Graphing Functions in the Form *y* = a *sin* k(*x* – d) + c and *y* = a *cos* k(*x* – d) + c

For the sinusoidal functions *y* = sin*x* and *y* = cos*x*, the point (*x*, *y*) maps onto the point ($\frac{1}{k}x + d$, $ay + c$) on the graphs of $y = a \sin k(x - d) + c$ or $y = a \cos k(x - d) + c$.

always be sure k is factored out

$$y = a \sin [k(x - d)] + c$$

- Vertical stretch or compression factor of a
- reflection across x-axis if a<0
- amplitude = |a|
- $(x, y) \rightarrow (x, ay)$

- horizontal stretch or compression factor by a factor of $\frac{1}{k}$
- $P = \frac{2\pi}{k}$ (or $\frac{360°}{k}$)
- $(x, y) \rightarrow (\frac{1}{k}x, y)$

- horizontal translation (phase shift)

- vertical translation "c" units up/down
- y=c is the equilibrium axis

$g(x) = -3\cos[3(x - \frac{\pi}{3})] + 1$

a) Describe the transformations that must be applied to *y* = cos*x* to obtain the graph of *g*(*x*) = – 3 cos (3*x* – π) + 1.

1. V.R. across the x-axis
2. V.E. by a factor of 3
3. H.C. by a factor of $\frac{1}{3}$
4. H.T. $\frac{\pi}{3}$ units right
5. V.T. 1 unit up

$\frac{2\pi}{6}$

b) State the amplitude and the period of *g*(*x*). A = 3 P = $\frac{2\pi}{3} = \frac{4\pi}{6}$

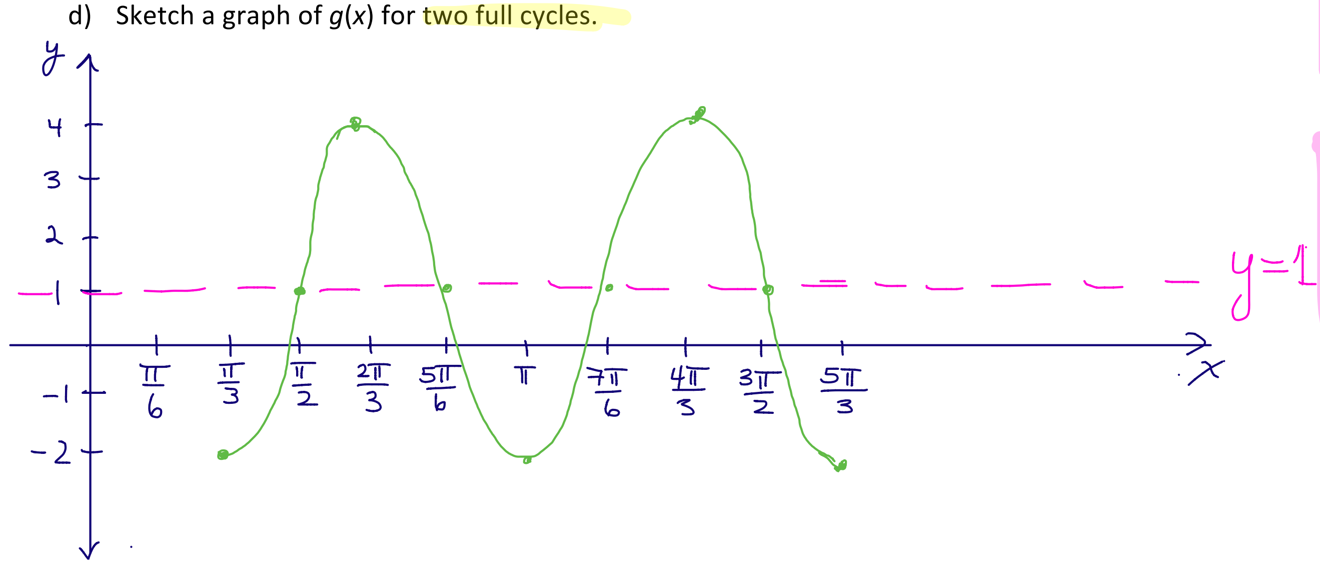
Find *P*:

$P = \frac{2\pi}{|k|}$

$P = \frac{2\pi}{3}$

c) Find the x-scale. ∴ x-scale is $\frac{\pi}{6}$

d) Sketch a graph of *g*(*x*) for two full cycles.



Find the intervals between key points:

$\frac{1}{4} \times P = \frac{1}{4} \cdot \frac{2\pi}{3} = \frac{\pi}{6}$

e) State the domain and range of *g*(*x*).

D = $\{x \in \mathbb{R} \mid \frac{\pi}{3} \le x \le \frac{5\pi}{3}\}$

R = $\{y \in \mathbb{R} \mid -2 \le y \le 4\}$

Sketch the graph of each of the following on the domain specified. State the range.

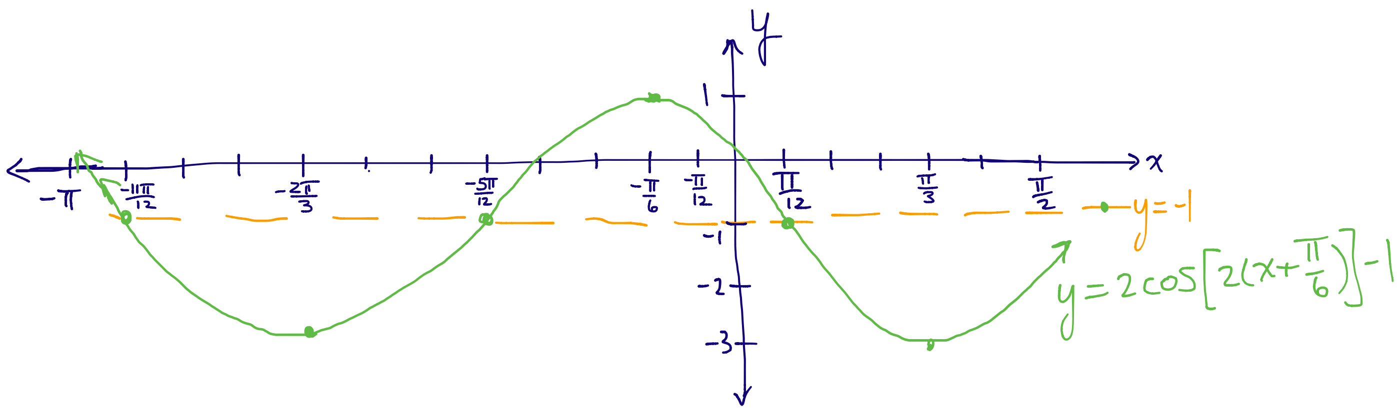
max $-1+2$
min $-1-2$

a. $y = 2\cos\left(2x + \frac{\pi}{3}\right) - 1$ for $-\pi \le x \le \frac{\pi}{2}$ R = $\{y \in \mathbb{R} \mid -3 \le y \le 1\}$

$y = 2\cos\left[2\left(x + \frac{\pi}{6}\right)\right] - 1$ $-\frac{12\pi}{12} \le x \le \frac{6\pi}{12}$

$P = \frac{2\pi}{2} = \pi$; $\frac{1}{4} \times \pi = \frac{\pi}{4} = \frac{3\pi}{12}$

P.S. $= \frac{\pi}{6}$ left $= \frac{2\pi}{12}$ $\therefore$ x-scale is $\frac{\pi}{12}$

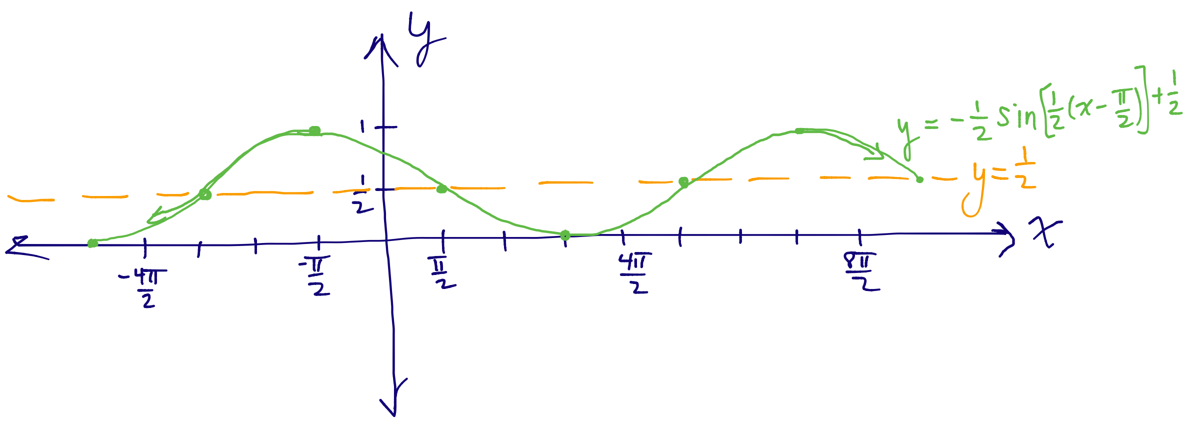


b. $y = -\frac{1}{2}\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) + \frac{1}{2}$ for $-2\pi \le x \le 4\pi$ R = $\{y \in \mathbb{R} \mid 0 \le y \le 1\}$

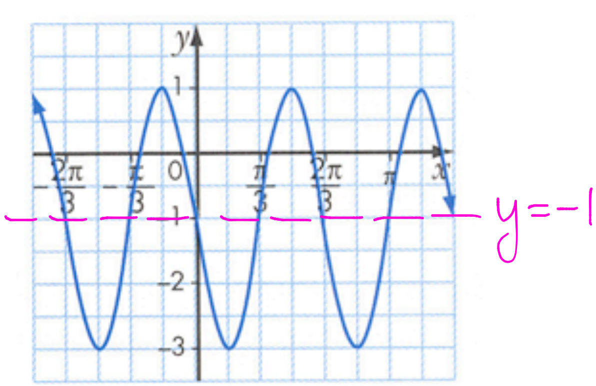
$y = -\frac{1}{2}\sin\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right] + \frac{1}{2}$ $-\frac{4\pi}{2} \le x \le \frac{8\pi}{2}$

$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$; $\frac{1}{4} \times 4\pi = \pi = \frac{2\pi}{2}$

P.S. $= \frac{\pi}{2}$ right $\therefore$ x-scale is $\frac{\pi}{2}$



Determine an equation of the form *y = a sin k(x – d) + c for the sine function graphed below.*



$A = 2$ & reflection $\rightarrow a = -2$

Phase Shift: none $\rightarrow d = 0$

$P = \frac{2\pi}{3} \rightarrow k = \frac{2\pi}{P}$

$= \frac{2\pi}{\left(\frac{2\pi}{3}\right)}$

$\therefore k = 3$

Equilibrium axis: $y = -1 \rightarrow c = -1$

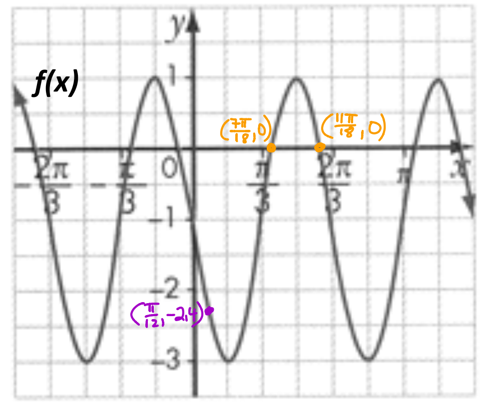
HW: p. 387-388 # 7ade, 8bcd, 10bcdef, 11

$\therefore$ the equation is $y = -2\sin 3x - 1$

$y = 2\sin\left[3\left(x - \frac{\pi}{3}\right)\right] - 1$

Applications of Trigonometric Functions

Part A: Finding Equations Given the Graph



1. Given the graph of *f(x):*
   a) Determine at least two equivalent equations for the function.

$f(x) = -2\sin 3x - 1$ (from lesson 4)

or

$f(x) = 2\cos\left[3\left(x + \frac{\pi}{6}\right)\right] - 1$

b) Use the equation from part a) to evaluate $f\left(\frac{\pi}{12}\right)$ to one decimal place and mark the point on the graph.

$f\left(\frac{\pi}{12}\right) = -2\sin\left[\cancel{3}^{1}\left(\frac{\pi}{\cancel{12}_{4}}\right)\right] - 1$

$f\left(\frac{\pi}{12}\right) = -2\underbrace{\sin\frac{\pi}{4}} - 1$

$f\left(\frac{\pi}{12}\right) = -2\left(\frac{1}{\sqrt{2}}\right) - 1$

$f\left(\frac{\pi}{12}\right) = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - 1$

$f\left(\frac{\pi}{12}\right) = \frac{-2\sqrt{2} - 2}{2}$

$f\left(\frac{\pi}{12}\right) = -\sqrt{2} - 1$

$f\left(\frac{\pi}{12}\right) \doteq -2.4$

c) Find the value(s) of x on the interval $0 \le x \le \frac{2\pi}{3}$ for $f(x) = 0$. Mark these points on the graph.

$0 = -2\sin 3x - 1$

$2\sin 3x = -1$

$\sin 3x = -\frac{1}{2}$

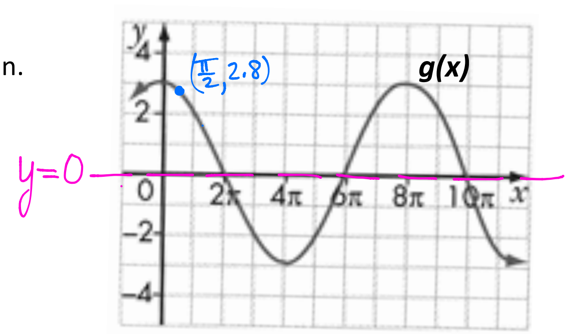
Let $3x = \theta$

$\sin\theta = -\frac{1}{2}$

raa $= \frac{\pi}{6}$

In QIII:	In QIV:
$\theta = \pi + \text{raa}$	$\theta = 2\pi - \text{raa}$
$\theta = \pi + \frac{\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6}$
$\theta = \frac{7\pi}{6}$	$\theta = \frac{11\pi}{6}$
$\frac{1}{3}(3x) = \left(\frac{7\pi}{6}\right)\frac{1}{3}$	$\frac{1}{3}(3x) = \left(\frac{11\pi}{6}\right)\frac{1}{3}$
$x = \frac{7\pi}{18}$	$x = \frac{11\pi}{18}$

2. Given the graph of *g(x):*
   a) Determine an equation for the sine function and the cosine function.



$\boxed{P = 8\pi \quad k = \frac{2\pi}{8\pi} = \frac{1}{4}}$

$c = 0$

$A = 3 \rightarrow a = \pm 3$

$g(x) = -3\sin\left[\frac{1}{4}(x - 2\pi)\right]$

or

$g(x) = 3\cos\left(\frac{1}{4}x\right)$

b) Use an equation to evaluate $g\left(\frac{\pi}{2}\right)$ to one decimal place and mark the point on the graph.

$g\left(\frac{\pi}{2}\right) = 3\cos\left[\left(\frac{1}{4}\right)\left(\frac{\pi}{2}\right)\right]$

$= 3\cos\left(\frac{\pi}{8}\right)$

$\therefore g\left(\frac{\pi}{2}\right) \doteq 2.8$