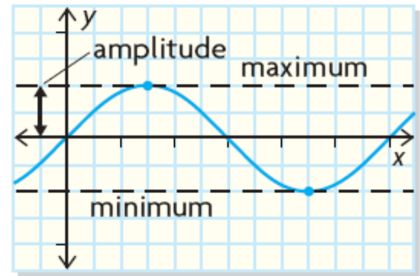
Graphing the Primary Trigonometric Functions

The graphs of the primary trigonometric functions are **periodic**. The sine and cosine functions have a distinct **wavelike** appearance (often referred to as a sinusoidal wave).

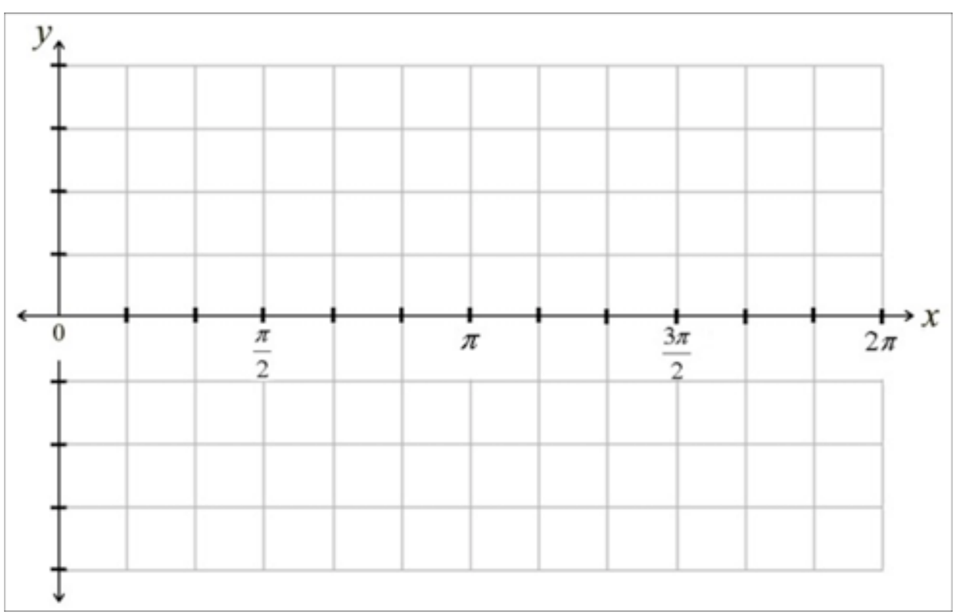
- the **period** is the interval of the independent variable needed for a repeating action to complete *one full cycle* (a cycle can begin at any point on the graph)
- the **equilibrium axis** is the equation of the horizontal line *halfway* between the maximum and the minimum value (calculated by finding $\frac{\max + \min}{2}$)
- the **amplitude** is the *distance* from the function's equilibrium axis to either the maximum or the minimum value (calculated by finding $\frac{\max - \min}{2}$)



A. The Graph of *y* = sin*x*

The **sine function** can be represented by the set of ordered pairs (*x*, sin*x*), where *x* is an angle in standard position measured in degrees or radians and $x \in R$. The equation of the sine function is written in the form *y* = sin*x* or *f(x)* = sin*x*. Graph the equation *y* = sin*x*, where *x* is an angle between 0 and 2π.

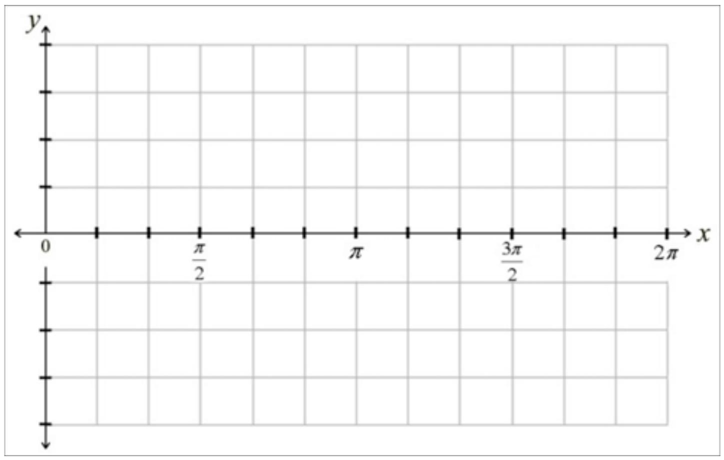
x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Exact value of sin*x*	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$													
Decimal value of sin*x*	0	0.5	0.7	0.9													



B. The Graph of *y* = cos*x*

Graph the equation *y* = cos*x*, where *x* is an angle between 0 and 2π.

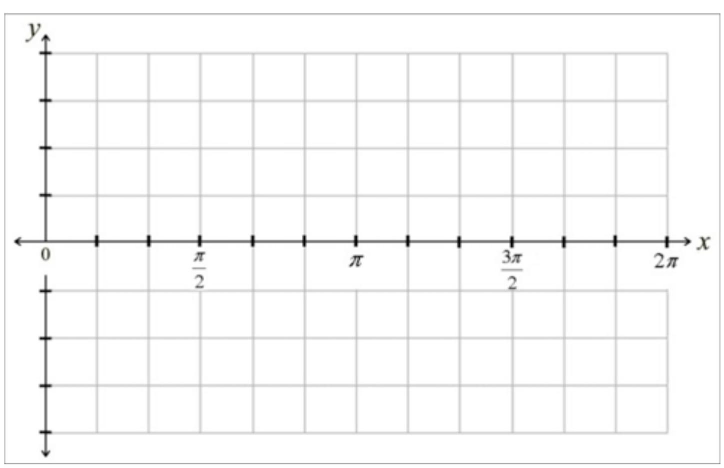
x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Exact value of cos*x*	1																
Decimal value of cos*x*	1																



C. The Graph of *y* = tan*x*

Graph the equation *y* = tan*x*, where *x* is an angle between 0 and 2π.

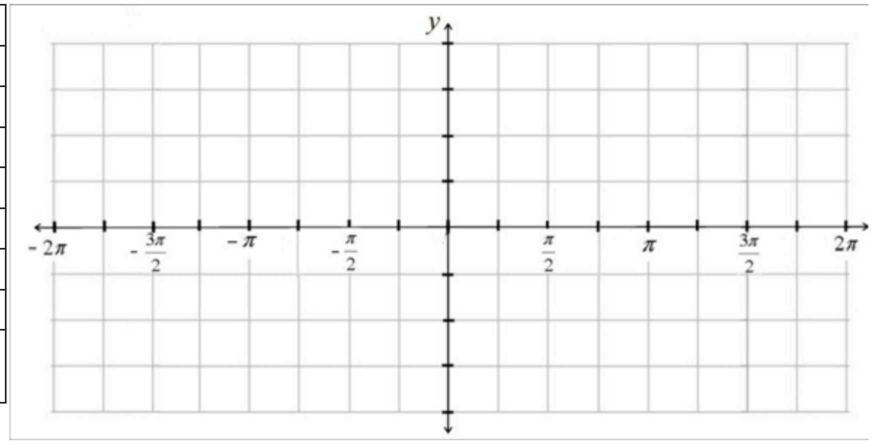
x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Exact value of tan*x*	0																
Decimal value of tan*x*	0																



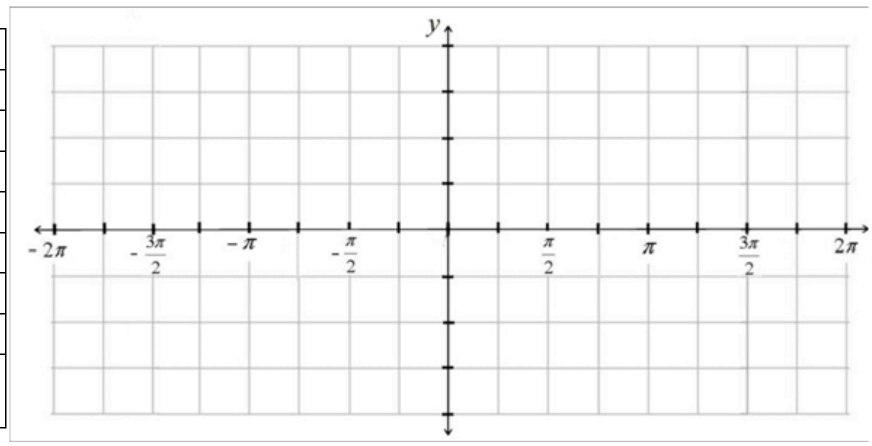
D. Key Features of Sinusoidal Functions

Sketch the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$ and fill in the following tables on the interval $0 \le x \le 2\pi$. For this unit, we will use the y-values of 0, 1 and –1 to plot the key points for each curve.

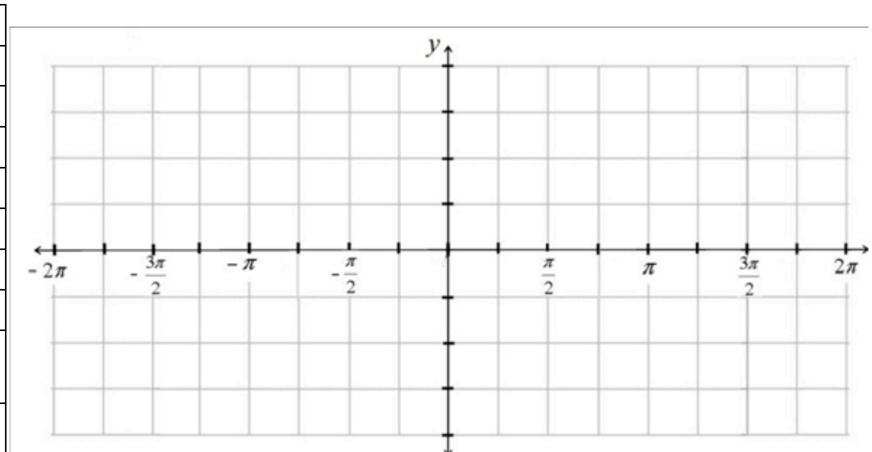
Key Features	***y* = sinx**
Maximum value	
Minimum value	
Amplitude	
Period	
Domain	
Range	
Zeros	
Equation of the equilibrium axis	



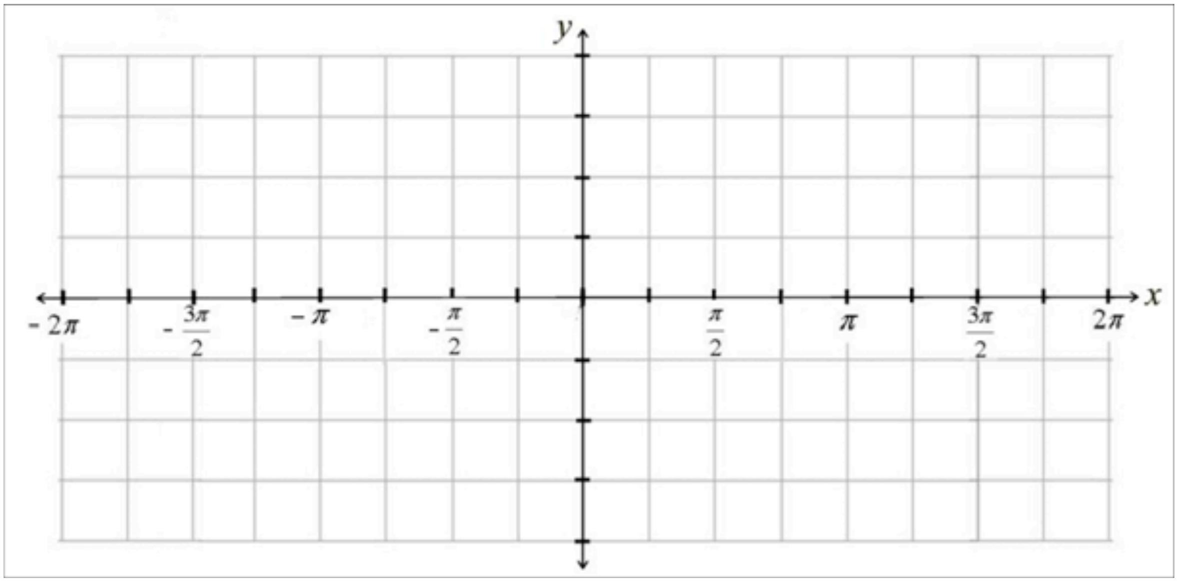
Key Features	***y* = cosx**
Maximum value	
Minimum value	
Amplitude	
Period	
Domain	
Range	
Zeros	
Equation of the equilibrium axis	

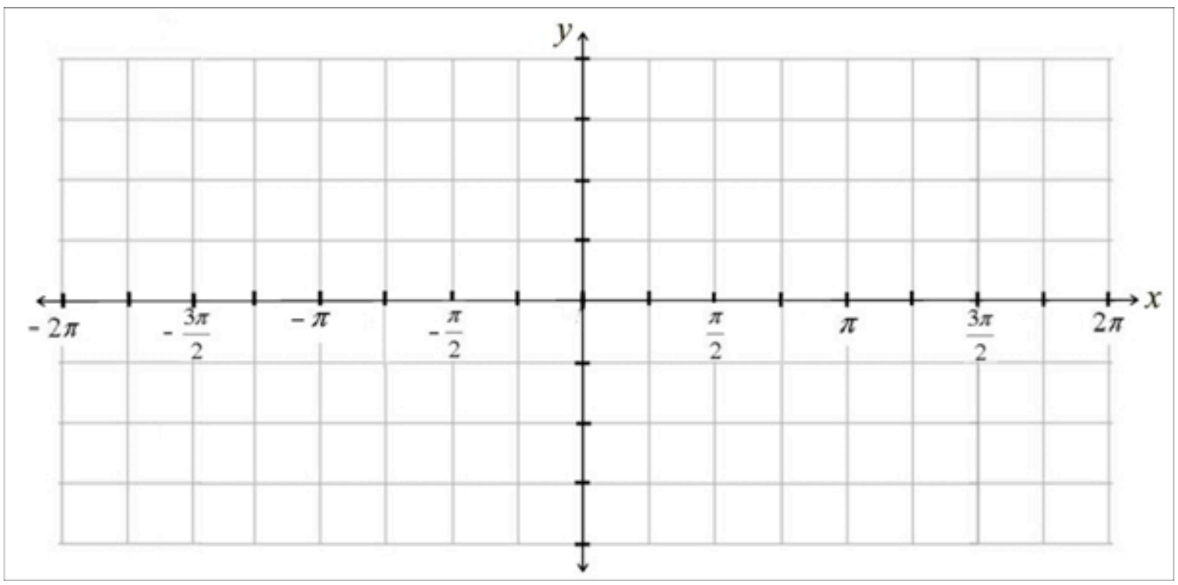


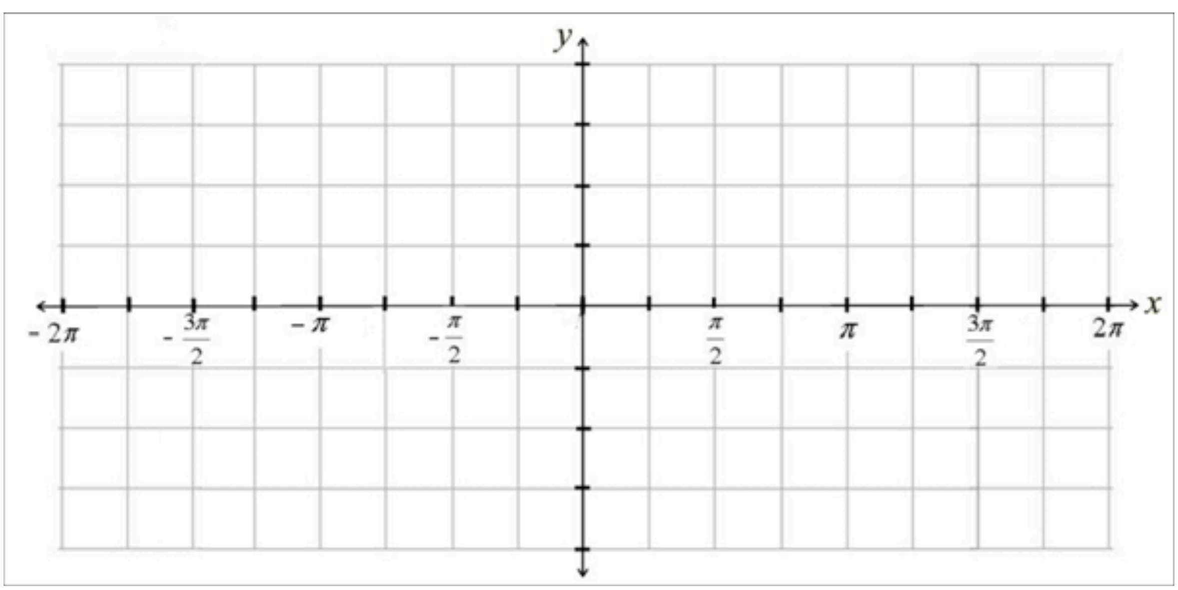
Key Features	***y* = tanx**
Maximum value	
Minimum value	
Amplitude	
Period	
Domain	
Range	
Zeros	
Equation of the equilibrium axis	
Equation of vertical asymptotes	



HW: Use the grids on the following page to sketch $y = \sin x$, $y = \cos x$, *and* $y = \tan x$ on the interval $-2\pi \le x \le 2\pi$. Where tan*x* is undefined, draw and label vertical asymptotes! Memorize these graphs!!


y
x
- 2π
- 3π/2
– π
- π/2
π/2
π
3π/2
2π


y
x
- 2π
- 3π/2
– π
- π/2
π/2
π
3π/2
2π


y
x
- 2π
- 3π/2
– π
- π/2
π/2
π
3π/2
2π

Stretches and Reflections of Periodic Functions

Like other functions, the stretches and reflections of sine and cosine functions can be summarized as follows:

Transformations	**Transformed Function**	**Effect on *y = sin x* or *y* = cos *x***
Vertical Reflection and Vertical Stretch	*y = a sin x* *y = a cos x*	• If $a < 0$, the graph is **vertically reflected** in the ***x*-axis.** • If $\lvert a\rvert > 1$, the graph is **vertically expanded** by a factor of $\lvert a\rvert$. • If $0 < \lvert a\rvert < 1$, the graph is **vertically compressed** by a factor of $\lvert a\rvert$. • The point (x, y) on $y = f(x)$ becomes the point (x, ay) on $y = a f(x)$. • The **AMPLITUDE** of the function is $A = \lvert a\rvert$.
Horizontal Reflection and Horizontal Stretch	*y = sin kx* *y = cos kx*	• If $k < 0$, the graph is **reflected** in the ***y*-axis.** • If $\lvert k\rvert > 1$, the graph is **horizontally compressed** by a factor of $\frac{1}{\lvert k\rvert}$. • If $0 < \lvert k\rvert < 1$, the graph is **horizontally expanded** by a factor of $\frac{1}{\lvert k\rvert}$. • The point (x, y) on $y = f(x)$ becomes the point $(\frac{1}{k}x, y)$ on $y = f(kx)$. • The **PERIOD** of the function is $P = \frac{2\pi}{\lvert k\rvert} = \frac{360°}{\lvert k\rvert}$.

A. Sketch the following using transformations on *y = sin x*:

a) $y = 2 \sin x$ $(x, y) \rightarrow$ (________ , _________) A = ____________ P = ____________

b) $y = -\frac{1}{2} \sin x$ $(x, y) \rightarrow$ (________ , _________) A = ____________ P = ____________

B. Sketch the following using transformations on *y = cos x*:

a) $y = \cos 2x$ $(x, y) \rightarrow$ (________ , ________) A = ____________ P = ______________

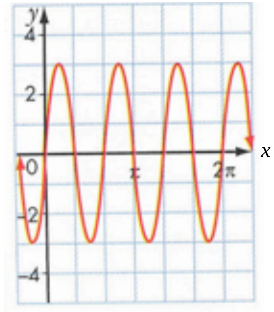
b) $y = \cos \frac{1}{2}x$ $(x, y) \rightarrow$ (________ , ________) A = ____________ P = ______________

How to set the scale for horizontal stretches given *k*:

1) Find the period length, P, using: $P = \frac{2\pi}{\lvert k\rvert}$.
2) Get the number of radians between each key point by calculating $\frac{1}{4} \times P$.
3) Add $\frac{1}{4} \times P$ to the first point in the cycle, and to each subsequent point until the cycle(s) are complete.
4) Label the *x*-axis accordingly.

C. Determine the equations for the following a) *sine* and b) *cosine* functions:

a)

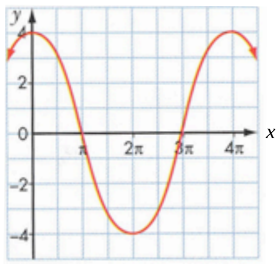


A = ________ P = ___________

k = ____________________________

$\therefore y =$ __________________________

b)



A = ________ P = ___________

k = ____________________________

$\therefore y =$ __________________________

How to find k given the period length, P:

1) Rearrange the equation for period length to isolate $|k|$:

$$P = \frac{2\pi}{|k|}$$

$$|k| = \frac{2\pi}{P}$$

2) Reduce the fraction to lowest terms.

D. Graph each function for one cycle and state the domain and range for your graph.

a) $y = -2 \sin x$

D = ____

R = ____

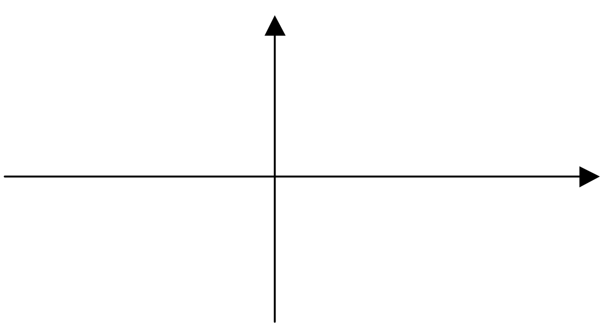
b) $y = \frac{1}{2}\cos 3x$

D = ____

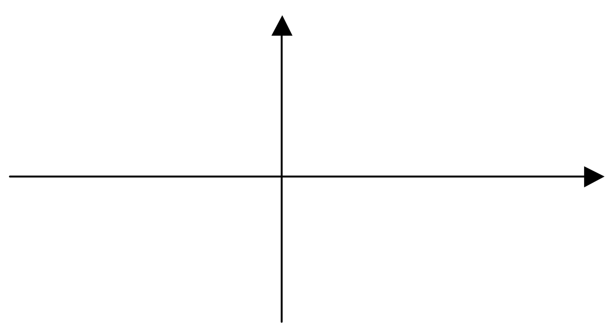
R = ____

E. Sketch the graph of the following functions.

a) $y = \frac{1}{2}\sin \frac{1}{2}x \quad for \ -2\pi \le x \le 2\pi$



b) $y = -3\cos 2x \quad for \ -\pi \le x \le \pi$



HW: p. 374-376 # 2, 4 and 5(equations only, no graphs), 6, 7b, 8a, 9cd, 12 (no graphs)

Translations of Periodic Functions

For the functions $y = \sin(x - d) + c$ and $y = \cos(x - d) + c$, **d** represents a phase shift (or horizontal translation) and **c** represents a vertical translation.

Transformation	**Transformed Function**	**Effect on $y = \sin x$ or $y = \cos x$**
Horizontal Translation	$y = \sin(x - d)$ $y = \cos(x - d)$	If $d > 0$, the graph is horizontally translated **right** $\lvert d \rvert$ units. If $d < 0$, the graph is horizontally translated **left** $\lvert d \rvert$ units. The **PHASE SHIFT** of the function is: P.S. = $\lvert d \rvert$ units right if $d > 0$ ***or*** P.S. = $\lvert d \rvert$ units left if $d < 0$
Vertical Translation	$y = \sin x + c$ $y = \cos x + c$	If $c > 0$, the graph is **vertically translated up** $\lvert c \rvert$ units If $c < 0$, the graph is **vertically translated down** $\lvert c \rvert$ units The **VERTICAL TRANSLATION** of the function is: V.T. = $\lvert c \rvert$ units up if $c > 0$ ***or*** V.T. = $\lvert c \rvert$ units down if $c < 0$ The equation of the **equilibrium axis** is $y = c$.

A. Graph the function $y = \sin x$ for one cycle. Then graph the following using transformations on $y = \sin x$:

$y = \sin x + 1$　　(x, y) → (________ , ________)　　A = _____;　P = _______;　P.S. = ___________ ;　V.T = _________

B. Graph the function $y = \cos x$ for one cycle. Then graph the following using transformations on $y = \cos x$:

$y = \cos\left(x - \frac{\pi}{2}\right)$　　(x, y) → (________ , ________)　　A = _____;　P = _______;　P.S. = ___________ ;　V.T = ________

C. Graph each function for one cycle. State the domain and range of the cycle.

a) $y = -3\sin\left(x + \frac{\pi}{3}\right)$

b) $y = \cos 2\left(x - \frac{\pi}{8}\right) - 1$

D = ____

R = ____

D = ____

R = ____

D. Determine the amplitude, period, phase shift and vertical translation for each function. Find the x-scale.

a) $y = -4\sin 2\left(x - \frac{\pi}{6}\right)$

b) $y = \frac{1}{2}\cos\frac{2}{3}\left(x + \frac{\pi}{4}\right) - \frac{1}{2}$

E. Write an equation for the function in the form *y = a sin k(x – d) + c* or *y = a cos k(x – d) + c*

a) *sine* function: A = 1 ; P = 4π ; P.S. = $\frac{\pi}{2}$ left ; V.T. = none

b) *cosine* function: A = 4 ; P = $\frac{\pi}{2}$; P.S. = none ; V.T. = up 3

HW: p. 387-388 # 1 and 2 (even parts), 3ab, 4ade, 5, 6, 9

Graphing Functions in the Form $y = a \sin k(x - d) + c$ and $y = a \cos k(x - d) + c$

For the sinusoidal functions $y = \sin x$ and $y = \cos x$, the point (x, y) maps onto the point $(\frac{1}{k}x + d, ay + c)$ on the graphs of $y = a \sin k(x - d) + c$ or $y = a \cos k(x - d) + c$.

$$y = a \sin k(x - d) + c$$

a) Describe the transformations that must be applied to $y = \cos x$ to obtain the graph of $g(x) = -3\cos(3x - \pi) + 1$.

1. ____
2. ____
3. ____
4. ____
5. ____

b) State the amplitude and the period of $g(x)$. A = __________ P = _____________

c) Find the x-scale.

d) Sketch a graph of $g(x)$ for two full cycles.

Find P:

$P = \frac{2\pi}{|k|}$

Find the intervals between key points:

$\frac{1}{4} \times P =$

e) State the domain and range of $g(x)$.

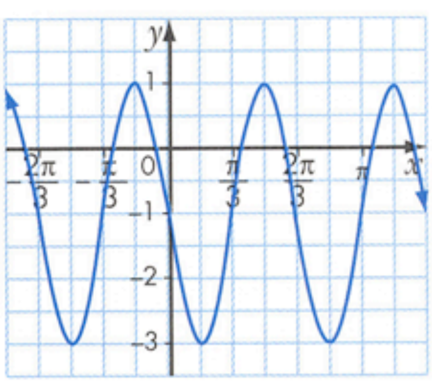
D = ____ R = ____

Sketch the graph of each of the following on the domain specified. State the range.

a. $y = 2\cos\left(2x + \frac{\pi}{3}\right) - 1$ for $-\pi \le x \le \frac{\pi}{2}$ R = ______________________

b. $y = -\frac{1}{2}\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) + \frac{1}{2}$ for $-2\pi \le x \le 4\pi$ R = ______________________

Determine an equation of the form ***y = a sin k(x – d) + c for the sine function graphed below.***

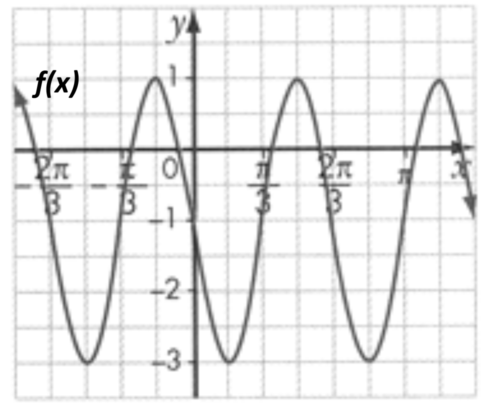


HW: p. 387-388 # 7ade, 8bcd, 10bcdef, 11

Applications of Trigonometric Functions

Part A: Finding Equations Given the Graph

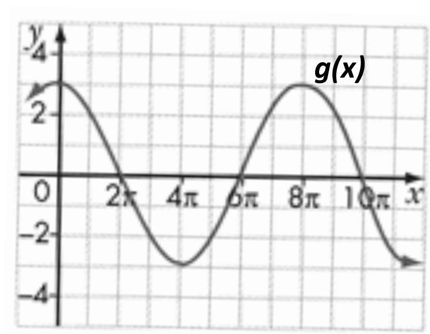
1. Given the graph of *f(x):*
   a) Determine at least two equivalent equations for the function.



b) Use the equation from part a) to evaluate $f\left(\frac{\pi}{12}\right)$ to one decimal place and mark the point on the graph.

c) Find the value(s) of *x* on the interval $0 \le x \le \frac{2\pi}{3}$ for *f*(x) = 0. Mark these points on the graph.

2. Given the graph of *g(x):*
   a) Determine an equation for the sine function and the cosine function.



b) Use an equation to evaluate $g\left(\frac{\pi}{2}\right)$ to one decimal place and mark the point on the graph.

Part B: Solving Ferris Wheel Problems!

3. A carnival Ferris wheel with a radius of 16m rotates once every 48 seconds. Passengers get on at the lowest point, which is 1 m above the ground.

   a) Sketch a graph to show how your height, *h*, above ground (in metres) varies with time, *t,* (in seconds) for two revolutions, starting when the you get onto the Ferris wheel at its lowest point.

   b) Write a cosine equation and a sine equation in function notation that model your height above the ground.

   c) Calculate your height above ground at 44 seconds, to one decimal place. Mark this point on your graph.

HW p. 375 #7cd, 8cd; p. 416 #30[express as a sine function; alternative answer is $y = -9.5\cos\frac{\pi}{5}t + 10.7$],

#31[label your horizontal axis "Time after 3:00 (hours)"; express as a sine function; alternative answer is $d(t) = 6\cos\frac{\pi}{6}t + 14$]

Review: Graphing Trigonometric Functions and Applications

The average depth, *d* in metres, of the water on a port in a tidal river is 4 m. At low tide, the depth of the water is 2 m. One cycle is completed approximately every 12 hours.

a) Find an equation of the depth, *d,* in metres, as a function of the time, *t* hours, after high tide. Give your answer as a sine function and a cosine function.

b) Draw a graph of the function for 24 hours after high tide, which occurred at 9:00.

c) Determine the depth of the water at 10:30. Mark and label this point on your graph.

Textbook Review: p. 418 #6, 7ce, 8bc; p. 388 #12 *(alternative answer for a)* $y = -5\cos\frac{\pi}{6}t + 16$*)*;

p. 389 #14 *(alternative answer for a) with respect to bottom:* $y = 1.5\cos\frac{\pi}{6}t + 4.5$ *or* $y = -1.5\sin\frac{\pi}{6}(t-3) + 4.5$*)*;

p. 390 #16 *(alternative answer for b)* $y = -7\cos\frac{\pi}{8}t + 8.5$*)*.

Trig Identities – Part I

An IDENTITY is an equation that is true for <u>all values</u> of the variable for which the expressions on both sides of the equation are defined.

I. Simplify: $\dfrac{\sin\theta}{\cos\theta}$

The Quotient Identity

II. From the Pythagorean Theorem:

$x^2 + y^2 = r^2$

The Pythagorean Identity

Recall:

III. Prove each of the following identities. Start with the more "complex" side and transform it algebraically into the exact form of the "simpler" side.

a. Prove: $1-\sin^2 x = \dfrac{\sin^2 x}{\tan^2 x}$

b. Prove: $\dfrac{\sin^2 x}{1-\cos x} = 1+\cos x$

c. Prove: $\cos x = \dfrac{1}{\cos x} - \sin x \tan x$

d. Prove: $\dfrac{\sin x}{\sin x + \cos x} = \dfrac{\tan x}{1 + \tan x}$

Recall:

e. Prove: $\cos^2 x = \sin^2 x + 2\cos^2 x - 1$

f. Prove: $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

g. Prove: $\dfrac{\sin x - 1}{\sin x + 1} = \dfrac{-\cos^2 x}{(\sin x + 1)^2}$

HW: p. 398-399 # 2acegik, 3bdf, 4acehi

Trig Identities – Part II

I. Reciprocal Trigonometric Ratios and Identities

$cosecant\,\theta = \frac{r}{y}$ $secant\,\theta = \frac{r}{x}$ $cotangent\,\theta = \frac{x}{y}$

$csc\,\theta = \frac{r}{y}$ $sec\,\theta = \frac{r}{x}$ $cot\,\theta = \frac{x}{y}$

$csc\,\theta = \frac{1}{sin\theta}$ $sec\,\theta = \frac{1}{cos\theta}$ $cot\,\theta = \frac{1}{tan\theta}$

Reciprocal Identities	Quotient Identities	Pythagorean Identities
$csc\theta = \frac{1}{sin\theta}$ $sec\theta = \frac{1}{cos\theta}$ $cot\theta = \frac{1}{tan\theta}$	$tan\theta = \frac{sin\theta}{cos\theta}$ $cot\theta = \frac{cos\theta}{sin\theta}$	$sin^2\theta + cos^2\theta = 1$ $sin^2\theta = 1 - cos^2\theta$ $cos^2\theta = 1 - sin^2\theta$

II. Prove each identity:

a. $csc\theta - \frac{cot\theta}{sec\theta} = sin\theta$

Recall:

b. $tan\theta + cot\theta = sec\theta csc\theta$

III. Prove the following identities for homework: #1 – 4 (bdf for all)

(A)

1. Prove each identity.
   a) $\tan\theta\cos\theta = \sin\theta$
   b) $\cot\theta\sec\theta = \csc\theta$
   c) $\sin\theta\cot\theta = \cos\theta$
   d) $\tan\theta\csc\theta = \sec\theta$
   e) $\sin\theta = \dfrac{\tan\theta}{\sec\theta}$
   f) $\dfrac{\cot\theta}{\csc\theta} = \cos\theta$

2. Prove each identity.
   a) $\csc\theta(1 + \sin\theta) = 1 + \csc\theta$
   b) $\sin\theta(1 + \csc\theta) = 1 + \sin\theta$
   c) $\cos\theta(\sec\theta - 1) = 1 - \cos\theta$
   d) $\sin\theta\sec\theta\cot\theta = 1$
   e) $\dfrac{1 - \tan\theta}{1 - \cot\theta} = -\tan\theta$
   f) $\cot\theta = \dfrac{1 + \cot\theta}{1 + \tan\theta}$

(B)

3. Prove each identity.
   a) $\sin\theta\tan\theta + \sec\theta = \dfrac{\sin^2\theta + 1}{\cos\theta}$
   b) $\dfrac{1 + \cos\theta}{1 - \cos\theta} = \dfrac{1 + \sec\theta}{\sec\theta - 1}$
   c) $\dfrac{1 + \sin\theta}{1 - \sin\theta} = \dfrac{\csc\theta + 1}{\csc\theta - 1}$
   d) $\dfrac{1 + \tan\theta}{1 + \cot\theta} = \dfrac{1 - \tan\theta}{\cot\theta - 1}$
   e) $\dfrac{1 + \sin\theta}{1 + \csc\theta} = \sin\theta$
   f) $\dfrac{\sin\theta + \tan\theta}{\cos\theta + 1} = \tan\theta$

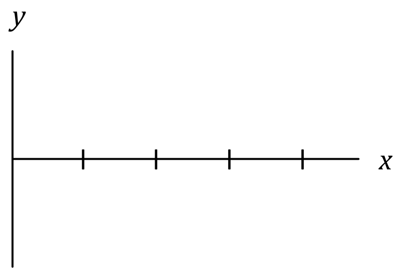
4. Prove each identity.
   a) $\sin^2\theta\cot^2\theta = 1 - \sin^2\theta$
   b) $\csc^2\theta - 1 = \csc^2\theta\cos^2\theta$
   c) $\sin^2\theta = \dfrac{\tan^2\theta}{1 + \tan^2\theta}$
   d) $\dfrac{\sin\theta + \cos\theta\cot\theta}{\cot\theta} = \sec\theta$
   e) $\sin\theta\cos\theta\tan\theta = 1 - \cos^2\theta$
   f) $\dfrac{\cos\theta}{1 + \sin\theta} + \dfrac{\cos\theta}{1 - \sin\theta} = 2\sec\theta$

Solving Trigonometric Equations I: Linear Trigonometric Equations

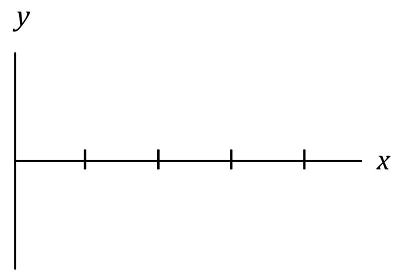
Part A: Using Graphs to Solve Trigonometric Equations

Review: Graph the primary trigonometric functions on the grids provided for $0 \le x \le 2\pi$.

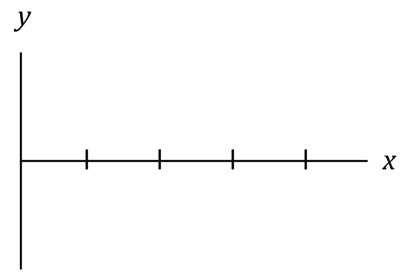
$\boldsymbol{y = \sin x}$



$\boldsymbol{y = \cos x}$



$\boldsymbol{y = \tan x}$



1. Use the graphs to solve for x, $0 \le x \le 2\pi$.

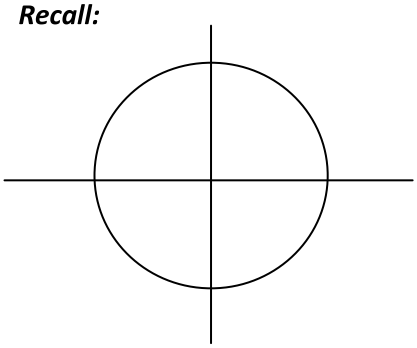
   a) $\sin x = 0$ b) $\cos x = -1$ c) $\tan x = 1$ d) $\sin x = 2$

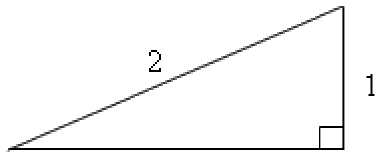
2. Use the graphs to solve for x, $0° \le x \le 360°$.

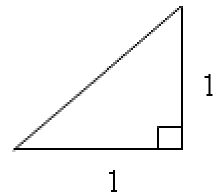
   a) $\cos x = 0$ b) $\sin x = 1$ c) $\tan x = 0$ d) $\cos x = -1.5$

Part B: Using Special Angles to Solve Trigonometric Equations

Recall:







1. Solve each equation for x, $0 \leq x \leq 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

   a) $\sqrt{2}\,cos\,x - 1 = 0$ b) $2\,sin\,x + \sqrt{3} = 0$ c) $10\,tan\,x + 3 = tan\,x$

2. Solve each equation for x, $0° \leq x \leq 360°$. Round approximate solutions to the nearest tenth of a degree.

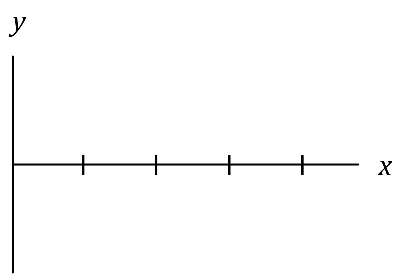
   a) $\sqrt{3}\,tan\,x - 1 = 0$ b) $5\,cos\,x + 1 = 3\,cos\,x$ c) $\frac{5\,sin\,x}{2} - \frac{1}{3} = \frac{1}{6}$

HW: p. 408 # 1, 2, 5ab; p. 417 #37ab

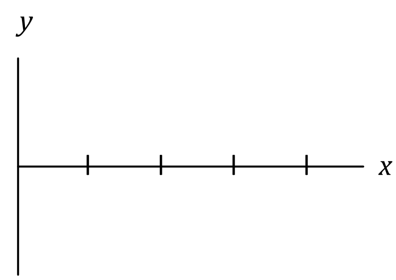
Solving Trigonometric Equations II: Quadratic Trigonometric Equations

Review (again): Graph the primary trigonometric functions on the grids provided for $0 \leq x \leq 2\pi$.

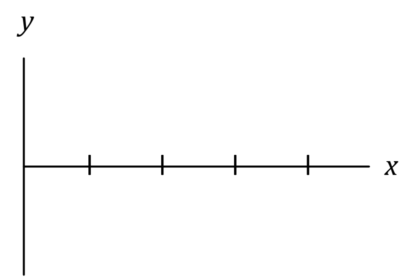
$\boldsymbol{y = sin\, x}$



$\boldsymbol{y = cos\, x}$



$\boldsymbol{y = tan\, x}$



1. Solve each equation for $0 \leq x \leq 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

   a) $sin^2x - sin\, x = 0$

   b) $cos^2x + 3cos\, x + 2 = 0$

   c) $4\, sin^2\, x - 3 = 0$

   d) $2\, tan^2x = tan\, x + 1$

2. Solve for x, $0° \leq x \leq 360°$. Round approximate solutions to the nearest tenth of a degree.

a) $cos\ x = 2\,cos\,x\,sin\,x$

b) $6\,cos^2x - sin\,x - 5 = 0$

HW: p. 408-409 # 3abde, 4abe, 5c, 6ab, 10ac

Solving Trigonometric Equations III

Part A: Solving Trigonometric Equations Involving the Reciprocal Identities

Recall: $csc\, x = \frac{1}{sin\, x}$ $sec\, x = \frac{1}{cos\, x}$ $cot\, x = \frac{1}{tan\, x}$

1. Solve each equation for $0 \leq x \leq 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

   a) $csc\, x = \frac{2}{\sqrt{3}}$

   b) $3\, cot\, x + 2 = 0$

   c) $4\, sec^2\, x - 8 = 0$

   d) $2\, csc^2 x + csc\, x - 1 = 0$

Part B: Solving a Variety of Trigonometric Equations

2. Solve each equation for $0 \leq x \leq 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

   a) $2\, sin^2 x = -cos\, x$

b) $tan^3x - tan\,x = 0$

c) $4\,sin^4x + 15\,sin^2x - 4 = 0$

d) $sin^3x + sin^2x + sin\,x + 1 = 0$

e) $4\,cot\,x\,cos\,x - cot\,x - 4\,cos\,x + 1 = 0$

HW: "Worksheet: Solving a Variety of Trigonometric Equations"

Worksheet: Solving a Variety of Trigonometric Equations

Solve each equation for $0 \leq x \leq 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

1) $3 + 10\, sec\, x - 1 = -18$

2) $3\, csc\, x + 16 = csc\, x$

3) $\frac{-5\, cot\, x}{2} + \frac{7}{3} = -\frac{1}{6}$

4) $3\, cos\, x\, cot^2 x - cos\, x = 0$

5) $3csc^2 x - 5csc\, x - 2 = 0$

6) $8\, sin^2 x - 10\, cos\, x - 11 = 0$

7) $3\, sin^2 x = 1 - sin\, x$

8) $cos^4 x - 5\, cos^2 x + 4 = 0$

9) $6\, tan^3 x + 3\, tan^2 x + 4\, tanx + 2 = 0$

10) $2\, sinx\, tanx - tanx - 2\, sinx + 1 = 0$

Answers

1) $\frac{2\pi}{3}, \frac{4\pi}{3}$

2) $3.27\,,\ 6.16$

3) $\frac{\pi}{4}, \frac{5\pi}{4}$

4) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

5) $\frac{\pi}{6}, \frac{5\pi}{6}$

6) $\frac{2\pi}{3}, 2.42\,, 3.86\,, \frac{4\pi}{3}$

7) $0.49\,, 2.69\,, 4.02\,, 5.41$

8) $0\,, \pi\,, 2\pi$

9) $2.68\,, 5.82$

10) $\frac{\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}$

Review: Proving Trig Identities and Solving Trig Equations

1. Prove the following identities:

   a) $\dfrac{\sin^2 x - 6\sin x + 9}{\sin^2 x - 9} = \dfrac{\sin x - 3}{\sin x + 3}$

   b) $(\sin x - \cos x)^2 = 1 - 2\sin x \cos x$

2. Solve the following equation for $0 \le x \le 2\pi$. Give the exact solution, where possible. Otherwise, round to the nearest hundredth of a radian.

   $9\cot^3 x + 6\cot^2 x - 3\cot x - 2 = 0$

Textbook Review: p. 419 #11, 12; p. 416-417 #32bceh, 36bdfh; Lesson 7 #3ae, 4a