

MCR3UI Final Exam Preparation

I. Polynomials, Factoring, Rationals, and Inequalities

A. Simplify. Where appropriate, state any restrictions on the variables.

$$1. \frac{3x-3y}{5x-5y} \quad 2. \frac{a-2}{2a-3} - \frac{a+3}{3-2a} \quad 3. \frac{3m}{8} + \frac{m}{6} - \frac{2m}{3} \quad 4. \frac{t^2-9}{4t^2+10t-6} \div \frac{2t^2+2t-24}{2t^2+7t-4}$$

$$5. \frac{x+1}{x^2+4x-32} - \frac{2x-1}{3x^2-11x-4} \quad 6. 2(k-1)(k^2-3k+2) - (2k^2-3k+4)(2k+3)$$

B. Solve.

$$7. 6-2x > 4 \quad 8. \frac{4m-3}{6} + \frac{2m}{3} < -\frac{9}{2}$$

II. Radicals, Complex Numbers and Quadratics

A. Simplify.

$$9. (2\sqrt{3} + \sqrt{5})(\sqrt{3} - 4\sqrt{5}) \quad 10. 8\sqrt{5} - 3\sqrt{7} + 7\sqrt{7} - 4\sqrt{5} \quad 11. \frac{-12 + \sqrt{48}}{4} \quad 12. \frac{-8 + \sqrt{-32}}{2}$$

$$13. (4\sqrt{7})(2\sqrt{14}) \quad 14. \frac{12\sqrt{60}}{3\sqrt{5}} \quad 15. \frac{2}{\sqrt{7}} \quad 16. 3\sqrt{72} - 2\sqrt{98} + \sqrt{108} + (2\sqrt{14})(3\sqrt{2})$$

$$17. \frac{2}{3i} \quad 18. \frac{1+i}{3-5i} \quad 19. (2i)(6i) \quad 20. 2(5-2i) - i(3-i)$$

B. Solve, where $x \in C$.

$$21. (x+3)^2 = 16 \quad 22. x^2 + 128 = 0 \quad 23. 12x^2 + 13x + 3 = 0$$

24. Find the minimum value of $y = x^2 - 5x$ and state the value of x when it occurs.

25. A rancher wishes to build a rectangular corral using 50 m of fencing. The corral is to be built with a wall of the barn as one side with the fencing enclosing the other three sides. What are the dimensions of the corral with the maximum possible area?

26. The function $h(t) = -5t^2 + 20t + 2$ gives the approximate height, h metres, of a thrown football as a function of time, t seconds, since it was thrown. The ball hit the ground before the receiver could get near it. How long was the ball in the air, to the nearest tenth of a second?

III. Transformations of Functions

27. Given $f(x) = 4x - 4x^2$, find $f(-1)$.

28. If $f(x) = x^2 + 6x$ and $f(x) = 16$, find the value(s) of x .

29. Given $f(x) = x^2 - 4x$, write the equation for:

a) $y = f(-x)$

b) $y = -f(x)$

30. Given $f(x) = x^2 - 3$,

a) find the equation of the inverse algebraically

d) state the domain and range of f^{-1}

b) graph f and f^{-1} on the same set of axes

e) restrict the domain of f so that f^{-1} is a function

c) determine, with reasons, whether the inverse is a function

31. The point (x, y) on $y = \sqrt{x}$ is transformed to what point on the graph of $y = -2\sqrt{x+3} - 4$?

$$(x, y) \rightarrow (\text{_____}, \text{_____})$$

32. The graph of $y = x^2$ is vertically stretched by a factor of 2, vertically translated downward 4 units, and horizontally translated 5 units to the right. What is the equation of the transformed function?

33. The graph of $y = \sqrt{x}$ is horizontally compressed by a factor of $\frac{1}{3}$, reflected in the y-axis, and horizontally translated 2 units to the left. What is the equation of the transformed function?

34. Sketch the graphs of each of the following functions using transformations. For each function, state the transformations in order on the base (parent) function, give the domain and range, and be sure to label any axes or asymptotes on the graph.

i) $y = -2(x+3)^2 - 2$

ii) $y = -\sqrt{2x+4} - 3$

iii) $y = \frac{1}{2}(3)^{-\frac{1}{2}x+2} + 4$

IV. Trigonometry

35. Solve $\triangle ABC$ where $\angle C = 95^\circ$, $a = 2.3$ cm and $b = 4.5$ cm.

36. Solve $\triangle XYZ$ where $x = 6.0$ m, $y = 7.3$ m and $z = 4.8$ m.

37. Solve $\triangle PQR$ where $\angle Q = 63^\circ$, $r = 17.0$ cm and $q = 20.0$ cm.

38. Solve $\triangle ABC$ where $\angle A = 28^\circ$, $c = 3.2$ mm and $a = 2.4$ mm.

39. Convert: a) 210° to radians b) $\frac{5\pi}{6}$ to degrees

40. The point $Q(12, -5)$ lies on the terminal arm of θ , an angle in standard position. Find $\cos \theta$ and $\angle \theta$, to the nearest degree.

41. Find the exact value of each of the following:

a) $\cos(-225^\circ)$ b) $\sin \frac{4\pi}{3}$ c) $\cos\left(-\frac{\pi}{6}\right)$
 d) $\tan(-2\pi)$ e) $\cos 120^\circ$ f) $\sin\left(\frac{\pi}{2}\right)$

42. If $\sin \beta = -0.8950$, find the value(s) of $\angle \beta$, an angle in standard position where $0 \leq \beta \leq 360^\circ$. Round to one decimal place.

43. If $\tan \theta = -\frac{1}{\sqrt{3}}$, find the value(s) of $\angle \theta$, an angle in standard position where $0 \leq \theta \leq 2\pi$. Give exact values.

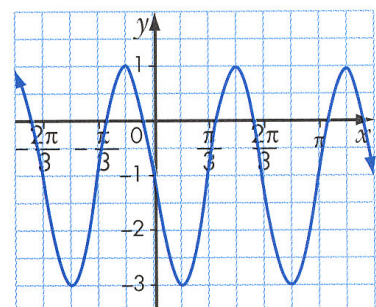
V. Trigonometric Functions

44. For $y = 2 \cos(3x - \pi) + 5$, state the:

a) amplitude: _____ b) period: _____ c) phase shift: _____ d) vertical translation: _____

45. Sketch $y = -3 \sin(2x + \frac{\pi}{2}) - 1$, $-2\pi \leq x \leq 2\pi$. Label the equilibrium axis and state the range for this function.

46. State a possible trigonometric equation for the graph provided to the right.



47. Prove each identity.

a) $\tan^2 x - 2 \tan x + 1 = \frac{1 - 2 \sin x \cos x}{\cos^2 x}$

b) $\frac{\cos x - \sin x}{\cos^2 x} = \frac{1 - \tan x}{\cos x}$

48. Solve for x , $0 \leq x \leq 2\pi$. Give exact solutions, where possible, otherwise round to the nearest hundredth of a radian.

a) $\sqrt{2} \sin x - 1 = 0$

b) $2 \sin x - \sin x \cos x = 0$

c) $4 \sin^2 x + 4 \cos x - 5 = 0$

VI. Exponential Functions

A. Simplify. No decimal answers (keep improper fractions reduced to lowest terms). All answers should have positive exponents.

49. $\frac{3^{-1}}{3^{-2} + 3^0}$

50. $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$

51. $\left(\frac{(2u^{-6}v^3)(8uv^4)}{-4u^2v^{-5}}\right)^{-2}$

52. $\left(\frac{(10x^{-3}y^4)^2}{5x^2y^{-2}}\right)^{-3}$

53. $\frac{\sqrt[3]{x^{-12}(x^2)^{-9}}}{(x^3)^{-\frac{2}{3}}}$

B. Solve for x , where $x \in R$.

54. $3^{3x+5} = \frac{1}{81}$

55. $\frac{125^{x+2}}{25^{x+3}} = 625^{x-3}$

56. $6^{x+2} + 6^x = 222$

C. Calculate:

57. A 200 g sample of radium-226 has a half-life of 1590 years. This means that every 1590 years, the amount of radium left in a sample is half of the original amount.

- a) Write an equation using function notation that models the mass M of radium, in grams, that remains after t years.
b) Determine the mass that remains after 5000 years, to one decimal place.

58. \$3750 is invested at 6% per annum, compounded quarterly. What is the accumulated value after five years?

VII. Sequences and Series

59. Given $t_n = \frac{n-2}{2}$, find t_{12} .

60. Write the first four terms for each of the following: a) $t_n = 4 + 3n$

b) $t_1 = 2; t_n = (t_{n-1})^2$

61. Find the general formula for the n^{th} term, t_n , for each of the following: a) 729, -243, 81, ...

b) 3, 7, 11, 15, ...

62. Write an explicit formula for the n^{th} term, t_n , for each of the following: a) $t_1 = 2; t_n = 3t_{n-1}$

b) $t_1 = 11; t_n = t_{n-1} + 14$

63. Find the **tenth** term for each of the following: a) -7, 5, 17, 29 ...

b) -1, 5, -25, 125, ...

64. Find S_6 , correct to the nearest hundredth of $8 + 2 + \frac{1}{2} + \frac{1}{8} + \dots$

65. Find the sum of $5 + 11 + 17 + 23 + \dots + 821$.

66. Expand and simplify each binomial power (draw and use Pascal's triangle and the pattern we developed for expanding a binomial):

a) $(x-1)^5$

b) $(3x+2y)^4$

Answers:

I. Polynomials, Factoring, Rationals, and Inequalities

A. *Simplify.* Where appropriate, state any restrictions on the variables.

1. $\frac{3}{5}, x \neq y$

2. $\frac{2a+1}{2a-3}, a \neq \frac{3}{2}$

3. $-\frac{m}{8}$

4. $\frac{1}{4}, t \neq -4, \pm 3, \frac{1}{2}$

5. $\frac{x^2 - 11x + 9}{(x+8)(x-4)(3x+1)}, x \neq -8, -\frac{1}{3}, 4$

6. $-2k^3 - 8k^2 + 11k - 16$

B. *Solve.*

7. $x < 1$

8. $m < -3$

II. Radicals, Complex Numbers and Quadratics

A. *Simplify.*

9. $-7(2 + \sqrt{15})$

10. $4(\sqrt{5} + \sqrt{7})$

11. $-3 + \sqrt{3}$

12. $-4 + 2i\sqrt{2}$

13. $56\sqrt{2}$

14. $8\sqrt{3}$

15. $\frac{2\sqrt{7}}{7}$

16. $4\sqrt{2} + 6\sqrt{3} + 12\sqrt{7}$

17. $\frac{-2i}{3}$

18. $\frac{-1+4i}{17}$

19. -12

20. $9 - 7i$

B. *Solve,* where $x \in \mathbb{C}$.

21. $x = -7, 1$

22. $x = \pm 8i\sqrt{2}$

23. $x = -\frac{3}{4}, -\frac{1}{3}$

24. $\min = -\frac{25}{4}$ at $x = \frac{5}{2}$

25. $25m$ by $12.5m$; $\text{Area} = 312.5m^2$

26. $t = 4.1$ s

III. Transformations of Functions

27. $f(-1) = -8$

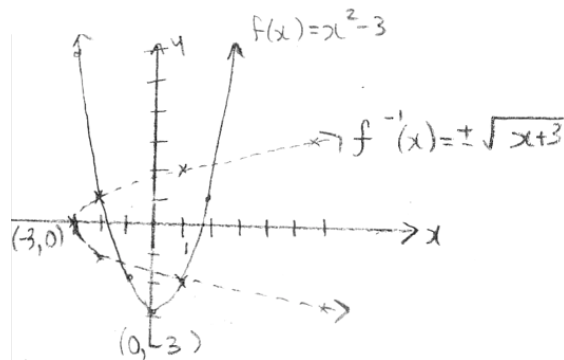
28. $x = -8, 2$

29. a) $y = x^2 + 4x$

29. b) $y = -x^2 + 4x$

30. a) $f^{-1} = \pm\sqrt{x+3}$

30. b)



30. c) *No. Fails the vertical line test.*

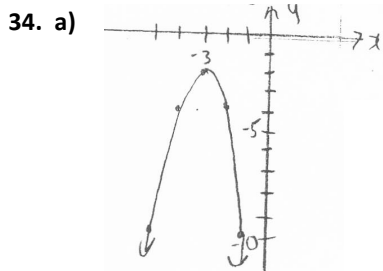
30. d) $D_{f^{-1}} = \{x \in \mathbb{R} \mid x \geq -3\}, R_{f^{-1}} = \{y \in \mathbb{R}\}$

30. e) $D_f = \{x \in \mathbb{R} \mid x \geq 0\}$ or $D_f = \{x \in \mathbb{R} \mid x \leq 0\}$

31. $(x, y) \rightarrow (x-3, -2y-4)$

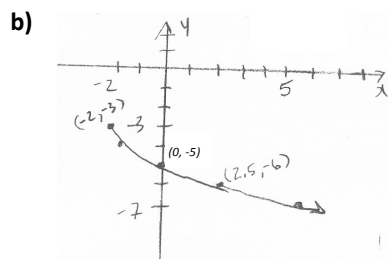
32. $y = 2(x-5)^2 - 4$

33. $y = \sqrt{-3(x+2)}$



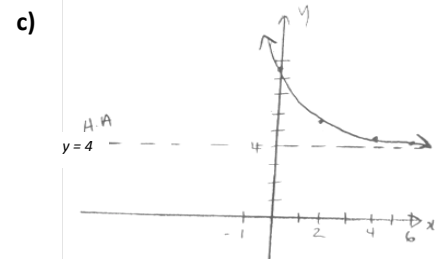
$D = \{x \mid x \in \mathbb{R}\}, R = \{y \mid y \in \mathbb{R}, y \leq -2\}$

- i) VR across the x-axis
- ii) VE by a factor of 2
- iii) HT 3 units left
- iv) VT 2 units down



$D = \{x \mid x \in \mathbb{R}, x \geq -2\}, R = \{y \mid y \in \mathbb{R}, y \leq -3\}$

- i) VR across the x-axis
- ii) HC by a factor of $\frac{1}{2}$
- iii) HT 2 units left
- iv) VT 3 units down



$D = \{x \mid x \in \mathbb{R}\}, R = \{y \mid y \in \mathbb{R}, y > 4\}$

- i) VC by a factor of $\frac{1}{2}$
- ii) HR across the y-axis
- iii) HE by a factor of 2
- iv) HT 4 units right
- v) VT 4 units up

IV. Trigonometry

35. $\angle A = 26^\circ$, $\angle B = 59^\circ$, $c = 5.2 \text{ cm}$

36. Find angle Y first (largest angle). $\angle Y = 84^\circ$, $\angle X = 55^\circ$, $\angle Z = 41^\circ$

37. $\angle R = 49^\circ$, $\angle P = 68^\circ$, $p = 20.8 \text{ cm}$

38. $\angle B = 113^\circ$, $\angle C = 39^\circ$, $b = 4.7 \text{ cm}$ or $\angle B = 11^\circ$, $\angle C = 141^\circ$, $b = 1.0 \text{ cm}$

39. a) $\frac{7\pi}{6}$ b) 150°

40. $\cos \theta = \frac{12}{13}$, $\theta = 337^\circ$

41. a) $-\frac{1}{\sqrt{2}}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 0 e) $-\frac{1}{2}$ f) 1

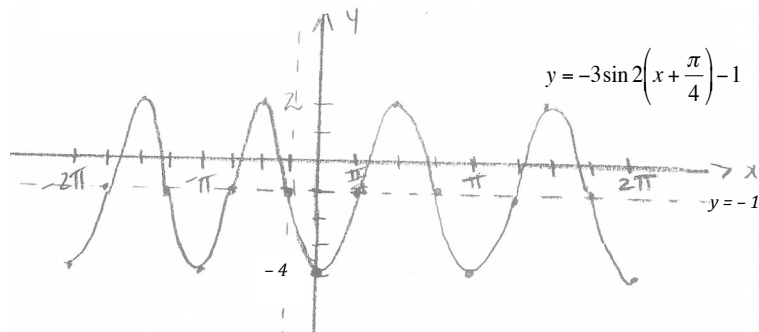
42. $\beta = 243.5^\circ$, 296.5°

43. $\tan \theta = -\frac{1}{\sqrt{3}}$, $\theta = \frac{5\pi}{6}$, $\frac{11\pi}{6}$

V. Trigonometric Functions

44. a) $A = 2$ b) $P = \frac{2\pi}{3}$ c) $PS = \frac{\pi}{3}$ radians right d) $VT = 5$ units up

45. $y = -3\sin 2\left(x + \frac{\pi}{4}\right) - 1$
 reflection: in x -axis
 amplitude: 3
 period: π
 phase shift: left $\frac{\pi}{4}$
 vertical translation: down 1
 our scale: $\frac{\pi}{4}$



$R = \{y \mid y \in \mathbb{R}, -4 \leq y \leq 2\}$

46. $y = -2\sin 3x - 1$ or $y = 2\sin 3\left(x - \frac{\pi}{3}\right) - 1$ or $y = -2\cos 3\left(x - \frac{\pi}{6}\right) - 1$ or $y = 2\cos 3\left(x - \frac{\pi}{2}\right) - 1$

47. a) L.S. = $\tan^2 x - 2\tan x + 1$ R.S. = $\frac{1 - 2\sin x \cos x}{\cos^2 x}$
 $= \frac{\sin^2 x}{\cos^2 x} - \frac{2 \cdot \sin x}{1 \cdot \cos x} + \frac{1}{1}$
 $= \frac{\sin^2 x - 2\sin x \cdot \cos x + \cos^2 x}{\cos^2 x}$
 $= \frac{(\sin^2 x + \cos^2 x) - 2\sin x \cdot \cos x}{\cos^2 x}$
 $= \frac{1 - 2\sin x \cdot \cos x}{\cos^2 x}$
 $\therefore \text{L.S.} = \text{R.S.}$
 $\therefore \text{Q.E.D.}$

b) L.S. = $\frac{\cos x - \sin x}{\cos^2 x}$ R.S. = $\frac{1 - \frac{\sin x}{\cos x}}{\cos x}$
 $= \left[\frac{1 - \frac{\sin x}{\cos x}}{1} \right] \cdot \frac{\cos x}{1}$
 $= \frac{\cos x - \sin x}{\cos x} \cdot \frac{1}{\cos x}$
 $= \frac{\cos x - \sin x}{\cos^2 x}$
 $\therefore \text{L.S.} = \text{R.S.}$
 $\therefore \text{Q.E.D.}$

48. a) $x = \frac{\pi}{4}$, $\frac{3\pi}{4}$

b) $x = 0, \pi, 2\pi$

c) $x = \frac{\pi}{3}, \frac{5\pi}{3}$

VI. Exponential Functions

A. Simplify.

49. $\frac{3}{10}$

50. $\frac{9}{4}$

51. $\frac{u^{14}}{16v^{24}}$

52. $\frac{x^{24}}{8000y^{30}}$

53. $\frac{1}{x^8}$

B. Solve for x , where $x \in R$.

54. $x = -3$

55. $x = 4$

56. $x = 1$

C. Calculate.

57. a) $M(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{1590}}$

b) $M(5000) = 22.6 \text{ g}$

58. \$5050.71

VII. Sequences and Series

59. $t_{12} = 5$

60. a) 7, 10, 13, 16

b) 2, 4, 16, 256

61. a) $t_n = 729\left(-\frac{1}{3}\right)^{n-1}$

b) $t_n = 4n - 1$

62. a) $t_n = 2(3)^{n-1}$

b) $t_n = 14n - 3$

63. a) $t_{10} = 101$

b) $t_{10} = 1 \ 953 \ 125$

64. $S_6 = \frac{1365}{128} \approx 10.66$

65. $n = 137, S_{137} = 56 \ 581$

66. a) $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$

b) $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$

The End

