Trig Identities – Part I

An IDENTITY is an equation that is true for all values of the variable for which the expressions on both sides of the equation are defined.

Recall: $\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

I. Simplify: $\dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\left(\frac{y}{r}\right)}{\left(\frac{x}{r}\right)}$$

$$= \frac{y}{\cancel{r}} \times \frac{\cancel{r}}{x}$$

$$= \frac{y}{x} \rightarrow = \boxed{\tan\theta}\ !$$

The Quotient Identity

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

squared version

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

II. From the Pythagorean Theorem:

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

The Pythagorean Identity

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

Rearranged & factored versions

$$\cos^2\theta = 1 - \sin^2\theta$$
$$= (1+\sin\theta)(1-\sin\theta)$$

$$\sin^2\theta = 1 - \cos^2\theta$$
$$= (1+\cos\theta)(1-\cos\theta)$$

III. Prove each of the following identities. Start with the more "complex" side and transform it algebraically into the exact form of the "simpler" side.

a. Prove: $1-\sin^2 x = \frac{\sin^2 x}{\tan^2 x}$

$RS = \frac{\sin^2 x}{\tan^2 x}$

Quotient Identity

$= \frac{\sin^2 x}{\left(\frac{\sin^2 x}{\cos^2 x}\right)}$

K.C.I.

$= \cancel{\sin^2 x} \cdot \frac{\cos^2 x}{\cancel{\sin^2 x}}$

Reduce

$= \cos^2 x$

$= 1 - \sin^2 x$ Pythagorean Identity

$\because LS = RS$

$\therefore$ Q.E.D.

b. Prove: $\frac{\sin^2 x}{1-\cos x} = 1+\cos x$

$LS = \frac{\sin^2 x}{1-\cos x}$

Pythagorean Identity

$= \frac{1-\cos^2 x}{1-\cos x}$

Factor using difference of squares

$= \frac{(1+\cos x)\cancel{(1-\cos x)}}{\cancel{(1-\cos x)}}$

Reduce

$= 1 + \cos x$

$\because LS = RS$

$\therefore$ Q.E.D.

c. Prove: $\cos x = \frac{1}{\cos x} - \sin x \tan x$

$$RS = \frac{1}{\cos x} - \sin x \tan x$$

$$= \frac{1}{\cos x} - \sin x \left(\frac{\sin x}{\cos x}\right)$$ Quotient Identity

$$= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$$

$$= \frac{1 - \sin^2 x}{\cos x}$$

Pythagorean Identity

$$= \frac{\cos^2 x}{\cos x}$$

$$= \frac{\cos x \cancel{\cos x}^{1}}{\cancel{\cos x}_{1}}$$ Reduce

$$= \cos x$$

$\therefore LS = RS$

$\therefore$ Q.E.D.

d. Prove: $\frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$

$$RS = \frac{\tan x}{1 + \tan x}$$ Quotient Identity

$$= \frac{\left(\frac{\sin x}{\cos x}\right)}{\frac{\cos x}{\cos x} \cdot \frac{1}{1} + \left(\frac{\sin x}{\cos x}\right)}$$ Find LCD

$$= \frac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{\cos x + \sin x}{\cos x}\right)}$$ K.C.I.

$$= \frac{\sin x}{\cancel{\cos x}_{1}} \cdot \frac{\cancel{\cos x}^{1}}{\cos x + \sin x}$$

$$= \frac{\sin x}{\cos x + \sin x}$$

$\therefore LS = RS$

$\therefore$ Q.E.D.

e. Prove: $\cos^2 x = \sin^2 x + 2\cos^2 x - 1$

$RS = \sin^2 x + 2\cos^2 x - 1$

$= (1 - \cos^2 x) + 2\cos^2 x - 1$ Pythagorean Identity

$= +1 - \cos^2 x + 2\cos^2 x - 1$ Gather like terms

$= \cos^2 x$

$\therefore LS = RS$

$\therefore$ Q.E.D.

f. Prove: $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

$LS = \dfrac{1+\tan^2 x}{1-\tan^2 x}$

$= \dfrac{\frac{\cos^2 x}{\cos^2 x}\cdot\frac{1}{1} + \left(\frac{\sin^2 x}{\cos^2 x}\right)}{\frac{\cos^2 x}{\cos^2 x}\cdot\frac{1}{1} - \left(\frac{\sin^2 x}{\cos^2 x}\right)}$ Quotient Identity

LCD

$= \dfrac{\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)}{\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x}\right)}$ K.C.I.

$= \dfrac{\cos^2 x + \sin^2 x}{\cancel{\cos^2 x}_{1}} \cdot \dfrac{\cancel{\cos^2 x}^{1}}{\cos^2 x - \sin^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \dfrac{1}{\cos^2 x - \sin^2 x}$ Pythagorean Identity

$\therefore LS = RS$

$\therefore$ Q.E.D.

g. Prove: $\dfrac{\sin x - 1}{\sin x + 1} = \dfrac{-\cos^2 x}{(\sin x + 1)^2}$

$RS = \dfrac{-\cos^2 x}{(\sin x + 1)^2}$

$= \dfrac{-(1 - \sin^2 x)}{(\sin x + 1)^2}$ Pythagorean Identity

$= \dfrac{\sin^2 x - 1}{(\sin x + 1)^2}$

$= \dfrac{\cancel{(\sin x + 1)}^{1}(\sin x - 1)}{\cancel{(\sin x + 1)}^{1}(\sin x + 1)}$ Factor & reduce

$= \dfrac{\sin x - 1}{\sin x + 1}$

$\therefore LS = RS$

$\therefore$ Q.E.D.

HW: p. 398-399 # 2acegik, 3bdf, 4acehi

Trig Identities – Part II

I. Reciprocal Trigonometric Ratios and Identities

$cosecant\,\theta = \frac{r}{y}$ $secant\,\theta = \frac{r}{x}$ $cotangent\,\theta = \frac{x}{y}$

$csc\,\theta = \frac{r}{y}$ $sec\,\theta = \frac{r}{x}$ $cot\,\theta = \frac{x}{y}$

$csc\,\theta = \frac{1}{sin\theta}$ $sec\,\theta = \frac{1}{cos\theta}$ $cot\,\theta = \frac{1}{tan\theta}$

Reciprocal Identities	Quotient Identities	Pythagorean Identities
$csc\theta = \frac{1}{sin\theta}$ $sec\theta = \frac{1}{cos\theta}$ $cot\theta = \frac{1}{tan\theta}$	$tan\theta = \frac{sin\theta}{cos\theta}$ $cot\theta = \frac{cos\theta}{sin\theta}$	$sin^2\theta + cos^2\theta = 1$ $sin^2\theta = 1 - cos^2\theta$ $cos^2\theta = 1 - sin^2\theta$

II. Prove each identity:

a. $csc\theta - \frac{cot\theta}{sec\theta} = sin\theta$

$$LS = \frac{1}{\sin\theta} - \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{1}{\cos\theta}\right)}$$ K.C.I.

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{1}$$

$$= \frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{1-\cos^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta}$$ Pythagorean Identity

$$= \frac{\cancel{\sin\theta}\,\sin\theta}{\cancel{\sin\theta}}$$ Reduce

$$= \sin\theta$$

$\therefore LS = RS \quad \therefore$ Q.E.D.

b. $tan\theta + cot\theta = sec\theta csc\theta$

$LS = \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta}\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\frac{\cos\theta}{\cos\theta}$

Find LCD

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

Pythagorean Identity

$= \frac{1}{\sin\theta\cos\theta}$

$RS = \sec\theta\csc\theta$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta}$

$\therefore LS = RS$

$\therefore$ Q.E.D.

III. Prove the following identities for homework: #1 – 4 (even parts)

(A)

1. Prove each identity.

a) $\tan\theta\cos\theta = \sin\theta$

b) $\cot\theta\sec\theta = \csc\theta$

c) $\sin\theta\cot\theta = \cos\theta$

d) $\tan\theta\csc\theta = \sec\theta$

e) $\sin\theta = \frac{\tan\theta}{\sec\theta}$

f) $\frac{\cot\theta}{\csc\theta} = \cos\theta$

2. Prove each identity.

a) $\csc\theta(1 + \sin\theta) = 1 + \csc\theta$

b) $\sin\theta(1 + \csc\theta) = 1 + \sin\theta$

c) $\cos\theta(\sec\theta - 1) = 1 - \cos\theta$

d) $\sin\theta\sec\theta\cot\theta = 1$

e) $\frac{1 - \tan\theta}{1 - \cot\theta} = -\tan\theta$

f) $\cot\theta = \frac{1 + \cot\theta}{1 + \tan\theta}$

(B)

3. Prove each identity.

a) $\sin\theta\tan\theta + \sec\theta = \frac{\sin^2\theta + 1}{\cos\theta}$

b) $\frac{1 + \cos\theta}{1 - \cos\theta} = \frac{1 + \sec\theta}{\sec\theta - 1}$

c) $\frac{1 + \sin\theta}{1 - \sin\theta} = \frac{\csc\theta + 1}{\csc\theta - 1}$

d) $\frac{1 + \tan\theta}{1 + \cot\theta} = \frac{1 - \tan\theta}{\cot\theta - 1}$

e) $\frac{1 + \sin\theta}{1 + \csc\theta} = \sin\theta$

f) $\frac{\sin\theta + \tan\theta}{\cos\theta + 1} = \tan\theta$

4. Prove each identity.

a) $\sin^2\theta\cot^2\theta = 1 - \sin^2\theta$

b) $\csc^2\theta - 1 = \csc^2\theta\cos^2\theta$

c) $\sin^2\theta = \frac{\tan^2\theta}{1 + \tan^2\theta}$

d) $\frac{\sin\theta + \cos\theta\cot\theta}{\cot\theta} = \sec\theta$

e) $\sin\theta\cos\theta\tan\theta = 1 - \cos^2\theta$

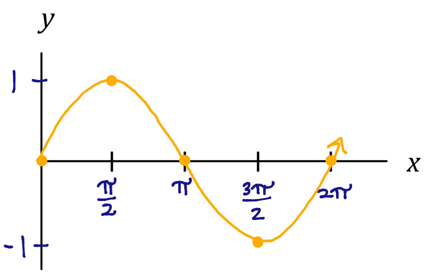
f) $\frac{\cos\theta}{1 + \sin\theta} + \frac{\cos\theta}{1 - \sin\theta} = 2\sec\theta$

Solving Trigonometric Equations I: Linear Trigonometric Equations

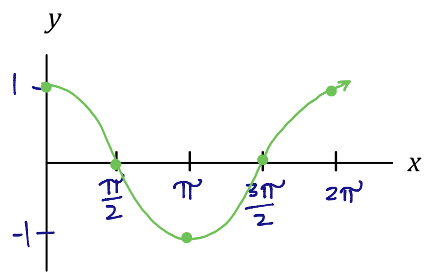
Part A: Using Graphs to Solve Trigonometric Equations

Preview: Graph the primary trigonometric functions on the grids provided for $0 \le x \le 2\pi$.

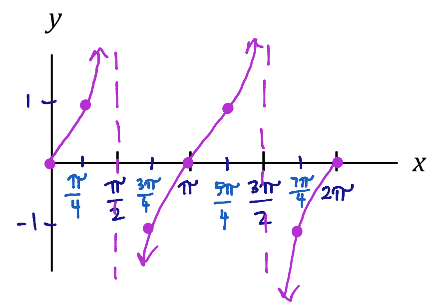
$y = \sin x$



$y = \cos x$



$y = \tan x$



1. Use the graphs to solve for x, $0 \le x \le 2\pi$.

   a) $\sin x = 0$

   $x = 0, \pi, 2\pi$

   b) $\cos x = -1$

   $x = \pi$

   c) $\tan x = 1$

   $x = \frac{\pi}{4}, \frac{5\pi}{4}$

   d) $\sin x = 2$

   no solution

2. Use the graphs to solve for x, $0° \le x \le 360°$.

   a) $\cos x = 0$

   $x = 90°, 270°$

   b) $\sin x = 1$

   $x = 90°$

   c) $\tan x = 0$

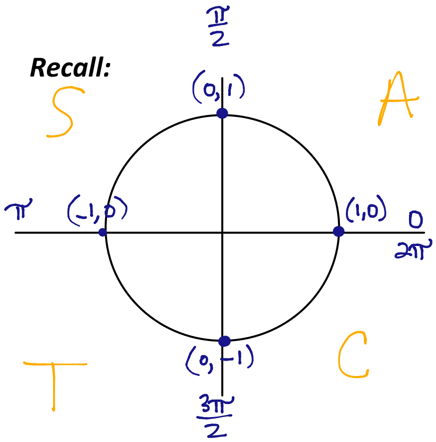
   $x = 0°, 180°, 360°$

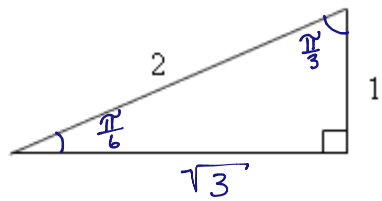
   d) $\cos x = -1.5$

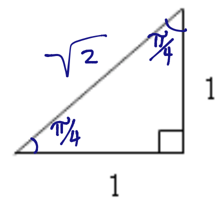
   no solution

Part B: Using Special Angles to Solve Trigonometric Equations

Recall:







1. Solve each equation for x, $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $\sqrt{2}\cos x - 1 = 0$

$\sqrt{2}\cos x = 1$

$\cos x = \frac{1}{\sqrt{2}}$

raa $= \frac{\pi}{4}$, QI & QIV

QI: $x = $ raa $= \frac{\pi}{4}$

QIV: $x = 2\pi - $ raa $= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$\boxed{\therefore x = \frac{\pi}{4}, \frac{7\pi}{4}}$

b) $2\sin x + \sqrt{3} = 0$

$2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$

raa $= \frac{\pi}{3}$, QIII & QIV

QIII: $x = \pi + $ raa $= \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

QIV: $x = 2\pi - $ raa $= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\boxed{\therefore x = \frac{4\pi}{3}, \frac{5\pi}{3}}$

c) $10\tan x + 3 = \tan x$

$10\tan x - \tan x = -3$

$9\tan x = -3$

$\tan x = -\frac{1}{3}$

raa $= \tan^{-1}(\frac{1}{3})$

raa $\doteq 0.32$, QII & QIV

QII: $x = \pi - $ raa $= \pi - 0.32 = 2.82$

QIV: $x = 2\pi - $ raa $= 2\pi - 0.32 = 5.96$

$\boxed{\therefore x = 2.82, 5.96}$

2. Solve each equation for x, $0° \le x \le 360°$. Round approximate solutions to the nearest tenth of a degree.

a) $\sqrt{3}\tan x - 1 = 0$

$\sqrt{3}\tan x = 1$

$\tan x = \frac{1}{\sqrt{3}}$

raa $= 30°$, QI & QIII

QI: $x = $ raa $= 30°$

QIII: $x = 180° + $ raa $= 180° + 30° = 210°$

$\boxed{\therefore x = 30°, 210°}$

b) $5\cos x + 1 = 3\cos x$

$5\cos x - 3\cos x = -1$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

raa $= 60°$, QII & QIII

QII: $x = 180° - $ raa $= 180° - 60 = 120°$

QIII: $x = 180° + $ raa $= 180° + 60° = 240°$

$\boxed{\therefore x = 120°, 240°}$

c) $\left(\frac{5\sin x}{2} - \frac{1}{3} = \frac{1}{6}\right)\frac{6}{1}$

$\frac{\cancel{6}^{3}}{1}\left(\frac{5\sin x}{\cancel{2}}\right) - \frac{\cancel{6}^{2}}{1}\left(\frac{1}{\cancel{3}}\right) = \frac{\cancel{6}^{1}}{1}\left(\frac{1}{\cancel{6}}\right)$

$15\sin x - 2 = 1$

$15\sin x = 3$

$\sin x = \frac{1}{5}$

raa $= \sin^{-1}(1/5)$

raa $\doteq 11.5°$, QI & QII

QI: $x = $ raa $= 11.5°$

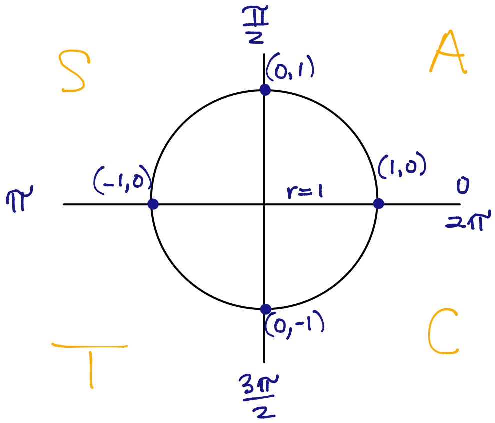
QII: $x = 180° - $ raa $= 180° - 11.5° = 168.5°$

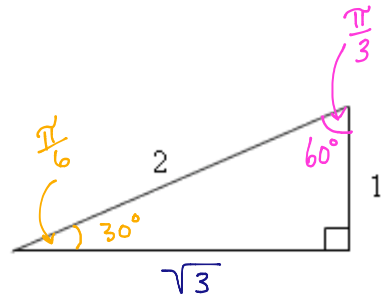
$\boxed{\therefore x = 11.5°, 168.5°}$

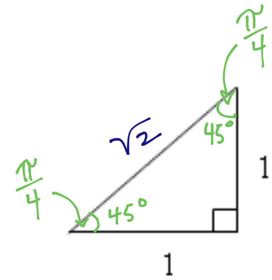
HW: p. 408 # 1, 2, 5ab; p. 417 #37ab

Solving Trigonometric Equations II: Quadratic Trigonometric Equations

Recall (again) ☺:







1. Solve each equation for $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $sin^2x - sin\,x = 0$

$\sin x(\sin x - 1) = 0$ common factor

$\sin x = 0$ $\sin x - 1 = 0$ set each factor equal to zero and solve

$x = 0, \pi, 2\pi$ $\sin x = 1$

$x = \frac{\pi}{2}$

$\boxed{\therefore x = 0, \frac{\pi}{2}, \pi, 2\pi}$

b) $cos^2x + 3cos\,x + 2 = 0$

$(\cos x + 1)(\cos x + 2) = 0$ trinomial factor

$\cos x + 1 = 0$ $\cos x + 2 = 0$ set each factor equal to zero and solve

$\cos x = -1$ $\cos x = -2$

$x = \pi$ no sol'n

$\boxed{\therefore x = \pi}$

c) $4\,sin^2\,x - 3 = 0$

$4\sin^2 x = 3$ rearrange to isolate $\sin^2 x$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\sqrt{\frac{3}{4}}$ square root both sides to isolate $\sin x$

$\sin x = \pm\frac{\sqrt{3}}{2}$

raa $= \frac{\pi}{3}$; QI, II, III, IV

QI: $x = \frac{\pi}{3}$ QII: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ QIII: $x = \pi + \frac{\pi}{3}$, $x = \frac{4\pi}{3}$ QIV: $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\boxed{\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

d) $2\,tan^2x = tan\,x + 1$

$2\tan^2 x - \tan x - 1 = 0$ tricky trinomial factor

$(2\tan x + 1)(\tan x - 1) = 0$

$2\tan x + 1 = 0$ $\tan x - 1 = 0$ set each factor equal to zero and solve

$2\tan x = -1$ $\tan x = 1$

$\tan x = -\frac{1}{2}$ raa $= \frac{\pi}{4}$; QI, III

raa $= \tan^{-1}(\frac{1}{2})$

raa $\doteq 0.46$; QII, IV

QII: $x = \pi - 0.46 = 2.68$ QIV: $x = 2\pi - 0.46 = 5.82$

QI: $x = \frac{\pi}{4}$ QIII: $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\boxed{\therefore x = \frac{\pi}{4}, \frac{5\pi}{4}, 2.68, 5.82}$

2. Solve for x, $0° \leq x \leq 360°$. Round approximate solutions to the nearest tenth of a degree.

a) $\cos x = 2\cos x \sin x$

$0 = 2\cos x \sin x - \cos x$

$0 = \cos x(2\sin x - 1)$

$\cos x = 0$

$x = 90°, 270°$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

raa $= 30°$; QI, II

QI:

$x = 30°$

QII:

$x = 180° - 30°$

$x = 150°$

$$\boxed{\therefore x = 30°, 90°, 150°, 270°}$$

must be the same!

b) $6\cos^2 x - \sin x - 5 = 0$

Use: $\cos^2 x = 1 - \sin^2 x$

$6(1 - \sin^2 x) - \sin x - 5 = 0$

$6 - 6\sin^2 x - \sin x - 5 = 0$

Expand & simplify

$[-6\sin^2 x - \sin x + 1 = 0] \times (-1)$

$6\sin^2 x + \sin x - 1 = 0$

Tricky trinomial factor

$(3\sin x - 1)(2\sin x + 1) = 0$

$3\sin x - 1 = 0$

$3\sin x = 1$

$\sin x = \frac{1}{3}$

raa $= \sin^{-1}(1/3)$

raa $\doteq 19.5°$; QI, II

QI:

$x = 19.5°$

QII:

$x = 180° - 19.5°$

$= 160.5°$

$2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

raa $= 30°$; QIII, IV

QIII:

$x = 180° + 30°$

$= 210°$

QIV:

$x = 360° - 30°$

$= 330°$

$$\boxed{\therefore x = 19.5°, 160.5°, 210°, 330°}$$

HW: p. 408-409 # 3abde, 4abe, 5c, 6ab, 10ac

Solving Trigonometric Equations III

Part A: Solving Trigonometric Equations Involving the Reciprocal Identities

Recall: $csc\,x = \frac{1}{sin\,x}$ $sec\,x = \frac{1}{cos\,x}$ $cot\,x = \frac{1}{tan\,x}$

1. Solve each equation for $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $csc\,x = \frac{2}{\sqrt{3}}$

$\frac{1}{\sin x} = \frac{2}{\sqrt{3}}$

Invert

$\sin x = \frac{\sqrt{3}}{2}$

raa $= \frac{\pi}{3}$; QI, II

$\boxed{\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}}$

b) $3\,cot\,x + 2 = 0$

$3\cot x = -2$

$\cot x = -\frac{2}{3}$

Invert

$\tan x = -\frac{3}{2}$

raa $= \tan^{-1}(3/2)$

raa $\doteq 0.98$; QII, IV

$\boxed{\therefore x = 2.16,\ 5.30}$

c) $4\,sec^2\,x - 8 = 0$

$4\sec^2 x = 8$

$\sec^2 x = \frac{8}{4}$

$\sec^2 x = \frac{2}{1}$

Invert

$\frac{1}{\cos^2 x} = \frac{2}{1}$

$\cos^2 x = \frac{1}{2}$

Square root both sides

$\cos x = \pm\frac{1}{\sqrt{2}}$

raa $= \frac{\pi}{3}$; QI → IV

$\boxed{\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

d) $2\,csc^2 x + csc\,x - 1 = 0$

$(2\csc x - 1)(\csc x + 1) = 0$

$2\csc x - 1 = 0$

$2\csc x = 1$

$\csc x = \frac{1}{2}$

$\frac{1}{\sin x} = \frac{1}{2}$

$\sin x = \frac{2}{1}$

no sol'n

$\csc x + 1 = 0$

$\csc x = -1$

$\frac{1}{\sin x} = \frac{1}{-1}$

$\sin x = -1$

$x = \frac{3\pi}{2}$

$\boxed{\therefore x = \frac{3\pi}{2}}$

Part B: Solving a Variety of Trigonometric Equations

2. Solve each equation for $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $2\,sin^2 x = -\,cos\,x$

$2\sin^2 x + \cos x = 0$

$2(1 - \cos^2 x) + \cos x = 0$

$2 - 2\cos^2 x + \cos x = 0$

$[-2\cos^2 x + \cos x + 2 = 0] \times (-1)$

$2\cos^2 x - \cos x - 2 = 0$

Does not factor ∴ use Q.F.

$a = 2 \quad b = -1 \quad c = -2$

$\cos x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-2)}}{2(2)}$

$\cos x = \frac{1 + \sqrt{17}}{4}$ or $\frac{1 - \sqrt{17}}{4}$

$\cos x \doteq 1.2808$ or $\boxed{-0.7808}$

inadmissable

raa $= \cos^{-1}(0.7808)$

raa $\doteq 0.67$; QII, III

$\boxed{\therefore x = 2.47,\ 3.81}$

b) $\tan^3 x - \tan x = 0$

$\tan x(\tan^2 x - 1) = 0$

$\tan x(\tan x + 1)(\tan x - 1) = 0$

$\tan x = 0$

$x = 0, \pi, 2\pi$

$\tan x = -1$

raa $= \frac{\pi}{4}$; QII, IV

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\tan x = 1$

raa $= \frac{\pi}{4}$; QI, III

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\boxed{\therefore x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi}$

c) $4\sin^4 x + 15\sin^2 x - 4 = 0$

$4\sin^4 x + 16\sin^2 x - \sin^2 x - 4 = 0$

$4\sin^2 x(\sin^2 x + 4) - 1(\sin^2 x + 4) = 0$

$(4\sin^2 x - 1)(\sin^2 x + 4) = 0$

$(2\sin x + 1)(2\sin x - 1)(\sin^2 x + 4) = 0$

$2\sin x + 1 = 0$

$\sin x = \frac{-1}{2}$

raa $= \frac{\pi}{6}$; QIII, IV

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

raa $= \frac{\pi}{6}$; QI, II

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin^2 x + 4 = 0$

$\sin^2 x = -4$

no real sol'n

$\boxed{\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

d) $\underbrace{\sin^3 x + \sin^2 x} + \underbrace{\sin x + 1} = 0$

$\sin^2 x(\sin x + 1) + 1(\sin x + 1) = 0$

$(\sin^2 x + 1)(\sin x + 1) = 0$

$\sin^2 x + 1 = 0$

$\sin^2 x = -1$

no real sol'n

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

$\boxed{\therefore x = \frac{3\pi}{2}}$

e) $\underbrace{4\cot x\cos x - \cot x} - \underbrace{4\cos x + 1} = 0$

$\cot x(4\cos x - 1) - 1(4\cos x - 1) = 0$

$(\cot x - 1)(4\cos x - 1) = 0$

$\cot x - 1 = 0$

$\cot x = 1$

$\tan x = 1$

raa $= \frac{\pi}{4}$; QI, III

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$4\cos x - 1 = 0$

$4\cos x = 1$

$\cos x = \frac{1}{4}$

raa $= \cos^{-1}\left(\frac{1}{4}\right)$

raa $= 1.32$; QI, IV

$x = 1.32, 4.96$

$\boxed{\therefore x = \frac{\pi}{4}, 1.32, \frac{5\pi}{4}, 4.96}$

HW: "Worksheet: Solving a Variety of Trigonometric Equations"