

### Reviewing the Exponent Laws

Exponent Laws	
$a^m \cdot a^n =$	$(ab)^m =$
$a^m \div a^n =$	$\left(\frac{a}{b}\right)^m =$
$a^0 =$	$a^{-m} =$
$(a^m)^n =$	$\left(\frac{a}{b}\right)^{-m} =$
$a^{m/n} =$	

**A. Simplify. Express answers with positive exponents.**

a)  $(5u^2v^4w^8)(-2uv^2w^{-3})$

b)  $(-3x^2y^4)^2$

c)  $\left(\frac{3x^3}{y^4}\right)^{-2}$

d)  $\left(\frac{36a^8b^6c^3}{-9a^3b^4c^2}\right)^2$

e)  $\frac{(-5m^{-2}n^{-1})(-4m^{-3}n^{-2})}{-10m^2n^{-3}}$

f)  $\frac{(2p^2q^{-3})^{-2}(4pq^{-1})^3}{(-3p^3q^{-2})^{-2}}$

B. Evaluate each of the following.

a)  $\left(\frac{2}{5}\right)^{-3}$

b)  $\frac{(-4)^0}{2^{-3}}$

c)  $\frac{3^{-1} + 3^{-2}}{3^{-3}}$

or

$$\frac{3^{-1} + 3^{-2}}{3^{-3}}$$

d)  $\frac{4^{-112} + 4^{-111}}{4^{-110} - 4^{-112}}$

e)  $3^x(3^{-x} - 3^{-3-x})$

## Rational Exponents

**Use a calculator to complete the following.**

i)  $9^{\frac{1}{2}} = \underline{\hspace{2cm}}$  and  $\sqrt{9} = \underline{\hspace{2cm}}$   $\therefore$

ii)  $64^{\frac{1}{3}} = \underline{\hspace{2cm}}$  and  $\sqrt[3]{64} = \underline{\hspace{2cm}}$   $\therefore$

iii)  $16^{\frac{1}{4}} = \underline{\hspace{2cm}}$  and  $\sqrt[4]{16} = \underline{\hspace{2cm}}$   $\therefore$

**Rules:** i)  $a^{\frac{1}{n}} = \sqrt[n]{a}$ , where  $\sqrt[n]{a}$  is called a **radical** and means the  $n^{\text{th}}$  root of  $a$ .

ii)  $a^{\frac{m}{n}} = \underline{\hspace{2cm}}$  or  $a^{\frac{m}{n}} = \underline{\hspace{2cm}}$

**Ex. 1:** Write in radical form.

a)  $x^{\frac{1}{2}}$

b)  $a^{\frac{4}{5}}$

c)  $125^{-\frac{2}{3}}$

**Ex. 2:** Write as a power using rational exponents and simplify.

a)  $\sqrt[3]{12}$

b)  $\frac{1}{\sqrt[5]{(-6)^3}}$

c)  $\sqrt[3]{\sqrt{2x^6}}$

d)  $\left(\sqrt[3]{x^2}\right)\left(\sqrt[4]{x^3}\right)$

**Ex. 3:** Evaluate. Show all steps. Give final answers as fractions in lowest terms, where applicable (no decimal answers).

a)  $625^{\frac{1}{4}}$

b)  $(-64)^{\frac{1}{3}}$

c)  $16^{0.75}$

d)  $\left(\frac{49}{25}\right)^{\frac{1}{2}}$

e)  $625^{-\frac{3}{2}}$

f)  $(-32)^{-\frac{2}{5}}$

g)  $\left(9^{\frac{3}{7}} \times 3^{\frac{1}{7}}\right)^{14}$

h)  $\sqrt[3]{\sqrt{64}}$

i)  $\left(\sqrt[3]{5^2}\right)\left(\sqrt[3]{5}\right)$

**Ex. 4:** Solve for  $x$ .

a)  $2^{x+1} = 16$

b)  $4^{3x} = \frac{1}{64}$

c)  $x^3 = -\frac{1}{216}$

d)  $2x^{\frac{4}{3}} = 162$

## Solving Exponential Equations

- In an exponential equation, the variables appear as exponents
- If  $a^x = a^y$ , then  $x = y$  for all  $a \neq 0, 1, -1$
- Method for solving exponential equations:
  1. Express both sides of the equation as single powers with the same base.
  2. Set exponents equal and solve.

1. Solve for  $x$  by rewriting the powers with the same base.

a)  $9^{x+1} = 3^{x-1}$

b)  $25^{3x+1} = 125^x$

c)  $2^{x^2+2x} = 2^{x+6}$

d)  $27^{x-1} = \left(\frac{1}{9}\right)^{2x+5}$

$$e) 81 = \frac{9^{x+4}}{27^{x-1}}$$

$$f) 36^{2x+4} = \sqrt{1296^x}$$

2. Solve the following system of exponential equations.

- i. reduce to a linear system of equations
- ii. solve the linear system algebraically **by elimination**

$$\begin{cases} 5^{-3x+2y} = \frac{1}{25} \\ 2^{-10x+3y} = 256 \end{cases}$$

HW: p. 23-25 #1-3 (ace), 4-10 (odd parts), 19ab, 20bc, 22ac, 23

**PLUS: Solve each system:**

i) $\begin{cases} 2^{2x+y} = 32 \\ 2^{x-3y} = \frac{1}{2} \end{cases}$	ii) $\begin{cases} 9^{x+2y} = \frac{1}{9} \\ 3^{2x+y} = 81 \end{cases}$
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**Answers:**                      i)  $(x = 2, y = 1)$                       ii)  $(x = 3, y = -2)$

**Solving *Advanced* Exponential Equations**

1. Solve for  $x$  algebraically by first common factoring the power with the lowest exponent.

Note:  $a^{x+b} - a^x = a^x(a^b - a^0)$

a)  $2^{x+2} - 2^x = 12$

b)  $8^{x-1} - 8^{x-2} = 7$

2. Solve each of the following *quadratic* exponential equations! Note:  $a^{2x} = (a^x)^2$

a)  $2^{2x} - 6(2^x) + 8 = 0$

b)  $5^{2x} = 125 - 20(5^x)$

**WORKSHEET: Solving *Advanced* Exponential Equations**

1. Solve for  $x$  algebraically by first removing a power as a common factor. **Note:**  $a^{x+b} - a^x = a^x(a^b - a^0)$ .

a)  $6^{x+1} + 6^{x+2} = 7$

b)  $3^{x+2} = 3^x + 216$

c)  $2^{x+2} - 2^x = 48$

d)  $4^{x+3} + 4^x - 260 = 0$

e)  $3^{x+3} - 3^{x+1} = 648$

f)  $10^{x+4} = 11 - 10^{x+3}$



2. Solve each of the following *quadratic* exponential equations! **Note:**  $a^{2x} = (a^x)^2$

a)  $3^{2x} - 12(3^x) + 27 = 0$

b)  $3(3^{2x}) - 10(3^x) + 3 = 0$

c)  $5^{2x} - 4(5^x) - 5 = 0$

d)  $6^{2x} - 7(6^x) + 6 = 0$

e)  $4^{2x} - 20(4^x) = -64$

f)  $5(5^{2x}) + 5 = 10(5^x)$

**Answers:**

1. a)  $x = -1$   
2. a)  $x = 1, 2$

b)  $x = 3$   
b)  $x = -1, 1$

c)  $x = 4$   
c)  $x = 1$

d)  $x = 1$   
d)  $x = 0, 1$

e)  $x = 3$   
e)  $x = 1, 2$

f)  $x = -3$   
f)  $x = 0$

**Review: Simplifying Exponential Expressions & Solving Exponential Equations**

1. *Textbook*: p. 85-86 #3, 6, 9, 10, 11; p. 90-91 #1, 2, 3, 5, 6

2. Simplify the following:

a)  $\sqrt[5]{\frac{1024(x^{-1})^{10}}{(2x^{-3})^5}}$

b)  $\frac{(8x^6y^{-3})^{\frac{1}{3}}}{(2xy)^3}$

c)  $\frac{2^{-1001} - 2^{-1002}}{2^{-1002} + 2^{-1001}}$

3. Solve the following:

a)  $\left(\frac{1}{9}\right)^{x-2} = \left(\frac{1}{27}\right)^{x+1}$

b)  $(5^{x-1})^x - 25 = 0$

c)  $-500 = 5^{x+1} - 5^{x+2}$

d)  $2^{2x} - 12(2^x) + 32 = 0$

4. Solve the following systems of exponential equations:

- i. reduce to a linear system of equations
- ii. solve the linear system algebraically **by elimination**

$$\mathbf{a)} \begin{cases} \left(\frac{1}{4}\right)^{y-x} = 16 \\ 3^{2x+3y} = \left(\frac{1}{27}\right)^{-3} \end{cases}$$

$$\mathbf{b)} \begin{cases} 27^{\frac{4}{3}y} = \left(\frac{1}{9}\right)^{-x-1} \\ 125^x = \left(\frac{1}{5}\right)^{2y-13} \end{cases}$$

**Answers:**

2. a)  $2x$

b)  $\frac{1}{4xy^4}$

c)  $\frac{1}{3}$

3. a)  $x = -7$

b)  $x = 2, x = -1$

c)  $x = 2$

d)  $x = 2, x = 3$

4. a)  $x = 3, y = 1$

b)  $x = 3, y = 2$