Properties of Exponential Functions

Exponential functions are curves that ***increase*** or ***decrease*** throughout their domains. They have the basic form $y = b^x$, where $b > 1$ *or* $0 < b < 1$. They model many different phenomena, including population growth and radioactive decay.

I. Comparing the graphs of $y = b^x$ when $b > 1$

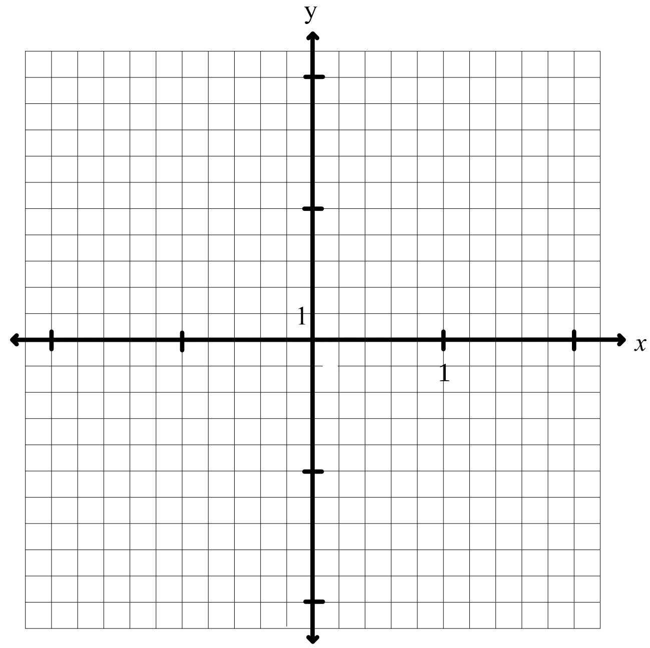
1. Create a table of values for each function, and graph them both on the same set of axes.

i) $y = 2^x$

x	y
-2	
-1	
0	
1	
2	

ii) $y = 3^x$

x	y
-2	
-1	
0	
1	
2	



2. Sketch the functions $y = 2^x$, $y = 5^x$ and $y = 10^x$ on the same set of axes. Summarize your findings below.

For $y = b^x$ when $b > 1$:

Domain = ___________________________________

Range = ____________________________________

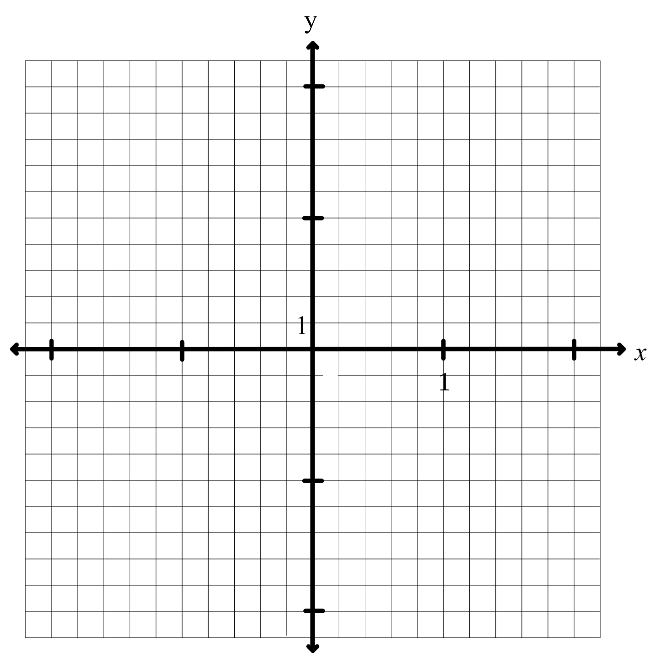
y-intercept = ________________________________

Horizontal Asymptote: _______________________

Increasing or decreasing? ____________________

As *b* ↑: _______________________________________

____



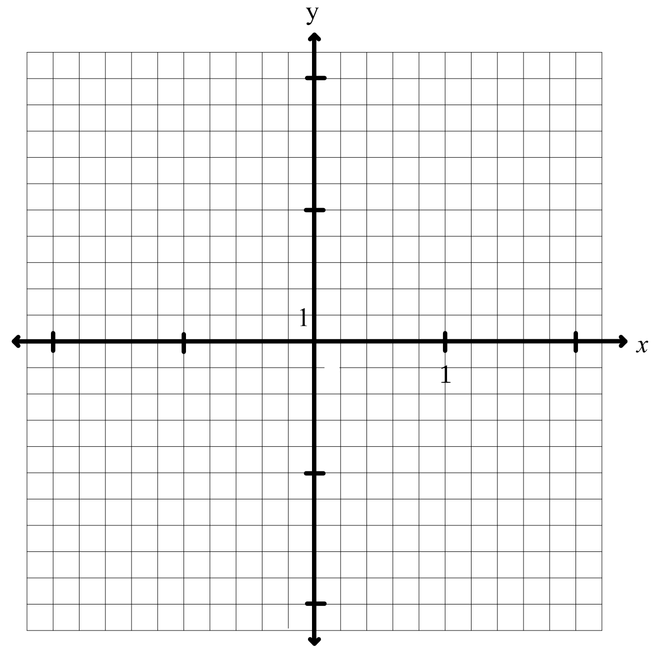
II. Comparing the graphs of $y = b^x$ when $0 < b < 1$

1. Create a table of values for each function, and graph them both on the same set of axes.

i) $y = \left(\frac{1}{2}\right)^x$ ii) $y = 2^x$

x	y
-2	
-1	
0	
1	
2	

x	y
-2	
-1	
0	
1	
2	



Note that the equation $y = \left(\frac{1}{2}\right)^x$ is equivalent to $y = 2^{-x}$! During transformations, when we make the x-value of a function negative, we cause a ______________________________ of the parent function across the _______-axis. Have we done that here, also? _________ Now, check that the graph of $y = \left(\frac{1}{2}\right)^{-x}$ is the same as $y = 2^x$!

2. Sketch the functions $y = \left(\frac{1}{2}\right)^x$, $y = \left(\frac{1}{5}\right)^x$ and $y = \left(\frac{1}{10}\right)^x$ on the same set of axes. Summarize your findings below.

For $y = b^x$ when $0 < b < 1$:

Domain = ________________________________

Range = _________________________________

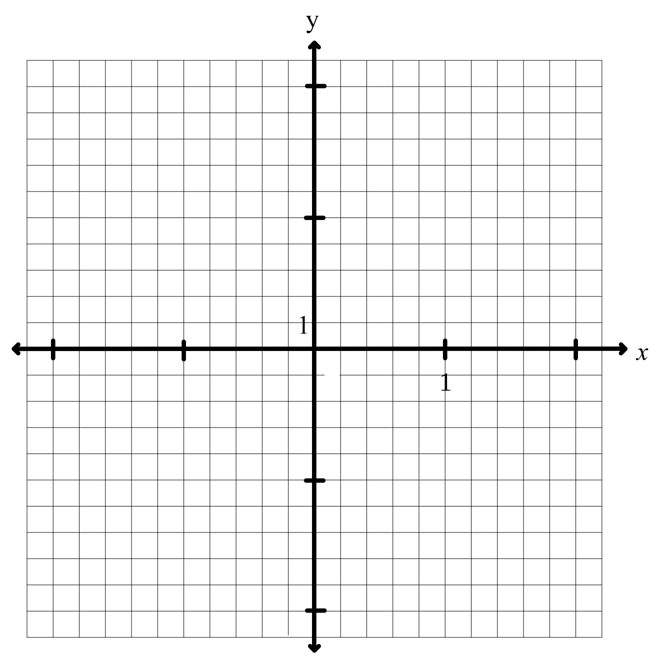
y-intercept = ____________________________

Horizontal Asymptote: ____________________

Increasing or decreasing? _________________

As *b* ↓*:* _________________________________

____



3. Extension:

a) Investigate why an exponential function should not have a base that is *negative*.

b) Investigate why an exponential function should not have a base that is *0 or 1.*

III. Comparing Linear, Quadratic, and Exponential Functions

Create a table of values for each function, and calculate their first and second differences and their common ratio.

i) $f(x) = -2x + 3$ ii) $g(x) = x^2 - 5$ iii) $h(x) = 3(2)^x$

x	y	Δy	Δ^2y
-3	9		
-2	7		
-1	5		
0	3		
1	1		
2	−1		
3	−3		

x	y	Δy	Δ^2y
-3	4		
-2	−1		
-1	−4		
0	−5		
1	−4		
2	−1		
3	4		

x	y	Δy	Δ^2y	Ratio
-3	$\frac{3}{8}$			
-2	$\frac{3}{4}$			
-1	$\frac{3}{2}$			
0	3			
1	6			
2	12			
3	24			

IV. Summary

i) ____ are constant for ___________________________________

(**linear**) polynomial functions of the form _________________________ .

ii) ____ are constant for ___________________________________

(**quadratic**) polynomial functions of the form _________________________________ .

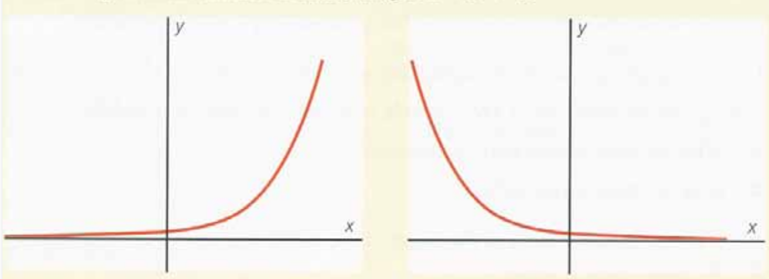
Note: _____________________________ are constant for _____________________ polynomial functions.

iii) ________________ of consecutive y-values are constant for _____________________________ functions

of the form _____________________________ .

Properties of the Exponential Function $f(x) = b^x$, $b > 0$

- The base b is positive.
- $f(0) = 1$. (The y-intercept of the graph is 1.)
- The x-axis is a horizontal asymptote.
- The domain is the set of real numbers R.
- The range is the set of positive real numbers R.
- The exponential function is increasing if $b > 1$.
- The exponential function is decreasing if $0 < b < 1$.



$y = b^x$, $b > 1$
The function is increasing as x increases.

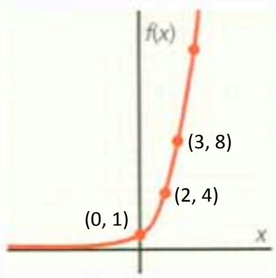
$y = b^x$, $0 < b < 1$
The function is decreasing as x increases.

Worksheet: Properties of Exponential Functions

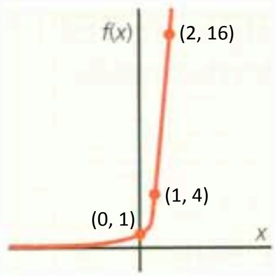
Part A:

1. The graphs below represent functions $f(x) = b^x$, $b > 0$.

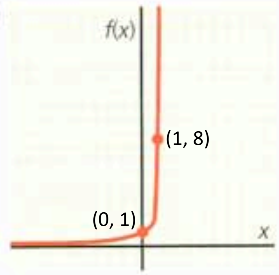
i)



ii)



iii)



Answer the following questions about each of the functions represented by these graphs. For all graphs, one square represents one unit.

a. What is the value of $f(0)$?

b. What is the value of $f(1)$?

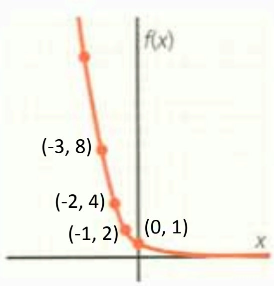
c. What is the value of b?

d. State the equation of the function.

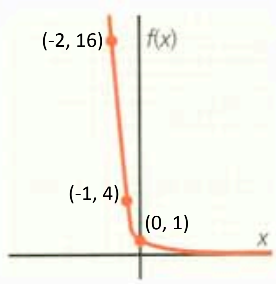
e. What are the values of $f(2)$, $f(3)$, $f(-1)$, and $f(-3)$?

2. The graphs below represent functions $f(x) = b^x$, $0 < b < 1$.

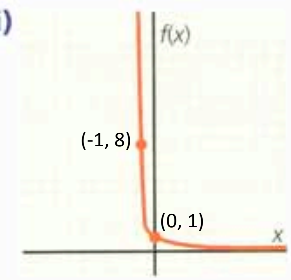
i)



ii)



iii)



Answer the following questions about each of the functions represented by these graphs. For all graphs, one square represents one unit.

a. What is the value of $f(0)$?

b. What is the value of $f(-1)$?

c. What is the value of b?

d. State the equation of the function.

e. What are the values of $f(2)$, $f(3)$, $f(1)$, and $f(-3)$?

3. Describe how you determine the equation of a function of the form $y = b^x$, $b > 0$, if you are given its graph.

Part A Answers:

1. i) a. 1 **b.** 2 **c.** 2 **d.** $y = 2^x$ **e.** 4, 8, $\frac{1}{2}$, $\frac{1}{8}$ **ii) a.** 1 **b.** 4 **c.** 4 **d.** $y = 4^x$ **e.** 16, 64, $\frac{1}{4}$, $\frac{1}{64}$ **iii) a.** 1 **b.** 8 **c.** 8 **d.** $y = 8^x$ **e.** 64, 512, $\frac{1}{8}$, $\frac{1}{512}$
2. i) a. 1 **b.** 2 **c.** $\frac{1}{2}$ **d.** $y = \left(\frac{1}{2}\right)^x$ **e.** $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{2}$, 8 **ii) a.** 1 **b.** 4 **c.** $\frac{1}{4}$ **d.** $y = \left(\frac{1}{4}\right)^x$ **e.** $\frac{1}{16}$, $\frac{1}{64}$, $\frac{1}{4}$, 64 **iii) a.** 1 **b.** 8 **c.** $\frac{1}{8}$ **d.** $y = \left(\frac{1}{8}\right)^x$ **e.** $\frac{1}{64}$, $\frac{1}{512}$, $\frac{1}{8}$, 512

Part B:

1. Use differences to identify the type of function represented by the table of values.

a)

x	y
−4	5
−3	8
−2	13
−1	20
0	29
1	40

b)

x	y
−5	32
−4	16
−3	8
−2	4
−1	2
0	1

c)

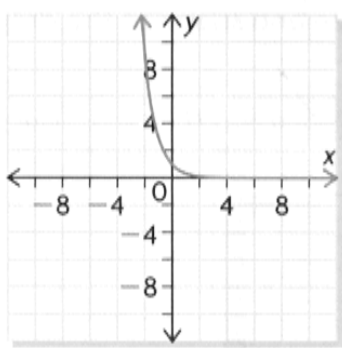
x	y
−2	−2.75
0	−2
2	1
4	13
6	61
8	253

d)

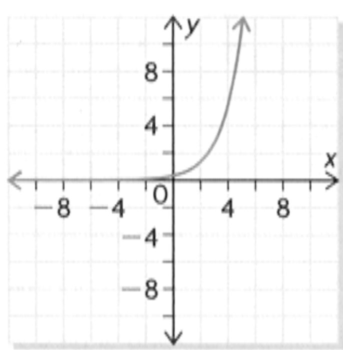
x	y
0.5	0.9
0.75	1.1
1	1.3
1.25	1.5
1.5	1.7
1.75	1.9

2. What type of function is represented in each graph? Explain how you know.

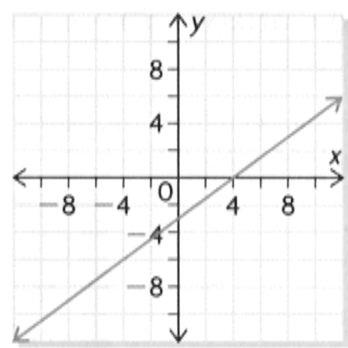
a)



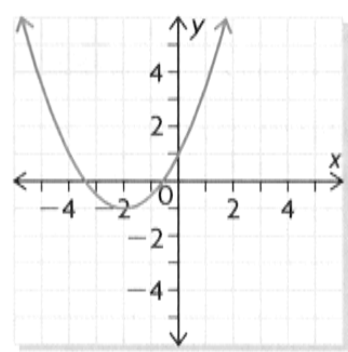
b)



c)



d)

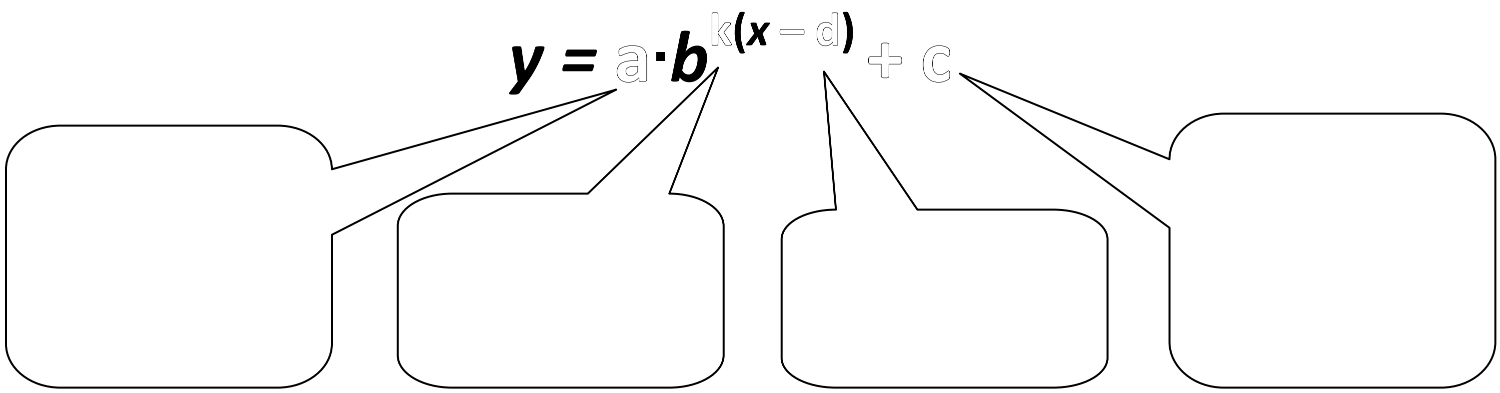


Part B Answers:

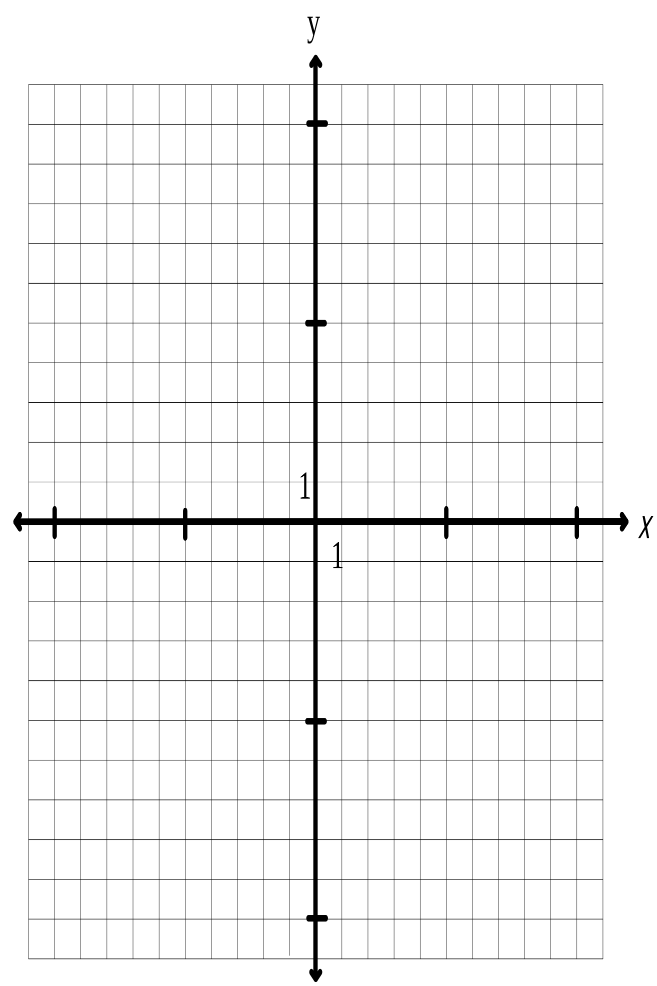
1. *a) Quadratic   b) Exponential   c) Not linear, quadratic, or exponential   d) Linear*
2. *a) Exponential; curve is decreasing at a decreasing rate across its entire domain; $0 < b < 1$*
   b) Exponential; curve is increasing at an increasing rate across its entire domain; $b > 1$
   c) Linear; line is increasing at a constant rate across its entire domain
   d) Quadratic; curve is decreasing then increasing across its domain; shape is a parabola that opens up with vertex (-2, -1)

Transformations of Exponential Functions of the Form $y = a \cdot b^{k(x-d)} + c$

Exponential functions of the form **$y = b^x$** can be transformed using the same algorithm as for other functions we have studied this semester, where the point (x, y) on $y = b^x$ maps onto the point $(\frac{1}{k}x + d, ay + c)$ on $y = a \cdot b^{k(x-d)} + c$.



1. Given the function $y = 3^{-(x+1)} - 2$, use the base function (_________) to graph the transformed function using the following steps:



a) List the transformations on $y = 3^x$ in order:

i)

ii)

iii)

b) Describe the transformation that maps a point on the base function to a point on the transformed function:

(x, y) → (___________, ___________)

c) Map the changes onto a few key points from the base function:

d) Sketch the transformed function and label the horizontal asymptote,_______________.

e) State the following:

Domain: ____

Range: ____

2. Given the function $y = -2^{2(x-3)} + 5$, use the base function (_________) to graph the transformed function using the following steps:

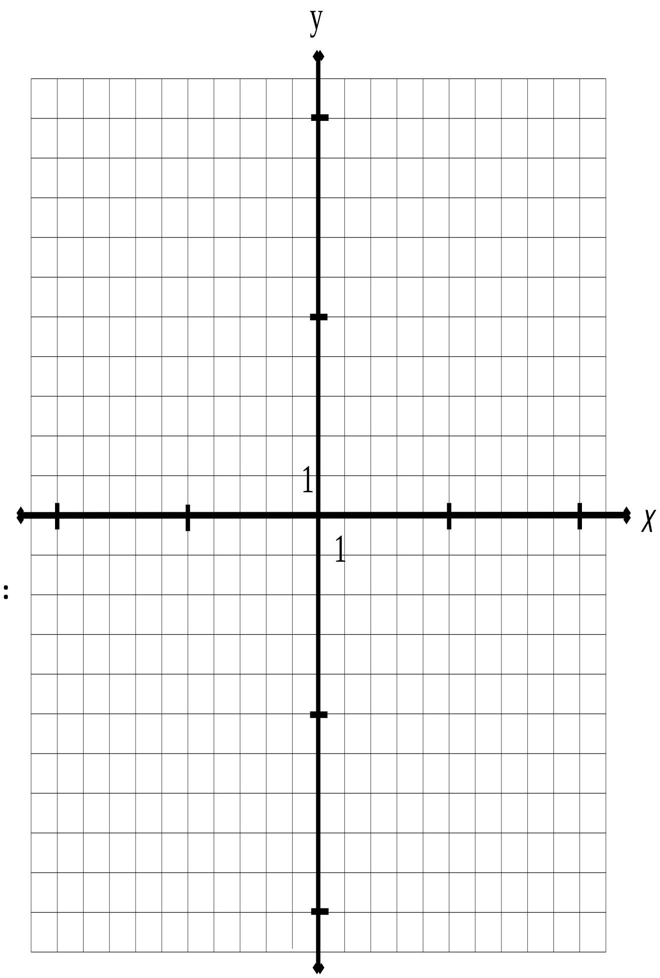
a) List the transformations on $y = 2^x$ in order:

i)

ii)

iii)

iv)



b) Describe the transformation that maps a point on the base function to a point on the transformed function:

(x, y) → (___________, ____________)

c) Map the changes onto a few key points from the base function:

d) Sketch the transformed function and label the horizontal asymptote,________________.

e) State the following:

Domain: ____

Range: ____

Summary:

- The horizontal asymptote changes when vertical translations (+/– c) are applied; h.a. equation is ***y = c***
- The domain is always D = $\{x \mid x \in R\}$.
- The range depends on the location of the horizontal asymptote and whether the function is above or below the asymptote. If it is above the asymptote, the range is R = $\{y \mid y \in R, y > c\}$ If it is below the asymptote, the range is R = $\{y \mid y \in R, y < c\}$.

HW: WORKSHEET – Transforming Exponential Functions

WORKSHEET: Transforming Exponential Functions

Name: _______________

Part 1: List the transformations for each function in order, and write the equation of the horizontal asymptote.

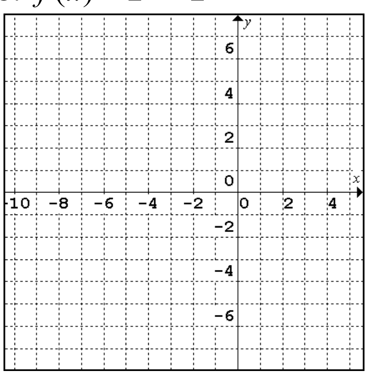
1. $f(x) = (3)4^{x} - 5$
2. $f(x) = \left(\frac{1}{4}\right)2^{-2x} + 11$
3. $f(x) = -7 - 3^{x}$
4. $f(x) + 3 = 2^{-x-5}$

Part 2: Write the equation of the horizontal asymptote for each exponential function.

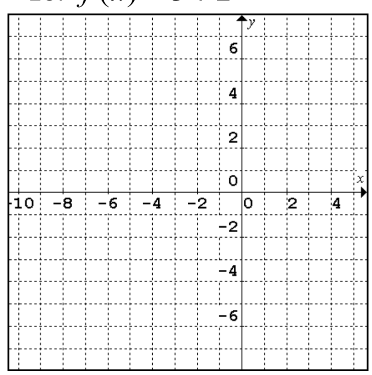
5. $f(x) = 2^{x}$
6. $f(x) = 2^{x} - 0.5$
7. $f(x) = 24.25 + 2^{x}$
8. $f(x) - 5 = 2^{x} - 8$

Part 3: Make an accurate sketch of each function.

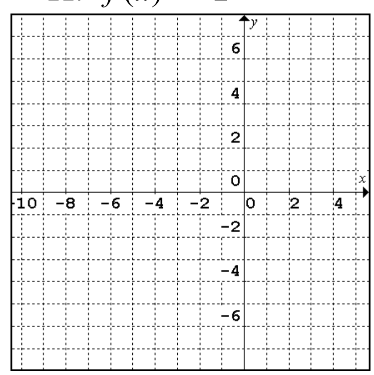
9. $f(x) = 2^{x} - 2$



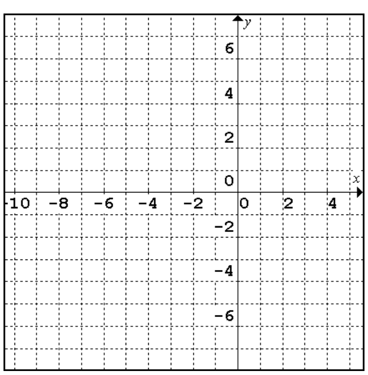
10. $f(x) = 3 + 2^{x}$



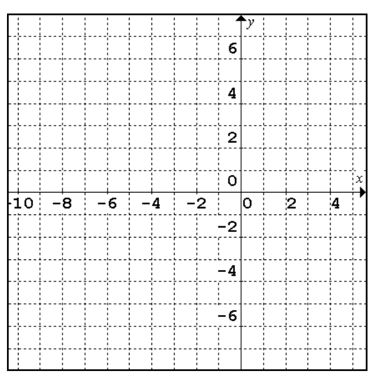
11. $f(x) = -2^{x}$



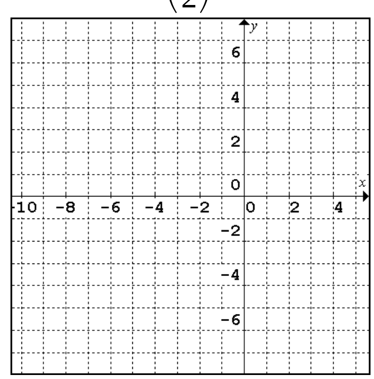
12. $f(x) = -2^{-x} + 5$



13. $f(x) = (-3)2^{x-2} - 1$



14. $f(x) = \left(\frac{1}{2}\right)2^{2x+8}$



Part 3: Write the equation resulting from the following transformations on $y = 6^{x}$:

15. a vertical translation up 1.71 units: ________________________________

16. a vertical translation down 1.6 units and a reflection across the x-axis: ________________________________

17. a vertical expansion by a factor of 5, a reflection across the y-axis, a horizontal expansion by a factor of 2, and a vertical translation up 4 units: ____

Applications Involving Exponential Functions

A. Exponential Growth and Decay

Exponential growth or ***decay*** occurs when quantities increase or decrease at a rate proportional to the initial quantity present. This ***growth*** or ***decay*** occurs in savings accounts, the size of populations, appreciation, depreciation, and with radioactive chemicals.

All situations of this type can be modeled using the **exponential function** $y = a \cdot b^x$ or $f(x) = a \cdot b^x$ where:

- y or $f(x)$ is the *final* value
- a is the *initial* or *original* value
- b is the *growth factor* if $b > 1$
- b is the *decay factor* if $0 < b < 1$
- *the growth or decay rate is* $|b - 1|$
- x is the *number of growth* or *decay periods*

Ex. 1: If the growth in population of the regional municipality of Wood Buffalo, Alberta since 1996 is given by the function $P(n) = 35\,000(1.08)^n$, determine the:

a) initial population of the municipality ____________________ **b)** growth factor __________

c) growth rate _______________ **d)** population in the year 2010 ________________________

Ex. 2: If the value of a car after it is purchased depreciates according to the formula $V = 30\,000(0.85)^n$, where V is the car's value in the n^{th} year since it was purchased, determine the:

a) purchase price of the car ________________________ **b)** decay factor _______________

c) annual rate of depreciation ____________ **d)** car's value at the end of 3 years ________________

e) car's value at the end of 30 months ______________________

Ex. 3: An antique vase was purchased in 2000 for $8 000 and appreciates in value by 6% per year.

a) Write an equation that represents the value of the vase, *V*, in dollars, *n* years after 2000 using function notation.

b) In what year did the owner double his investment?

Ex. 4: A colony of insects has a current population of 80 insects and doubles every 10 days.

a) Write an equation that represents the population, *P*, after *t* days using function notation.

b) Determine the number of insects in the colony 3 days ago.

c) Exactly how long would it take for the population to reach 40 960?

Ex. 5: A sample of radioactive polonium-210 has a *half-life* of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount.

a) If the original sample was 200 g, write an equation that represents the mass, *M*, of polonium, in grams, that remains after *t* days. Use this equation to determine exactly how long it takes for this sample to decay to 12.5 grams.

b) If there are 80 grams remaining after 365 days, what was the mass of the original sample (to the nearest gram)?

B. Compound interest

Compound interest is a type of exponential growth and can be modeled by the function $A = P(1+i)^n$ where:

- *A* is the final amount that the investment is worth.
- *P* is the principal (the original amount of money invested).
- *i* is the interest rate *per compounding period.* [*i = (annual interest rate) ÷ (# of compounding periods/year)]*
- *n* is the number of times interest is calculated over the length of the investment.
  [*n = (number of years) × (# of compounding periods/year)]*

Ex. 6: $3 500 is invested at 6% per annum. What is the accumulated amount after 5 years if the interest was compounded:

a) *quarterly?* **b)** *monthly?*

HW: "WORKSHEET: Applications Involving Exponential Functions" #1cd, 2-6, 8-12 [for #10: when using percents, *a* = 100; when using fractions, *a* = 1]; Textbook p. 508 #3; p. 573 #6

WORKSHEET: Applications Involving Exponential Functions

CHECK *Your Understanding*

1. Solve each exponential equation. Express answers to the nearest hundredth of a unit.

a) $A = 250(1.05)^{10}$

b) $P = 9000\left(\frac{1}{2}\right)^{8}$

c) $500 = N_0(1.25)^{1.25}$

d) $625 = P(0.71)^{9}$

2. Complete the table.

	Function	Exponential Growth or Decay?	Initial Value	Growth or Decay Rate
a)	$V(t) = 20(1.02)^t$			
b)	$P(n) = (0.8)^n$			
c)	$A(x) = 0.5(3)^x$			
d)	$Q(w) = 600\left(\frac{5}{8}\right)^w$			

3. The growth in population of a small town since 1996 is given by the function $P(n) = 1250(1.03)^n$.

a) What is the initial population? Explain how you know.

b) What is the growth rate? Explain how you know.

c) Determine the population in the year 2007.

d) In which year does the population reach 2000 people?

4. A computer loses its value each month after it is purchased. Its value as a function of time, in months, is modelled by $V(m) = 1500(0.95)^m$.

a) What is the initial value of the computer? Explain how you know.

b) What is the rate of depreciation? Explain how you know.

c) Determine the value of the computer after 2 years.

d) In which month after it is purchased does the computer's worth fall below $900?

PRACTISING

5. In 1990, a sum of $1000 is invested at a rate of 6% per year for 15 years.

a) What is the growth rate?

b) What is the initial amount?

c) How many growth periods are there?

d) Write an equation that models the growth of the investment, and use it to determine the value of the investment after 15 years.

6. A species of bacteria has a population of 500 at noon. It doubles every 10 h.
K The function that models the growth of the population, P, at any hour, t, is

$$P(t) = 500\left(2^{\frac{t}{10}}\right).$$

a) Why is the exponent $\frac{t}{10}$?

b) Why is the base 2?

c) Why is the multiplier 500?

d) Determine the population at midnight.

e) Determine the population at noon the next day.

f) Determine the time at which the population first exceeds 2000.

---

1. a) 407.22 b) 35.16 c) 378.30 d) 13 631.85

2.

	[illegible]	Exponential Growth or Decay?	Initial Value	[illegible]
a)	$V = 20(1.02)^t$	growth	20	2%
b)	$P = (0.8)^n$	decay	1	20%
c)	$A = 0.5(3)^x$	growth	0.5	200%
d)	$Q = 600\left(\frac{5}{8}\right)^w$	decay	600	37.5%

3. a) 1250 persons; it is the value for *a* in the general exponential function.
b) 3%; it is the base of the exponent minus 1. c) 1730 d) 2012

4. a) $1500; it is the value for *a* in the general exponential function.
b) 5%; it is the base of the exponent minus 1.
c) $437.98
d) 10th month after purchase

5. a) 6% c) 15
b) $1000 d) $V = 1000(1.06)^n$, $2396.56

6. a) The doubling period is 10 hours.
b) 2 represents the fact that the population is doubling in number (100% growth rate).
c) 500 is the initial population.
d) 1149 bacteria
e) 2639 bacteria
f) The population is 2000 at 8 a.m. the next day.

7. Which of these functions describe exponential decay? Explain.
   a) $g(x) = -4(3)^x$
   b) $h(x) = 0.8(1.2)^x$
   c) $j(x) = 3(0.8)^{2x}$
   d) $k(x) = \frac{1}{3}(0.9)^{\frac{x}{2}}$

8. A town with a population of 12 000 has been growing at an average rate of 2.5% for the last 10 years. Suppose this growth rate will be maintained in the future. The function that models the town's growth is

$$P(n) = 12(1.025^n)$$

where $P(n)$ represents the population (in thousands) and n is the number of years from now.
   a) Determine the population of the town in 10 years.
   b) Determine the number of years until the population doubles.
   c) Use this equation (or another method) to determine the number of years ago that the population was 8000. Answer to the nearest year.
   d) What are the domain and range of the function?

9. **A** A student records the internal temperature of a hot sandwich that has been left to cool on a kitchen counter. The room temperature is 19 °C. An equation that models this situation is

$$T(t) = 63(0.5)^{\frac{t}{10}} + 19$$

where T is the temperature in degrees Celsius and t is the time in minutes.
   a) What was the temperature of the sandwich when she began to record its temperature?
   b) Determine the temperature, to the nearest degree, of the sandwich after 20 min.
   c) How much time did it take for the sandwich to reach an internal temperature of 30 °C?

10. In each case, write an equation that models the situation described. Explain what each part of each equation represents.
   a) the percent of colour left if blue jeans lose 1% of their colour every time they are washed
   b) the population if a town had 2500 residents in 1990 and grew at a rate of 0.5% each year after that for t years
   c) the population of a colony if a single bacterium takes 1 day to divide into two; the population is P after t days

11. A population of yeast cells can double in as little as 1 h. Assume an initial population of 80 cells.
   a) What is the growth rate, in percent per hour, of this colony of yeast cells?
   b) Write an equation that can be used to determine the population of cells at t hours.
   c) Use your equation to determine the population after 6 h.
   d) Use your equation to determine the population after 90 min.
   e) Approximately how many hours would it take for the population to reach 1 million cells?
   f) What are the domain and range for this situation?

12. A collector's hockey card is purchased in 1990 for $5. The value increases by 6% every year.
   a) Write an equation that models the value of the card, given the number of years since 1990.
   b) Determine the increase in value of the card in the 4th year after it was purchased (from year 3 to year 4).
   c) Determine the increase in value of the card in the 20th year after it was purchased.

8. a) 15361
   b) During the 29th year the population will double.
   c) 17 years ago
   d) $\{n \in \mathbf{R}\}$; $\{P \in \mathbf{R} \mid P \geq 0\}$

9. a) 82 °C b) 35 °C c) after 25 min

10. a) $C = 100(0.99)^w$.
   100 refers to the percent of the colour at the beginning.
   99 refers to the fact that 1% of the colour is lost during every wash.
   w refers to the number of washes.
   b) $P = 2500(1.005)^t$
   2500 refers to the initial population.
   1.005 refers to the fact that the population grows 0.5% every year.
   t refers to the number of years after 1990.
   c) $P = P_0(2)^t$
   2 refers to the fact that the population doubles in one day.
   t refers to the number of days.

11. a) 100% d) 226
   b) $P = 80(2^t)$ e) 13.6 h
   c) 5120 f) $\{t \in \mathbf{R} \mid t \geq 0\}$; $\{P \in \mathbf{R} \mid p \geq 80\}$

12. a) $V = 5(1.06^t)$ b) $0.36 c) $0.91

Review: Exponential Functions and Applications

A. Complete the following review questions using your textbook.

a) p. 91 #12[*recall*: when using percents, *a* = 100; when using fractions, *a* = 1]
b) p. 24 # 13
c) p. 454 # 10, 11[*recall*: when using percents, *a* = 100; when using fractions, *a* = 1]
d) p. 86 #12
e) p. 572-573 #1ab, 8

B. For each exponential function, state the base function, $y = b^x$. Then state the transformations, in order, that map the base function onto the given function. State the domain, range, and equation of the horizontal asymptote. Use the transformations to sketch each graph.

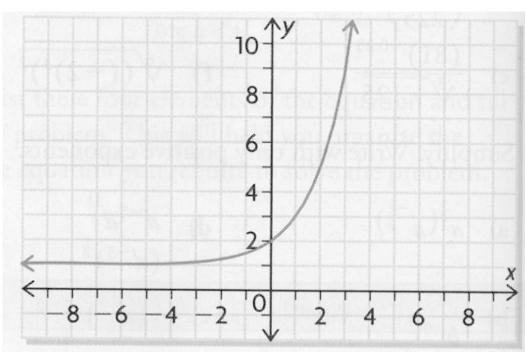
a) $y = \left(\frac{1}{2}\right)^{\frac{x}{2}} - 3$

b) $y = \frac{1}{4}(2)^{-x} + 1$

c) $y = -2(3)^{2x+4}$

d) $y = -\frac{1}{10}(5)^{3x-9} + 10$

C. The exponential function shown has been reflected in the y-axis and translated vertically. State its y-intercept, its asymptote, and a possible equation.



D. Complete the table:

Function	Exponential Growth or Decay?	Initial Value (y-intercept)	Growth or Decay Factor	Growth or Decay Rate
$V(t) = 100(1.08)^t$				
$P(n) = 32(0.95)^n$				
$A(x) = 10(4)^x$				
$Q(n) = 500\left(\frac{7}{8}\right)^n$				

Answers:

Part B: a) $y = \left(\frac{1}{2}\right)^x$; $HE\ by\ 2,\ VT\ down\ 3\ ;\ D = \{x \in R\},\ R = \{y \in R, y > -3\};\ HA: y = -3$

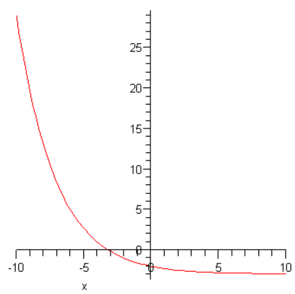
b) $y = (2)^x;\ VC\ by\ \frac{1}{4},\ ref.\ across\ the\ y - axis,\ VT\ up\ 1\ ;\ D = \{x \in R\},\ R = \{y \in R,\ y > 1\};\ HA: y = 1$

c) $y = (3)^x;\ refl.\ across\ the\ x - axis,\ VE\ by\ 2,\ HC\ by\ \frac{1}{2},\ HT\ left\ 2\ ;\ D = \{x \in R\}, R = \{y \in R,\ y < 0\}\ ;\ HA: y = 0$

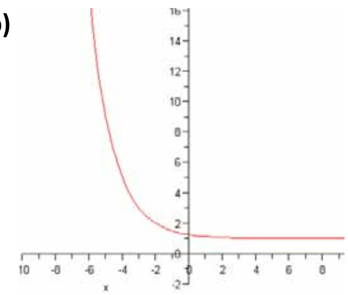
d) $y = (5)^x;\ refl.\ across\ the\ x - axis,\ VC\ by\ \frac{1}{10}, HC\ by\ \frac{1}{3}, HT\ right\ 3, VT\ up\ 10\ ; D = \{x \in R\}, R = \{y \in R, y < 10\}\ ; HA: y = 10$

Graphs:

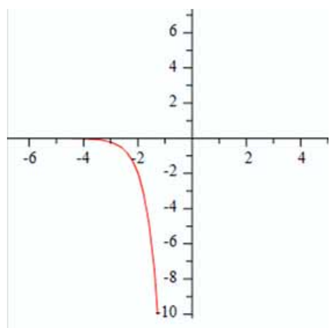
a)



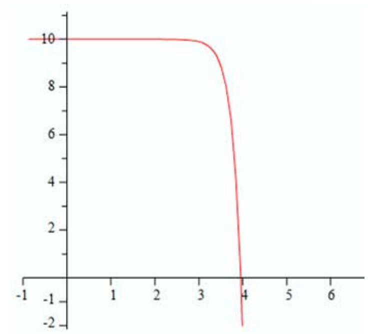
b)



c)



d)



Part C: *y-intercept: (0, 2); HA: y = 1; Possible equation:* $y = \left(\frac{1}{2}\right)^{-x} + 1$

Part D:

Function	Exponential Growth or Decay?	Initial Value (y-intercept)	Growth or Decay Factor	Growth or Decay Rate
$V(t) = 100(1.08)^t$	Growth	100	1.08	8%
$P(n) = 32(0.95)^n$	Decay	32	0.95	5%
$A(x) = 10(4)^x$	Growth	10	4	300%
$Q(n) = 500\left(\frac{7}{8}\right)^n$	Decay	500	7/8	12.5%

WORKSHEET: Transforming Exponential Functions

Name: Solutions

Part 1: List the transformations for each function in order, and write the equation of the horizontal asymptote.

1. $f(x)=(3)4^{x}-5$
   - VE by a factor of 3
   - VT down 5 units
   - ha: $y=-5$

2. $f(x)=\left(\frac{1}{4}\right)2^{-2x}+11$
   - VC by a factor of 1/4
   - HR across y-axis
   - HC by a factor of 1/2
   - VT up 11 units
   - ha: $y=11$

3. $f(x)=-7-3^{x}$
   - $f(x)=-3^{x}-7$
   - VR across x-axis
   - VT down 7 units
   - ha: $y=-7$

4. $f(x)+3=2^{-x-5}$
   - $f(x)=2^{-(x+5)}-3$
   - HR across y-axis
   - HT 5 units left
   - VT 3 units down
   - ha: $y=-3$

Part 2: Write the equation of the horizontal asymptote for each exponential function.

5. $f(x)=2^{x}$
   ha: $y=0$

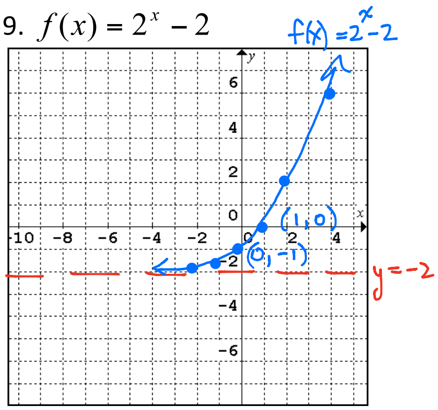
6. $f(x)=2^{x}-0.5$
   ha: $y=-0.5$

7. $f(x)=24.25+2^{x}$
   ha: $y=24.25$

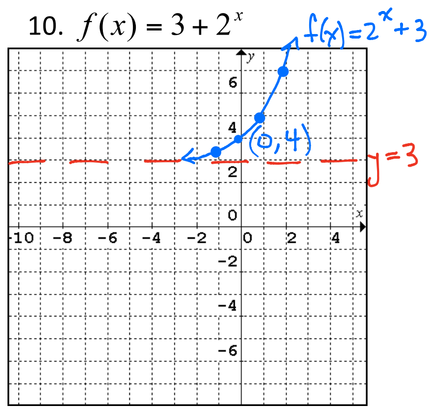
8. $f(x)-5=2^{x}-8$
   $f(x)=2^{x}-8+5$
   ha: $y=-3$

Part 3: Make an accurate sketch of each function.

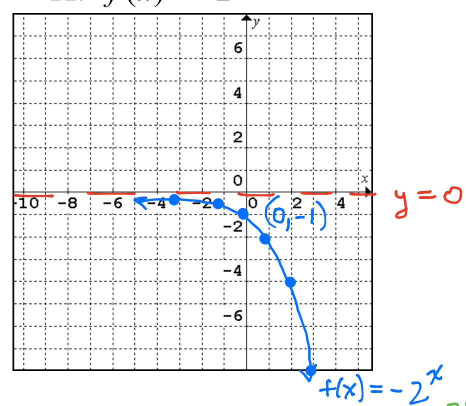
9. $f(x)=2^{x}-2$



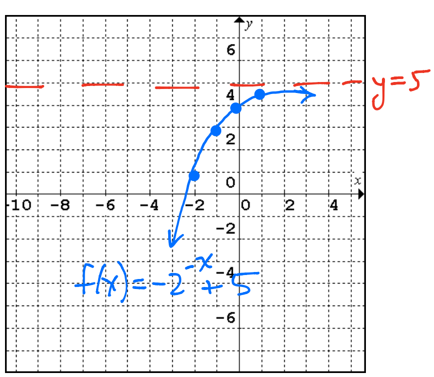
10. $f(x)=3+2^{x}$



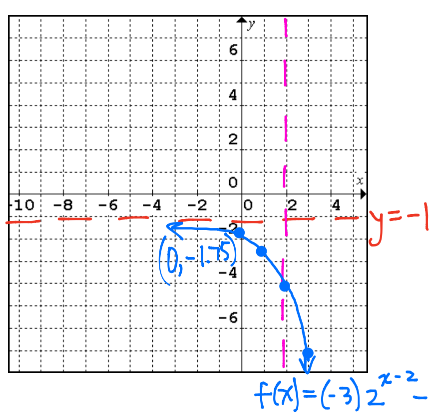
11. $f(x)=-2^{x}$



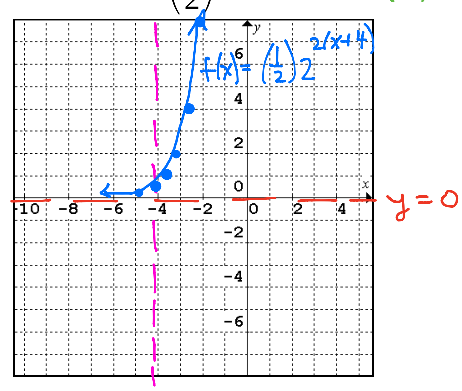
12. $f(x)=-2^{-x}+5$



13. $f(x)=(-3)2^{x-2}-1$



14. $f(x)=\left(\frac{1}{2}\right)2^{2x+8}$ $f(x)=\left(\frac{1}{2}\right)2^{2(x+4)}$



Part 3: Write the equation resulting from the following transformations on $y=6^{x}$:

15. a vertical translation up 1.71 units: $y=6^{x}+1.71$

16. a vertical translation down 1.6 units and a reflection across the x-axis: $y=-6^{x}-1.6$

17. a vertical expansion by a factor of 5, a reflection across the y-axis, a horizontal expansion by a factor of 2, and a vertical translation up 4 units: $y=(5)6^{-\frac{1}{2}x}+4$