3^{5} 5 ← exponent, 3 ← base, Power

Date: _______________

Reviewing the Exponent Laws

Exponent Laws	
Product Rule $a^m \cdot a^n = a^{m+n}$	$(ab)^m = a^m b^m$
Quotient Rule $a^m \div a^n = a^{m-n}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
$a^0 = 1$	$a^{-m} = \frac{1}{a^m}$
"Power of a Power" Rule $(a^m)^n = a^{m \times n}$	$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ ← next lesson . . . ☺	

A. Simplify. Express answers with positive exponents.

a) $(5u^2v^4w^8)(-2uv^2w^{-3})$

$= 5(-2)u^{2+1}v^{4+2}w^{8+(-3)}$

$= -10u^3v^6w^5$

b) $(-3x^2y^4)^2$

$= (-3)^2(x^2)^2(y^4)^2$

$= 9x^4y^8$

c) $\left(\frac{3x^3}{y^4}\right)^{-2}$

$= \left(\frac{y^4}{3x^3}\right)^2$

$= \frac{y^8}{9x^6}$

d) $\left(\frac{36a^8b^6c^3}{-9a^3b^4c^2}\right)^2$

$= (-4a^5b^2c)^2$

$= 16a^{10}b^4c^2$

e) $\frac{(-5m^{-2}n^{-1})(-4m^{-3}n^{-2})}{-10m^2n^{-3}}$

$= \frac{20m^{-5}n^{-3}}{-10m^2n^{-3}}$

$= -2m^{-7}n^0$

$= -2\left(\frac{1}{m^7}\right)(1)$

$= -\frac{2}{m^7}$

f) $\frac{(2p^2q^{-3})^{-2}(4pq^{-1})^3}{(-3p^3q^{-2})^{-2}}$

$= \frac{2^{-2}p^{-4}q^{6} \cdot 4^3p^3q^{-3}}{(-3)^{-2}p^{-6}q^4}$

$= \frac{\frac{1}{4}(4^3)p^{-1}q^3}{\frac{1}{(-3)^2}p^{-6}q^4}$

$= 144p^5q^{-1}$

$= \frac{144p^5}{q}$

B. Evaluate each of the following.

a) $\left(\frac{2}{5}\right)^{-3}$ $=\left(\frac{5}{2}\right)^{3}$

$=\frac{5^3}{2^3}$

$=\frac{125}{8}$

b) $\frac{(-4)^0}{2^{-3}}$ $=\frac{1}{2^{-3}}$

$=2^3$

$=8$

c) $\frac{3^{-1}+3^{-2}}{3^{-3}}$

$=\frac{\frac{1}{3}+\frac{1}{3^2}}{\frac{1}{3^3}}$

$=\frac{\left(\frac{1}{3}+\frac{1}{9}\right)}{\left(\frac{1}{27}\right)}$

$=\frac{\left(\frac{4}{9}\right)}{\left(\frac{1}{27}\right)}$

$=\frac{4}{\cancel{9}_{1}}\times\frac{\cancel{27}^{3}}{1}$

$=12$

or

$\frac{3^{-1}+3^{-2}}{3^{-3}}\times\frac{3^3}{3^3}$ ← "Clear" the negative exponents

$=\frac{3^2+3^1}{3^0}$

$=\frac{9+3}{1}$

$=12$

d) $\frac{(4^{-112}+4^{-111})}{(4^{-110}-4^{-112})}\times\frac{4^{112}}{4^{112}}$ ← "Clear" the negative exponents

$=\frac{4^0+4^1}{4^2-4^0}$

$=\frac{1+4}{16-1}$

$=\frac{5}{15}$

$=\frac{1}{3}$

e) $3^x(3^{-x}-3^{-3-x})$

$=3^{-x+x}-3^{-3-x+x}$

$=3^0-3^{-3}$

$=1-\frac{1}{3^3}$

$=\frac{1^{\times 27}}{1_{\times 27}}-\frac{1}{27}$

$=\frac{26}{27}$

HW: p. 9-10 #1ghi, 2, 3fghij, 4bcd, 5bef, 6cde, 7dfjmnop, 8abdfhjl, 9, 14

Rational Exponents

Use a calculator to complete the following.

i) $9^{\frac{1}{2}}$ = 3 *and* $\sqrt[2]{9^1}$ = 3 ∴

ii) $64^{\frac{1}{3}}$ = 4 *and* $\sqrt[3]{64^1}$ = 4 ∴

iii) $16^{\frac{1}{4}}$ = 2 *and* $\sqrt[4]{16^1}$ = 2 ∴

$a^{1/n} = \sqrt[n]{a^1}$

Rules: **i)** $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $\sqrt[n]{a}$ is called a **radical** and means the n^{th} root of *a.*

ii) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ *or* $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

Ex. 1: Write in radical form.

a) $x^{\frac{1}{2}} = \sqrt{x}$

b) $a^{\frac{4}{5}} = \left(\sqrt[5]{a}\right)^4$

$= \sqrt[5]{a^4}$

c) $125^{-\frac{2}{3}} = \left(\sqrt[3]{125}\right)^{-2}$

$= \left(\frac{1}{\sqrt[3]{125}}\right)^2$

Ex. 2: Write as a power using rational exponents and simplify.

a) $\sqrt[3]{12} = 12^{1/3}$

b) $\frac{1}{\sqrt[5]{(-6)^3}} = \frac{1}{(-6)^{3/5}}$

c) $\sqrt[3]{\sqrt[2]{2x^6}} = \left(\left(2x^6\right)^{1/2}\right)^{1/3}$

$= \left(2^1x^6\right)^{1/6}$

$= 2^{1/6}x^1$

$= 2^{1/6}x$

d) $\left(\sqrt[3]{x^2}\right)\left(\sqrt[4]{x^3}\right) = \left(x^{2/3}\right)\left(x^{3/4}\right)$

$= \left(x^{\frac{8}{12}}\right)\left(x^{\frac{9}{12}}\right)$

$= x^{\frac{17}{12}}$

Ex. 3: Evaluate. Show all steps. Give final answers as fractions in lowest terms, where applicable (no decimal answers).

a) $625^{\frac{1}{4}} = \sqrt[4]{625}$

$= 5$

b) $(-64)^{\frac{1}{3}} = \sqrt[3]{-64}$

$= -4$

c) $16^{0.75} = 16^{\frac{3}{4}}$

$= \left(\sqrt[4]{16}\right)^3$

$= (2)^3$

$= 8$

d) $\left(\frac{49}{25}\right)^{\frac{1}{2}} = \frac{\sqrt{49}}{\sqrt{25}}$

$= \frac{7}{5}$

e) $625^{-\frac{3}{2}} = \left(\frac{1}{625}\right)^{\frac{3}{2}}$

$= \left(\frac{1}{25}\right)^3$

$= \frac{1}{15625}$

f) $(-32)^{-\frac{2}{5}} = \left(\frac{1}{-32}\right)^{\frac{2}{5}}$

$= \left(\frac{1}{-2}\right)^2$

$= \frac{1}{4}$

g) $\left(9^{\frac{3}{7}} \times 3^{\frac{1}{7}}\right)^{14} = 9^{\frac{3}{\cancel{7}_1}\cdot \cancel{14}^2} \times 3^{\frac{1}{\cancel{7}_1}\cdot \cancel{14}^2}$

$= 9^6 \times 3^2$

$= 9^6 \times 9$

$= 9^7$

$= 4\ 782\ 969$

h) $\sqrt[3]{\sqrt[2]{64}} = \left[\left(64\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}$

$= [8]^{\frac{1}{3}}$

$= 2$

i) $\left(\sqrt[3]{5^2}\right)\left(\sqrt[3]{5^1}\right) = 5^{\frac{2}{3}} \cdot 5^{\frac{1}{3}}$

$= 5^{\frac{3}{3}}$

$= 5$

Ex. 4: Solve for *x*.

a) $2^{x+1} = 16$

rewrite LS and RS with same base

$2^{x+1} = 2^4$

$\because$ bases are equal

$x+1 = 4$

$x = 4-1$

$\boxed{\therefore x = 3}$

b) $4^{3x} = \frac{1}{64}$

rewrite LS and RS with same base

$4^{3x} = 4^{-3}$

$\because$ bases are equal

$3x = -3$

$\boxed{\therefore x = -1}$

need to isolate base of x^1

c) $x^3 = -\frac{1}{216}$

$\left(x^3\right)^{\frac{1}{3}} = \left(-\frac{1}{216}\right)^{\frac{1}{3}}$

$x^1 = \frac{\sqrt[3]{-1}}{\sqrt[3]{216}}$

$x = \frac{-1}{6}$

$\boxed{\therefore x = -\frac{1}{6}}$

d) $2x^{\frac{4}{3}} = 162$

$\frac{\cancel{2}^1 x^{\frac{4}{3}}}{\cancel{2}_1} = \frac{162}{2}$

$x^{\frac{4}{3}} = 81$

$\left(x^{\frac{4}{3}}\right)^{\frac{3}{4}} = (81)^{\frac{3}{4}}$

$x^1 = (3)^3$

$\boxed{\therefore x = 27}$

HW: p. 16-17 #1 to 6 odd parts, 13; p. 10 #16

Solving Exponential Equations

- In an exponential equation, the variables appear as exponents
- If $a^x = a^y$, then x = y for all a ≠ 0, 1, -1
- Methods for solving exponential equations:
  - rewrite the powers with the same base
  - remove a power as a common factor

1. Solve for *x* by rewriting the powers with the same base.

a) $9^{x+1} = 3^{x-1}$

$3^{2(x+1)} = 3^{x-1}$

$3^{2x+2} = 3^{x-1}$

∵ bases are equal

$2x+2 = x-1$

$2x-x = -1-2$

$\boxed{\therefore x = -3}$

b) $25^{3x+1} = 125^{x}$

$5^{2(3x+1)} = 5^{3(x)}$

$5^{6x+2} = 5^{3x}$

∵ bases are equal

$6x+2 = 3x$

$6x-3x = -2$

$3x = -2$

$\boxed{\therefore x = \frac{-2}{3}}$

c) $2^{x^2+2x} = 2^{x+6}$

∵ bases are equal

$x^2+2x = x+6$

$x^2+2x-x-6=0$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$\boxed{\therefore x = -3, 2}$

d) $27^{x-1} = \left(\frac{1}{9}\right)^{2x+5}$

$3^{3(x-1)} = (3^{-2})^{2x+5}$

$3^{3x-3} = 3^{-4x-10}$

∵ bases are equal

$3x-3 = -4x-10$

$3x+4x = -10+3$

$7x = -7$

$\boxed{\therefore x = -1}$

e) $81 = \frac{9^{x+4}}{27^{x-1}}$

$3^4 = \frac{3^{2(x+4)}}{3^{3(x-1)}}$

$3^4 = \frac{3^{2x+8}}{3^{3x-3}}$

$3^4 = 3^{(2x+8)-(3x-3)}$

$\because$ bases are equal

$4 = (2x+8)-(3x-3)$

$4 = 2x+8-3x+3$

$4-8-3 = 2x-3x$

$-7 = -x$

$\boxed{\therefore x = 7}$

f) $36^{2x+4} = \sqrt{1296^x}$

$6^{2(2x+4)} = \sqrt{6^{4x}}$

$6^{4x+8} = \left(6^{4x}\right)^{\frac{1}{2}}$

$6^{4x+8} = 6^{2x}$

$\therefore$ bases are equal

$4x+8 = 2x$

$4x-2x = -8$

$2x = -8$

$\boxed{\therefore x = -4}$

2. Solve the following system of exponential equations.
   i. reduce to a linear system of equations
   ii. solve the linear system algebraically ***by elimination***

$$\begin{cases} 5^{-3x+2y} = \frac{1}{25} \ ① \\ 2^{-10x+3y} = 256 \ ② \end{cases}$$

① $\longrightarrow$ $5^{-3x+2y} = 5^{-2}$ $\therefore$ bases are equal $\therefore -3x+2y=-2$ ③

② $\longrightarrow$ $2^{-10x+3y} = 2^{8}$ $\therefore$ bases are equal $\therefore -10x+3y = 8$ ④

$[-3x+2y=-2]$ ③ $\times 3$

$[-10x+3y=8]$ ④ $\times 2$

Subtract to eliminate y:

$$\begin{array}{rl} & -9x+6y=-6 \\ - & -20x+6y=16 \\ \hline & 11x+0y=-22 \end{array}$$

Solve for x:

$11x = -22$

$\therefore x = -2$

Sub $x=-2$ into ③:

$-3(-2)+2y = -2$

$6+2y = -2$

$2y = -2-6$

$2y = -8$

$\therefore y = -4$

$\therefore$ the solution to the system is $(-2,-4)$

HW: p. 23-25 #1-3 (ace), 4-10 (odd parts), 19ab, 20bc, 22ac, 23

PLUS: ***<u>Solve each system:</u>*** i) $\begin{cases} 2^{2x+y} = 32 \\ 2^{x-3y} = \frac{1}{2} \end{cases}$ ii) $\begin{cases} 9^{x+2y} = \frac{1}{9} \\ 3^{2x+y} = 81 \end{cases}$

<u>Answers</u> i) $(x=2, y=1)$ ii) $(x=3, y=-2)$

Solving *Advanced* Exponential Equations

1. Solve for *x* algebraically by removing a power as a common factor. *Note*: using the *law for multiplying powers with the same base* in reverse, we can see that $a^{x+b} = (a^x)(a^b)$.

a) $2^{x+2} - 2^{x} = 12$

$2^x(2^2 - 1) = 12$ ← common factor using the lowest exponent... i.e. 2^x

$2^x(4-1) = 12$

$\frac{2^x(\cancel{3})}{\cancel{3}} = \frac{12}{3}$

$2^x = 4$

$2^x = 2^2$

∵ bases are equal

$\boxed{\therefore x = 2}$

b) $8^{x-1} - 8^{x-2} = 7$

$8^{x-2}(8^1 - 1) = 7$ ← common factor using the lowest exponent... i.e. 8^{x-2}

$\frac{8^{x-2}(\cancel{7})}{\cancel{7}} = \frac{7}{7}$

$8^{x-2} = 1$

$8^{x-2} = 8^0$

∵ bases are equal

$x - 2 = 0$

$\therefore x = 2$

2. Solve each of the following *quadratic* exponential equations! *Note*: using the *power of a power rule* in reverse, we can see that $a^{bx} = (a^b)^x$ *or* $(a^x)^b$.

a) $2^{2x} - 6(2^x) + 8 = 0$ ← quadratic

$(2^x)^2 - 6(2^x) + 8 = 0$ Let $a = 2^x$

$a^2 - 6a + 8 = 0$

$(a-2)(a-4) = 0$

$a = 2$ or $a = 4$

$2^x = 2$ or $2^x = 4$

$2^x = 2^1$ or $2^x = 2^2$

∵ bases are equal

$\boxed{\therefore x = 1, 2}$

b) $5^{2x} = 125 - 20(5^x)$ ← quadratic

$(5^x)^2 + 20(5^x) - 125 = 0$ Let $b = 5^x$

$b^2 + 20b - 125 = 0$

$(b+25)(b-5) = 0$

$b = -25$ or $b = 5$

$5^x = -25$ or $5^x = 5$

no sol'n ∵ bases are equal

$\boxed{\therefore x = 1}$

HW: p. 27 #2gh (solve algebraically); WORKSHEET: "Solving *Advanced* Exponential Equations"