MCV4UI

Grade 12 University Level Calculus & Vectors

This Book Belongs To:

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V1

Date:______________

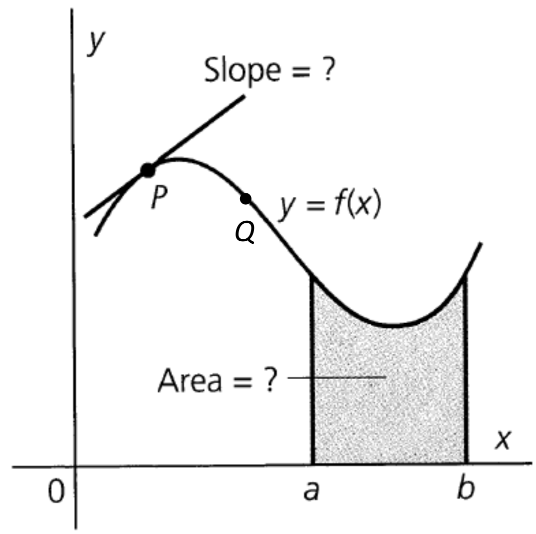
UNIT 1 – INTRODUCTION TO CALCULUS

Section 3.1 – The Slope of a Tangent

What Is Calculus?

Two simple geometric problems originally led to the development of what is now called calculus. Both problems can be stated in terms of the graph of a function $y = f(x)$.

- **The problem of tangents:** What is the value of the slope of the tangent to the graph of a function at a given point P?
- **The problem of areas:** What is the area under a graph of a function $y = f(x)$ between $x = a$ and $x = b$?



REVIEW: Secant Lines and Tangent Lines

The ***tangent line*** touches the curve at one point, and is the straight line that most resembles the graph near that point. Its slope tells how steep the graph is at the point of tangency.

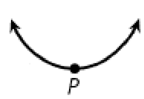
The ***secant line*** passes through more than one point on the curve.

$$m_{secant} = \frac{y_2 - y_1}{x_2 - x_1} \qquad or \qquad m_{secant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Ex. 1. Draw an approximate tangent for each curve at point P.

a.

b.



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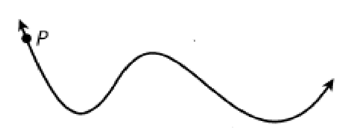

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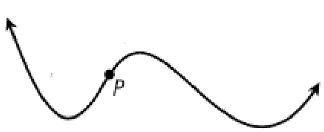

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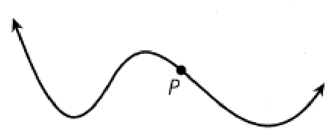
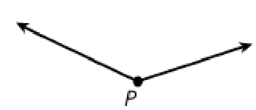


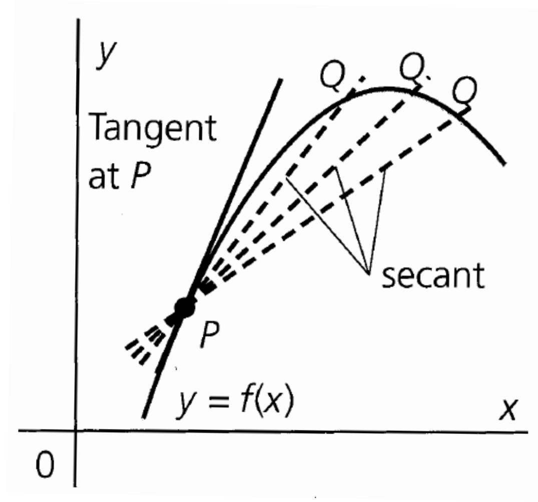
f.



GOAL: Develop a method for determining the **slope of a tangent** at a given point on a curve.

To find the equation of a tangent to a curve at a given point, we first need to know the slope of the tangent. What can we do when we only have one point?
We proceed as follows:

Consider a curve $y = f(x)$ and a point *P* that lies on the curve. Now consider another point *Q* on the curve. The line joining *P* to *Q* is called a **secant**. Think of *Q* as a moving point that slides along the curve towards *P*, so that the slope of the secant *PQ* becomes a progressively better estimate of the slope of the tangent at *P*.

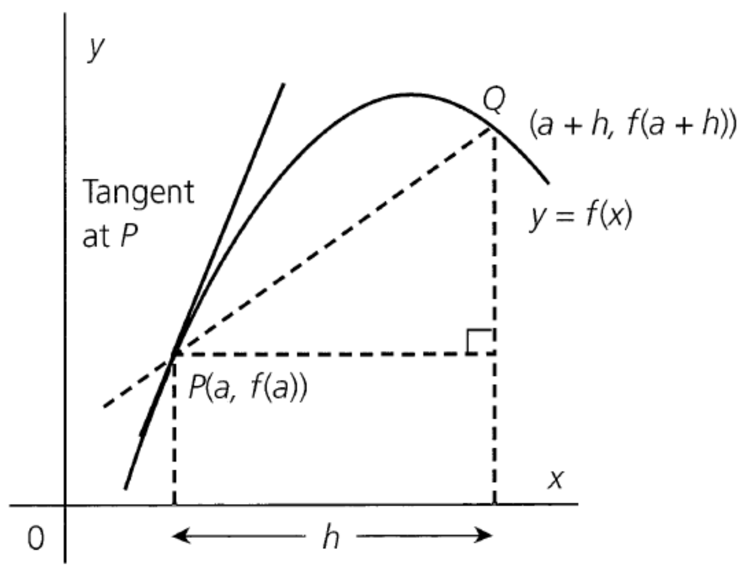


This suggests the following definition of the slope of the tangent:

> The **slope of the tangent** to a curve at a point *P* is the limiting slope of the secant *PQ* as the point *Q* slides along the curve towards *P*. In other words, the **slope of the tangent** is said to be the **limit** of the **slope of the secant** as *Q* approaches *P* along the curve

The Slope of a Tangent at an Arbitrary Point

We can now generalize the method used above to derive a formula for the slope of the tangent to the graph of any function $y = f(x)$.

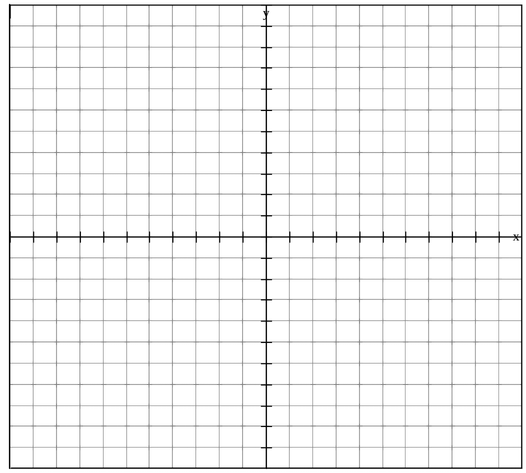


The **slope of the tangent** to the graph $y = f(x)$ at a specific point $P(a, f(a))$ is:

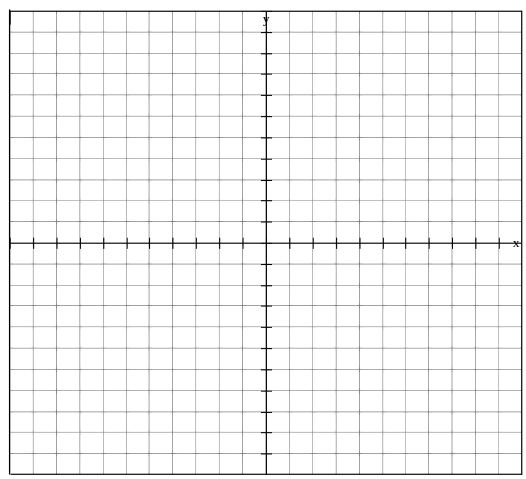
$$m_t = \lim_{h \to o} \frac{f(a+h) - f(a)}{h}$$

Ex. 2. Find the slope of the tangent to each curve at the point whose *x*-value is given. Illustrate your solution graphically.

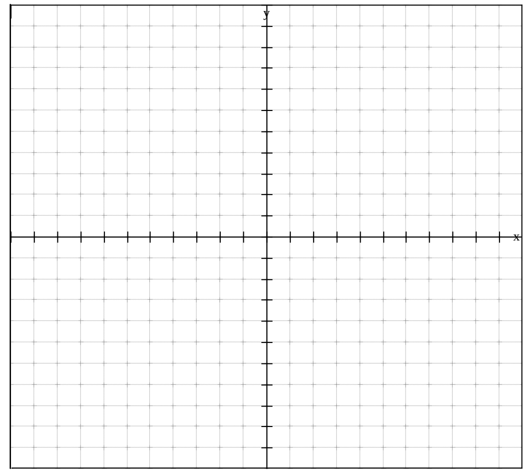
a) $f(x) = -x^2,\ at\ (2, -4)$



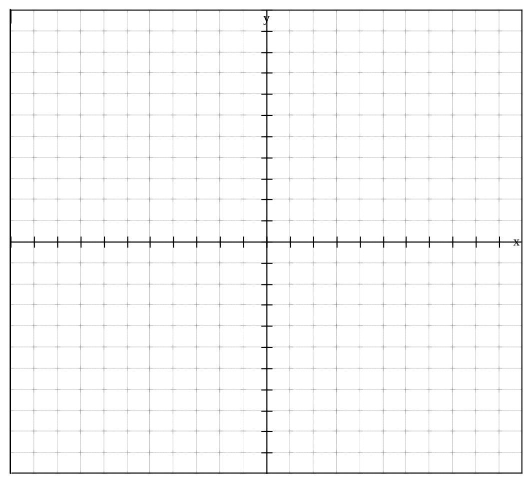
b) $y = 2\sqrt{x+4},\ at\ x = -3$



c) $y = \frac{1}{x}$, $at\ x = 1$



Ex. 3. Find the slope and equation of the tangent to $y = x^3 - 4x$, $at\ x = -2$.
Illustrate your solution graphically.



HW: p. 84 #8ac, 9ac, 10ac, *11abefgh (For #11, find the equation of the tangent & graph)

Date:_______________ **Warm-up**

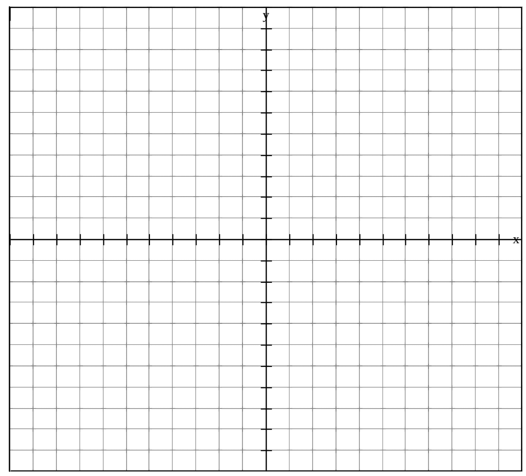
Recall:

The **slope of the tangent** to the graph $y = f(x)$ at a specific point $P(a, f(a))$ is:

$$m_t = \lim_{h \to o} \frac{f(a+h) - f(a)}{h}$$

Ex. 1. Find the equation of the tangent and normal to $g(x) = \dfrac{2}{3\sqrt{1-x}}$, $at\ x = -3$.

Illustrate graphically.



We can also find an expression for the slope of the tangent to a curve at any point in terms of *x*.

The **slope of the tangent** to the graph $y = f(x)$ at any point $P(x, f(x))$ is:

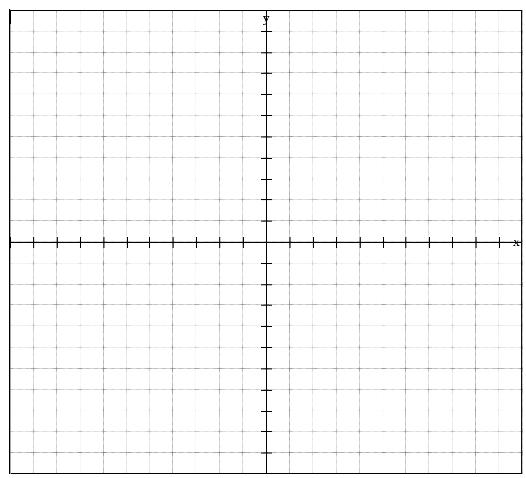
$$m_t = \lim_{h \to o} \frac{f(x+h) - f(x)}{h}$$

Ex. 2. Graph the function $h(x) = -2x^2 - 8x - 6$, and determine the following:

a) the slopes of the tangents to the curve at the points whose *x*-coordinates are given

i) x **ii)** 0 **iii)** -2

b) the equation of the tangent with a slope of 4



HW: Worksheet on Tangents and Slopes of Tangents #1efgh, 2acdf, 5abd, *6bc, *7, *8a, *9
***Follow Ex. 2. from the note.**

Date:______________ **Worksheet on Tangents and Slopes of Tangents**

1. Find the slope of the tangent to the curve at the given point.

(a) $y = x^2 + 2$ at $(2,6)$

(b) $f(x) = x^2 - 6x + 9$ at $(1,4)$

(c) $g(x) = \sqrt{x-3}$ at $(7,2)$

(d) $y = \frac{1}{x}$ at $\left(2, \frac{1}{2}\right)$

(e) $h(x) = \frac{1}{x-2}$ at $(3,1)$

(f) $y = \frac{1}{x^2}$ at $(1,1)$

(g) $y = \frac{1}{1-x}$ at $(2,-1)$

(h) $f(x) = \frac{1}{\sqrt{x}}$ at $(1,1)$

2. Find the equation of the tangent to the curve at the given point.

(a) $y = x^2 - x + 3$ at $(-1,5)$

(b) $f(x) = 4 - x^2$ at $(-2,0)$

(c) $g(x) = \frac{1}{x+3}$ at $(-4,-1)$

(d) $f(x) = \sqrt{3-x}$ at $(-1,2)$

(e) $g(x) = \frac{2x+1}{x-1}$ at $(2,5)$

(f) $h(x) = \frac{1}{\sqrt{x-1}}$ at $(2,1)$

3. Find the slopes of the tangents to the parabola $y = x^2 - 8x + 12$ at the points whose x-coordinates are:

(a) 0 (b) 2 (c) 4

4. For the following curves: (i) find the slope of the tangent at the given point;
(ii) find the equation of the tangent at the given point;
(iii) graph the curve and the tangent.

(a) $f(x) = x^2 - 9$ at $(3,0)$

(b) $h(x) = x^2 + 4x - 1$ at $(-2,-5)$

(c) $j(x) = 4 - x^2$ at $(2,0)$

(d) $f(x) = x^3 - 2$ at $(2,6)$

(e) $y = \frac{1}{1-x}$ at $(2,-1)$

(f) $f(x) = \sqrt{x+2}$ at $(2,2)$

5. Find the equation of the tangent at the given point.

(a) $f(x) = 2x^2 - 3x + 2$ at $(1,1)$

(b) $h(x) = \frac{x+1}{x-1}$ at $(2,3)$

(c) $y = \frac{x^2-4}{x+4}$ at $(0,-1)$

(d) $h(x) = \sqrt{x^2+9}$ at $(4,5)$

6. For each curve, find the slopes of the tangents at the points whose x-coordinates are $x, -2, 0 \& 1$.

(a) $y = x^3$ (b) $f(x) = x^2 + 2x - 3$ (c) $g(x) = \frac{6}{x-3}$

7. (a) Find an expression for the slope of the tangent to the parabola $y = 2x^2 + 3x$ at any point (x, y).

(b) At what point on the parabola is the tangent parallel to the line $y = 14x - 6$?

8. (a) Find the equation of the tangent to $y = x^2 + 6x + 9$ that has a slope of 4.

(b) Find equations of the tangents to $y = x^3$ that have a slope of 12.

9. Find the equations of the tangents to the graph of $y = x^3 - x$ at the intersection of the graph with the x-axis. Sketch the graph and the tangents.

Answers: 1. a) 4 b) -4 c) $\frac{1}{4}$ d) $-\frac{1}{4}$ e) -1 f) -2 g) 1 h) $-\frac{1}{2}$

2. a) $y = -3x + 2$ b) $y = 4x + 8$ c) $y = -x - 5$ d) $x + 4y - 7 = 0$
e) $y = -3x + 11$ f) $x + 2y - 4 = 0$

3. a) -8 b) -4 c) 0

4. a) i) 6 ii) $y = 6x - 18$ b) i) 0 ii) $y = -5$ c) i) -4 ii) $y = -4x + 8$
d) i) 12 ii) $y = 12x - 18$ e) i) 1 ii) $y = x - 3$ f) i) $\frac{1}{4}$ ii) $y = \frac{1}{4}x + \frac{3}{2}$

5. a) $y = x$ b) $y = -2x + 7$ c) $y = \frac{1}{4}x - 1$ d) $y = \frac{4}{5}x + \frac{9}{5}$

6. a) $3x^2$, 12, 0, 3 b) $2x + 2$, $-2, 2, 4$ c) $-\frac{6}{(x-3)^2}, -\frac{6}{25}, -\frac{2}{3}, -\frac{3}{2}$

7. a) $4x + 3$ b) $\left(\frac{11}{4}, \frac{187}{8}\right)$

8. a) $y = 4x + 8$ b) $y = 12x + 16$, $y = 12x - 16$

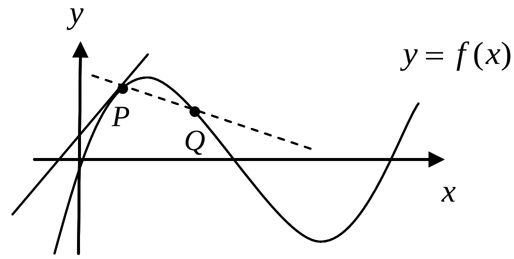
9. At $x = -1$, $y = 2x + 2$; at $x = 0$, $y = -x$; at $x = 1$, $y = 2x - 2$

Date:_______________ **Section 3.2 – Rates of Change**

Recall:

$$m_t = \lim_{h \to 0} m_{PQ}$$

$$m_t = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

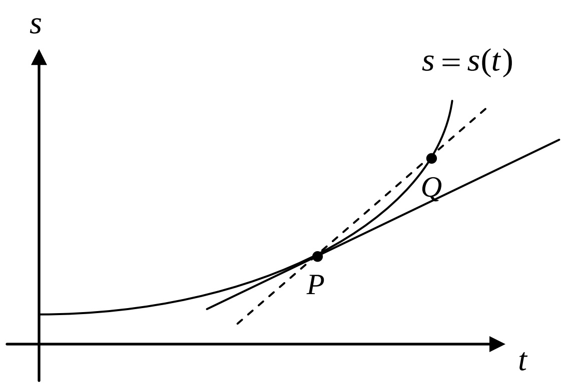


Note: ***The slope of the secant is the <u>average rate of change</u> of y with respect to x, whereas, the slope of the tangent is the instantaneous <u>rate of change</u> of y with respect to x.***

We begin by considering a familiar ***rate of change*** – the ***velocity*** of a moving object.
For example, a truck that travels a ***distance*** of 300 km in a ***time*** of 4 h has an ***average velocity*** of

The ***velocity*** at an instant of time, called the ***instantaneous velocity*** will often vary.
This ***velocity*** would be the *speedometer reading* at an instant of time.

Note: The ***speed*** of an object is the absolute value of its velocity. It indicates how fast an object is moving, whereas ***velocity*** indicates both ***speed*** and ***direction*** relative to a given coordinate system.



Average Velocity

$$average\ velocity = m_{PQ}$$

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

$$v_{avg} = \frac{s_2 - s_1}{t_2 - t_1}$$

$$\therefore v_{avg} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Instantaneous Velocity

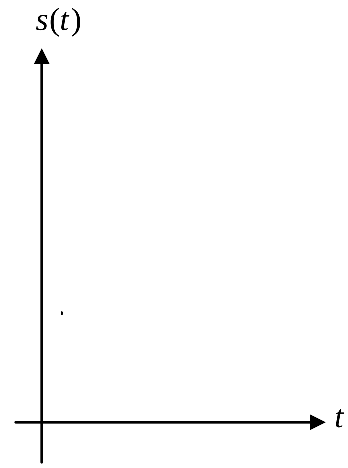
$$velocity = m_t$$

$$v = \lim_{h \to 0} m_{PQ}$$

$$\therefore v = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

*The **average velocity** is the **average rate of change** of distance, s, with respect to time, t.*
*The **velocity** is the **rate of change** of distance, s, with respect to time, t.*

Ex. 1. A pebble is dropped from a cliff of height 80 m. After t seconds, it is s metres above the ground, where $s(t) = 80 - 5t^2$, $0 \le t \le 4$. Find the pebbles's:



a) ***average velocity***

i) for $t \in [2, 4]$

ii) during the third second

b) ***velocity*** at $t = 2$

t	$s(t)$

Ex. 2. A dragster races down a 400 m strip in 8 s. Its distance in metres from the starting line after t seconds is $s(t) = 6t^2 + 2t$.

a) Find its ***average velocity*** over the first 4 seconds.

b) Find its ***velocity*** as it crosses the finish line.

c) Determine when its ***velocity*** is 38 m/s.

Ex. 3. When a certain drug is injected into a muscle, the muscle contracts an amount, $C(x)$ in millimetres, for an amount, x millitres, of the drug, where $C(x) = \dfrac{4}{12+2x}$.

Find the ***exact*** rate of change of the amount of muscle contraction if 50 ml of the drug is injected.

HW: p. 92 #1 to 3, 7 to 10, 12, 13

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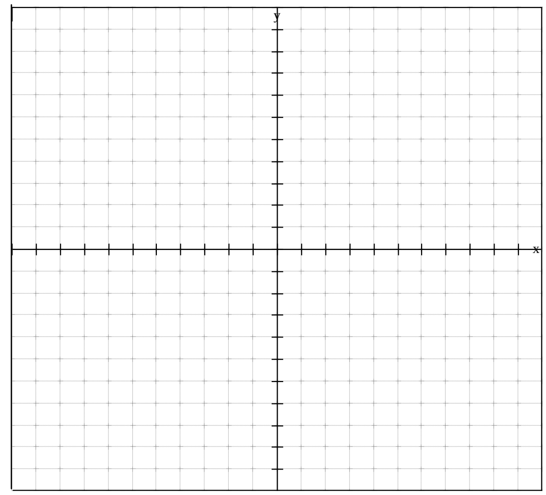
Section 3.3 – The Limit of a Function

Warm-up



1. Graph $f(x) = 2x - 4$.

2. Use the ***graph*** to evaluate the following limit.

$\lim_{x \to 3} f(x)$

3. Evaluate the same limit ***algebraically***.

Ex. 1. Evaluate the following limits by using the ***direct substitution technique***.

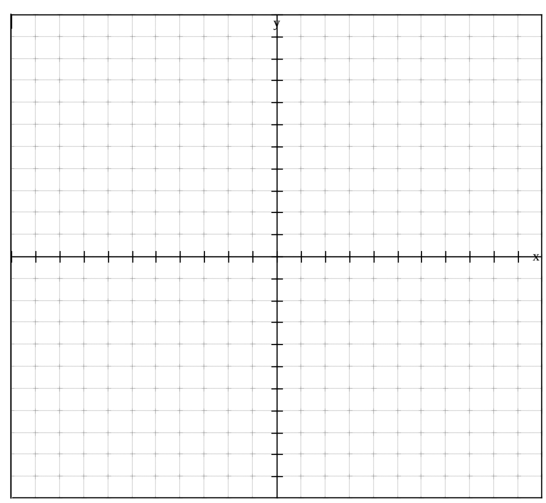
a) $\lim_{x \to -1} (-x^2 + 2x - 4)$ **b)** $\lim_{x \to 2} \frac{-4}{x + 4}$ **c)** $\lim_{x \to 3^+} \sqrt{x^2 - 9}$

Evaluating the Limit of a Function Using One-Sided Limits

1. Graph $f(x) = \begin{cases} x - 1, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ x + 1, & \text{if } x > 1 \end{cases}$

If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x)$ does not exist.

If $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x) = L$.



2. Use the ***graph*** to evaluate ***one-sided limits***, in order to find the indicated limit if it exists.

$\lim_{x \to 1} f(x)$

Left-sided limit $\lim_{x \to 1^-} f(x)$

Right-sided limit $\lim_{x \to 1^+} f(x)$

3. Evaluate the same limit ***algebraically***, using ***one-sided limits***.

$\lim_{x \to 1^-} f(x)$ $\lim_{x \to 1^+} f(x)$

KEY CONCEPTS

- The limit of a function is written as $\lim_{x\to a} f(x) = L$, which is read as "the limit of $f(x)$ as x approaches a, equals L".
- If the values of $f(x)$ approach L more and more closely as x approaches a more and more closely (from either side of a), but $x \neq a$, then $\lim_{x\to a} f(x) = L$.
- The left-sided limit of a function is written as $\lim_{x\to a^-} f(x)$, which is read as "the limit of $f(x)$ as x approaches a from the left".
- If the values of $f(x)$ approach L more and more closely as x approaches a more and more closely, with $x < a$, then $\lim_{x\to a^-} f(x) = L$.
- The right-sided limit of a function is written as $\lim_{x\to a^+} f(x)$, which is read as "the limit of $f(x)$ as x approaches a from the right".
- If the values of $f(x)$ approach L more and more closely as x approaches a more and more closely, with $x > a$, then $\lim_{x\to a^+} f(x) = L$.
- In order for $\lim_{x\to a} f(x)$ to exist, the one-sided limits $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ must both exist and be equal. That is,

  If $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$, then $\lim_{x\to a} f(x)$ does not exist.

  If $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$, then $\lim_{x\to a} f(x) = L$.
- To check that a function $f(x)$ is continuous at $x = a$, check that the following three conditions are satisfied:

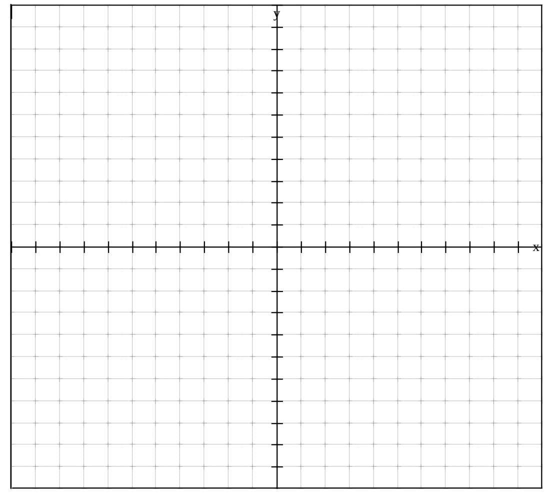
  a) $f(a)$ is defined (a is in the domain of $f(x)$)
  b) $\lim_{x\to a} f(x)$ exists
  c) $\lim_{x\to a} f(x) = f(a)$

  Also, if $f(x)$ is continuous at $x = a$, then $\lim_{x\to a} f(x) = f(a)$.
- Every polynomial P is continuous at every number, that is, $\lim_{x\to a} P(x) = P(a)$.
- Every rational function $f(x) = \dfrac{P(x)}{Q(x)}$ where P and Q are polynomials, is continuous at every number a for which $Q(a) \neq 0$, that is, $\lim_{x\to a} \dfrac{P(x)}{Q(x)} = \dfrac{P(a)}{Q(a)}, Q(a) \neq 0$.
- Discontinuous:
  a) If a function $f(x)$ has a removable discontinuity at $x = a$, then $\lim_{x\to a} f(x) = L$ exists, and the discontinuity can be removed by (re)defining $f(x) = L$ at the single point a.
  b) If a function has a jump discontinuity, the function "jumps" from one value to another.
  c) If a function $f(x)$ has in infinite discontinuity at $x = a$, the absolute values of the function become larger and larger as x approaches a.

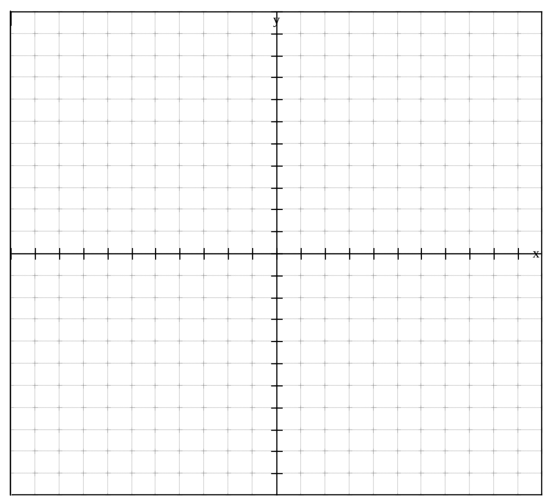
Ex. 2. By evaluating ***one-sided limits***, find the indicated limit if it exists. Graph the function and state whether the function is continuous or discontinuous with reasons.

Note: A function that is ***continuous*** has no breaks in its graph.
A function that is ***discontinuous*** has some type of break in its graph. This break is the result of a ***hole***, ***jump***, or ***vertical asymptote***.

a) $f(x)=\begin{cases} 2x+4, & if \;\; x<-2 \\ 4-x^2, & if \;\; x\geq -2 \end{cases}; \;\; \lim\limits_{x\to -2} f(x)$

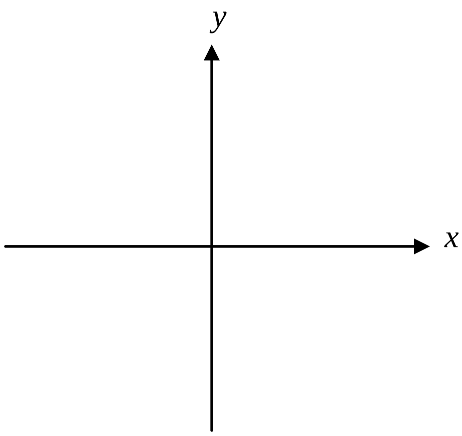


b) $g(x)=\begin{cases} -x+3, & if \;\; x\geq 1 \\ x^3, & if \;\; x<1 \end{cases}; \;\; \lim\limits_{x\to 1} g(x)$

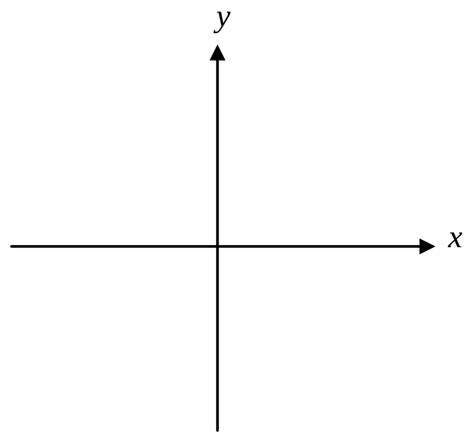


Ex. 3. Sketch the graph of any function that satisfies the given conditions in each case.

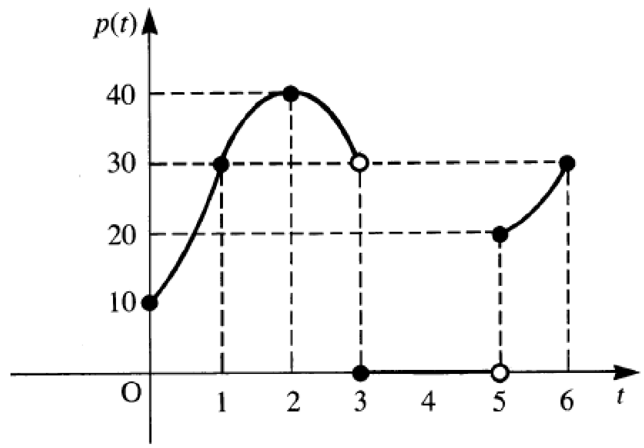
a) $\lim_{x \to -3} f(x) = 3, \ f(-3) = 0$



b) $\lim_{x \to 2^-} f(x) = -1, \ \lim_{x \to 2^+} f(x) = 1$



Ex. 4. The function $p(t)$ describes the production of unleaded gasoline in a refinery, in thousands of litres, where the time t, is measured in days.



a) Evaluate $\lim_{t \to 1} p(t)$.

b) Evaluate $\lim_{t \to 3^-} p(t)$ and $\lim_{t \to 3^+} p(t)$.

c) When was the refinery shut down for repairs and when did production begin again?

d) At what times is the production function $p(t)$ discontinuous?

e) At what time was the production highest and what was the rate of change of production at this time?

HW: p. 98 #4 to 8, 10 to 14

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Warm-up

1. Evaluate each limit, if possible, using the ***direct substitution technique*** for evaluating limits.

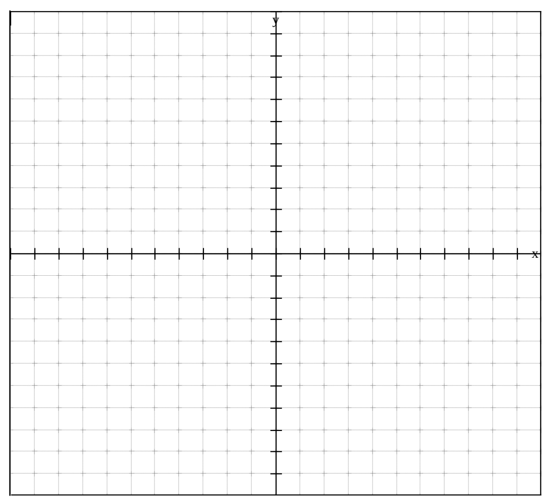
a) $\lim\limits_{x\to 4}\left(\sqrt{x}+2\right)^2$ **b)** $\lim\limits_{x\to 0}\left(\sqrt{1+\sqrt{1+x}}\right)$ **c)** $\lim\limits_{x\to a}\dfrac{(x+a)^2}{x^2+a^2}$ **d)** $\lim\limits_{x\to -2}\dfrac{1}{x+2}$

e) $\lim\limits_{x\to 4} f(x)$ if $f(x)=\begin{cases} x+3, & if \;\; x>2 \\ \dfrac{4}{x}, & if \;\; x\le 2 \end{cases}$

f) $\lim\limits_{x\to -\frac{1}{2}} f(x)$ if $f(x)=\begin{cases} x+3, & if \;\; x>2 \\ \dfrac{4}{x}, & if \;\; x\le 2 \end{cases}$

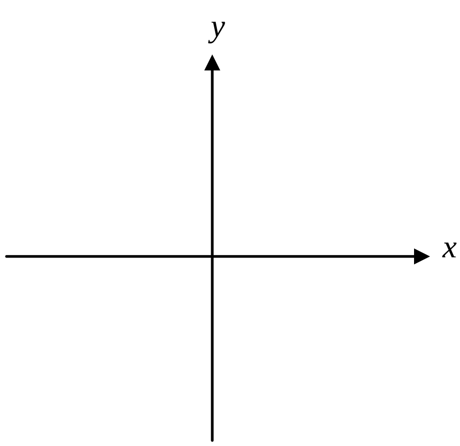
2. By evaluating ***one-sided limits***, find the indicated limit if it exists.
Graph the function and state whether the function is continuous.

$f(x)=\begin{cases} x+3, & if \;\; x>2 \\ \dfrac{4}{x}, & if \;\; x\le 2 \end{cases}$; $\lim\limits_{x\to 2} f(x)$

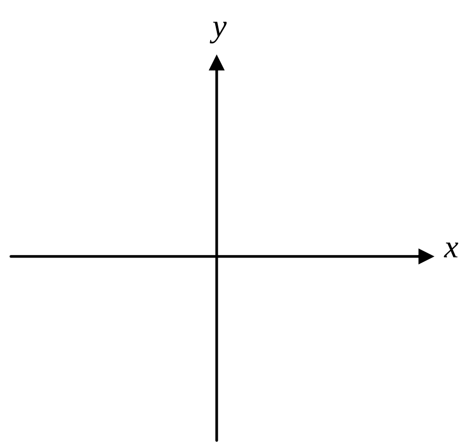


3. Sketch the graph of any function that satisfies the given conditions in each case.

a) $\lim\limits_{x\to 3} f(x)=-2$, $f(x)$ is discontinuous at $x=3$



b) $\lim\limits_{x\to 0} g(x)=1$, $g(x)$ is continuous at $x=0$



Date:______________

Section 3.4 – Limits of Indeterminate Forms

Sometimes the limit of $f(x)$ as x approaches a cannot be found by direct substitution. This is of special interest when direct substitution results in an ***indeterminate form*** $\left(\frac{0}{0}\right)$. In such cases, we look for an equivalent function that agrees with f for all values except the troublesome value at $x = a$.

Recall: Factoring

Difference of squares	***Difference of cubes***	***Sum of cubes***
$a^2 - b^2$	$a^3 - b^3$	$a^3 + b^3$

Ex. 1. Evaluate the limit of each ***indeterminate*** quotient by using ***factoring*** or ***rationalizing techniques***.

a) $\lim\limits_{x \to -5} \dfrac{x^2 - 25}{x + 5}$

b) $\lim\limits_{x \to -3} \dfrac{x^3 + 27}{3x + 9}$

c) $\lim\limits_{x \to 0} \dfrac{3^{2x} - 3^x}{1 - 3^x}$

d) $\lim\limits_{x \to \frac{3}{2}} \dfrac{8x^3 - 27}{2x^3 - 9x^2 + x + 12}$

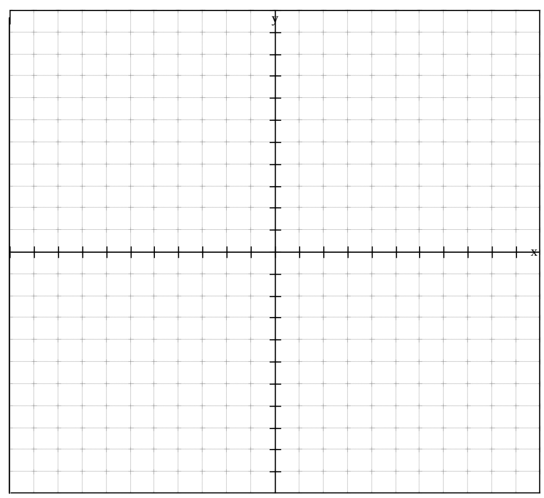
e) $\lim\limits_{x \to 0} \dfrac{1 - \sqrt{1 + 3x}}{x}$

Ex. 2. Evaluate the following limit using the ***change of variable technique***.

$$\lim_{x \to 1} \frac{x^{\frac{1}{6}} - 1}{x - 1}$$

Ex. 3. Evaluate the following limit using the ***one-sided limits technique***.
Illustrate your results graphically.

$$\lim_{x \to 2} \frac{x|x-2|}{x-2}$$



HW: p. 105 #1, 4, 7 to 10

Date:_______________ **Section 3.4 – Properties of Limits**

The Limit Laws

Suppose that the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and c is a constant.

Then, we have the following limit laws.

1. $\lim_{x\to a} c = a$

2. $\lim_{x\to a} x = x$

3. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ -The limit of a sum is the sum of the limits.

4. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ -The limit of a difference is the difference of the limits.

5. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$ -The limit of a constant times a function is the constant times the limit of the function.

6. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\lim_{x\to a} g(x)$ -The limit of a product is the product of the limits.

7. $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ *if* $\lim_{x\to a} g(x) \neq 0$ -The limit of a quotient is the quotient of the limits.

8. $\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$ -The limit of a power is the power of the limit.

9. $\lim_{x\to a} x^n = a^n$

10. $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ -The limit of a root is the root of the limit.

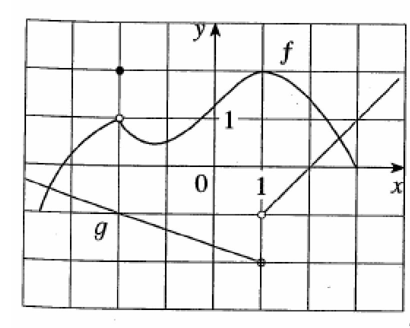
11. $\lim_{x\to a}\sqrt[n]{x} = \sqrt[n]{a}$

Ex. 1. If $\lim_{x\to -2} f(x) = 9$, state and use the ***properties of limits*** to evaluate each limit.

a) $\lim_{x\to -2} 2[f(x)]^2$

b) $\lim_{x\to -2}\dfrac{\sqrt{f(x)}}{f(x)-x^2}$

Ex. 2. Given the graphs of f and g, use and state the ***Limit Laws*** to evaluate the following limits if they exist.

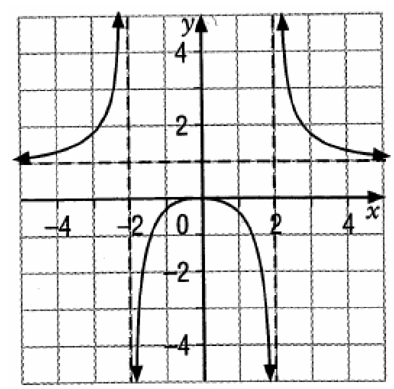


a) $\lim_{x\to 2} \frac{f(x)}{g(x)}$

b) $\lim_{x\to -2} [3f(x)+g(x)]$

c) $\lim_{x\to 1^-} [\, f(x)\; g(x)]$

Ex. 3. Evaluate the following limits if possible. (Use $+\infty$ or $-\infty$ if appropriate.)



a) $\lim_{x\to 4} \frac{x^2}{x^2-4}$

b) $\lim_{x\to -\infty} \frac{x^2}{x^2-4}$

c) $\lim_{x\to +\infty} \frac{x^2}{x^2-4}$

d) $\lim_{x\to -2^-} \frac{x^2}{x^2-4}$

e) $\lim_{x\to -2^+} \frac{x^2}{x^2-4}$

f) $\lim_{x\to -2} \frac{x^2}{x^2-4}$

Ex. 4. Evaluate the following limits if possible. (Use $+\infty$ or $-\infty$ if appropriate.)

a) $\lim_{x\to \infty} \frac{6x^2+3x-2}{(3x-2)^2}$

b) $\lim_{x\to -\infty} \frac{x^2+x}{3-x}$

c) $\lim_{x\to \infty} \sqrt{x^2+1}$

d) $\lim_{x\to 3} \frac{x^2-9}{x^2+2x-3}$

e) $\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3}$

f) $\lim_{x\to 1^+} \frac{x^2-9}{x^2+2x-3}$

HW: p. 107 #13, 16 to 19 & Additional Limits Worksheet #1 to 3 odd parts

Date:_______________

Additional Limits Worksheet

1. Given that

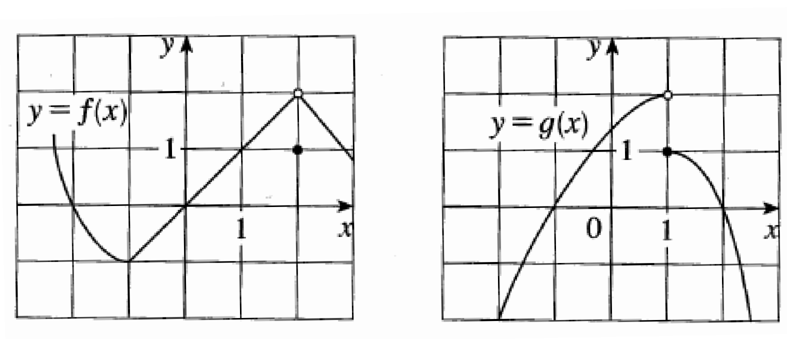
$$\lim_{x\to 2} f(x) = 4, \qquad \lim_{x\to 2} g(x) = -2, \qquad \text{and} \qquad \lim_{x\to 2} h(x) = 0$$

use and state the ***Limit Laws*** to find the limits that exist. If the limit does not exist, explain why.

a) $\lim_{x\to 2}[f(x)+5g(x)]$ **b)** $\lim_{x\to 2}[g(x)]^3$ **c)** $\lim_{x\to 2}\sqrt{f(x)}$

d) $\lim_{x\to 2}\frac{3f(x)}{g(x)}$ **e)** $\lim_{x\to 2}\frac{g(x)}{f(x)}$ **f)** $\lim_{x\to 2}\frac{g(x)h(x)}{f(x)}$

2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why. **Note:** State the ***Limit Laws*** used in each step of your solution.



a) $\lim_{x\to 2}[f(x)+g(x)]$ **b)** $\lim_{x\to 1^+}[f(x)+g(x)]$ **c)** $\lim_{x\to 0}[f(x)g(x)]$

d) $\lim_{x\to -1}\frac{f(x)}{g(x)}$ **e)** $\lim_{x\to 2}[x^3 f(x)]$ **f)** $\lim_{x\to 1}\sqrt{3+f(x)}$

3. Evaluate the following limits if possible. (Use $+\infty$ or $-\infty$ where appropriate.)

a) $\lim_{x\to 3^-}\frac{-1}{(x-3)^2}$ **b)** $\lim_{x\to 3^+}\frac{-1}{(x-3)^2}$ **c)** $\lim_{x\to\infty}\frac{-1}{(x-3)^2}$

d) $\lim_{x\to\infty}\frac{4x^2}{16-x^2}$ **e)** $\lim_{x\to 4^-}\frac{4x^2}{16-x^2}$ **f)** $\lim_{x\to -4^-}\frac{4x^2}{16-x^2}$

g) $\lim_{x\to\infty}\frac{2x^3-3x^2}{16+4x^3}$ **h)** $\lim_{x\to -\frac{1}{2}^-}\frac{x-3}{2x+1}$ **i)** $\lim_{x\to -\frac{1}{2}^+}\frac{x-3}{2x+1}$

j) $\lim_{x\to 0^+}\frac{x-3}{2x+1}$ **k)** $\lim_{x\to -\infty}\frac{(1-2x^2)(1+x^2)}{5+x-3x^4}$ **l)** $\lim_{x\to -\infty}\frac{1+x^6}{x^3+1}$

m) $\lim_{x\to\infty}(x^4+x^5)$ **n)** $\lim_{x\to 2}\frac{x^3-8}{x^3-2x^2-x+2}$ **o)** $\lim_{x\to 1^+}\frac{x^3-8}{x^3-2x^2-x+2}$

Answers:

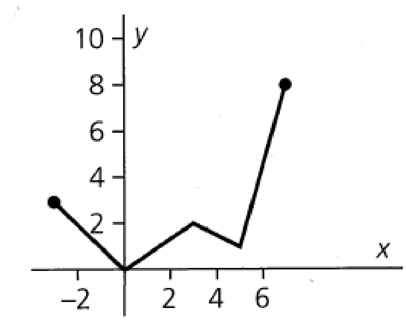
1. a) -6 **b)** -8 **c)** 2 **d)** -6 **e)** $-\frac{1}{2}$ **f)** 0   **2. a)** 2 **b)** 2 **c)** 0 **d)** d.n.e. **e)** 16 **f)** 2

3. a) $-\infty$ **b)** $-\infty$ **c)** 0 **d)** -4 **e)** ∞ **f)** $-\infty$ **g)** $\frac{1}{2}$ **h)** ∞ **i)** $-\infty$ **j)** -3 **k)** $\frac{2}{3}$ **l)** $-\infty$ **m)** ∞ **n)** 4 **o)** ∞

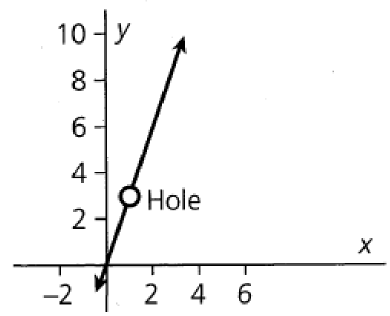
Date:______________ **Section 3.5 – Continuity**

When we talk about a function being ***continuous at a point***, we mean that the graph passes through the point without a break.
A graph that is ***not continuous at a point*** (sometimes referred to as being ***discontinuous at a point***) has a break of some type at the point. The following graphs illustrate these ideas.

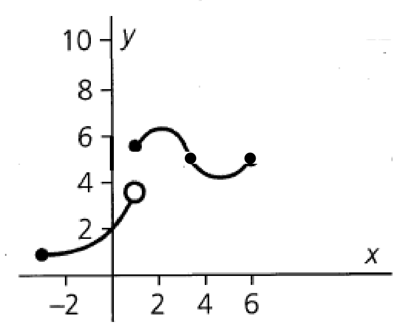
a. Continuous for all values of the domain



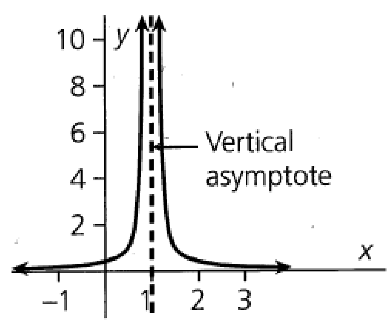
b. Discontinuous at $x=1$ (removable discontinuity)



c. Discontinuous at $x=1$ (jump discontinuity)



d. Discontinuous at $x=1$ (infinite discontinuity)

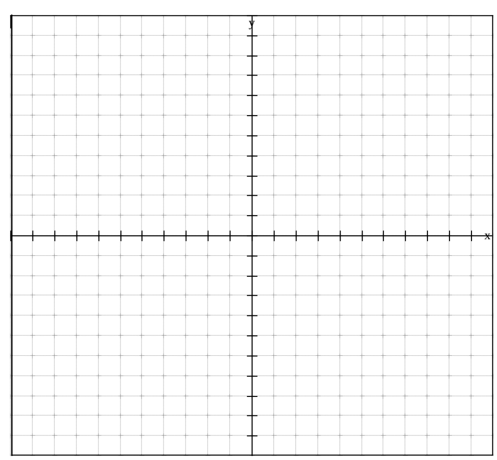


A function f is ***continuous*** at $x=a$ if <u>all three</u> of the following conditions are satisfied: • $f(a)$ is defined • $\lim_{x\to a} f(x)$ exists • $\lim_{x\to a} f(x) = f(a)$

Ex. 1. Given $f(x)=\begin{cases} x^2-3, & if \;\; x<-1 \\ x-1, & if \;\; x\geq -1 \end{cases}$,

a) graph the function.

b) determine $\lim_{x\to -1} f(x)$.

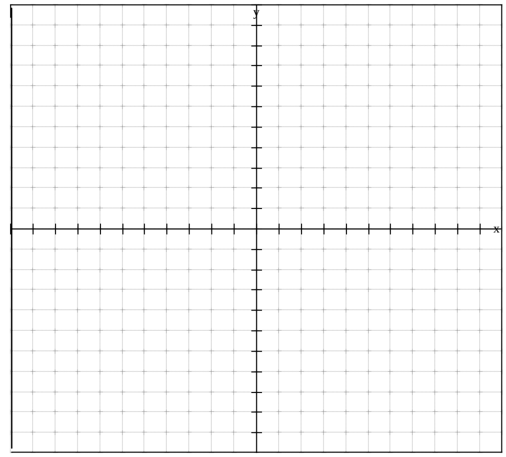


c) determine $f(-1)$.

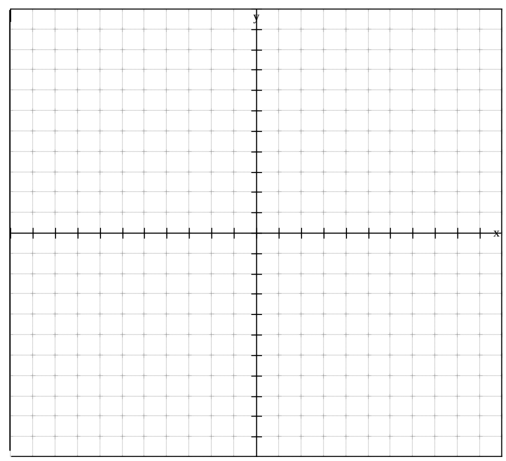
d) is f continuous at $x=-1$? Explain.

Ex. 2. Test the continuity of each of the following functions at $x = 2$.
If the function is not continuous at $x = 2$, give a reason why it is not continuous.
Illustrate graphically.

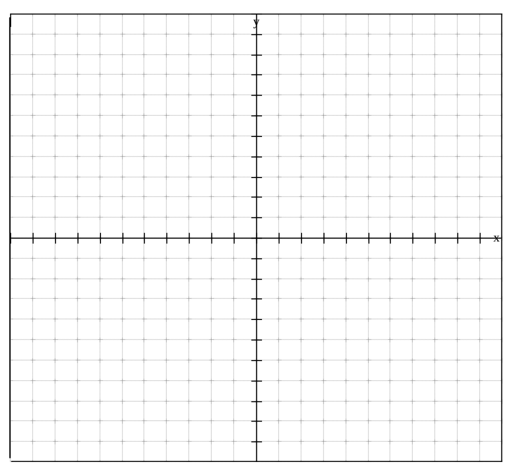
a) $g(x) = \begin{cases} 4 - x^2, & if \;\; x < 2 \\ 3 & , if \;\; x \geq 2 \end{cases}$



b) $f(x) = \dfrac{1}{(x-2)^2}$



c) $h(x) = \dfrac{x^2 - x - 2}{x - 2}$, $x \neq 2$ and $h(2) = 2$



HW: p. 110 #1, 4 to 8, 10 to 13
REVIEW for TEST: p. 115 #1 to 12, 13 to 15 (Evaluate using factoring or rationalizing techniques), 16 to 18;
Day 6 Worksheet #1 to 3 even parts; p. 119 #4, 9, 11 to 17