Date:____________ **UNIT 2: QUADRATIC EQUATIONS & APPLICATIONS**

2.1 Solving Quadratic Equations: Two More Methods

1. Solve using ***inverse operations*** to isolate the variable when it appears only once, $x \in C$.

a) $x^2 + 36 = 0$

$x^2 = -36$

$x = \pm\sqrt{-36}$

$x = \pm 6i$

b) $-2x^2 + 24 = 0$

$-2x^2 = -24$

$x^2 = 12$

$x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

c) $36(x+2)^2 - 4 = 0$

$36(x+2)^2 = 4$

$(x+2)^2 = \frac{1}{9}$

$x+2 = \pm\frac{1}{3}$

$x+2 = -\frac{1}{3}$ or $x+2 = \frac{1}{3}$

$x = -2\frac{1}{3}$ $\quad$ $x = \frac{1}{3} - \frac{6}{3}$

$x = -\frac{5}{3}$

2. Solve using the ***quadratic formula*** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $ax^2 + bx + c = 0$ and $x \in C$.

a) $\frac{x-1}{1} = \frac{2x+3}{x-1}$, $x \neq 1$ crossmultiply

$(x-1)(x-1) = 1(2x+3)$

$x^2 - 2x + 1 = 2x + 3$

$x^2 - 4x - 2 = 0$

$a=1,\ b=-4,\ c=-2$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{4 \pm \sqrt{24}}{2}$

$x = \frac{4 \pm 2\sqrt{6}}{2}$

$x = 2 \pm \sqrt{6}$

b) $\frac{x^2}{2} + \frac{x}{3} = -1$

$\times 6)$ $3x^2 + 2x = -6$

$3x^2 + 2x + 6 = 0$

$a=3,\ b=2,\ c=6$

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(6)}}{2(3)}$

$x = \frac{-2 \pm \sqrt{-68}}{6}$

$x = \frac{-2 \pm 2i\sqrt{17}}{6}$

$x = \frac{\cancel{2}^{1}(-1 \pm i\sqrt{17})}{\cancel{6}_{3}}$

$x = \frac{-1 \pm i\sqrt{17}}{3}$

3. Write a ***quadratic equation*** in expanded form with integer coefficients having the given roots. ← x-intercepts or solutions

a) $-3,\ \frac{3}{4}$

$x = -3$ or $x = \frac{3}{4}$

$x+3=0$ or $4x-3=0$

$(x+3)(4x-3)=0$

$\therefore 4x^2 + 9x - 9 = 0$ is the quadratic equation with the given roots.

b) $-1-2\sqrt{3}\,i,\ -1+2\sqrt{3}\,i$

$x = -1-2\sqrt{3}i$ or $x = -1+2\sqrt{3}i$

$x+1+2\sqrt{3}i = 0$ or $x+1-2\sqrt{3}i = 0$

$(\underbrace{x+1}_{a} \underbrace{+}_{+} \underbrace{2\sqrt{3}i}_{b})(\underbrace{x+1}_{a} \underbrace{-}_{-} \underbrace{2\sqrt{3}i}_{b}) = 0$

$(x+1)^2 - (2\sqrt{3}i)^2 = 0$

$(x+1)(x+1) - (2)^2(\sqrt{3})^2(i)^2 = 0$

$x^2 + 2x + 1 - 4(3)(-1) = 0$

$x^2 + 2x + 1 + 12 = 0$

$\therefore x^2 + 2x + 13 = 0$ is the required equation.

HW. Exercise 2.1

Date:____________ **2.2 Solving Cubic and Quartic Equations**

Note: *A polynomial of the n^{th} degree has n roots.*

Ex. Solve for x in each of the following, $x \in C$.

a) $-4x^3 - 18x^2 + 10x = 0$ ← 3 roots

$-2x(2x^2 + 9x - 5) = 0$

$\therefore x = 0$ or $2x^2 + 9x - 5 = 0$

$(2x-1)(x+5) = 0$

$\therefore x = \frac{1}{2}$ or $x = -5$

$\therefore$ the solutions are $x = -5$ or $x = 0$ or $x = \frac{1}{2}$

b) $x^4 - 24x^2 - 25 = 0$ ← 4 roots

$(x^2 - 25)(x^2 + 1) = 0$

$x^2 - 25 = 0$ or $x^2 + 1 = 0$

$x^2 = 25$ $\quad x^2 = -1$

$x = \pm 5$ $\quad x = \pm\sqrt{-1}$

$x = \pm i$

$\therefore$ the solutions are $x = -5$ or $x = 5$ or $x = -i$ or $x = i$

c) $\underbrace{3x^3 + x^2}\underbrace{+24x + 8} = 0$ ← 3 roots

$x^2(3x+1) + 8(3x+1) = 0$

$(3x+1)(x^2+8) = 0$

$3x + 1 = 0$ or $x^2 + 8 = 0$

$x = -\frac{1}{3}$ $\quad x^2 = -8$

$x = \pm\sqrt{-8}$

$x = \pm 2i\sqrt{2}$

$\therefore$ the solutions are $x = -\frac{1}{3}$ or $x = -2i\sqrt{2}$ or $x = 2i\sqrt{2}$

d) $(x^2 - 5x)^2 - 2(x^2 - 5x) - 24 = 0$

Let $y = x^2 - 5x$

$y^2 - 2y - 24 = 0$

$(y-6)(y+4) = 0$

$\therefore y - 6 = 0$ or $y + 4 = 0$

$x^2 - 5x - 6 = 0$ $\quad x^2 - 5x + 4 = 0$

$(x-6)(x+1) = 0$ $\quad (x-4)(x-1) = 0$

$\therefore x = -1$ or $x = 1$ or $x = 4$ or $x = 6$

e) $\left(x + \frac{1}{x}\right)^2 - 7\left(x + \frac{1}{x}\right) + 12 = 0$

Let $a = x + \frac{1}{x}$

$a^2 - 7a + 12 = 0$

$(a-4)(a-3) = 0$

$a - 4 = 0$ or $a - 3 = 0$

$x + \frac{1}{x} - 4 = 0$ $\quad x + \frac{1}{x} - 3 = 0$

$\cdot x)$ $x^2 - 4x + 1 = 0$

$a=1,\ b=-4,\ c=1$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{12}}{2}$

$x = \frac{4 \pm 2\sqrt{3}}{2}$

$x = 2 \pm \sqrt{3}$

$\therefore x = 2 - \sqrt{3}$ or $x = 2 + \sqrt{3}$

$\cdot x)$ $x^2 - 3x + 1 = 0$

$a=1,\ b=-3,\ c=1$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{3 \pm \sqrt{5}}{2}$

$\therefore x = \frac{3 - \sqrt{5}}{2}$ or $x = \frac{3 + \sqrt{5}}{2}$

HW. Exercise 2.2

Date:______________ **2.3 Solving Linear and Quadratic Inequalities**

Complete the table for the ***inequality* 9 > 6**:

Operation:	Add 3	Subtract 3	Multiply by 3	Divide by 3	Multiply by -3	Divide by -3
Resulting Inequality:	$12>9$	$6<3$	$27>18$	$3>2$	$-27<-18$	$-3<-2$

Rule:
*When **multiplying** or **dividing** an inequality by a **negative**, **change the direction** of the inequality sign.*

1. Solve each of the following ***linear inequalities*** and ***graph*** the solution on a number line.

a) $4x-1<11$

$4x<12$

$\therefore x<3$

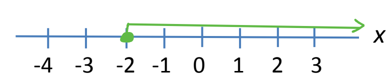
b) $3(2-x)+1\le 13$

$6-3x+1\le 13$

$-3x+7\le 13$

$-3x\le 6$

$\therefore x\ge -2$

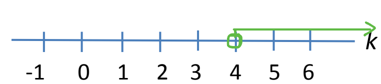


c) $\frac{3}{4}k+\frac{1}{2}k>5$

$\cdot 4)$ $3k+2k>20$

$5k>20$

$\therefore k>4$



d) $\frac{z-1}{5}\ge\frac{z+2}{4}-1$

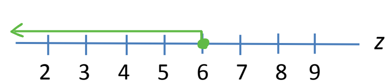
$\cdot 20)$ $4(z-1)\ge 5(z+2)-20$

$4z-4\ge 5z+10-20$

$4z-4\ge 5z-10$

$-z\ge -6$

$z\le 6$

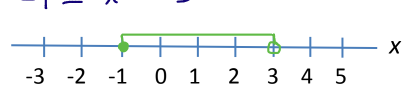


e) $-3\le 2x-1<5$

$-3+1\le 2x-1+1<5+1$

$-2\le 2x<6$

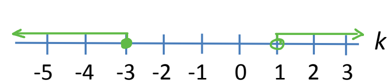
$-1\le x<3$



f) $2-k\ge 5$ *or* $2-3k<-1$

$-k\ge 3$ or $-3k<-3$

$\therefore k\le -3$ $k>1$



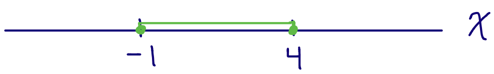
2. Solve each of the following ***quadratic inequalities*** by following the steps below and ***graph*** the solution on a number line.

- *Rearrange to compare* ax^2+bx+c *to* 0, with $a>0$.
- *Find the zeros of the corresponding quadratic equation and graph accordingly.*
- *If* $ax^2+bx+c<0$, *the solution is between the zeros.*
- *If* $ax^2+bx+c>0$, *the solution is outside the zeros.*

a) $x^2-3x-4\le 0$

$(x-4)(x+1)\le 0$

zeros are $x=-1$ and $x=4$



$\therefore -1\le x\le 4$

b) $k^2+5k>0$

$k(k+5)>0$

zeros are $k=-5$ and $k=0$

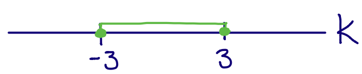
$\therefore k<-5$ or $k>0$

c) $-2k^2+18>0$

$k^2-9<0$

$(k-3)(k+3)<0$

zeros are $k=-3$ and $k=3$



$\therefore -3\le k\le 3$

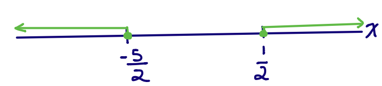
d) $-4x^2-8x\le -5$

$-4x^2-8x+5\le 0$

$4x^2+8x-5\ge 0$

$(2x+5)(2x-1)\ge 0$

zeros are $x=-\frac{5}{2}$ and $x=\frac{1}{2}$



$\therefore x\le -\frac{5}{2}$ or $x\ge \frac{1}{2}$

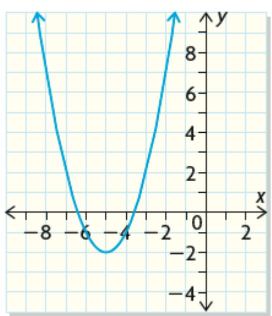
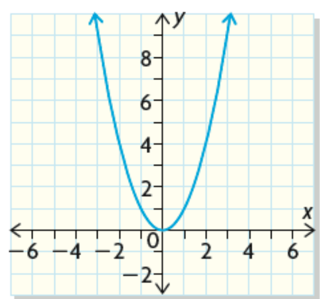
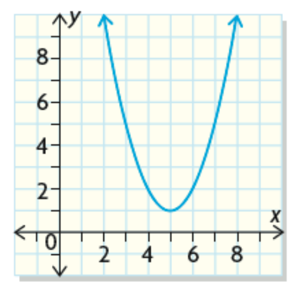
HW. Exercise 2.3

Date:______________

2.4 The Discriminant

Recall: *If* $ax^2+bx+c=0$, *and* $a\neq 0$, *then* $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ ⟵ ***Quadratic Formula***

Definition: In the quadratic formula, the ***discriminant*** is the quantity b^2-4ac under the radical sign. The ***discriminant***, b^2-4ac can determine the nature of the roots to the corresponding quadratic equation and the number of x-intercepts to the corresponding quadratic relation. The ***nature*** of the two roots can be one of three types based on the value of the ***discriminant***, b^2-4ac.

two distinct (different) real roots	two equal (same) real roots	two non-real (imaginary) roots
$b^2-4ac>0$	$b^2-4ac=0$	$b^2-4ac<0$
two *x*-intercepts	**one *x*-intercept**	**no *x*-intercepts**
		

Note: *If the **discriminant**,* b^2-4ac, *is a positive perfect square, the quadratic equation can be solved by factoring.*

1. Use the ***discriminant*** to determine the nature of the roots for each of the following.

a) $16x^2-16x+4=0$

$\div 4)$ $4x^2-4x+1=0$

$a=4,\ b=-4,\ c=1$

$b^2-4ac=(-4)^2-4(4)(1)$

$=16-16$

$=0$

$\because b^2-4ac=0$

$\therefore$ there are two equal real roots.

b) $1-2x-3x^2=0$

$-3x^2-2x+1=0$

$a=-3,\ b=-2,\ c=1$

$b^2-4ac=(-2)^2-4(-3)(1)$

$=4+12$

$=16$

$\because b^2-4ac>0$

$\therefore$ there are two distinct real roots.

c) $3=2\sqrt{2}x-x^2$

$x^2-2\sqrt{2}x+3=0$

$a=1,\ b=-2\sqrt{2},\ c=3$

$b^2-4ac=(-2\sqrt{2})^2-4(1)(3)$

$=8-12$

$=-4$

$\because b^2-4ac<0$

$\therefore$ there are two non-real roots.

2. Determine the value(s) of k that will give the indicated types of roots.

a) $kx^2 - 2x + 1 = 0$; *distinct real roots*

$b^2 - 4ac > 0$ $\quad a = k,\ b = -2,\ c = 1$

$(-2)^2 - 4(k)(1) > 0$

$4 - 4k > 0$

$-4k > -4$

$k < 1$

b) $(1-3k)x^2 + 4x - 4 = 0$; *equal real roots*

$b^2 - 4ac = 0$ $\quad a = 1-3k,\ b = 4,\ c = -4$

$(4)^2 - 4(1-3k)(-4) = 0$

$16 + 16(1-3k) = 0$

$16 + 16 - 48k = 0$

$32 - 48k = 0$

$-48k = 32$

$k = -\frac{32}{48}$

$k = -\frac{2}{3}$

c) $2x^2 - kx + 8 = 0$; *non-real roots*

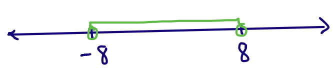
$b^2 - 4ac < 0$ $\quad a = 2,\ b = -k,\ c = 8$

$(-k)^2 - 4(2)(8) < 0$

$k^2 - 64 < 0$

$(k-8)(k+8) < 0$

zeros are $k = -8$ and $k = 8$



$\therefore -8 < k < 8$

d) $(2k-1)x^2 + (3k+2)x + (k-1) = 0$; *real roots*

$b^2 - 4ac \geq 0$ $\quad a = 2k-1,\ b = 3k+2,\ c = k-1$

$(3k+2)^2 - 4(2k-1)(k-1) \geq 0$

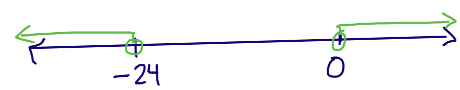
$9k^2 + 12k + 4 - 4(2k^2 - 3k + 1) \geq 0$

$9k^2 + 12k + 4 - 8k^2 + 12k - 4 \geq 0$

$k^2 + 24k \geq 0$

$k(k+24) \geq 0$

zeros are $k = -24$ and $k = 0$



$\therefore k < -24$ or $k > 0$.

HW. Exercise 2.4