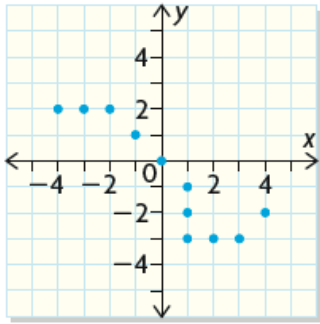


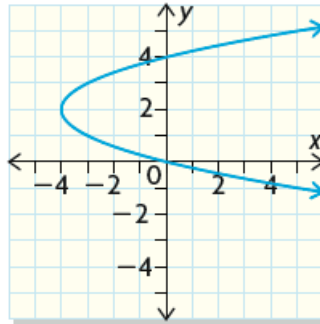
Date: \_\_\_\_\_ **3.1 Relations, Domain, Range and Functions**

1. State the domain and range of each of the following and determine if the relation is a function.

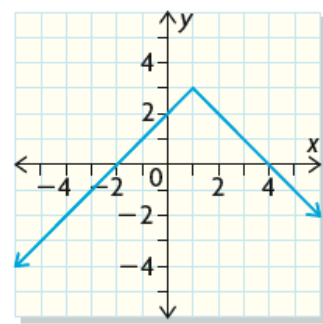
a)



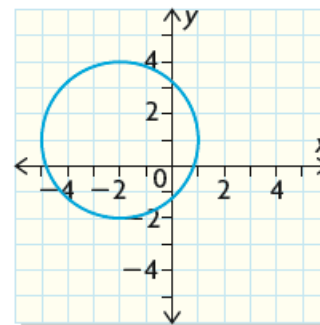
b)



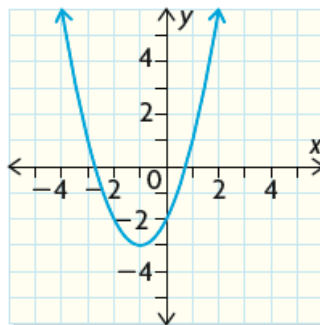
c)



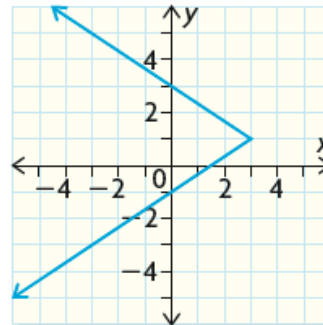
d)



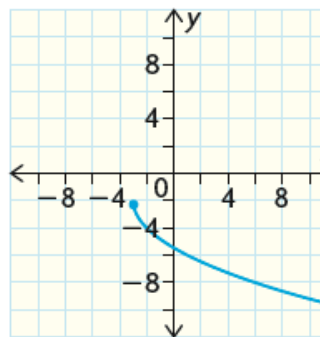
e)



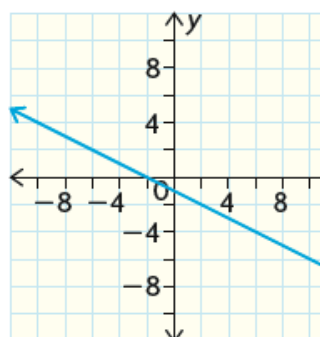
f)



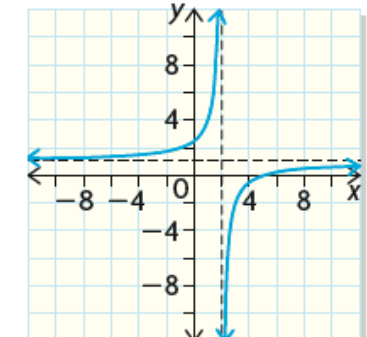
g)



h)



i)



2. State the domain and range of each of the following and determine if the relation is a function. Illustrate graphically.

a)  $y = x(x-3)$

b)  $x^2 + y^2 = 36$

c)  $2x - y - 4 = 0$

d)  $y = -\frac{1}{3}x^2 - 2x + 1$

e)  $4x + 2 = 0$

f)  $x = 1 - y^2$

3. State the domain and range of each of the following and determine if the relation is a function.

a)  $\{(-4,1), (0,5), (0,3), (3,2), (4,-1)\}$

b)  $y = -2\sqrt{3x+6}$

c)  $y = \sqrt{16-x^2}$

d)  $y = \frac{1}{6x^2 - 11x - 10}$

e)  $y = \frac{x^2 - 6x + 9}{3-x}$

f)  $y = \frac{1}{\sqrt{1-0.5x}}$

### 3.1 Answers

1. a)  $D = \{x \in I \mid -4 \leq x \leq 4\}$  ,  $R = \{y \in I \mid -3 \leq y \leq 2\}$  , *not a function*   b)  $D = \{x \in \mathfrak{R} \mid x \geq 4\}$  ,  $R = \{y \in \mathfrak{R}\}$  , *not a function*  
c)  $D = \{x \in \mathfrak{R}\}$  ,  $R = \{y \in \mathfrak{R} \mid y \leq 3\}$  , *is a function*   d)  $D = \{x \in \mathfrak{R} \mid -5 \leq x \leq 1\}$  ,  $R = \{y \in \mathfrak{R} \mid -2 \leq y \leq 4\}$  , *not a function*  
e)  $D = \{x \in \mathfrak{R}\}$  ,  $R = \{y \in \mathfrak{R} \mid y \geq -3\}$  , *is a function*   f)  $D = \{x \in \mathfrak{R} \mid x \leq 3\}$  ,  $R = \{y \in \mathfrak{R}\}$  , *not a function*  
g)  $D = \{x \in \mathfrak{R} \mid x \geq -3\}$  ,  $R = \{y \in \mathfrak{R} \mid y \leq -2\}$  , *is a function*   h)  $D = \{x \in \mathfrak{R}\}$  ,  $R = \{y \in \mathfrak{R}\}$  , *is a function*  
i)  $D = \{x \in \mathfrak{R} \mid x \neq 2\}$  or  $D = \{x \in \mathfrak{R} \mid x < 2 \text{ or } x > 2\}$  ,  $R = \{y \in \mathfrak{R} \mid y \neq 1\}$  or  $R = \{y \in \mathfrak{R} \mid y < 1 \text{ or } y > 1\}$  , *is a function*

2. Check the accuracy of your graphs by using Desmos or other graphing technology or software.

- a)  $D = \{x \in \mathfrak{R}\}$  ,  $R = \left\{y \in \mathfrak{R} \mid y \geq -2\frac{1}{4}\right\}$  , *is a function*   b)  $D = \{x \in \mathfrak{R} \mid -6 \leq x \leq 6\}$  ,  $R = \{y \in \mathfrak{R} \mid -6 \leq y \leq 6\}$  , *not a function*  
c)  $D = \{x \in \mathfrak{R}\}$  ,  $R = \{y \in \mathfrak{R}\}$  , *is a function*   d)  $D = \{x \in \mathfrak{R}\}$  ,  $R = \{y \in \mathfrak{R} \mid y \leq 1\}$  , *is a function*  
e)  $D = \left\{-\frac{1}{2}\right\}$  or  $D = \left\{x \in \mathfrak{R} \mid x = -\frac{1}{2}\right\}$  ,  $R = \{y \in \mathfrak{R}\}$  , *not a function*   f)  $D = \{x \in \mathfrak{R} \mid x \leq 1\}$  ,  $R = \{y \in \mathfrak{R}\}$  , *not a function*

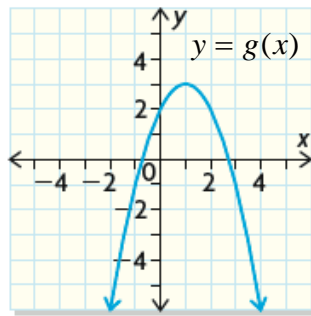
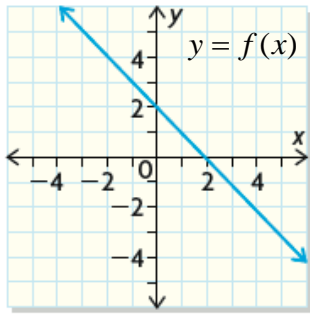
3. Graphing using Desmos or other graphing technology or software will enhance your understanding.

- a)  $D = \{-4, 0, 3, 4\}$  ,  $R = \{-1, 1, 2, 3, 5\}$  , *not a function*   b)  $D = \{x \in \mathfrak{R} \mid x \geq -2\}$  ,  $R = \{y \in \mathfrak{R} \mid y \leq 0\}$  , *is a function*  
c)  $D = \{x \in \mathfrak{R} \mid -4 \leq x \leq 4\}$  ,  $R = \{y \in \mathfrak{R} \mid 0 \leq y \leq 4\}$  , *is a function*  
d)  $D = \left\{x \in \mathfrak{R} \mid x \neq -\frac{2}{3}, \frac{5}{2}\right\}$  ,  $R = \{y \in \mathfrak{R} \mid y \neq 0\}$  , *is a function*   e)  $D = \{x \in \mathfrak{R} \mid x \neq 3\}$  ,  $R = \{y \in \mathfrak{R} \mid y \neq 0\}$  , *is a function*  
f)  $D = \{x \in \mathfrak{R} \mid x < 2\}$  ,  $R = \{y \in \mathfrak{R} \mid y > 0\}$  , *is a function*

Date: \_\_\_\_\_

**3.2 Function Notation**

1. The graphs of  $y = f(x)$  and  $y = g(x)$  are shown.



Using the graphs, determine

- a) the domain and range of  $f$   
 b) the domain and range of  $g$   
 c)  $f(-3)$     d)  $g(3)$     e)  $f(2) - g(-1)$   
 f)  $x$ , if  $f(x) = -2$     g)  $x$ , if  $g(x) = 2$   
 h) the equation of the linear function  
 i) the equation of the quadratic function  
 j)  $x$ , if  $f(x) = g(x)$   
 k)  $x$ , if  $g(x) = 0$

2. If  $f(x) = -2x + 4$ , determine

- a)  $f(0)$     b)  $f(3)$     c)  $f\left(-\frac{1}{2}\right)$     d)  $f(2x-5)$   
 e)  $x$ , if  $f(x) = 0$     f)  $x$ , if  $f(x) = -\frac{3}{4}$     g)  $x$ , if  $f(2x^2) = 3$

3. If  $g(x) = \sqrt{x+5}$ , determine

- a)  $g(4)$     b)  $g(-5)$     c)  $g(0) + g(15)$     d)  $g(5-2x)$   
 e)  $g(4x^6 - 5)$     f)  $x$ , if  $g(x) = 1$     g) the domain of  $g$     h) the range of  $g$

4. If  $h(x) = -x^2 - 2x + 3$ , determine

- a)  $h(0)$     b)  $h(-4)$     c)  $h\left(\frac{1}{2}\right)$     d)  $h(x+1)$     e)  $x$ , if  $h(x) = 0$   
 f)  $x$ , if  $h(-x^2) = 4$     g)  $x$ , if  $h(3x-2) = 0$     h) the domain of  $h$     i) the range of  $h$

5. If  $f(x) = 5 - 3x$  and  $g(x) = 4x + 1$ , determine

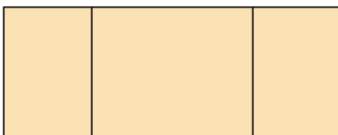
- a)  $f(3) - g(-1)$     b)  $x$ , if  $f(x) = g(-x)$     c)  $x$ , if  $f(2x+2) = g(x^2)$

6. If  $f(x) = \frac{x}{1+x}$ , determine

- a)  $f(2) + f\left(\frac{1}{2}\right)$     b)  $f(3) + f\left(\frac{1}{3}\right)$     c)  $f(n) + f\left(\frac{1}{n}\right)$     d)  $n$ , if  $f(n) = \frac{4}{n}$

7. A farmer has 600 m of fencing to enclose a rectangular area and divide it into three sections as shown.

- a) Express the rectangular area,  $A$ , in  $\text{m}^2$  as a function of its width,  $w$ , in m.  
 b) Determine the domain and range of this area function.  
 c) Determine the dimensions that give the maximum area.



### 3.2 Answers

1. a)  $D_f = \{x \in \mathfrak{R}\}$  ,  $R_f = \{y \in \mathfrak{R}\}$  b)  $D_g = \{x \in \mathfrak{R}\}$  ,  $R_g = \{y \in \mathfrak{R} | y \leq 3\}$  c) 5 d) -1 e) 1 f)  $x = 4$  g)  $x = 0$  or  $x = 2$   
h)  $f(x) = -x + 2$  i)  $g(x) = -(x-1)^2 + 3$  j)  $x = 0$  or  $x = 3$  k)  $x = 1 - \sqrt{3}$  or  $x = 1 + \sqrt{3}$
2. a) 4 b) -2 c) 5 d)  $-4x + 14$  e)  $x = 2$  f)  $2\frac{3}{8}$  g)  $x = -\frac{1}{2}$  or  $x = \frac{1}{2}$
3. a) 3 b) 0 c)  $3\sqrt{5}$  d)  $\sqrt{10-2x}$  e)  $2x^3$  f) -4 g)  $D_g = \{x \in \mathfrak{R} | x \geq -5\}$  h)  $R_g = \{y \in \mathfrak{R} | y \geq 0\}$
4. a) 3 b) -5 c)  $1\frac{3}{4}$  d)  $-x^2 - 4x$  e)  $x = -3$  or  $x = 1$  f)  $x = -1, -1, 1, 1$  g)  $x = -\frac{1}{3}$  or  $x = 1$   
h)  $D_h = \{x \in \mathfrak{R}\}$  i)  $R_h = \{y \in \mathfrak{R} | y \leq 4\}$
5. a) -1 b)  $x = -4$  c)  $x = -1$  or  $x = -\frac{1}{2}$  6. a) 1 b) 1 c) 1 d)  $x = 2 - 2\sqrt{2}$  or  $x = 2 + 2\sqrt{2}$
7. a)  $A(w) = -2w^2 + 300w$  b)  $D = \{w \in \mathfrak{R} | 0 < w < 150\}$  ,  $R = \{A(w) \in \mathfrak{R} | 0 < A(w) \leq 11250\}$  c) 150 m by 75 m

Date: \_\_\_\_\_

**3.2 Function Notation Continued**

Given  $f(x) = -2x^2 + 8$ ,  $g(x) = 2x - 4$  and  $h(x) = \frac{2x}{x-2}$ ,

1. State the domain and range for each function.

2. Determine exact values for the following.

a)  $f(-3)$     b)  $h(-4)$     c)  $g\left(\frac{3}{2}\right)$     d)  $h\left(\frac{3}{4}\right)$     e)  $f(0) + g(0)$     f)  $2[g(3) - h(1)]$

3. Determine simplified expressions for the following. State any restrictions on the variable.

a)  $f(-x)$     b)  $g(-x)$     c)  $h(-x)$     d)  $-[f(x)]$   
 e)  $f(\sqrt{2x+4})$     f)  $f(x-3)$     g)  $g\left(\frac{1}{2}x+2\right)$     h)  $h(2x^2)$   
 i)  $g(x) - f(x)$     j)  $\frac{g(x)}{f(x)}$     k)  $g(x) - h(x)$     l)  $[g(x)] \cdot [h(x)]$

4. Determine exact values of  $x$  for the following.

a)  $f(x) = 0$     b)  $h(x) = \frac{2}{3}$     c)  $f(x+2) = -8$     d)  $f(x) = g(x)$   
 e)  $g(x) = h(x)$     f)  $f(x) + 2[g(x)] = 0$     g)  $f(3x-1) = -12$     h)  $g(x^3) + f(x) = 4x$

**3.2 Continued Answers**

1.  $D_f = \{x \in \mathbb{R}\}$ ,  $R_f = \{y \in \mathbb{R} \mid y \leq 8\}$ ,  $D_g = \{x \in \mathbb{R}\}$ ,  $R_g = \{y \in \mathbb{R}\}$ ,  $D_h = \{x \in \mathbb{R} \mid x \neq 2\}$ ,  $R_h = \{y \in \mathbb{R} \mid y \neq 2\}$  (from Desmos)

2. a)  $-10$     b)  $\frac{4}{3}$  or  $1\frac{1}{3}$     c)  $-1$     d)  $-\frac{6}{5}$  or  $-1\frac{1}{5}$     e)  $4$     f)  $8$

3. a)  $-2x^2 + 8$     b)  $-2x - 4$     c)  $\frac{2x}{x+2}$ ,  $x \neq -2$     d)  $2x^2 - 8$     e)  $-4x$ ,  $x \geq -2$     f)  $-2x^2 + 12x - 10$     g)  $x$

h)  $\frac{2x^2}{x^2-1}$ ,  $x \neq -1, 1$     i)  $2x^2 + 2x - 12$     j)  $\frac{-1}{x+2}$ ,  $x \neq -2, 2$     k)  $\frac{2(x-4)(x-1)}{x-2}$ ,  $x \neq 2$     l)  $4x$ ,  $x \neq 2$

4. a)  $x = -2$  or  $x = 2$     b)  $x = -1$     c)  $x = -2 - 2\sqrt{2}$  or  $x = -2 + 2\sqrt{2}$     d)  $x = -3$  or  $x = 2$     e)  $x = 1$  or  $x = 4$

f)  $x = 0$  or  $x = 2$     g)  $x = \frac{1-\sqrt{10}}{3}$  or  $x = \frac{1+\sqrt{10}}{3}$     h)  $x = -\sqrt{2}, 1, \sqrt{2}$