Date:______________ **2.5 Applications: Quadratic Equations**

1. The height, h, in metres of an object thrown off the top of a cliff after t seconds is given by the following equation: $h = -5t^2 + 20t + 80$

a) Use the ***discriminant*** to determine if the object ever reaches a height of 110 m.

Find t if $h = 110$:

$110 = -5t^2 + 20t + 80$

$5t^2 - 20t + 30 = 0$

$a = 5,\ b = -20,\ c = 30$

$b^2 - 4ac$

$= (-20)^2 - 4(5)(30)$

$= 400 - 600$

$= -200$

$\because b^2 - 4ac < 0$

$\therefore$ no real roots

$\therefore$ the object will not reach 110m.

b) Exactly when does the object hit the ground?

Find t if $h = 0$:

$0 = -5t^2 + 20t + 80$

$0 = -5(t^2 - 4t - 16)$

$a = 1,\ b = -4,\ c = -16$

$t = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$

$t = \dfrac{4 \pm \sqrt{80}}{2}$

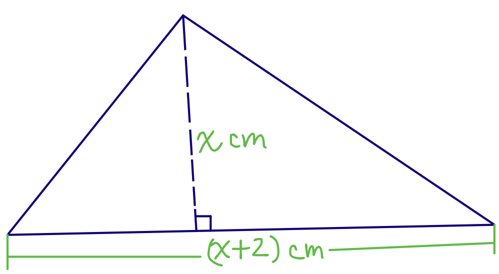
$t = \dfrac{4 \pm 4\sqrt{5}}{2}$

$t = 2 \pm 2\sqrt{\ }$

$\therefore \underbrace{t = 2 - 2\sqrt{5}}_{\text{inadmissable}}$ or $t = 2 + 2\sqrt{5}$

$\therefore$ the object hits the ground at $t = 2 + 2\sqrt{5}$ seconds.

2. The base of a triangle is 2 cm more than the height. If the area is 5 cm^2, find the exact length of the base.



Let x cm represent the height of a triangle and $(x+2)$ cm represent the base.

$A = \frac{1}{2}bh$

$5 = \frac{1}{2}(x)(x+2)$

$5 = \frac{1}{2}(x^2 + 2x)$

$10 = x^2 + 2x$

$0 = x^2 + 2x - 10$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-10)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{44}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{11}}{2}$

$x = -1 \pm \sqrt{11}$

$\underbrace{x = -1 - \sqrt{11}}_{\text{inadmissable}}$ or $x = -1 + \sqrt{11}$

$\because x \geq 0$

If $x = -1 + \sqrt{11}$ then $b = x + 2$

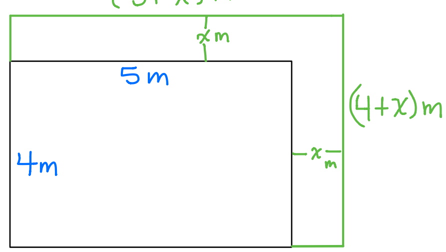
$= -1 + \sqrt{11} + 2$

$= 1 + \sqrt{11}$

$\therefore$ the base is $1 + \sqrt{11}$ cm.

3. A dining room measures 5 m by 4 m. A strip of uniform width is added to two adjacent sides to increase the area by 5 m^2. Find the width of the strip to 1 decimal place.

Let x represent the width of the strip, in m.



$A_{original} = 20 m^2$

$A_{final} = 25 m^2$

$A = lw$

$A_{final} = l_{final} \times w_{final}$

$25 = (4+x)(5+x)$

$25 = 20 + 4x + 5x + x^2$

$0 = x^2 + 9x - 5$

$a = 1,\ b = 9,\ c = -5$

$x = \frac{-9 \pm \sqrt{(9)^2 - 4(1)(-5)}}{2(1)}$

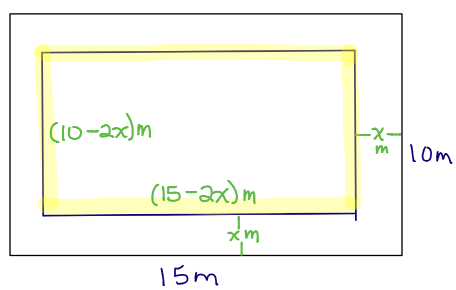
$x = \frac{-9 \pm \sqrt{101}}{2}$

$x \doteq 0.5,\ \underbrace{x \doteq -9.5}_{\text{inadmissable}}$

$\therefore$ the width of the strip is approx 0.5m

4. A uniform-width boardwalk is built around the inside edge of a rectangular park that measures 15 m by 10 m. If the boardwalk takes up 20% of the lot, how wide is the boardwalk to the nearest centimetre?

Let x represent the width of the boardwalk, in m.



$A_{lot} = 150 m^2$

20% of $A_{lot} = 0.2 \times 150$
$= 30 m^2$

$A_{inner} = 150 - 30$
$= 120 m^2$

$A_{inner} = l_{inner} \times w_{inner}$

$120 = (15-2x)(10-2x)$

$120 = 150 - 50x + 4x$

$0 = 4x^2 - 50x + 30$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{50 \pm \sqrt{(-50)^2 - 4(4)(30)}}{2(4)}$

$x = \frac{50 \pm \sqrt{2020}}{8}$

$\therefore x \doteq 0.63,\ \underbrace{x \doteq 11.87}_{\text{inadmissable}}$

$\therefore$ the width of the boardwalk is approx. 63 cm.

HW. Exercise 2.5

Date:______________ **2.6 Graphing Quadratic Relations Given Any Form**

A. Vertex Form of a Quadratic Relation: $y = a(x-h)^2 + k$

- *The vertex is* (h, k).
- *The parabola opens up if* $a > 0$ *and the vertex is a minimum.*
- *The parabola opens down if* $a < 0$ *and the vertex is a maximum.*
- *The parabola is congruent to* $y = |a|x^2$.

1. For each of the following quadratic relations, state the vertex of the parabola, the direction of opening, the equation of the parabola it's congruent to and the maximum or minimum value of the relation and when it occurs. Graph each relation on the grids below.

a) $y = (x+2)^2 - 4$

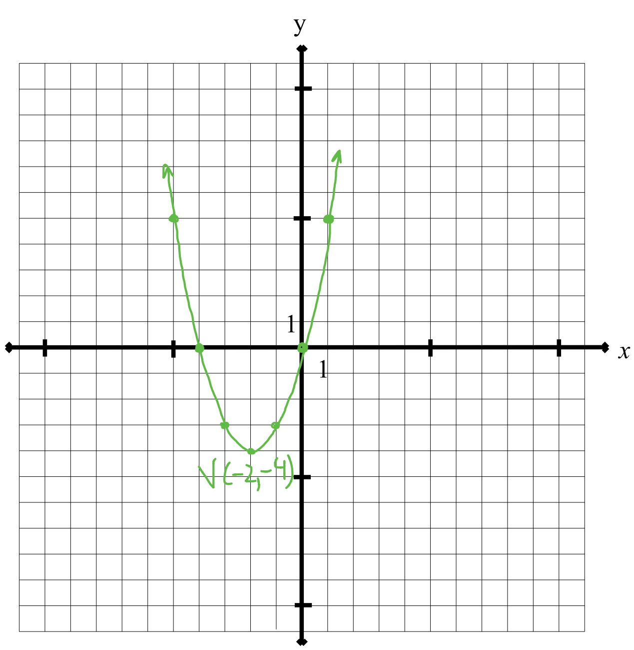
i) *vertex:* $(-2, -4)$

ii) *opens:* up

iii) *congruent to:* $y = x^2$

iv) *The* minimum *value of* y *is* -4 *when* $x =$ -2.

From the vertex $(-2,-4)$
1 over 1^2 up
2 over 2^2 up
3 over 3^2 up



b) $y = -3(x-5)^2 + 1$

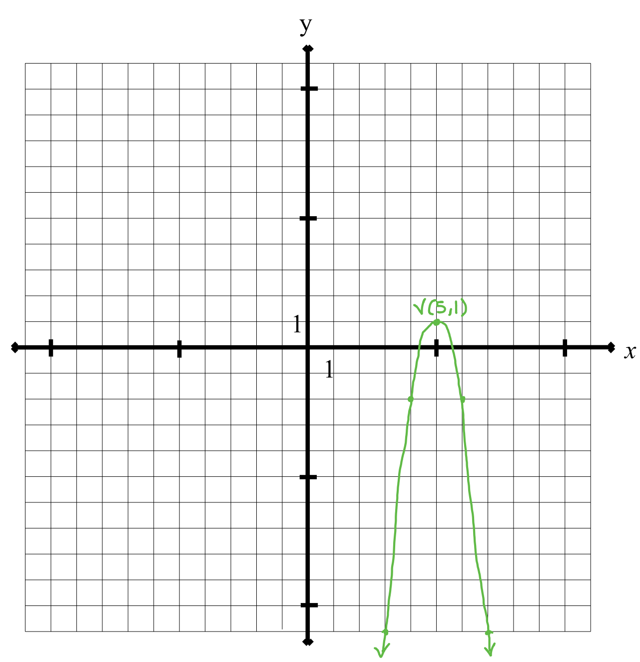
i) *vertex:* $(5, 1)$

ii) *opens:* down

iii) *congruent to:* $y = 3x^2$

iv) *The* maximum *value of* y *is* 1 *when* $x =$ 5.

From the vertex
1 over 3×1^2 down
2 over 3×2^2 down
3 over 3×3^2 down



B. Factored Form of a Quadratic Relation: $y = a(x-r)(x-s)$

- *The x-intercepts are r and s.*
- *At the vertex,* (x, y), $x = \dfrac{r+s}{2}$.
- *The parabola opens up if* $a > 0$ *and the vertex is a minimum.*
- *The parabola opens down if* $a < 0$ *and the vertex is a maximum.*
- *The parabola is congruent to* $y = |a|x^2$.

2. For each of the following quadratic relations, state the x-intercepts and vertex of the parabola, the direction of opening, the equation of the parabola it's congruent to and the maximum or minimum value of the relation and when it occurs. Graph each relation on the grids below.

a) $y = 2(x-3)(x-7)$

i) *x-intercepts:* 3 & 7

ii) *vertex:* (5, −8)

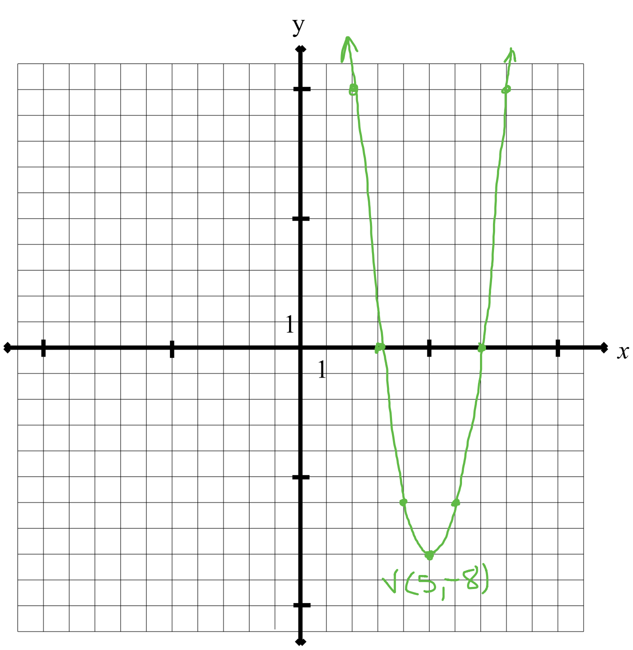
$x = \frac{3+7}{2}$
$x = 5$

$y = 2(5-3)(5-7)$
$y = 2(2)(-2)$
$y = -8$

iii) *opens:* up

iv) *congruent to:* $y = 2x^2$

v) *The* minimum *value of* y *is* −8 *when* $x =$ 5.



b) $y = -\frac{1}{2}x(x+4)$

i) *x-intercepts:* −4 & 0

ii) *vertex:* (−2, 2)

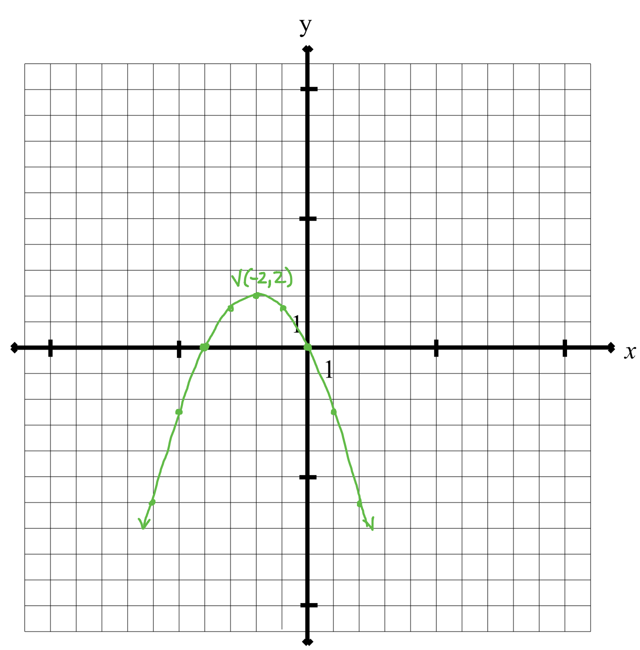
$x = \frac{-4+0}{2}$
$x = -2$

$y = -\frac{1}{2}(-2)(-2+4)$
$y = 1(2)$
$y = 2$

iii) *opens:* down

iv) *congruent to:* $y = \frac{1}{2}x^2$

v) *The* maximum *value of* y *is* 2 *when* $x =$ −2.



C. Standard Form of a Quadratic Relation: $y = ax^2 + bx + c$

- *Complete the square to express the relation in* ***Vertex Form*** .
- *The parabola opens up if* $a > 0$ *and the vertex is a minimum.*
- *The parabola opens down if* $a < 0$ *and the vertex is a maximum.*
- *The parabola is congruent to* $y = |a|x^2$.

3. For each of the following quadratic relations, express the relation in vertex form by completing the square, state the vertex of the parabola, the direction of opening, the equation of the parabola it's congruent to and the maximum or minimum value of the relation and when it occurs. Graph each relation on the grids below.

Vertex Form is $y = a(x-h)^2 + k$

a) $y = -x^2 + 4x - 7$

$y = -(x^2 - 4x) - 7$

$y = -(\underbrace{x^2 - 4x + 4}_{} - 4) - 7$

$y = -[(x-2)^2 - 4] - 7$

$y = -(x-2)^2 + 4 - 7$

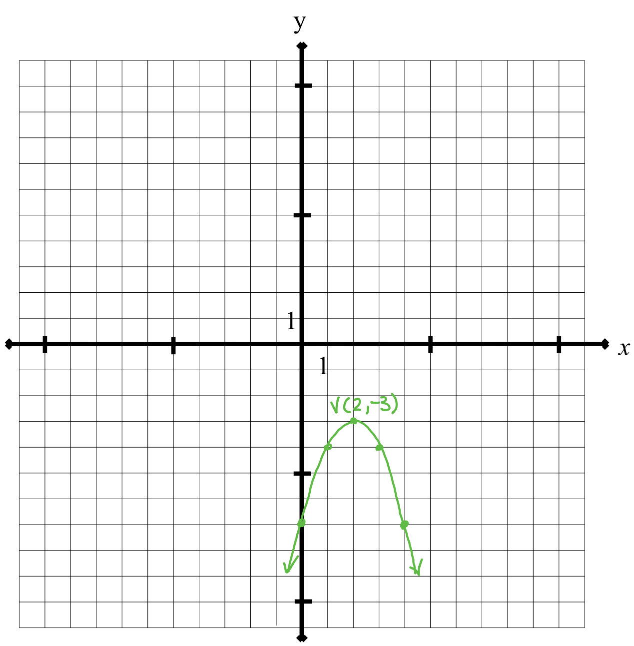
$y = -(x-2)^2 - 3$

i) *vertex:* $(2, -3)$

ii) *opens:* down

iii) *congruent to:* $y = x^2$

iv) *The* maximum *value of y is* -3 *when* $x =$ 2 .



b) $y = \frac{1}{3}x^2 + 2x + 3$

$y = \frac{1}{3}(x^2 + 6x) + 3$

$y = \frac{1}{3}(x^2 + 6x + 9 - 9) + 3$

$y = \frac{1}{3}[(x+3)^2 - 9] + 3$

$y = \frac{1}{3}(x+3)^2 - 3 + 3$

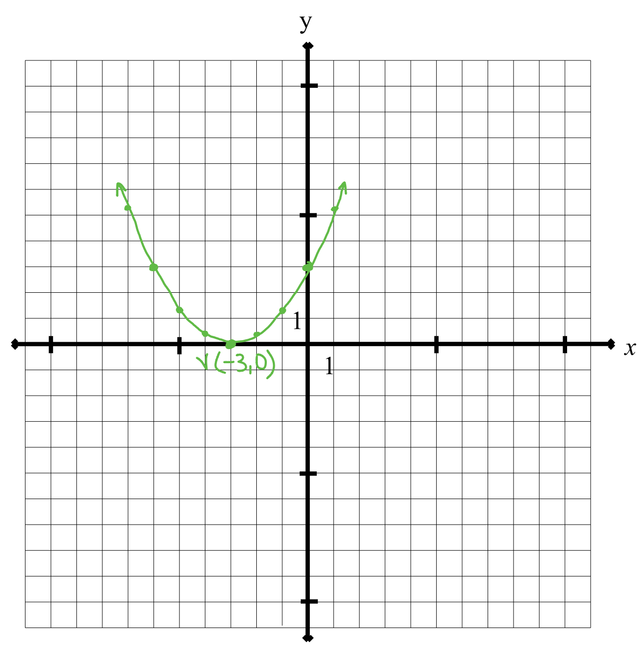
$y = \frac{1}{3}(x+3)^2$

i) *vertex:* $(-3, 0)$

ii) *opens:* up

iii) *congruent to:* $y = \frac{1}{3}x^2$

iv) *The* minimum *value of y is* 0 *when* $x =$ -3 .



HW. Exercise 2.6

Date:_____________

2.7 Applications: Maximum and Minimum Values

1. Determine the maximum or minimum value of each relation and when it occurs.

a) $y = 0.3x^2 + 1.2x + 4.5$

$y = 0.3(x^2 + 4\ \ ) + 4.5$

$y = 0.3(x^2 + 4x + 4 - 4) + 4.5$

$y = 0.3[(x+2)^2 - 4] + 4.5$

$y = 0.3(x+2)^2 - 1.2 + 4.5$

$y = 0.3(x+2)^2 + 3.3$

$V(\underset{x}{-2}, \underset{y}{3.3})$

∴ the minimum value of y is 3.3 when $x = -2$.

b) $A = -3w^2 + 5w - 1$

$A = -3(w^2 - \frac{5}{3}w) - 1$

$A = -3(w^2 - \frac{5}{3}w + \frac{25}{36} - \frac{25}{36}) - 1$

$A = -3(w - \frac{5}{6})^2 + \cancel{3}^{1} \times \frac{25}{\cancel{36}_{12}} - \frac{12}{12}$

$A = -3(w - \frac{5}{6})^2 + \frac{13}{12}$

$V(\underset{w}{\frac{5}{6}}, \underset{A}{\frac{13}{12}})$

∴ the maximum value of A is $\frac{13}{12}$ when $w = \frac{5}{6}$

2. A rocket is fired down a practice range. The height in metres after t seconds is given by $h = -\frac{1}{4}t^2 + 3t + 45$. Find the maximum height attained by the rocket and when it occurs.

$h = -\frac{1}{4}t^2 + 3t + 45$

$h = -\frac{1}{4}(t^2 - 12t) + 45$

$h = -\frac{1}{4}(t^2 - 12t + 36 - 36) + 45$

$h = -\frac{1}{4}(t-6)^2 + 9 + 45$

$h = -\frac{1}{4}(t-6)^2 + 54$

$V(\underset{t}{6}, \underset{h}{54})$

∴ the maximum height of the rocket is 54m when t=6 seconds

3. The concentration of bacteria in a pool, t days after treatment is $C = 30t^2 - 240t + 500$, where C is the concentration of bacteria per cm^3.

Find the lowest concentration of bacteria and the day on which it occurs.

$C = 30t^2 - 240t + 500$

$C = 30(t^2 - 8t) + 500$

$C = 30(t^2 - 8t + 16 - 16) + 500$

$C = 30(t-4)^2 - 480 + 500$

$C = 30(t-4)^2 + 20$

$V(\underset{t}{4}, \underset{C}{20})$

∴ the lowest concentration of bacteria is 20 per cm^3 on the 4th day.

4. Find two positive quantities whose sum is 18, if the sum of their squares is a minimum.

Let x and y represent the two positive quantities.
Let S represent the sum of their squares.

$x+y=18$ ①
$S=x^2+y^2$ ②

Rearrange ① & sub into ②:
$y=-x+18$

$S=x^2+(-x+18)^2$
$S=x^2+x^2-36x+324$
$S=2x^2-36x+324$
$S=2(x^2-18x)+324$
$S=2(x^2-18x+81-81)+324$
$S=2(x-9)^2-162+324$
$S=2(x-9)^2+162$
$V(9,162)$
$\therefore x=9$ & $x+y=18$
$y=9$

$\therefore$ the two positive values are 9 and 9.

5. A magazine producer can sell 600 of her magazines at \$6.00 each. A marketing survey shows her that for every \$0.50 she increases the price, she will lose 30 sales. What price should she set to obtain the greatest revenue?

Maximize the revenue, R, in \$.
Let x represent the number of price changes/increases.

Revenue = (cost per item) × (# of items sold)

$R=(6.00+0.5x)(600-30x)$
$R=3600+300x-180x-15x^2$
$R=-15x^2+120x+3600$
$R=-15(x^2-8x)+3600$
$R=-15(x^2-8x+16-16)+3600$
$R=-15(x-4)^2+240+3600$
$R=-15(x-4)^2+3840$
$V(4,3840)$

Find the price if $x=4$:
Price $=6.00+0.5(4)$
$=6.00+2.00$
$=8.00$

$\therefore$ the new price will be \$8.00.

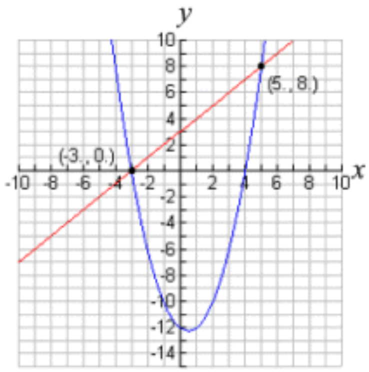
HW. Exercise 2.7

Date:______________ **2.8 Solving Systems of Equations**

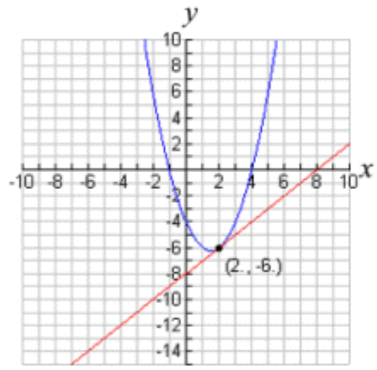
A. Linear-Quadratic Systems

Graphing a line and a parabola on the same set of axes yields one of three possible scenarios.

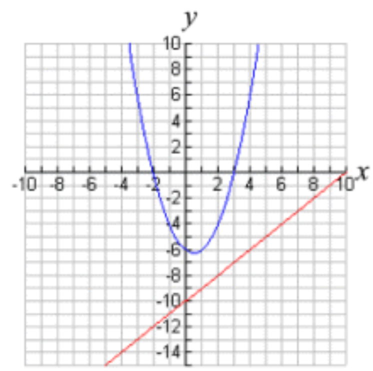
i) *Two points of intersection*



ii) *One point of intersection*



iii) *No points of intersection*



1. Solve the following linear-quadratic systems ***algebraically*** and illustrate ***graphically***.

$y = -(x+3)^2 + 2$ ①

$y = \frac{3}{2}x + 4$ ②

sub ① into ②:

$-(x+3)^2 + 2 = \frac{3}{2}x + 4$

$-x^2 - 6x - 9 + 2 = \frac{3}{2}x + 4$

$-x^2 - 6x - 7 = \frac{3}{2}x + 4$

×2) $-2x^2 - 12x - 14 = 3x + 8$

$0 = 2x^2 + 15x + 22$

$0 = (2x + 11)(x + 2)$

$\therefore x = -\frac{11}{2}$ or $x = -2$

Sub $x = -\frac{11}{2}$ into ②

$y = \frac{3}{2}\left(-\frac{11}{2}\right) + \frac{4}{1}$

$y = -\frac{33}{4} + \frac{16}{4}$

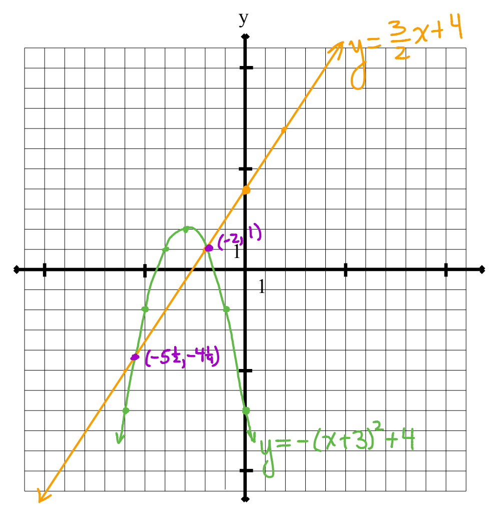
$y = \frac{-17}{4}$

Sub $x = -2$ into ②:

$y = \frac{3}{2}(-2) + 4$

$y = -3 + 4$

$y = 1$



$\therefore$ the line intersects the parabola at the points $(-2, 1)$ and $(-5\frac{1}{2}, -4\frac{1}{4})$

2. *Without solving* determine the number of points of intersection of the quadratic and linear relations $y = (x-1)^2$ and $3 = x - y$ by using the ***discriminant***. Illustrate your results ***graphically***.

$y = (x-1)^2$ ①

$3 = x - y$ ② → Rearrange ② to graph: $y = x - 3$

Sub ① into ②:

$3 = x - (x-1)^2$

$3 = x - [(x-1)(x-1)]$

$3 = x - x^2 + 2x - 1$

$3 = -x^2 + 3x - 1$

$x^2 - 3x + 4 = 0$

$a = 1,\ b = -3,\ c = 4$

$b^2 - 4ac$

$= (-3)^2 - 4(1)(4)$

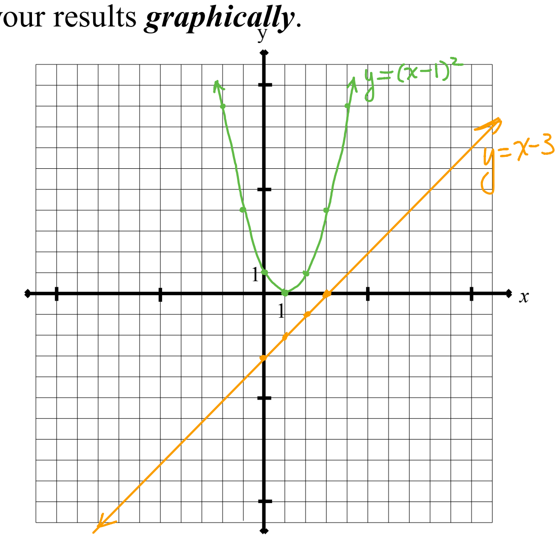
$= 9 - 16$

$= -7$

$\because b^2 - 4ac < 0$

$\therefore$ no real roots

$\therefore$ the line does not intersect the parabola.



B. Other Systems of Equations

3. Solve the following systems ***algebraically*** and illustrate ***graphically***.

a) $x^2 + y^2 = 25$ ①
$3x - 4y = 0$ ②

Rearrange ② for y

$3x - 4y = 0$
$-4y = -3x$
$y = \frac{3}{4}x$

and sub into ①

$x^2 + (\frac{3}{4}x)^2 = 25$
$x^2 + \frac{9}{16}x^2 = 25$
×16) $16x^2 + 9x^2 = 400$
$25x^2 = 400$
$x^2 = 16$
$x = \pm 4$

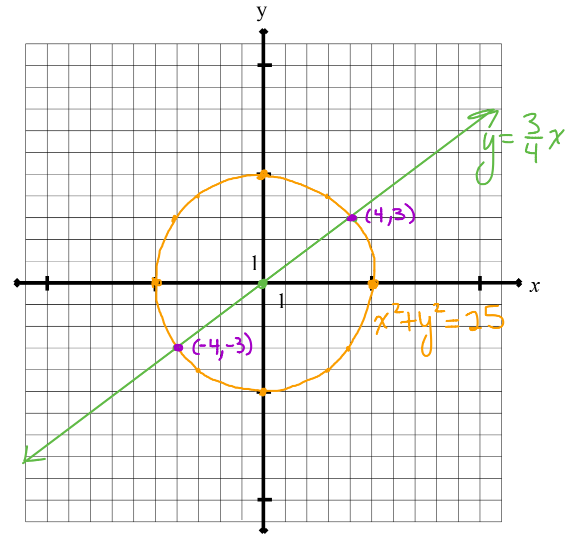
Sub $x = -4$ into ②

$y = \frac{3}{4}(-4)$
$y = -3$

Sub $x = 4$ into ②

$y = \frac{3}{4}(4)$
$y = 3$

∴ the line intersects the circle at the points $(-4,-3)$ and $(4,3)$



b) $y = -\frac{1}{2}x^2 - x + \frac{3}{2}$ ①
$y = x^2 - 4x + 3$ ②

Sub ① into ②:

$-\frac{1}{2}x^2 - x + \frac{3}{2} = x^2 - 4x + 3$
×2) $-x^2 - 2x + 3 = 2x^2 - 8x + 6$
$0 = 3x^2 - 6x + 3$
$0 = 3(x^2 - 2x + 1)$
$0 = 3(x-1)(x-1)$
∴ $x = 1$ or $x = 1$

Sub $x = 1$ into ②

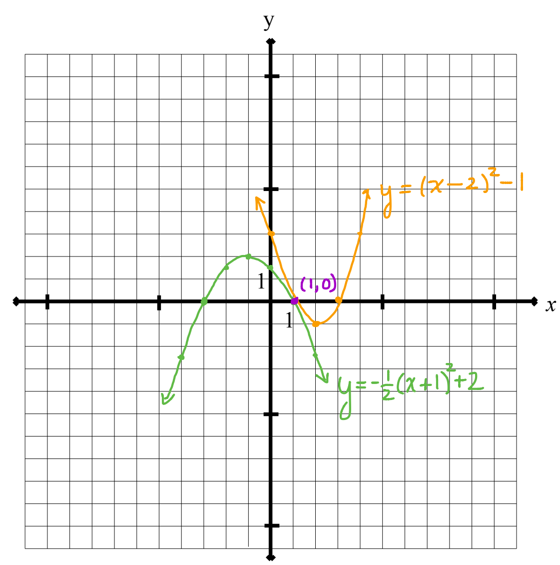
$y = (1)^2 - 4(1) + 3$
$y = 1 - 4 + 3$
$y = 0$

∴ the parabolas intersect at the point $(1, 0)$

For the graphs:

$y = -\frac{1}{2}x^2 - x + \frac{3}{2}$
$y = -\frac{1}{2}(x^2 + 2x) + \frac{3}{2}$
$y = -\frac{1}{2}(x^2 + 2x + 1 - 1) + \frac{3}{2}$
$y = -\frac{1}{2}(x+1)^2 + \frac{1}{2} + \frac{3}{2}$
$y = -\frac{1}{2}(x+1)^2 + 2$
$V(-1, 2)$ congruent to $y = \frac{1}{2}x^2$

$y = x^2 - 4x + 3$
$y = (x^2 - 4x + 4 - 4) + 3$
$y = (x-2)^2 - 4 + 3$
$y = (x-2)^2 - 1$
$V(2, -1)$ congruent to $y = x^2$



HW. Exercise 2.8
For Unit 2 Test: do Unit 2 Review Exercise