Review: Simplifying Exponential Expressions & Solving Exponential Equations

1. *Textbook*: p. 85-86 #3, 6, 9, 10, 11; p. 90-91 #1, 2, 3, 5, 6

2. Simplify the following:

a) $\sqrt[5]{\dfrac{1024(x^{-1})^{10}}{(2x^{-3})^5}}$

$= \left(\dfrac{2^{10}x^{-10}}{2^5x^{-15}}\right)^{\frac{1}{5}}$

$= (2^5x^5)^{\frac{1}{5}}$

$= (2^5)^{\frac{1}{5}}(x^5)^{\frac{1}{5}}$

$= 2x$

b) $\dfrac{(8x^6y^{-3})^{\frac{1}{3}}}{(2xy)^3}$

$= \dfrac{2^1x^2y^{-1}}{2^3x^3y^3}$

$= 2^{-2}x^{-1}y^{-4}$

$= \dfrac{1}{4}\cdot\dfrac{1}{x}\cdot\dfrac{1}{y^4}$

$= \dfrac{1}{4xy^4}$

c) $\dfrac{2^{-1001}-2^{-1002}}{2^{-1002}+2^{-1001}} \cdot \dfrac{2^{1002}}{2^{1002}}$

$= \dfrac{2^1-1}{1+2^1}$

$= \dfrac{1}{3}$

3. Solve the following:

a) $\left(\frac{1}{9}\right)^{x-2} = \left(\frac{1}{27}\right)^{x+1}$

$\left(\frac{1}{3}\right)^{2(x-2)} = \left(\frac{1}{3}\right)^{3(x+1)}$

$\left(\frac{1}{3}\right)^{2x-4} = \left(\frac{1}{3}\right)^{3x+3}$

$\because$ bases are equal

$2x-4 = 3x+3$

$-x = 7$

$\boxed{\therefore x = -7}$

b) $(5^{x-1})^x - 25 = 0$

$5^{x^2-x} = 5^2$

$\because$ bases are equal

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{\therefore x = 2 \text{ or } x = -1}$

c) $-500 = 5^{x+1} - 5^{x+2}$

$-500 = 5^{x+1}(1-5^1)$

$\dfrac{-500}{-4} = \dfrac{5^{x+1}(-4)}{-4}$

$125 = 5^{x+1}$

$5^3 = 5^{x+1}$

$\because$ bases are equal

$3 = x+1$

$\boxed{\therefore x = 2}$

d) $2^{2x} - 12(2^x) + 32 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$

Let $y = 2^x$

$y^2 - 12y + 32 = 0$

$(y-8)(y-4) = 0$

$y = 8$ or $y = 4$

$2^x = 2^3$ $\quad$ $2^x = 2^2$

$\because$ bases are equal

$\boxed{\therefore x = 3 \text{ or } x = 2}$

4. Solve the following systems of exponential equations:
   i. reduce to a linear system of equations
   ii. solve the linear system algebraically ***by elimination***

a) $\begin{cases} \left(\frac{1}{4}\right)^{y-x} = 16 \\ 3^{2x+3y} = \left(\frac{1}{27}\right)^{-3} \end{cases}$

$\left(\frac{1}{4}\right)^{y-x} = 16 \rightarrow 4^{-y+x} = 4^2$

$3^{2x+3y} = \left(\frac{1}{27}\right)^{-3} \rightarrow 3^{2x+3y} = 3^9$

$\because$ bases are equal

$x - y = 2$

$2x + 3y = 9$

Solve $x - y = 2$ ①

$2x + 3y = 9$ ②

①×2: $2x - 2y = 4$

②: $2x + 3y = 9$

Subtract: $-5y = -5$

$\boxed{\therefore y = 1}$

$\rightarrow$ Sub $y = 1$ into ①:

$x - 1 = 2$

$\boxed{\therefore x = 3}$

$\therefore$ the solution (x, y) is $(3, 1)$.

b) $\begin{cases} 27^{\frac{4}{3}y} = \left(\frac{1}{9}\right)^{-x-1} \\ 125^x = \left(\frac{1}{5}\right)^{2y-13} \end{cases}$

$27^{\frac{4}{3}y} = \left(\frac{1}{9}\right)^{-x-1} \rightarrow 3^{4y} = 3^{2x+2}$

$5^{3x} = 5^{-2y+13}$

$\because$ bases are equal

$4y = 2x + 2$

$3x = -2y + 13$

Solve: $2x - 4y = -2$ ①

$3x + 2y = 13$ ②

①: $2x - 4y = -2$

②×2: $6x + 4y = 26$

Add: $8x = 24$

$\boxed{\therefore x = 3}$

$\rightarrow$ Sub $x = 3$ into ①:

$2(3) - 4y = -2$

$-4y = -8$

$\boxed{\therefore y = 2}$

$\therefore$ the solution (x, y) is $(3, 2)$.

Answers:

2. a) $2x$ b) $\frac{1}{4xy^4}$ c) $\frac{1}{3}$

3. a) $x = -7$ b) $x = 2,\ x = -1$ c) $x = 2$ d) $x = 2,\ x = 3$

4. a) $x = 3, y = 1$ b) $x = 3, y = 2$