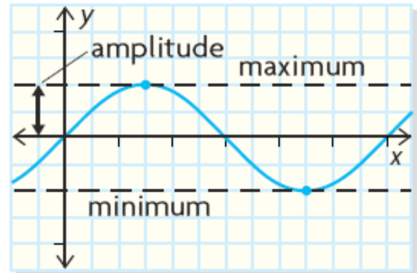
Graphing the Primary Trigonometric Functions

The graphs of the primary trigonometric functions are **periodic**. The sine and cosine functions have a distinct **wavelike** appearance (often referred to as a sinusoidal wave).



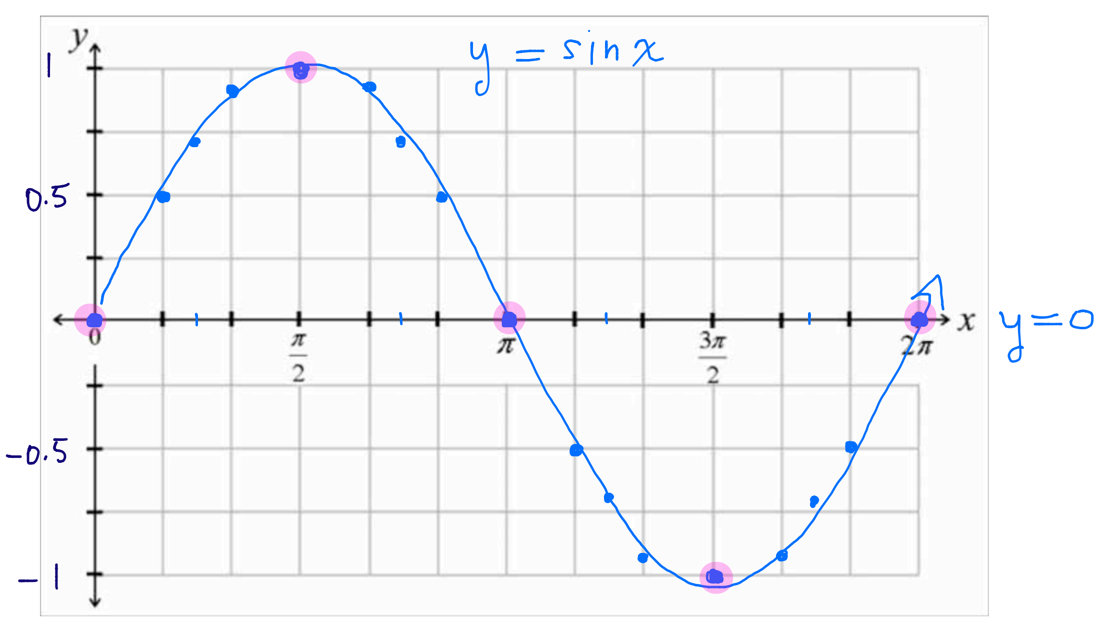
- the **period** is the interval of the independent variable needed for a repeating action to complete *one full cycle* (a cycle can begin at any point on the graph)

  → also called the average height
- the **equilibrium axis** is the equation of the horizontal line *halfway* between the maximum and the minimum value (calculated by finding $\frac{\max + \min}{2}$)
- the **amplitude** is the *distance* from the function's equilibrium axis to either the maximum or the minimum value (calculated by finding $\frac{\max - \min}{2}$)

A. The Graph of *y* = sinx

The **sine function** can be represented by the set of ordered pairs (*x*, sin*x*), where *x* is an angle in standard position measured in degrees or radians and $x \in R$. The equation of the sine function is written in the form *y* = sin*x* or *f(x)* = sin*x*. Graph the equation *y* = sin*x*, where *x* is an angle between 0 and 2π.

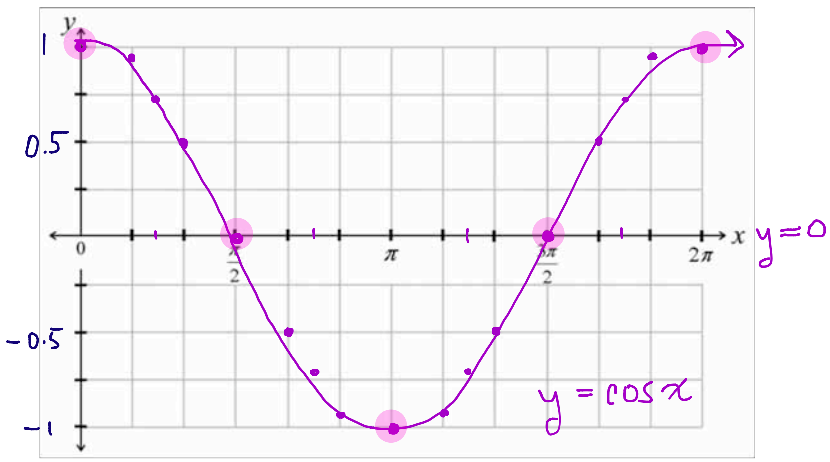
x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Exact value of sinx	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
Decimal value of sinx	0	0.5	0.7	0.9	1	0.9	0.7	0.5	0	−0.5	−0.7	−0.9	−1	−0.9	−0.7	−0.5	0



B. The Graph of $y = \cos x$

Graph the equation $y = \cos x$, where x is an angle between 0 and 2π.

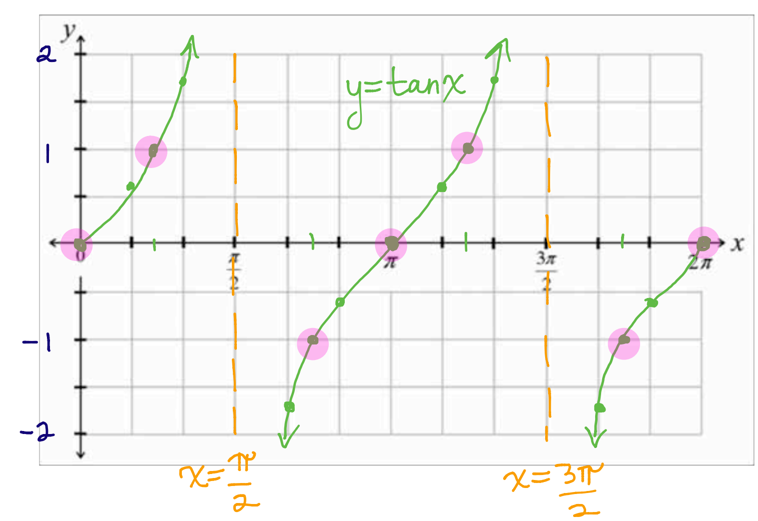
x	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Exact value of $\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Decimal value of $\cos x$	1	0.9	0.7	0.5	0	-0.5	-0.7	-0.9	-1	-0.9	-0.7	-0.5	0	0.5	0.7	0.9	1



C. The Graph of $y = \tan x$

Graph the equation $y = \tan x$, where x is an angle between 0 and 2π.

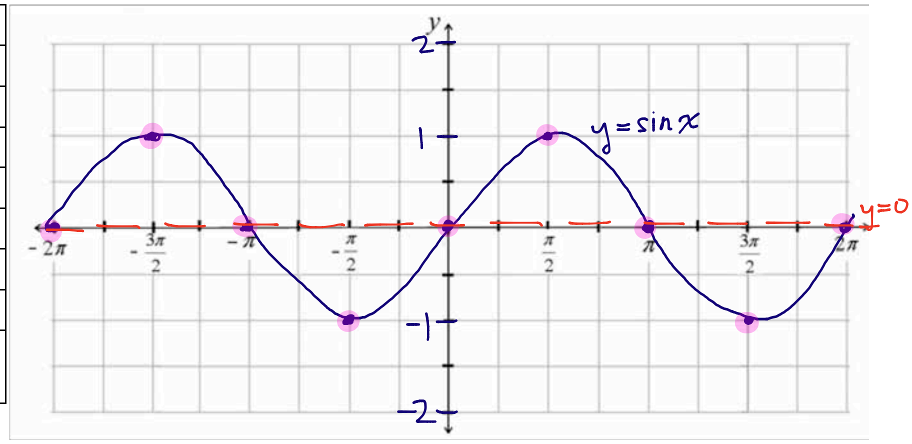
x	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Exact value of $\tan x$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{1}$	$\frac{\sqrt{3}}{1}$	$\frac{1}{0}$	$\frac{\sqrt{3}}{-1}$	$\frac{1}{-1}$	$\frac{1}{-\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{1}$	$\frac{\sqrt{3}}{1}$	$\frac{1}{0}$	$\frac{-\sqrt{3}}{1}$	$\frac{-1}{1}$	$\frac{-1}{\sqrt{3}}$	0
Decimal value of $\tan x$	0	0.6	1	1.7	DNE	-1.7	-1	-0.6	0	0.6	1	1.7	DNE	-1.7	-1	-0.6	0



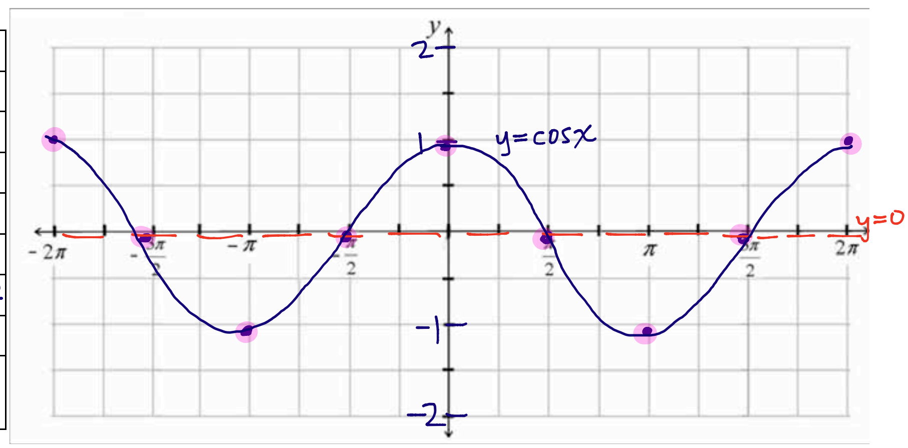
D. Key Features of Sinusoidal Functions

Sketch the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$ and fill in the following tables on the interval $0 \le x \le 2\pi$. For this unit, we will use the y-values of 0, 1 and –1 to plot the key points for each curve.

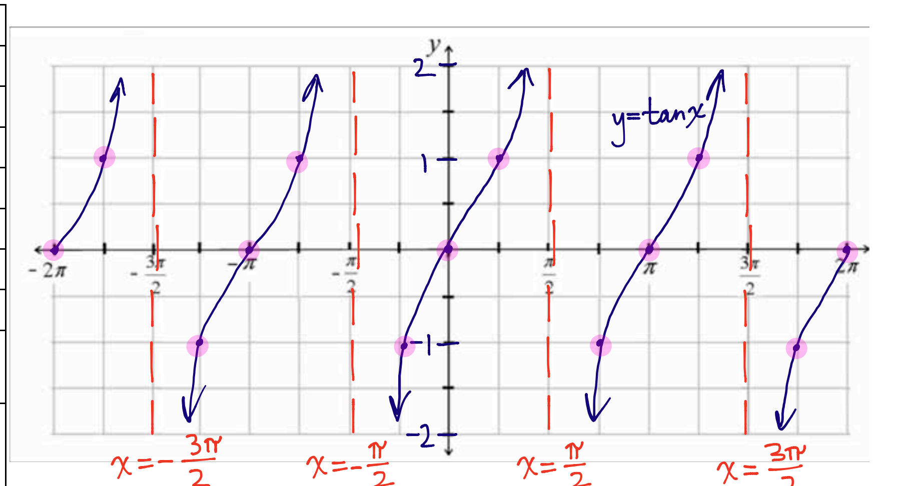
Key Features	**y = sinx**
Maximum value	1
Minimum value	-1
Amplitude	1
Period	2π
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}, -1 \le y \le 1\}$
Zeros	$x = 0, \pi, 2\pi, \ldots$
Equation of the equilibrium axis	$y = 0$



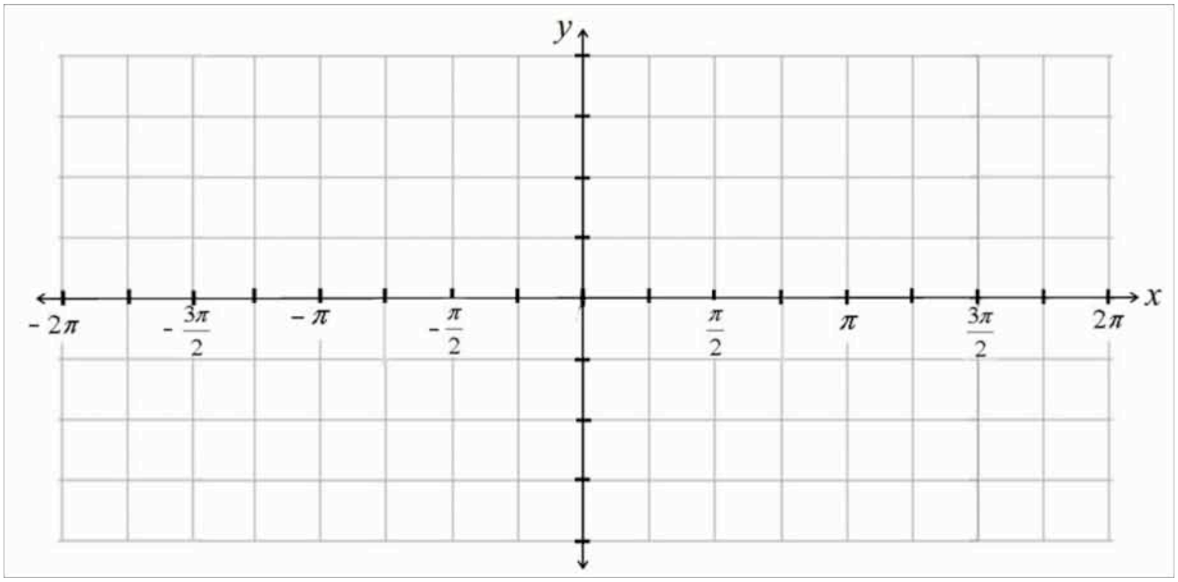
Key Features	**y = cosx**
Maximum value	1
Minimum value	-1
Amplitude	1
Period	2π
Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y \in \mathbb{R}, -1 \le y \le 1\}$
Zeros	$x = \pi/2, 3\pi/2, \ldots$
Equation of the equilibrium axis	$y = 0$

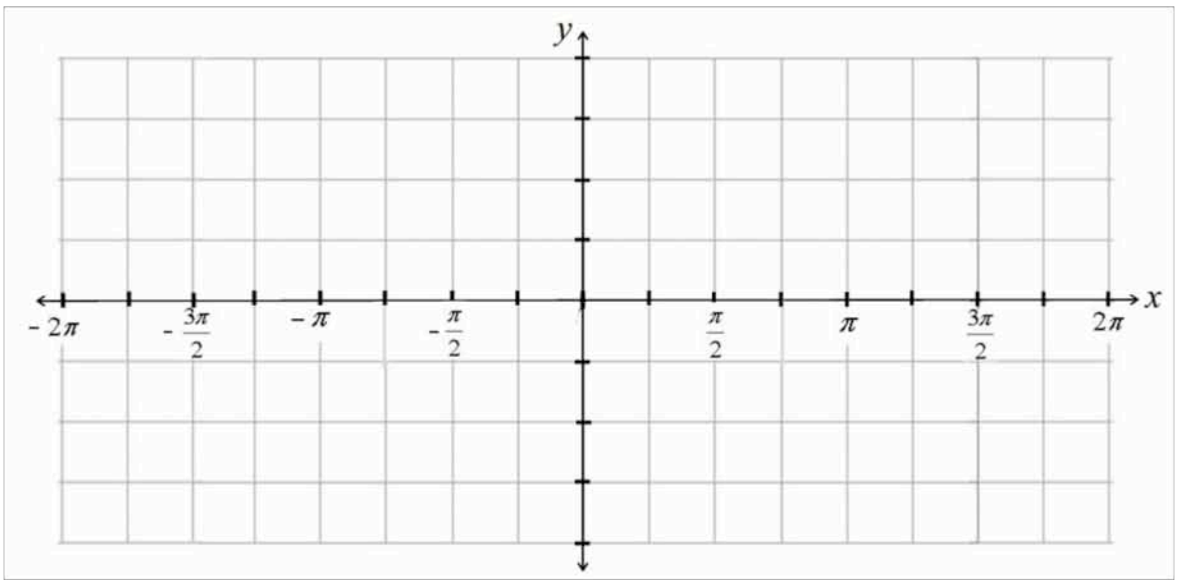


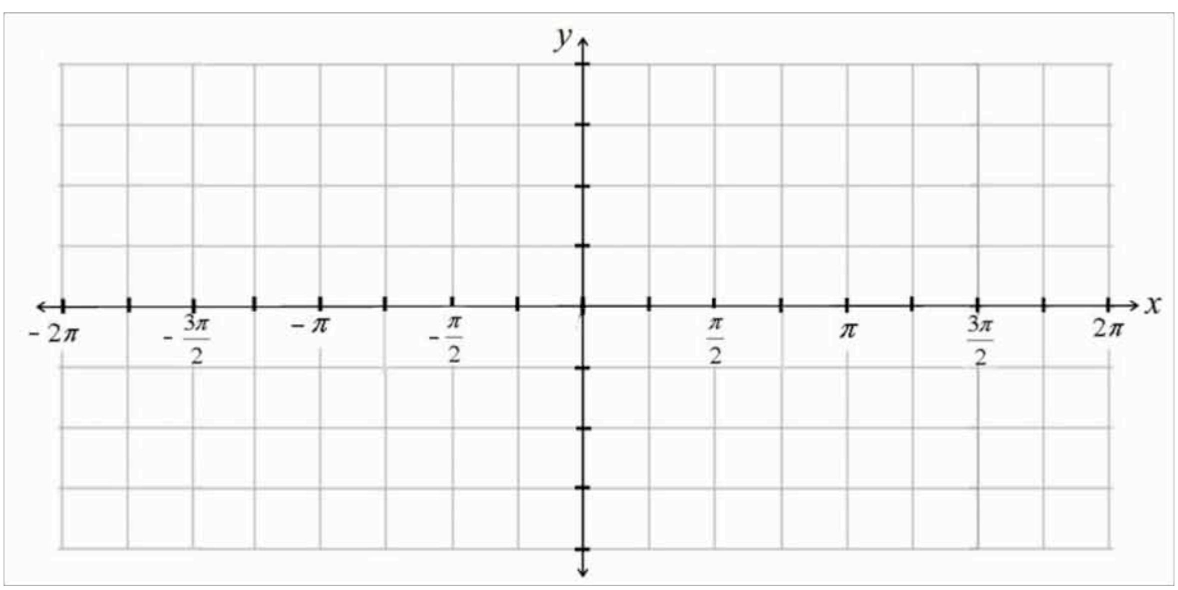
Key Features	**y = tanx**
Maximum value	∞
Minimum value	$-\infty$
Amplitude	n/a
Period	π
Domain	$\{x \mid x \in \mathbb{R}, x \ne \frac{\pi}{2} + \pi n, n \in I\}$
Range	$\{y \mid y \in \mathbb{R}\}$
Zeros	$x = 0, \pi, 2\pi, \ldots$
Equation of the equilibrium axis	n/a
Equation of vertical asymptotes	$x = \frac{\pi}{2}, x = \frac{3\pi}{2}, \ldots$



HW: Use the grids on the following page to sketch $y = \sin x$, $y = \cos x$, *and* $y = \tan x$ on the interval $-2\pi \le x \le 2\pi$. Where $\tan x$ is undefined, draw and label vertical asymptotes! Memorize these graphs!!


y
x
- 2π
- 3π/2
– π
- π/2
π/2
π
3π/2
2π


y
x
- 2π
- 3π/2
– π
- π/2
π/2
π
3π/2
2π


y
x
- 2π
- 3π/2
– π
- π/2
π/2
π
3π/2
2π

Stretches and Reflections of Periodic Functions

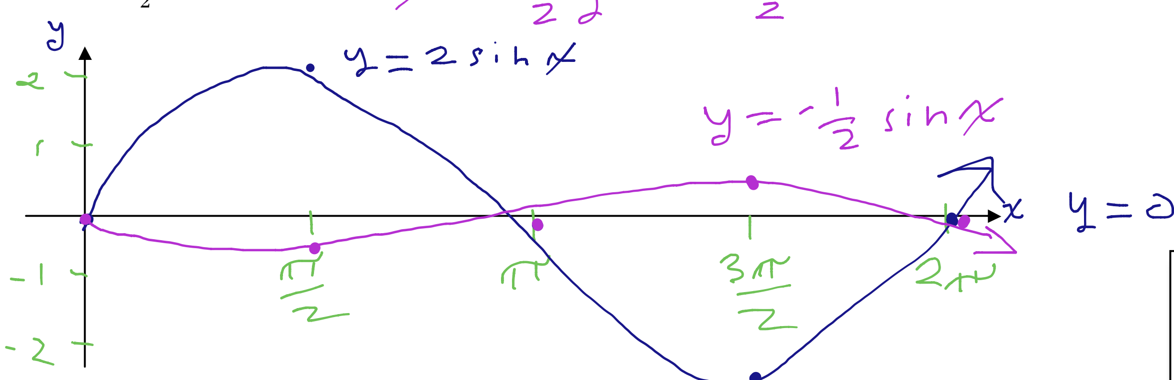
Like other functions, the stretches and reflections of sine and cosine functions can be summarized as follows:

Transformations	**Transformed Function**	**Effect on** $y = \sin x$ **or** $y = \cos x$
Vertical Reflection and Vertical Stretch	$y = a \sin x$ $y = a \cos x$	• If $a < 0$, the graph is **vertically reflected** in the **x-axis.** • If $\lvert a \rvert > 1$, the graph is **vertically expanded** by a factor of $\lvert a \rvert$. • If $0 < \lvert a \rvert < 1$, the graph is **vertically compressed** by a factor of $\lvert a \rvert$. • The point (x, y) on $y = f(x)$ becomes the point (x, ay) on $y = a f(x)$. • The **AMPLITUDE** of the function is A = $\lvert a \rvert$.
Horizontal Reflection and Horizontal Stretch	$y = \sin kx$ $y = \cos kx$	• If $k < 0$, the graph is **reflected** in the **y-axis.** • If $\lvert k \rvert > 1$, the graph is **horizontally compressed** by a factor of $\frac{1}{\lvert k \rvert}$. • If $0 < \lvert k \rvert < 1$, the graph is **horizontally expanded** by a factor of $\frac{1}{\lvert k \rvert}$. • The point (x, y) on $y = f(x)$ becomes the point $(\frac{1}{k}x, y)$ on $y = f(kx)$. • The **PERIOD** of the function is P = $\frac{2\pi}{\lvert k \rvert} = \frac{360°}{\lvert k \rvert}$.

A. Sketch the following using transformations on $y = \sin x$**:**

a) $y = 2 \sin x$ $(x, y) \rightarrow (\ x\ ,\ 2y\ )$ A = 2 P = 2π

b) $y = -\frac{1}{2} \sin x$ $(x, y) \rightarrow (\ x\ ,\ -\frac{1}{2}y\ )$ A = $\frac{1}{2}$ P = 2π



How to set the scale for horizontal stretches given k**:**

1) Find the period length, P, using: P = $\frac{2\pi}{\lvert k \rvert}$.

2) Get the number of radians between each key point by calculating $P \times \frac{1}{4}$.

3) Add $P \times \frac{1}{4}$ to the first point in the cycle, and to each subsequent point until the cycle(s) are complete.

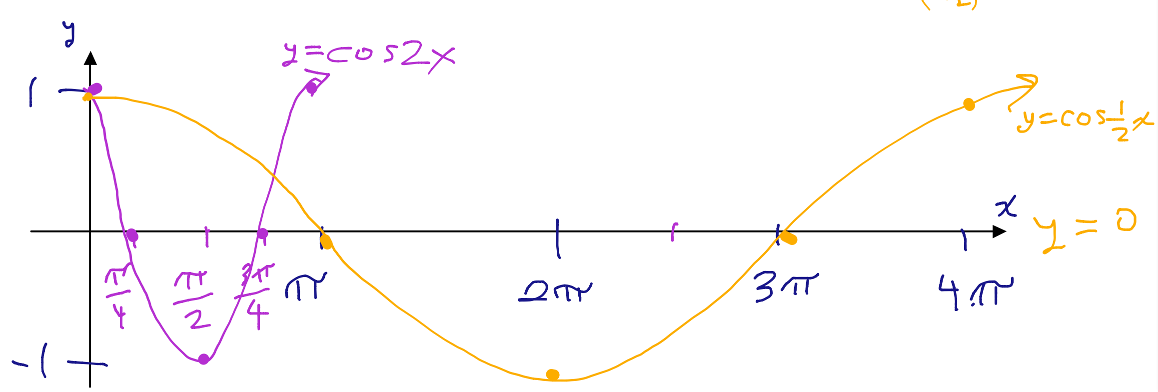
4) Label the x-axis accordingly.

B. Sketch the following using transformations on $y = \cos x$**:**

$\frac{2\pi}{k} = \frac{2\pi}{2} = \pi$

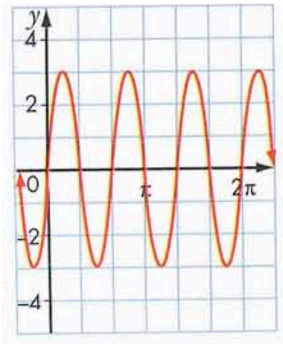
a) $y = \cos 2x$ $(x, y) \rightarrow (\ \frac{1}{2}x\ ,\ y\ )$ A = 1 P = ____________

b) $y = \cos \frac{1}{2}x$ $(x, y) \rightarrow (\ 2x\ ,\ y\ )$ A = 1 P = $\frac{2\pi}{(1/2)} = 4\pi$



C. Determine the equations for the following a) *sine* and b) *cosine* functions:

a)

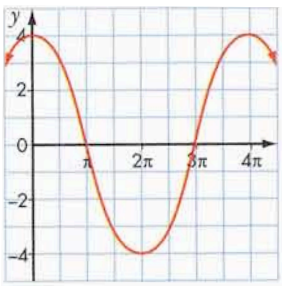


A = 3 P = $\frac{2\pi}{3}$

k = $\frac{2\pi}{P} = \frac{2\pi}{\left(\frac{2\pi}{3}\right)} = 3$

$\therefore y =$ $3 \sin 3x$

b)



A = 4 P = 4π

k = $\frac{2\pi}{P} = \frac{2\pi}{4\pi} = \frac{1}{2}$

$\therefore y =$ $4 \cos \frac{1}{2} x$

How to find *k* given the period length, *P*:

1) Rearrange the equation for period length to isolate $|k|$:

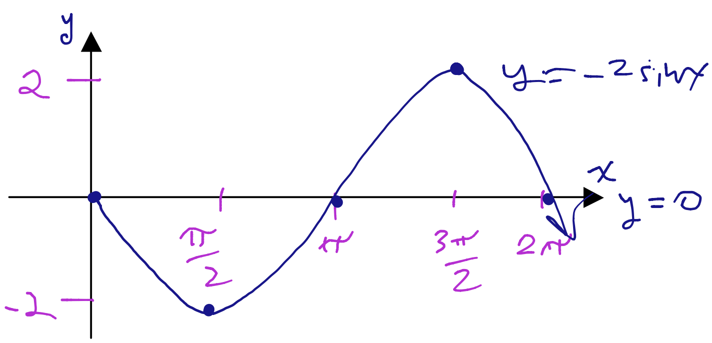
$$P = \frac{2\pi}{|k|}$$

$$|k| = \frac{2\pi}{P}$$

2) Reduce the fraction to lowest terms.

D. Graph each function for one cycle and state the domain and range for your graph.

a) $y = -2 \sin x$

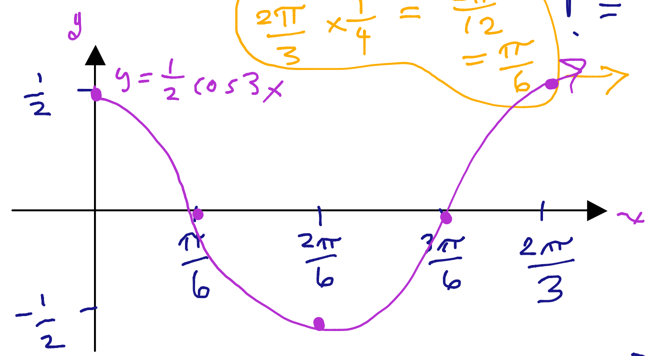


$D = \{x \in \mathbb{R}\}$ $R = \{y \in \mathbb{R}, -2 \le y \le 2\}$

b) $y = \frac{1}{2} \cos 3x$

$k = 3$ $P = \frac{2\pi}{k}$ $P = \frac{2\pi}{3}$

$\frac{2\pi}{3} \times \frac{1}{4} = \frac{2\pi}{12} = \frac{\pi}{6}$ → intervals btw pts



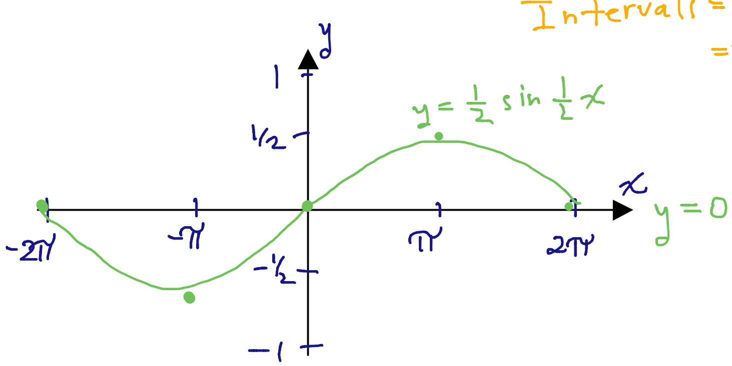
$R = \{y \mid y \in \mathbb{R}, -\frac{1}{2} \le y \le \frac{1}{2}\}$

E. Sketch the graph of the following functions.

a) $y = \frac{1}{2} \sin \frac{1}{2} x$ *for* $-2\pi \le x \le 2\pi$

$P = \frac{2\pi}{k} = \frac{2\pi}{1/2} = 4\pi$

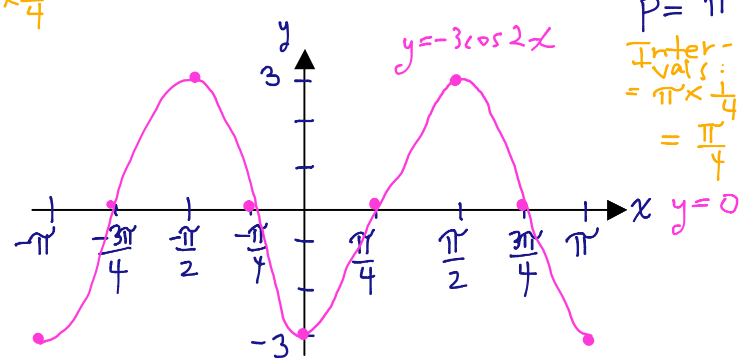
Intervals $= 4\pi \times \frac{1}{4} = \pi$



b) $y = -3 \cos 2x$ *for* $-\pi \le x \le \pi$

$A = 3$ $k = 2$ $P = \pi$

Intervals: $= \pi \times \frac{1}{4} = \frac{\pi}{4}$



HW: p. 374-376 # 2, 4 and 5(equations only, no graphs), 6, 7abc, 8acd, 9cd, 12 (no graphs)

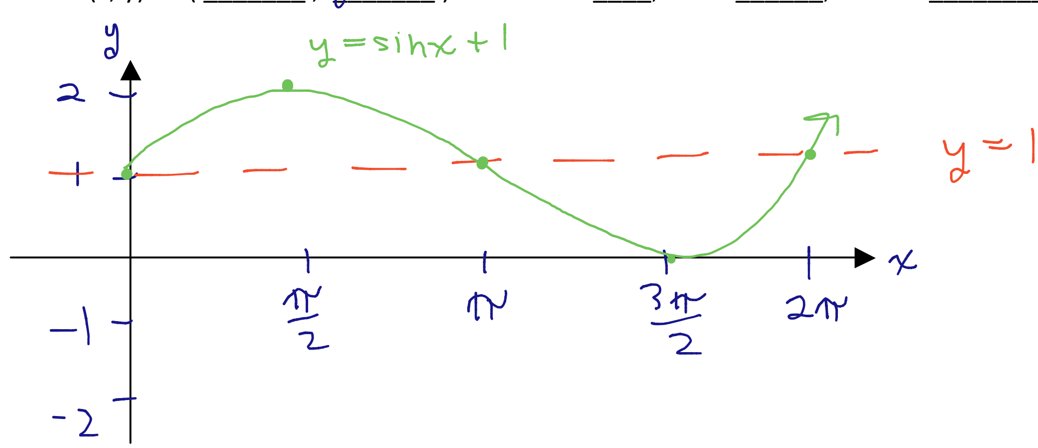
Translations of Periodic Functions

For the functions $y = \sin(x - d) + c$ and $y = \cos(x - d) + c$, **d** represents a phase shift (or horizontal translation) and **c** represents a vertical translation.

Transformation	Transformed Function	Effect on $y = \sin x$ or $y = \cos x$
Horizontal Translation	$y = \sin(x - d)$ $y = \cos(x - d)$ eg: $y = \sin\left(x + \frac{\pi}{4}\right)$ Left $\frac{\pi}{4}$ radians	If **$d > 0$**, the graph is horizontally translated **right** $\lvert d\rvert$ units. If **$d < 0$**, the graph is horizontally translated **left** $\lvert d\rvert$ units. The **PHASE SHIFT** of the function is: P.S. = $\lvert d\rvert$ units right if $d > 0$ ***or*** P.S. = $\lvert d\rvert$ units left if $d < 0$
Vertical Translation	$y = \sin x + c$ $y = \cos x + c$	If **$c > 0$**, the graph is **vertically translated up** $\lvert c\rvert$ units If **$c < 0$**, the graph is **vertically translated down** $\lvert c\rvert$ units The **VERTICAL TRANSLATION** of the function is: V.T. = $\lvert c\rvert$ units up if $c > 0$ ***or*** V.T. = $\lvert c\rvert$ units down if $c < 0$ The equation of the **equilibrium axis** is **$y = c$**.

A. Graph the function $y = \sin x$ for one cycle. Then graph the following using transformations on $y = \sin x$:

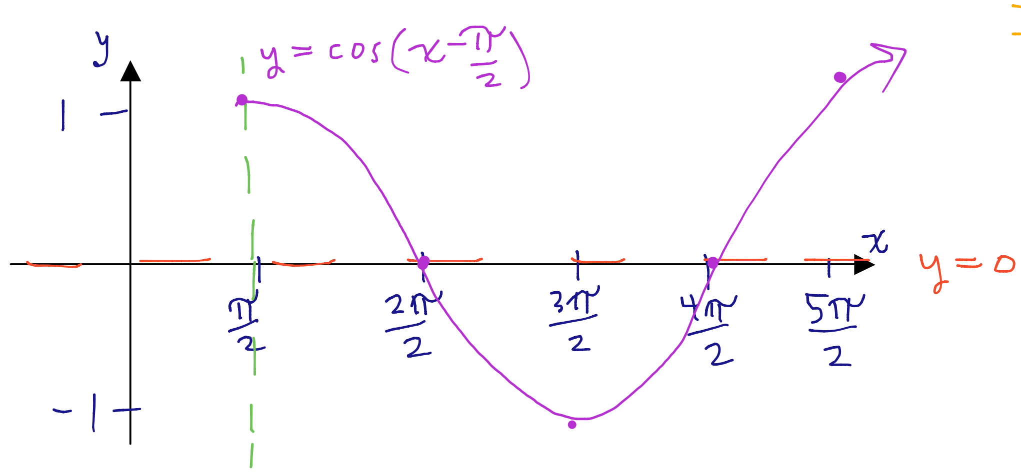
$y = \sin x + 1$ $(x, y) \rightarrow (\ x\ ,\ y+1\ )$. A = 1; P = 2π; P.S. = none; V.T = 1 unit up



B. Graph the function $y = \cos x$ for one cycle. Then graph the following using transformations on $y = \cos x$:

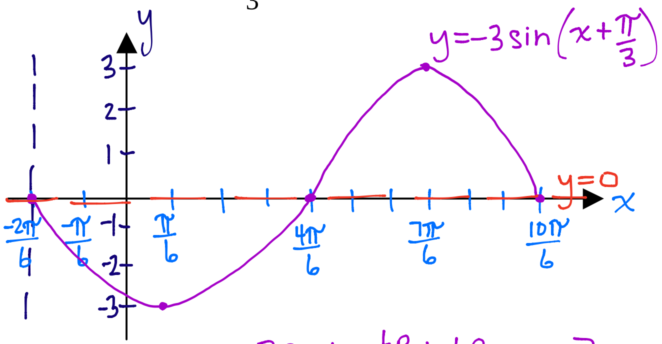
$y = \cos\left(x - \frac{\pi}{2}\right)$ $(x, y) \rightarrow \left(x + \frac{\pi}{2},\ y\right)$. A = 1; P = 2π; P.S. = $\frac{\pi}{2}$ right; V.T = none

Intervals: $P \times \frac{1}{4}$
$= 2\pi \times \frac{1}{4}$
$= \frac{\pi}{2}$



C. Graph each function for one cycle. State the domain and range of the cycle.

a) $y = -3\sin\left(x + \frac{\pi}{3}\right)$



VR in x-axis

$A = 3$

$PS = \frac{\pi}{3}$ left

$P = 2\pi$

$\frac{1}{4}P = \frac{1}{4}(2\pi)$

$= \frac{\pi}{2}$

PS + $\frac{1}{4}P + \frac{1}{4}P + \ldots$ } 5 Key points

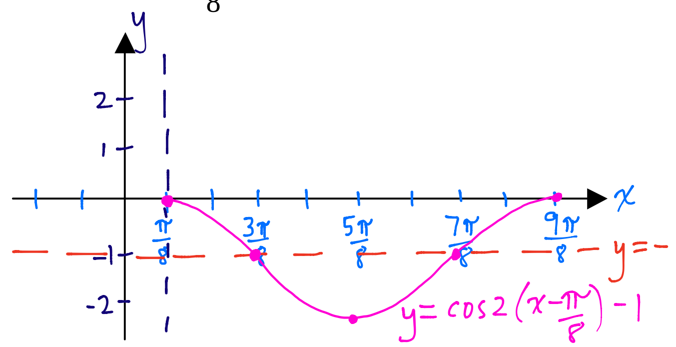
$\frac{-\pi}{3} + \frac{\pi}{2} + \frac{\pi}{2} + \ldots$ LCD=6

$= \frac{-2\pi}{6} + \frac{3\pi}{6} + \frac{3\pi}{6} + \ldots$ } $\therefore$ x-scale is $\frac{\pi}{6}$

D = $\{x \mid x \in \mathbb{R},\ 0 \le x \le 2\pi\}$

R = $\{y \mid y \in \mathbb{R},\ -3 \le y \le 3\}$

b) $y = \cos 2\left(x - \frac{\pi}{8}\right) - 1$



$A = 1$; VT down 1

$PS = \frac{\pi}{8}$ right

$P = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi$

$\frac{1}{4}P = \frac{1}{4}(\pi) = \frac{\pi}{4}$

x-scale = $PS + \frac{1}{4}P$

$= \frac{\pi}{8} + \frac{\pi}{4}^{\times 2}_{\times 2}$

$= \frac{\pi}{8} + \frac{2\pi}{8}$

D = $\{x \mid x \in \mathbb{R},\ 0 \le x \le \pi\}$

R = $\{y \mid y \in \mathbb{R},\ -2 \le y \le 0\}$

D. Determine the amplitude, period, phase shift and vertical translation for each function. Find the x-scale.

a) $y = -4\sin 2\left(x - \frac{\pi}{6}\right)$

$A = 4$

VR in x-axis

$P = \frac{2\pi}{2} = \pi$

$PS = \frac{\pi}{6}$ right

$\frac{1}{4}P = \frac{1}{4}\pi = \frac{\pi}{4}$

x-scale = $PS + \frac{1}{4}P$

$= \frac{\pi}{6}^{\times 2}_{\times 2} + \frac{\pi}{4}^{\times 3}_{\times 3}$

$= \frac{2\pi}{12} + \frac{3\pi}{12}$

$\therefore$ each tick is $\frac{\pi}{12}$

b) $y = \frac{1}{2}\cos\frac{2}{3}\left(x + \frac{\pi}{4}\right) - \frac{1}{2}$

$A = \frac{1}{2}$

$P = \frac{2\pi^{\times 3}}{\left(\frac{2}{3}\right)_{\times 3}} = \frac{6\pi}{2} = 3\pi$

$PS = \frac{\pi}{4}$ left

$\frac{1}{4}P = \frac{1}{4}3\pi = \frac{3\pi}{4}$

$VT = \frac{1}{2}$ unit down

x-scale = $PS + \frac{1}{4}P$

$= -\frac{\pi}{4} + \frac{3\pi}{4}$

$\therefore$ each tick is $\frac{\pi}{4}$

E. Write an equation for the function in the form *y = a sin k(x – d) + c* or *y = a cos k(x – d) + c*

a) *sine* function: A = 1 ; P = 4π ; P.S. = $\frac{\pi}{2}$ left ; V.T. = none

$k = \frac{2\pi}{P}$

$= \frac{\cancel{2\pi}}{\cancel{4\pi}}$ $\therefore k = \frac{1}{2}$

$y = \sin\frac{1}{2}\left(x + \frac{\pi}{2}\right)$

b) *cosine* function: A = 4 ; P = $\frac{\pi}{2}$; $k = \frac{2\pi}{P}$ P.S. = none ; V.T. = up 3

$= \frac{2\cancel{\pi}^{\times 2}}{\left(\frac{\cancel{\pi}}{2}\right)^{\times 2}}$ $\therefore k = 4$

$y = 4\cos 4x + 3$

HW: p. 387-388 # 1 and 2 (even parts), 3ab, 4ade, 5, 6, 9

Graphing Functions in the Form *y* = a *sin* k(*x* – d) + c and *y* = a *cos* k(*x* – d) + c

For the sinusoidal functions $y = \sin x$ and $y = \cos x$, the point (x, y) maps onto the point $(\frac{1}{k}x + d, ay + c)$ on the graphs of $y = a \sin k(x - d) + c$ or $y = a \cos k(x - d) + c$.

$$y = a \sin k(x - d) + c$$

a:
- Vertical stretch or compression by a factor of 'a'
- reflection if 'a' < 0
- amplitude = |a|
- $(x, y) \to (x, ay)$

k:
- horizontal stretch or compression by a factor of $\frac{1}{k}$
- $P = \frac{2\pi}{k} = \frac{360°}{k}$
- $(x, y) \to (\frac{1}{k}x, y)$

d:
- horizontal translation (phase shift)
- eg $y = \sin(x + \frac{\pi}{4})$ P.S. $= \frac{\pi}{4}$ left

c:
- vertical translation c units up/down
- y = c is the eq'n of the equilibrium axis
- $(x, y) \to (x, y + c)$

$g(x) = -3\cos 3(x - \frac{\pi}{3}) + 1$

a) Describe the transformations that must be applied to *y* = sin*x* to obtain the graph of *g(x) = – 3 cos (3x – π) + 1*.

1. Reflection across x-axis
2. V.E. by a factor of 3
3. H.C. by a factor of 1/3
4. $\frac{\pi}{3}$ radians to the right $\leftarrow \frac{2\pi}{6}$
5. V.T. 1 unit up

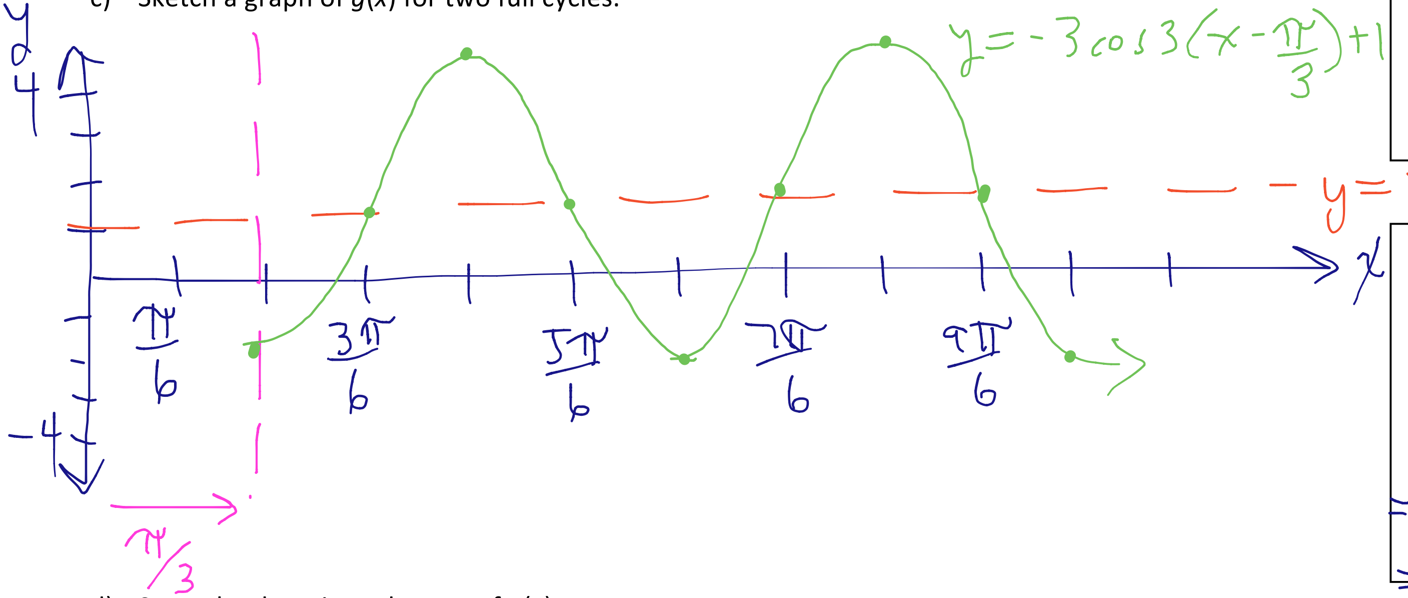
b) State the amplitude and the period of *g*(*x*). A = 3 P = $\frac{2\pi}{3} = \frac{4\pi}{6}$

Find *P*:

$P = \frac{2\pi}{|k|}$

$= \frac{2\pi}{3}$

c) Sketch a graph of *g*(*x*) for two full cycles.



Find the intervals between key points:

$P \times \frac{1}{4} =$

$\frac{2\pi}{3} \times \frac{1}{4}$

$= \frac{2\pi}{12}$

$= \frac{\pi}{6}$

d) State the domain and range of *g*(*x*).

D = $\{x \mid x \in \mathbb{R}, \frac{\pi}{3} \le x \le \frac{5\pi}{3}\}$

R = $\{y \mid y \in \mathbb{R}, -2 \le y \le 4\}$

Sketch the graph of each of the following on the domain specified. State the range.

a. $y = 2\cos\left(2x + \frac{\pi}{3}\right) - 1$ for $-\pi \le x \le \frac{\pi}{2}$

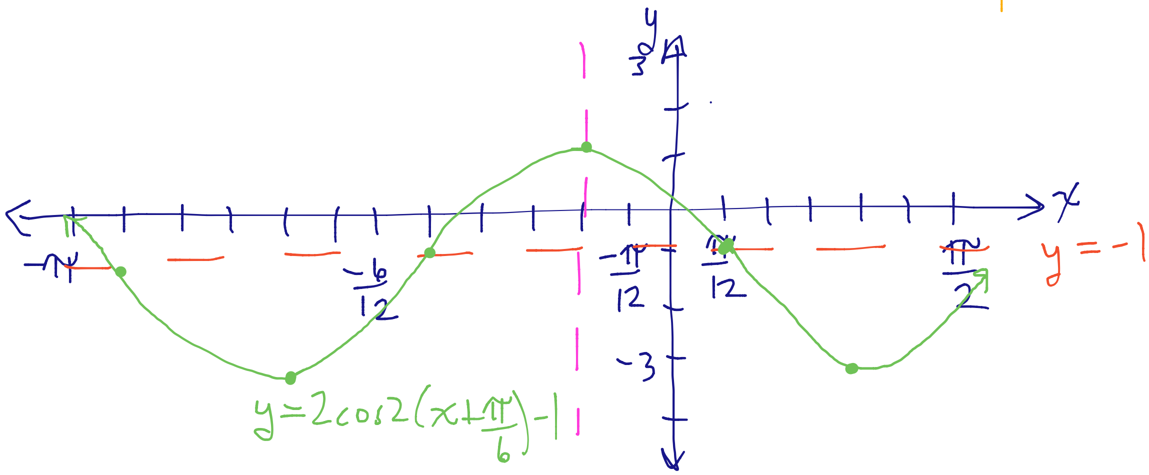
$y = 2\cos 2\left(x + \frac{\pi}{6}\right) - 1$

factor the "K" out!

$P = \frac{2\pi}{2} = \pi$

Intervals: $\pi \times \frac{1}{4} = \frac{\pi}{4} = \frac{3\pi}{12}$

P.S. $= \frac{\pi}{6}$ left $= \frac{2\pi}{12}$



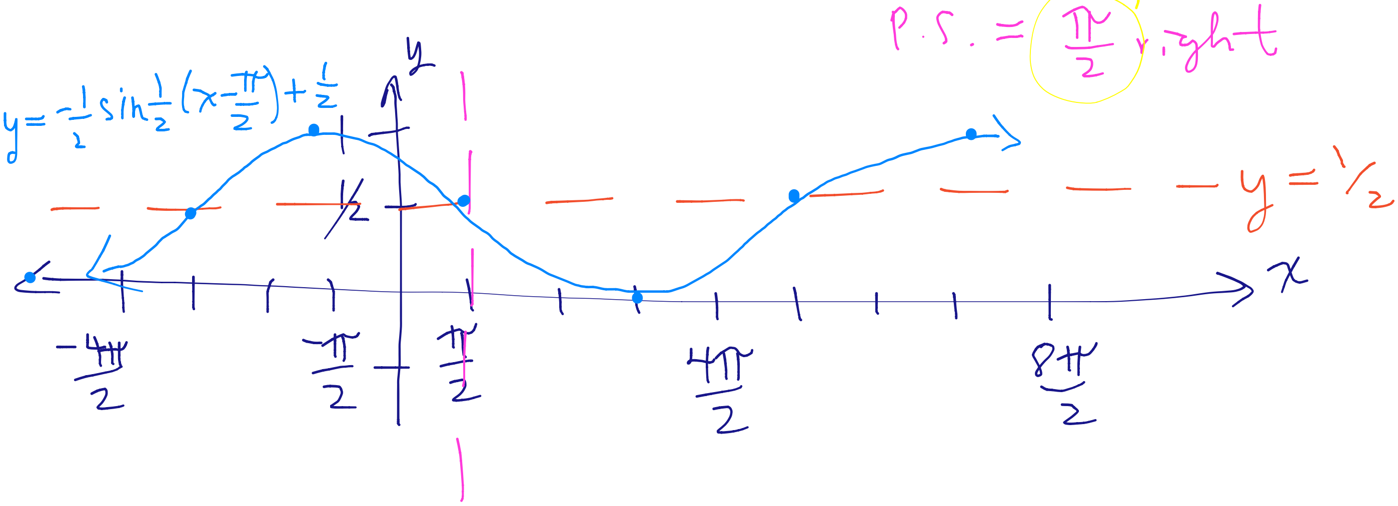
b. $y = -\frac{1}{2}\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) + \frac{1}{2}$ for $-2\pi \le x \le 4\pi$

$y = -\frac{1}{2}\sin\frac{1}{2}\left(x - \frac{\pi}{2}\right) + \frac{1}{2}$

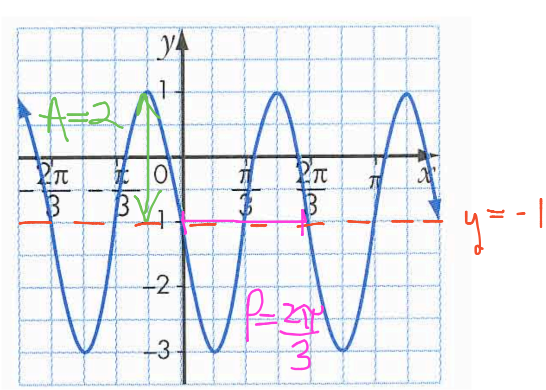
$P = \frac{2\pi}{1/2} = 4\pi = \frac{8\pi}{2}$

Intervals: $4\pi \times \frac{1}{4} = \pi = \frac{2\pi}{2}$

P.S. $= \frac{\pi}{2}$ right



Determine an equation of the form *y = a sin k(x – d) + c* for the sine function graphed below.



$A = 2 \rightarrow$ reflection $\therefore \boxed{a = -2}$

$d = n/a$

$P = \frac{2\pi}{3} \rightarrow K = \frac{2\pi}{P}$

$\boxed{c = -1}$

$K = \frac{2\pi}{(2\pi/3)}$

$\boxed{K = 3}$

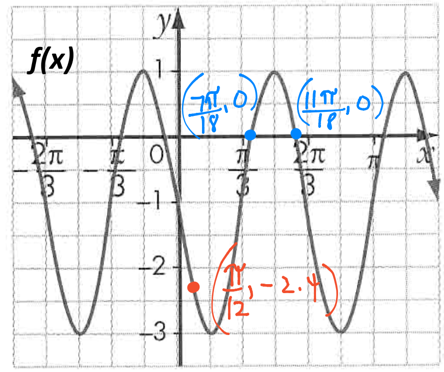
$\boxed{\therefore \text{an equation is } y = -2\sin 3x - 1}$

HW: p. 387-388 # 7ade, 8bcd, 10bcdef, 11

Applications of Trigonometric Functions

Part A: Finding Equations Given the Graph

1. Given the graph of *f(x):*



a) Determine an equation for the function.

From last class:

$$f(x) = -2\sin 3x - 1$$

b) Use the equation from part a) to evaluate $f\left(\frac{\pi}{12}\right)$ to one decimal place and mark the point on the graph.

$$f\left(\frac{\pi}{12}\right) = -2\sin 3\left(\frac{\pi}{12}\right) - 1$$

$$= -2\sin \frac{3\pi}{12} - 1$$

$$= \underbrace{\underbrace{-2\underbrace{\sin\left(\frac{\pi}{4}\right)}_{1}}_{2} - 1}_{3}$$

watch the order of operations!

$$= -2\left(\frac{1}{\sqrt{2}}\right) - 1$$

$$\boxed{f\left(\frac{\pi}{12}\right) \doteq -2.4 \text{ units}}$$

c) Find the value(s) of *x* on the interval $0 \le x \le \frac{2\pi}{3}$ for *f*(x) = 0. Mark these points on the graph.

$$0 = -2\sin 3x - 1$$

$$1 = -2\sin 3x$$

$$-\frac{1}{2} = \sin 3x \qquad \text{Let } (3x) = \theta$$

$$\sin\theta = -\frac{1}{2}$$

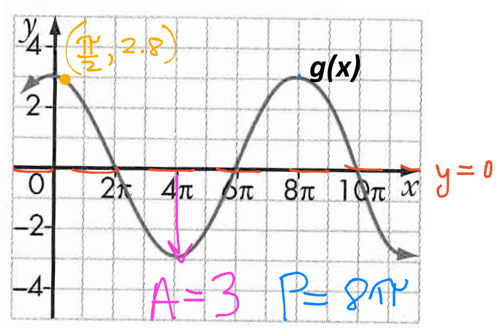
$$\text{raa} = \frac{\pi}{6}; \text{ QIII, QIV}$$

$$\theta = \frac{7\pi}{6} \quad \text{or} \quad \theta = \frac{11\pi}{6}$$

$$3x = \frac{7\pi}{6} \quad \text{or} \quad 3x = \frac{11\pi}{6}$$

$$\therefore \boxed{x = \frac{7\pi}{18} \quad \text{or} \quad x = \frac{11\pi}{18}}$$

2. Given the graph of *g(x):*



a) Determine an equation for the sine function and the cosine function.

$P = 8\pi$

$k = \frac{2\pi}{8\pi} = \frac{1}{4}$

$A = 3$

$c = 0$

Sine: P.S. = 2π right & refl.

$\therefore g(x) = -3\sin\frac{1}{4}(x - 2\pi)$

Cosine: P.S. = 0 & no refl.

$\therefore g(x) = 3\cos\frac{1}{4}x$

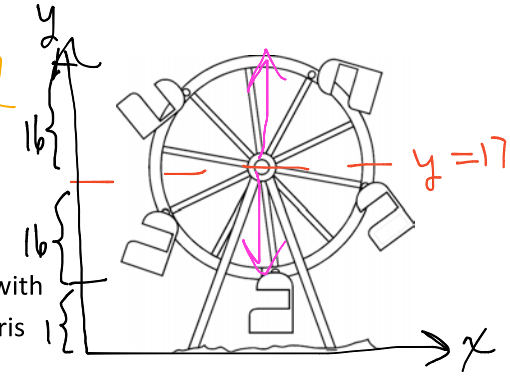
b) Use an equation to evaluate $g\left(\frac{\pi}{2}\right)$ to one decimal place and mark the point on the graph.

$$g\left(\frac{\pi}{2}\right) = 3\cos\frac{1}{4}\left(\frac{\pi}{2}\right)$$

$$= 3\cos\left(\frac{\pi}{8}\right) \quad \doteq 2.8 \text{ units}$$

Part B: Solving Ferris Wheel Problems!

Amplitude Period

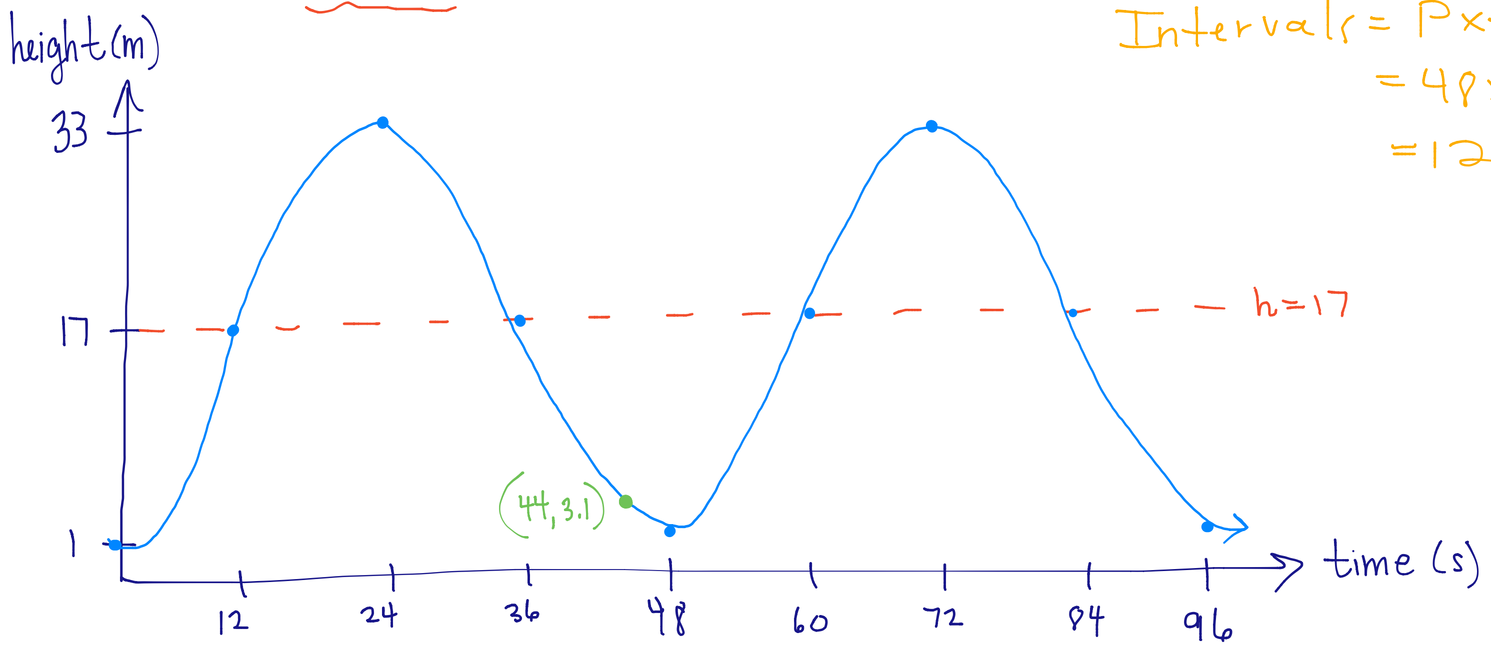


3. A carnival Ferris wheel with a radius of 16m rotates once every 48 seconds. Passengers get on at the lowest point, which is 1 m above the ground.

minimum

a) Sketch a graph to show how your height above ground (in metres) varies with time (in seconds) for two revolutions, starting when the you get onto the Ferris wheel at its lowest point.

Intervals = $P \times \frac{1}{4}$
$= 48 \times \frac{1}{4}$
$= 12\text{ s}$



b) Write an equation which expresses a your height as a function of time on the ride.

- $A = 16$
- Reflection across x-axis
- $P = 48$, $K = \frac{2\pi}{48}$
  $K = \frac{\pi}{24}$
- $c = 17$
- $P.S. = 0$

$$\boxed{\therefore\ h(t) = -16\cos\frac{\pi}{24}t + 17}$$

c) Calculate your height above ground at 44 seconds, to one decimal place. Mark this point on your graph.

$$h(44) = -16\cos\frac{\pi}{24}(44) + 17$$

$$= -16\cos\frac{44\pi}{24} + 17$$

$$\doteq 3.1 \text{ metres}$$

Unit 5 Test Review: p. 375 #7bcd, 8cd; p. 416 #30, 31; p. 418-419 #6, 7ce, 8bc, 11bd, 12