)(x-3)}{3}, \quad x \neq 2, -2, 3$

d) $\dfrac{2x-1}{x^2-6x+9} + \dfrac{2x}{3x-x^2}$ LCD $= (x-3)^2$

$= \dfrac{(2x-1)}{(x-3)(x-3)} + \dfrac{2\cancel{x}}{\cancel{x}(3-x)}$

$= \dfrac{(2x-1)}{(x-3)(x-3)} + \dfrac{2(x-3)}{-(x-3)(x-3)}$

$= \dfrac{(2x-1)}{(x-3)(x-3)} - \dfrac{(2x-6)}{(x-3)(x-3)}$

$= \dfrac{2x-1-2x+6}{(x-3)^2}$

$= \dfrac{5}{(x-3)^2}, \quad x \neq 3$

$x \neq 0$

HW: p. 87-89 #21dfij, 23cdhi, 25hi, 27acef; p. 158-159 #1, 2, 11, 12, 13, 15