Fibonacci

It's as easy as 1, 1, 2, 3

MCR3UI

Unit 7: Sequences and Series

Solutions!

Sequences

Part I: Arithmetic Sequences

+3 +3 +3

A sequence like **2, 5, 8, 11, . . .** where the difference between consecutive terms is a __constant__, is called an __arithmetic__ __sequence__. The first differences of an arithmetic sequence tell you the common difference between each term. ∴ linear relations

- In an arithmetic sequence, the **first term, t_1,** is the initial value and is denoted by the letter __a__ ;
- The **common difference** (when the first differences are constant) is denoted by the letter __d__. ***The common difference of an arithmetic sequence is:*** $\boxed{d = t_2 - t_1 = t_3 - t_2 = t_n - t_{n-1}}$

how to find "d" given 3 or more terms...

A. Writing the terms of a sequence: Given $t_n = 3n - 1$ is the formula for the nth term of an arithmetic sequence, find the first 4 terms.

First term: $t_1 = 3(1) - 1$
$= 2$

Second term: $t_2 = 3(2) - 1$
$= 5$

Third term: $t_3 = 3(3) - 1$
$= 8$

different ways to find terms...

n	t_n
1	2
2	5
3	8
4	11

∴ the first 4 terms of the sequence are: 2, 5, 8, 11

B. Finding the general formula for an arithmetic sequence with first term *a* and common difference *d*:

$t_1 = a$

$t_2 = a + d$

$t_3 = (a + d) + d$
$= a + 2d$

$t_4 = (a + 2d) + d$
$= a + 3d$

∴ $t_n = a + (n-1)d$

This is an equation in 4 variables...

The **General Formula** for an **Arithmetic Sequence** is:

$$t_n = a + (n-1)d \quad \text{or} \quad t_n = a + d(n-1), \quad n \in \mathbb{N}$$

where ***a*** is the **first term**, ***d*** is the **common difference** between any 2 consecutive terms,
and $d = t_2 - t_1 = t_3 - t_2 = t_n - t_{n-1}$

$9-4 = 14-9 = 19-14 = 5$

For the sequence 4, 9, 14, 19, . . . find t_n and t_{123}.

$a = 4 \quad d = 5$

a) $t_n = a + (n-1)d$
$= 4 + (n-1)(5)$
$= 4 + 5n - 5$
$= 5n - 1$

∴ $t_n = 5n - 1$

b) $t_{123} = 5(123) - 1$
$= 614$

∴ $t_{123} = 614$

C. Finding the number of terms: How many terms are in the sequence 12, 2, -8, -18, . . . , -298 ?

$a = 12$ $d = 2 - 12 = (-10)$ $t_n = -298$ $n = ?$

$t_n = a + (n-1)(d)$

$-298 = 12 + (n-1)(-10)$

$-298 = 12 - 10n + 10$

$10n = 12 + 10 + 298$

$\frac{10n}{10} = \frac{320}{10}$

$n = 32$

∴ there are 32 terms in the sequence

Part II: Geometric Sequences

A sequence like **2, 6, 18, 54, . . .** where each term is found by multiplying the previous term by a constant is called a geometric sequence. The **constant multiple** of a geometric sequence is called the **common ratio**. (like an exponential function)

- In a geometric sequence, the **first term, t_1,** is the initial value and is denoted by the letter a ;
- The **common ratio** (common multiple) is denoted by the letter r . ***The common ratio for a geometric sequence is:***

$$\boxed{r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}}}$$ ← how to find the common ratio given at least 3 terms

A. Writing the terms of a sequence: Given $t_n = 5(3)^{n-1}$ is the formula for the nth term of a geometric sequence, find the first 4 terms.

n	t_n
1	5
2	15
3	45
4	135

$t_1 = 5(3)^{1-1} = 5(3)^0 = 5$

$t_2 = 5(3)^{2-1} = 5(3) = 15$

$t_3 = 5(3)^{3-1} = 5(3)^2 = 45$

$\therefore$ the first 4 terms are: 5, 15, 45, 135

B. Finding the general formula for a geometric sequence with first term *a* and common ratio *r*:

$t_1 = a$

$t_2 = a(r)$

$t_3 = ar(r) = ar^2$

$t_4 = ar^2(r) = ar^3$

$\therefore t_n = ar^{n-1}$

Equation is in 4 variables...

The **General Formula** for a **Geometric Sequence** is:

$$t_n = a(r)^{n-1}, \quad n \in \mathbb{N}, \quad r \neq 0$$

where ***a*** is the **first term**, ***r*** is the **common ratio** between any 2 consecutive terms, and $r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_n}{t_{n-1}}$

For 1, 3, 9, 27, . . . find t_n and t_7

$a = 1, \quad r = \frac{3}{1} = \frac{9}{3} = \frac{27}{9} = 3$

a) $t_n = a(r)^{n-1}$

$= 1(3)^{n-1}$

$= 3^{n-1}$

$\therefore t_n = 3^{n-1}$

b) $t_7 = 3^{7-1}$

$= 3^6$

$= 729$

$\therefore t_7 = 729$

C. Finding the number of terms: How many terms are in the geometric sequence 3, −12, 48, −192, . . . , −3221225472?

$n = ?$ $\quad a = 3$ $\quad r = \frac{-12}{3} = \frac{48}{-12} = (-4)$ $\quad t_n$

$t_n = a(r)^{n-1}$

$\frac{-3\,221\,225\,472}{3} = \frac{3(-4)^{n-1}}{3}$

$-1\,073\,741\,824 = (-4)^{n-1}$

trial & error

$(-4)^{15} = (-4)^{n-1}$

$\because$ bases are equal

$15 = n - 1$

$\therefore n = 16$

$\therefore$ there are 16 terms in this sequence

HW: p. 441-442 # 2ace, 3abe, 4acdeg, 5cdf, 6ace, 7ace; p. 452-453 #1ace, 2aceg, 3ace, 4ace, 5adef, 6ace

Series

Part I: Arithmetic Series

2, 4, 6, . . . is called an **arithmetic sequence**; where a = 2 and d = 2 and $t_n = a + (n-1)d$.

2 + 4 + 6 + . . . is called an **arithmetic series**. A series is the sum of the terms in a sequence. The sum of the first n terms of a series is denoted S_n. From p. 466 of the text:

"The sum of the first n terms" in an **Arithmetic Series** is

$$S_n = \frac{n}{2}[\underset{\text{1st term}}{a} + \underset{\text{last term}}{t_n}] \quad \text{or} \quad S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}[2a + d(n-1)]$$

A. Sum of a series given the first terms.

1) Given the arithmetic series 1 + 6 + 11 + 16 + . . . , find the **sum** of the first twenty terms.

$a=1,\ d=5,\ n=20,\ S_n = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2(1) + (20-1)(5)]$

$= 10[2 + (19)(5)]$

$S_{20} = 10[97]$

$= 970$

$\therefore$ the sum of the first 20 terms is 970

2) Given the arithmetic series: −7 −11 −15 − . . . , find the **sum** of the first fifty-four terms.

$a = (-7),\ d = (-11) - (-7) = (-11) + 7 = (-4),\ n = 54,\ S_{54} = ?$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{54} = \frac{54}{2}[2(-7) + (54-1)(-4)]$

$= 27[-14 + 53(-4)]$

$S_{54} = 27[-226]$

$= -6102$

$\therefore$ the sum of the first 54 terms is -6102

B. Sum of a series given the first and the last terms.

1) Given the arithmetic series: 2 + 7 + 12 + . . . + 62 , find the **sum**. (Hint: to find S_n, we must first determine n).

$a=2,\ d=5,\ t_n=62,\ n=?,\ S_n=?$

① Find n:

$t_n = a + (n-1)d$

$62 = 2 + (n-1)(5)$

$62 = 2 + 5n - 5$

$62 - 2 + 5 = 5n$

$\frac{65}{5} = \frac{5n}{5}$

$\boxed{\therefore n = 13}$

② Find $S_n \Rightarrow S_{13}$:

$S_n = \frac{n}{2}[a + t_n]$

$S_{13} = \frac{13}{2}[2 + 62]$

$= \frac{13}{\cancel{2}}[\cancel{64}^{32}]$

$= 416$

$\therefore$ the sum of this series is 416

Part II: Geometric Series

5, 10, 20, . . . is called a **geometric sequence** ; where a = 5 and r = 2 and $t_n = a \cdot r^{n-1}$

5 + 10 + 20 + . . . is called a **geometric series** . The sum of the first n terms of a series is denoted S_n. From p. 473 of the text:

The sum of the first n terms in a **Geometric Series** is

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad r \neq 1$$

A. Sum of a series given the first terms.

1) Given the geometric series 3 + 6 + 12 + 24 + . . . , find the **sum** of the first eight terms.

$a = 3, \quad r = 2, \quad n = 8, \quad S_8 = ?$

$$S_8 = \frac{3(2^8 - 1)}{2-1}$$

$$S_8 = \frac{3(256-1)}{1}$$

$$S_8 = 765$$

$\therefore$ the sum of the first 8 terms is 765

2) Given the geometric series 2 – 8 + 32 –128 + . . . , find the **sum** of the first thirteen terms.

$a = 2, \quad r = (-4), \quad n = 13, \quad S_{13} = ?$

$$S_{13} = \frac{2((-4)^{13} - 1)}{-4-1}$$

$$= \frac{-134\ 217\ 730}{-5}$$

$$= 26\ 843\ 546$$

$\therefore$ the sum of the first 13 terms is 26 843 546

B. Sum of a series given the first and the last terms.

1) Given the geometric series: 3 – 9 + 27 –. . . –6561 , find the **sum**. (Hint: to find S_n, we must first determine n).

$a = 3, \quad r = (-3), \quad t_n = -6561, \quad n = ?, \quad S_n = ?$

① Find n:

$$t_n = a(r)^{n-1}$$

$$\underbrace{-6561}_{3} = \underbrace{3}_{3}(-3)^{n-1}$$

$$-2187 = (-3)^{n-1}$$

$$(-3)^7 = (-3)^{n-1}$$

$\because$ bases are equal

$$7 = n - 1$$

$\therefore 8 = n$

② Find S_8:

$$S_8 = \frac{3((-3)^8 - 1)}{-3-1}$$

$$S_8 = \frac{19\ 680}{-4}$$

$\therefore S_8 = -4920$

HW: p. 469 #1bcd, 2bcd, 3bcd, 4bd, 5bc; p. 476 #1ace, 2ace, 3ace

Recursion Formulas

Recall: The formula $t_n = 3n - 1$ determines the **arithmetic sequence** 2, 5, 8, 11, 14.

The formula $t_n = 2^{n-1}$ determines the **geometric sequence** 1, 2, 4, 8, 16.

These formulas are called explicit formulas because they can be used to calculate the n^{th} term (any term) of a sequence without knowing the previous term(s).

A. What is a recursion formula?

$t_1 = 5$; $t_n = t_{n-1} + 2$ is an example of a ***recursion formula***, where you need to know the previous term to calculate the next term.

Recursion formulas consist of at least **two parts:**

$$\begin{cases} t_1 = 5 & \text{the first term} \\ t_n = t_{n-1} + 2 & \text{instructions on how to use the previous term } (t_{n-1}) \text{ to get the next term } (t_n) \end{cases}$$

Find the first four terms of this sequence:

$t_1 = 5$

$t_2 = t_1 + 2$
$= 5 + 2$
$= 7$

$t_3 = t_2 + 2$
$= 7 + 2$
$= 9$

$t_4 = t_3 + 2$
$= 9 + 2$
$= 11$

$\therefore$ the first 4 terms are 5, 7, 9, 11

B. Use a recursion formula to write the first four terms of each sequence.

a) $t_1 = 48$; $t_n = 0.5t_{n-1}$

$t_2 = 0.5t_{2-1}$
$= 0.5t_1$
$= 0.5(48)$
$= 24$

$t_3 = 0.5t_{3-1}$
$= 0.5t_2$
$= 0.5(24)$
$= 12$

$t_4 = 0.5t_{4-1}$
$= 0.5t_3$
$= 0.5(12)$
$= 6$

$\therefore$ the first 4 terms are 48, 24, 12, 6

b) $t_1 = 2$; $t_n = t_{n-1} - 2n + 1$

$t_2 = t_{2-1} - 2(2) + 1$
$= t_1 - 4 + 1$
$= (2) - 4 + 1$
$= -1$

$t_3 = t_{3-1} - 2(3) + 1$
$= t_2 - 6 + 1$
$= (-1) - 6 + 1$
$= -6$

$t_4 = t_{4-1} - 2(4) + 1$
$= t_3 - 8 + 1$
$= (-6) - 8 + 1$
$= -13$

$\therefore$ the first 4 terms are 2, −1, −6, −13

c) $t_1 = -2$; $t_2 = 3$; $t_n = 2t_{n-2} + t_{n-1}$

$t_3 = 2t_{3-2} + t_{3-1}$
$= 2t_1 + t_2$
$= 2(-2) + (3)$
$= -1$

$t_4 = 2t_{4-2} + t_{4-1}$
$= 2t_2 + t_3$
$= 2(3) + (-1)$
$= 5$

$\therefore$ the first 4 terms are −2, 3, −1, 5

C. Write an explicit formula for each sequence based on its recursion formula.

a) Given $t_1 = 6$; $t_n = t_{n-1} + 4$, write the first four terms and write an **explicit formula** for the sequence.

$t_2 = t_1 + 4$
$= 6 + 4$
$= 10$

$t_3 = t_2 + 4$
$= 10 + 4$
$= 14$

$t_4 = t_3 + 4$
$= 14 + 4$
$= 18$

$\therefore$ the first 4 terms are 6, 10, 14, 18

$a = 6$, $d = 4$; arithmetic

$t_n = a + (n-1)d$

$t_n = 6 + (n-1)(4)$

$t_n = 6 + 4n - 4$

$\boxed{\therefore t_n = 4n + 2}$

b) Given $t_1 = 3$; $t_n = -2t_{n-1}$, write the first four terms and write an **explicit formula** for the sequence.

$t_2 = -2(t_1)$
$= -2(3)$
$= -6$

$t_3 = -2(t_2)$
$= -2(-6)$
$= 12$

$t_4 = -2(t_3)$
$= -2(12)$
$= -24$

$\therefore$ the first 4 terms are 3, −6, 12, −24

$a = 3$, $r = (-2)$; geometric

$t_n = a(r)^{n-1}$

$\boxed{\therefore t_n = 3(-2)^{n-1}}$

D. Write an explicit formula for the n^{th} term of each sequence using only the recursion formula.

a) $t_1 = 5$; $t_n = 2t_{n-1}$

$a = 5$ $r = 2$
(geometric)

$t_n = a(r)^{n-1} \longrightarrow \boxed{\therefore t_n = 5(2)^{n-1}}$

b) $t_1 = 52$; $t_n = t_{n-1} - 3$

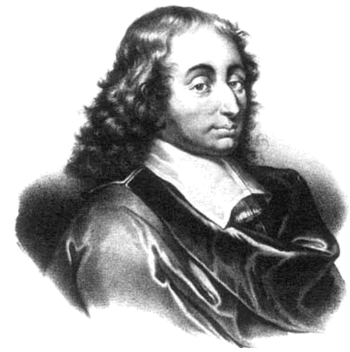
$a = 52$ $d = (-3)$
(arithmetic)

$t_n = a + (n-1)d$
$= 52 + (n-1)(-3)$
$= 52 - 3n + 3$

$\boxed{\therefore t_n = -3n + 55}$

HW: p. 461-463 #1ace, 2ace, 4ac, 5abc, 6, 7, 8, 9, 10, 17, 18

Pascal's Triangle and Binomial Expansions



Blaise Pascal, 1623-1662

A. Binomial Expansions of the Form *(a + b)^n* where *n* ∈ W.

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

$$\begin{aligned}(a+b)^3 &= (a+b)^2(a+b)\\ &= (a^2+2ab+b^2)(a+b)\\ &= a^3+a^2b+2a^2b+2ab^2+ab^2+b^3\\ &= a^3+3a^2b+3ab^2+b^3\end{aligned}$$

$$\begin{aligned}(a+b)^4 &= (a+b)^3(a+b)\\ &= (a^3+3a^2b+3ab^2+b^3)(a+b)\\ &= a^4+a^3b+3a^3b+3a^2b^2+3a^2b^2+3ab^3+ab^3+b^4\\ &= a^4+4a^3b+6a^2b^2+4ab^3+b^4\end{aligned}$$

$$\begin{aligned}(a+b)^5 &= (a+b)^4(a+b)\\ &= (a^4+4a^3b+6a^2b^2+4ab^3+b^4)(a+b)\\ &= a^5+a^4b+4a^4b+4a^3b^2+6a^3b^2+6a^2b^3+4a^2b^3+4ab^4+ab^4+b^5\\ &= a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\end{aligned}$$

B. Pascal's Triangle

1	← 0^{th} row
1 1	← 1^{st} row
1 2 1	← 2^{nd} row
1 3 3 1	← 3^{rd} row
1 4 6 4 1	← 4^{th} row
1 5 10 10 5 1	← 5^{th} row
1 6 15 20 15 6 1	← 6^{th} row
1 7 21 35 35 21 7 1	← 7^{th} row
1 8 28 56 70 56 28 8 1	← 8^{th} row

Key Ideas

- The arrangement of numbers shown below is called Pascal's triangle. Each row is generated by calculating the sum of pairs of consecutive terms in the previous row.

1	0th row
1 1	1st row
1 2 1	2nd row
1 3 3 1	3rd row
1 4 6 4 1	4th row
1 5 10 10 5 1	5th row

- The numbers in Pascal's triangle correspond to the coefficients in the expansion of binomials raised to whole-number exponents.

Need to Know

- Pascal's triangle has many interesting relationships among its numbers. Some of these relationships are recursive.
  - For example, down the sides are constant sequences: 1, 1, 1, ...
  - The diagonal beside that is the counting numbers, 1, 2, 3, ... , which form an arithmetic sequence.
  - The next diagonal is the triangular numbers, 1, 3, 6, 10, ... , which can be defined by the recursive formula

$$t_1 = 1,\ t_n = t_{n-1} + n, \quad \text{where } n \in \mathbf{N} \text{ and } n > 1.$$

- There are patterns in the expansions of a binomial $(a + b)^n$:
  - Each term in the expansion is the product of a number from Pascal's triangle, a power of a, and a power of b.
  - The coefficients in the expansion correspond to the numbers in the nth row in Pascal's triangle.
  - In the expansion, the exponents of a start at n and decrease by 1 down to zero, while the exponents of b start at zero and increase by 1 up to n.
  - In each term, the sum of the exponents of a and b is always n.

C. Expand and simplify each binomial power.

a) $(x-2)^5$ $= 1(x)^5(-2)^0 + 5(x)^4(-2)^1 + 10(x)^3(-2)^2 + 10(x)^2(-2)^3 + 5(x)^1(-2)^4 + 1(x)^0(-2)^5$

$a = x$

$b = (-2)$ $= 1x^5 + 5x^4(-2) + 10x^3(4) + 10x^2(-8) + 5x(16) + 1(-32)$

5^{th} row $= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

b) $(3x + 2y)^3$ $= 1(3x)^3(2y)^0 + 3(3x)^2(2y)^1 + 3(3x)^1(2y)^2 + 1(3x)^0(2y)^3$

$a = (3x)$

$= 1(27x^3) + 3(9x^2)(2y) + 3(3x)(4y^2) + 1(8y^3)$

$b = (2y)$

3^{rd} row $= 27x^3 + 54x^2y + 36xy^2 + 8y^3$

D. Expand and simplify *the first three terms* of each binomial power.

a) $(2x + 3)^6$ $= 1(2x)^6(3)^0 + 6(2x)^5(3)^1 + 15(2x)^4(3)^2 + \ldots$

$a = (2x)$ $= 1(64x^6) + 6(32x^5)(3) + 15(16x^4)(9) + \ldots$

$b = (3)$

6^{th} row $= 64x^6 + 576x^5 + 2160x^4 + \ldots$

b) $(5x^3 - 3y^2)^7$ $= 1(5x^3)^7(-3y^2)^0 + 7(5x^3)^6(-3y^2)^1 + 21(5x^3)^5(-3y^2)^2 + \ldots$

$a = (5x^3)$

$b = (-3y^2)$ $= 1(78125x^{21}) + 7(15625x^{18})(-3y^2) + 21(3125x^{15})(9y^4) + \ldots$

7^{th} row $= 78125x^{21} - 328125x^{18}y^2 + 590625x^{15}y^4 - \ldots$

HW: WORKSHEET: Pascal's Triangle and Binomial Expansions

WORKSHEET: Pascal's Triangle and Binomial Expansions

1. Complete Pascal's triangle to the tenth level/row (for use in the following questions).
2. The first four entries of the 12^{th} row of Pascal's triangle are 1, 12, 66, and 220. Determine the first four entries of the 13^{th} row of the triangle.
3. Expand and simplify each binomial power.
   a. $(x+2)^5$
   b. $(x-1)^6$
   c. $(2x-3)^3$
4. Expand and simplify the ***first three terms*** of each binomial power.
   a. $(x+5)^{10}$
   b. $(x-2)^8$
   c. $(2x-7)^9$
5. Expand and simplify each binomial power.
   a. $(y-5)^6$
   b. $(2x+7y)^3$
   c. $(2z^3-3y^2)^5$
6. Expand and simplify the ***first three terms*** of each binomial power.
   a. $(x-2)^7$
   b. $\left(3b^2-\frac{2}{b}\right)^8$

Answers:

2. The first four entries of the 13^{th} row are: 1, 13, 78, 286
3. a) $x^5+10x^4+40x^3+80x^2+80x+32$
   b) $x^6-6x^5+15x^4-20x^3+15x^2-6x+1$
   c) $8x^3-36x^2+54x-27$
4. a) $x^{10}+50x^9+1125x^8$
   b) $x^8-16x^7+112x^6$
   c) $512x^9-16128x^8+225792x^7$
5. a) $y^6-30y^5+375y^4-2500y^3+9375y^2-18750y+15625$
   b) $8x^3+84x^2y+294xy^2+343y^3$
   c) $-243y^{10}+810y^8z^3-1080y^6z^6+720y^4z^9-240y^2z^{12}+32z^{15}$
6. a) $x^7-14x^6+84x^5$
   b) $6561b^{16}-34992b^{13}+81648b^{10}$

Unit 7 Review

1. **i)** Determine the first five terms of each sequence, where $n \in \mathbf{N}$.
ii) Determine whether each sequence is arithmetic, geometric, or neither.

a) $t_n = 5 \times 3^{n+1}$

b) $t_n = \dfrac{3n + 2}{2n + 1}$

c) $t_n = 5n$

d) $t_1 = 5$, $t_n = 7t_{n-1}$, where $n > 1$

e) $t_1 = 19$, $t_n = 1 - t_{n-1}$, where $n > 1$

f) $t_1 = 7$, $t_2 = 13$, $t_n = 2t_{n-1} - t_{n-2}$, where $n > 2$

2. For each sequence, determine
i) the general term
ii) the recursive formula

a) a geometric sequence with $a = -9$ and $r = -11$

b) an arithmetic sequence with second term 123 and third term -456

3. Determine the number of terms in each sequence.

a) 18, 25, 32, ... , 193

b) 2, -10, 50, ... , $-156\,250$

4. Expand and simplify each binomial power.

a) $(x - 5)^4$

b) $(2x + 3y)^3$

5. Calculate the sum of each series.

a) 19 + 33 + 47 + ... + 439

b) the first 10 terms of the series 10 000 + 12 000 + 14 400 + ...

6. A sequence is defined by the recursive formula $t_1 = 4$, $t_2 = 5$, $t_n = \dfrac{t_{n-1} + 1}{t_{n-2}}$, where $n \in \mathbf{N}$ and $n > 2$. Determine t_{123}. Explain your reasoning.

7. Your grandparents put aside \$100 for you on your first birthday. Every following year, they put away \$75 more than they did the previous year. How much money will have been put aside by the time you are 21?

Answers:

1. a) i) the first five terms are 45, 135, 405, 1215, 3645 ii) it is geometric, r = 3
   b) i) the first five terms are $\frac{5}{3}, \frac{8}{5}, \frac{11}{7}, \frac{14}{9}, \frac{17}{11}$ ii) it is neither arithmetic nor geometric
   c) i) the first five terms are 5, 10, 15, 20, 25 ii) it is arithmetic, d = 5
   d) i) the first five terms are 5, 35, 245, 1715, 12 005 ii) it is geometric, r = 7
   e) i) the first five terms are 19, -18, 19, -18, 19 ii) it is neither arithmetic nor geometric
   f) i) the first five terms are 7, 13, 19, 25, 31 ii) it is arithmetic, d = 6

2. a) i) $t_n = (-9)(-11)^{n-1}$ ii) $t_1 = -9$; $t_n = -11t_{n-1}$
   b) i) $t_n = -579n + 1281$ ii) $t_1 = 702$; $t_n = t_{n-1} - 579$

3. a) n = 26
   b) n = 8

4. a) $(x - 5)^4 = x^4 - 20x^3 + 150x^2 - 500x + 625$
   b) $(2x + 3y)^3 = 8x^3 + 36x^2y + 54xy^2 + 27y^3$

5. a) n = 31, $S_{31} = 7099$
   b) $S_{10} = 259\ 586.8$

6. Find the first 7 terms and you will see a pattern repeating such that $t_{123} = 1.5$

7. a = 100, d= 75, n = 20, therefore $S_{20} = 16\ 250$