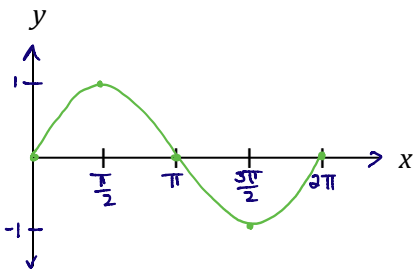
Solving Trigonometric Equations I: Linear Trigonometric Equations

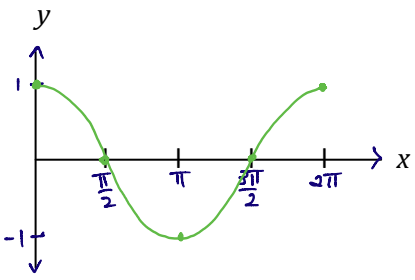
Part A: Using Graphs to Solve Trigonometric Equations

Review: Graph the primary trigonometric functions on the grids provided for $0 \leq x \leq 2\pi$.

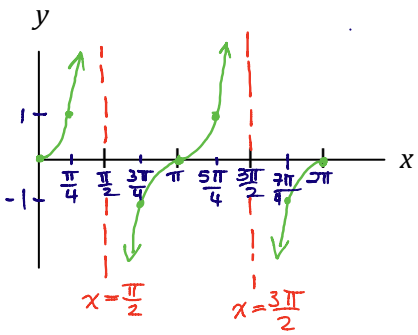
$y = \sin x$



$y = \cos x$



$y = \tan x$



1. Use the graphs to solve for x, $0 \leq x \leq 2\pi$.

a) $\sin x = 0$

$x = 0, \pi, 2\pi$

(from graph)

b) $\cos x = -1$

$x = \pi$

(from graph)

c) $\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

(from graph)

d) $\sin x = 2$

no solution

$\because -1 \leq \sin x \leq 1$

2. Use the graphs to solve for x, $0° \leq x \leq 360°$.

a) $\cos x = 0$

$x = 90°, 270°$

(from graph)

b) $\sin x = 1$

$x = 90°$

(from graph)

c) $\tan x = 0$

$x = 0°, 180°, 360°$

(from graph)

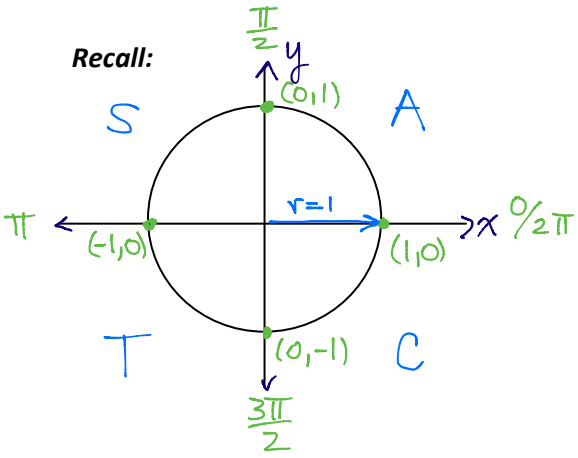
d) $\cos x = -1.5$

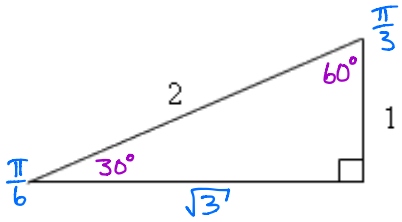
no solution

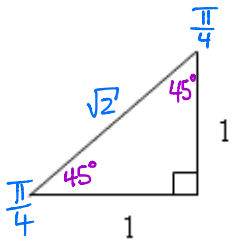
$\because -1 \leq \cos x \leq 1$

Part B: Using Special Angles to Solve Trigonometric Equations

Recall:







1. Solve each equation for $x,\ 0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $\sqrt{2}\cos x - 1 = 0$

$\cos x = \frac{1}{\sqrt{2}}$

$raa = \frac{\pi}{4}$

In QI: $x = raa$

$\therefore x = \frac{\pi}{4}$

In QIV: $x = 2\pi - raa$

$x = 2\pi - \frac{\pi}{4}$

$x = \frac{8\pi}{4} - \frac{\pi}{4}$

$\therefore x = \frac{7\pi}{4}$ ←

b) $2\sin x + \sqrt{3} = 0$

$\sin x = \frac{-\sqrt{3}}{2}$

$raa = \frac{\pi}{3}$

In QIII: $x = \pi + raa$

$x = \pi + \frac{\pi}{3}$

$x = \frac{3\pi}{3} + \frac{\pi}{3}$

$\therefore x = \frac{4\pi}{3}$ ←

In QIV: $x = 2\pi - raa$

$x = 2\pi - \frac{\pi}{3}$

$x = \frac{6\pi}{3} - \frac{\pi}{3}$

$\therefore x = \frac{5\pi}{3}$ ←

c) $10\tan x + 3 = \tan x$

$10\tan x - \tan x = -3$

$9\tan x = -3$

$\tan x = -\frac{1}{3}$

$raa = \tan^{-1}\left(+\frac{1}{3}\right)$

$\doteq 0.32$ rad.

In QII: $x = \pi - raa$

$x = \pi - 0.32$

$x \doteq 2.82$

In QIV: $x = 2\pi - raa$

$x = 2\pi - 0.32$

$x \doteq 5.96$

2. Solve each equation for $x,\ 0° \le x \le 360°$. Round approximate solutions to the nearest tenth of a degree.

a) $\sqrt{3}\tan x - 1 = 0$

$\tan x = \frac{1}{\sqrt{3}}$

$raa = 30°$

In QI: $x = 30°$

In QIII: $x = 180° + raa$

$x = 180° + 30°$

$x = 210°$

∴ the solutions are $x = 30°, 210°$

b) $5\cos x + 1 = 3\cos x$

$5\cos x - 3\cos x = -1$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$raa = 60°$

In QII: $x = 180° - raa$

$x = 180° - 60°$

$x = 120°$

In QIII: $x = 180° + raa$

$x = 180° + 60°$

$x = 240°$

∴ the solutions are $x = 120°, 240°$

c) $\frac{5\sin x}{2} - \frac{1}{3} = \frac{1}{6}$

$\frac{3}{1}\left(\frac{5\sin x}{2_1}\right) - \frac{6^2}{1}\left(\frac{1}{3_1}\right) = \frac{6^1}{1}\left(\frac{1}{6_1}\right)$

$15\sin x - 2 = 1$

$15\sin x = 3$

$\sin x = \frac{1}{5}$

$\therefore raa \doteq 11.5°$

In QI: $x = raa$

$x \doteq 11.5°$

In QII: $x = 180° - raa$

$x = 180° - 11.5°$

$x \doteq 168.5°$

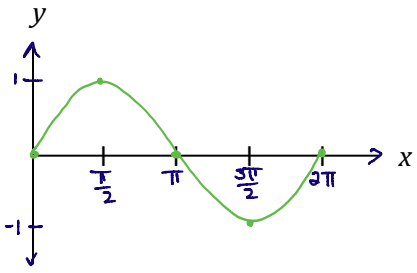
∴ the solutions are $x \doteq 11.5°, 168.5°$

HW: p. 408 # 1, 2, 5ab; p. 417 #37ab

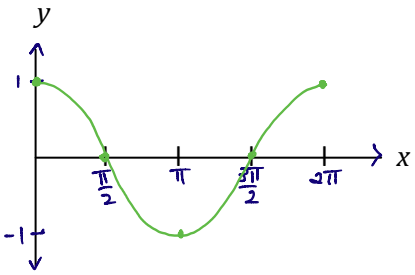
Solving Trigonometric Equations II: Quadratic Trigonometric Equations

Review (again): Graph the primary trigonometric functions on the grids provided for $0 \leq x \leq 2\pi$.

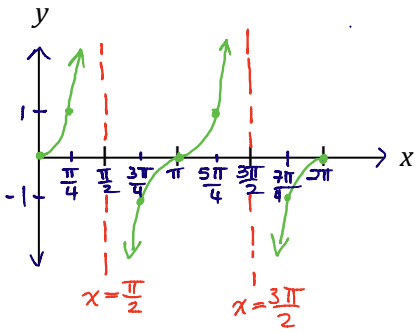
$y = \sin x$



$y = \cos x$



$y = \tan x$



1. Solve each equation for $0 \leq x \leq 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\quad$ $\sin x - 1 = 0$

$x = 0, \pi, 2\pi$ (from graph) $\quad$ $\sin x = 1$

$x = \frac{\pi}{2}$ (from graph)

b) $\cos^2 x + 3\cos x + 2 = 0$ → let $\cos x = a$

$a^2 + 3a + 2 = 0$

$(a+2)(a+1) = 0$

$(\cos x + 2)(\cos x + 1) = 0$

$\cos x + 2 = 0$ $\quad$ $\cos x + 1 = 0$

$\cos x = -2$ $\quad$ $\cos x = -1$

no solution $\quad$ $x = \pi$ (from graph)

$\because -1 \leq \cos x \leq 1$

c) $4\sin^2 x - 3 = 0$

$\sqrt{\sin^2 x} = \pm\sqrt{\frac{3}{4}}$

$\sin x = \pm\sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$\text{raa} = \frac{\pi}{3}$

In QI: $x = \frac{\pi}{3}$ $\quad$ In QII: $x = \frac{2\pi}{3}$

In QIII: $x = \frac{4\pi}{3}$ $\quad$ In QIV: $x = \frac{5\pi}{3}$

d) $2\tan^2 x = \tan x + 1$

$2\tan^2 x - \tan x - 1 = 0$

$(2\tan x + 1)(\tan x - 1) = 0$

$2\tan x + 1 = 0$ $\quad$ $\tan x - 1 = 0$

$\tan x = -\frac{1}{2}$ $\quad$ $\tan x = 1$

$\text{raa} = \tan^{-1}\left(+\frac{1}{2}\right)$ $\quad$ $\text{raa} = \frac{\pi}{4}$

$\doteq 0.46$

In QI: $x = \frac{\pi}{4}$ $\quad$ In QIII: $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

In QII: $x = \pi - 0.46 \doteq 2.68$ $\quad$ In QIV: $x = 2\pi - 0.46 \doteq 5.82$

2. Solve for x, $0° \leq x \leq 360°$. Round approximate solutions to the nearest tenth of a degree.

a) $\cos x = 2\cos x \sin x$

$0 = 2\cos x \sin x - \cos x$

$0 = \cos x(2\sin x - 1)$

$\cos x = 0$

$x = 90°, 270°$

(graph)

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

raa $= 30°$

QI: $x = 30°$

QII: $x = 180° - 30° = 150°$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

b) $6\cos^2 x - \sin x - 5 = 0$

$6(1 - \sin^2 x) - \sin x - 5 = 0$

$6 - 6\sin^2 x - \sin x - 5 = 0$

$\frac{-6\sin^2 x}{-1} - \frac{\sin x}{-1} + \frac{1}{-1} = \frac{0}{-1}$

$6\sin^2 x + \sin x - 1 = 0$

$(3\sin x - 1)(2\sin x + 1) = 0$

$3\sin x - 1 = 0$ or $2\sin x + 1 = 0$

$\sin x = \frac{1}{3}$ $\sin x = -\frac{1}{2}$

raa $= \sin^{-1}(\frac{1}{3}) \doteq 19.5°$

raa $= 30°$

In QI: $x = 19.5°$

In QII: $x = 180° - 19.5° = 160.5°$

In QIII: $x = 180° + 30° = 210°$

In QIV: $x = 360° - 30° = 330°$

HW: p. 408-409 # 3abde, 4abe, 5c, 6ab, 10ac

Solving Trigonometric Equations III

Part A: Solving Trigonometric Equations Involving the Reciprocal Identities

Recall: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

$\sin x = \frac{1}{\csc x}$ $\cos x = \frac{1}{\sec x}$ $\tan x = \frac{1}{\cot x}$

1. Solve each equation for $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $\csc x = \frac{2}{\sqrt{3}}$

$\frac{1}{\sin x} = \frac{2}{\sqrt{3}}$

$\frac{1}{\csc x} = \frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2}$

$raa = \frac{\pi}{3}$

In QI: $x = \frac{\pi}{3}$ In QII: $x = \frac{2\pi}{3}$

b) $3\cot x + 2 = 0$

$3\cot x = -2$

$\cot x = -\frac{2}{3}$

$\frac{1}{\tan x} = -\frac{2}{3}$

$\tan x = -\frac{3}{2}$

$raa = \tan^{-1}(+\frac{3}{2})$

$\doteq 0.98$ rad

In QII: $x \doteq 2.16$ In QIV: $x \doteq 5.30$

c) $4\sec^2 x - 8 = 0$

$\sec^2 x = \frac{8}{4}$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$raa = \frac{\pi}{4}$

In QI: $x = \frac{\pi}{4}$ In QII: $x = \frac{3\pi}{4}$

In QIII: $x = \frac{5\pi}{4}$ In QIV: $x = \frac{7\pi}{4}$

d) $2\csc^2 x + \csc x - 1 = 0$

$(2\csc x - 1)(\csc x + 1) = 0$

$2\csc x - 1 = 0$

$\csc x = \frac{1}{2}$

$\sin x = 2$

no solution

$\because -1 \le \sin x \le 1$

$\csc x + 1 = 0$

$\csc x = -1$

$\sin x = -1$

$x = \frac{3\pi}{2}$ (from graph)

Part B: Solving a Variety of Trigonometric Equations

2. Solve each equation for $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

a) $2\sin^2 x = -\cos x$

$2\sin^2 x + \cos x = 0$

$2(1 - \cos^2 x) + \cos x = 0$

$2 - 2\cos^2 x + \cos x = 0$

$-2\cos^2 x + \cos x + 2 = 0$

$2\cos^2 x - \cos x - 2 = 0$

does not factor (use formula)

$\cos x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-2)}}{2(2)}$

$\cos x = \frac{1 + \sqrt{17}}{4}$ or $\cos x = \frac{1 - \sqrt{17}}{4}$

$\cos x \doteq 1.2808$

no solution

$\because -1 \le \cos x \le 1$

$\cos x \doteq -0.7808$

$raa = \cos^{-1}(+0.7808)$

$\doteq 0.67$

In QII: $x \doteq 2.47$ rad In QIII: $x \doteq 3.81$ rad

b) $\tan^3 x - \tan x = 0$

$\tan x(\tan^2 x - 1) = 0$

$\tan x(\tan x - 1)(\tan x + 1) = 0$

$\tan x = 0$ or $\tan x = 1$ or $\tan x = -1$

raa $= \frac{\pi}{4}$ raa $= \frac{\pi}{4}$

$x = 0, \pi, 2\pi$ (graph)

QI: $x = \frac{\pi}{4}$ QIII: $x = \frac{5\pi}{4}$ QII: $x = \frac{3\pi}{4}$ QIV: $x = \frac{7\pi}{4}$

c) $4\sin^4 x + 15\sin^2 x - 4 = 0$

$(4\sin^2 x - 1)(\sin^2 x + 4) = 0$

$(2\sin x - 1)(2\sin x + 1)(\sin^2 x + 4) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$ or $\sin^2 x = -4$

raa $= \frac{\pi}{6}$ no real solutions

QI: $x = \frac{\pi}{6}$ QII: $x = \frac{5\pi}{6}$ QIII: $x = \frac{7\pi}{6}$ QIV: $x = \frac{11\pi}{6}$

d) $\sin^3 x + \sin^2 x + \sin x + 1 = 0$

$\sin^2 x(\sin x + 1) + 1(\sin x + 1) = 0$

$(\sin x + 1)(\sin^2 x + 1) = 0$

$\sin x = -1$ or $\sin^2 x = -1$

no real solutions

$x = \frac{3\pi}{2}$

(graph)

∴ the solution is $x = \frac{3\pi}{2}$

e) $4\cot x \cos x - \cot x - 4\cos x + 1 = 0$

$\cot x(4\cos x - 1) - 1(4\cos x - 1) = 0$

$(4\cos x - 1)(\cot x - 1) = 0$

$\cos x = \frac{1}{4}$ or $\cot x = 1$

raa $\doteq 1.32$ $\frac{1}{\tan x} = \frac{1}{1}$

In QI: $x \doteq 1.32$ In QIV: $x \doteq 4.96$ $\tan x = 1$

raa $= \frac{\pi}{4}$

In QI: $x = \frac{\pi}{4}$ In QIII: $x = \frac{5\pi}{4}$

HW: "Worksheet: Solving a Variety of Trigonometric Equations"

Worksheet: Solving a Variety of Trigonometric Equations

Solve each equation for $0 \leq x \leq 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

1) $3 + 10\ sec\ x - 1 = -18$

2) $3\ csc\ x + 16 = csc\ x$

3) $\frac{-5\ cot\ x}{2} + \frac{7}{3} = -\frac{1}{6}$

4) $3\ cos\ x\ cot^2 x - cos\ x = 0$

5) $3csc^2 x - 5csc\ x - 2 = 0$

6) $8\ sin^2 x - 10\ cos\ x - 11 = 0$

7) $3\ sin^2 x = 1 - sin\ x$

8) $cos^4 x - 5\ cos^2 x + 4 = 0$

9) $6\ tan^3 x + 3\ tan^2 x + 4\ tanx + 2 = 0$

10) $2\ sinx\ tanx - tanx - 2\ sinx + 1 = 0$

Answers

1) $\frac{2\pi}{3}, \frac{4\pi}{3}$

2) $3.27\ ,\ 6.16$

3) $\frac{\pi}{4}, \frac{5\pi}{4}$

4) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

5) $\frac{\pi}{6}, \frac{5\pi}{6}$

6) $\frac{2\pi}{3},\ 2.42\ ,\ 3.86\ ,\ \frac{4\pi}{3}$

7) $0.49\ ,\ 2.69\ ,\ 4.02\ ,\ 5.41$

8) $0\ ,\ \pi\ ,\ 2\pi$

9) $2.68\ ,\ 5.82$

10) $\frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}$

Review: Proving Trig Identities and Solving Trig Equations

1. Prove the following identities:

a) $\dfrac{\sin^2 x - 6\sin x + 9}{\sin^2 x - 9} = \dfrac{\sin x - 3}{\sin x + 3}$

LS $= \dfrac{\sin^2 x - 6\sin x + 9}{\sin^2 x - 9}$

$= \dfrac{(\sin x - 3)(\sin x - 3)}{(\sin x - 3)(\sin x + 3)}$

$= \dfrac{\sin x - 3}{\sin x + 3}$

RS $= \dfrac{\sin x - 3}{\sin x + 3}$

$\therefore$ LS = RS

$\therefore$ QED

b) $(\sin x - \cos x)^2 = 1 - 2\sin x \cos x$

L $= (\sin x - \cos x)^2$

$= (\sin x - \cos x)(\sin x - \cos x)$

$= \sin^2 x - \sin x \cos x - \cos x \sin x + \cos^2 x$

$= \sin^2 x + \cos^2 x - 2\sin x \cos x$

$= 1 - 2\sin x \cos x$

RS $= 1 - 2\sin x \cos x$

$\therefore$ LS = RS

$\therefore$ Q.E.D

2. Solve the following equation for $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

$9\cot^3 x + 6\cot^2 x - 3\cot x - 2 = 0$

$3\cot^2 x(3\cot x + 2) - (3\cot x + 2) = 0$

$(3\cot x + 2)(3\cot^2 x - 1) = 0$

$\cot x = -\frac{2}{3}$

$\tan x = -\frac{3}{2}$

raa $\doteq 0.98$

In QII: $x \doteq 2.16$

In QIV: $x \doteq 5.30$

$\cot^2 x = \frac{1}{3}$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

raa $= \frac{\pi}{3}$

In QI: $x = \frac{\pi}{3}$

In QII: $x = \frac{2\pi}{3}$

In QIII: $x = \frac{4\pi}{3}$

In QIV: $x = \frac{5\pi}{3}$

Textbook Review: p. 419 #11, 12; p. 416-417 #32bceh, 36bdfh; Lesson 7 #3ae, 4a