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UNIT 4 – DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

The Fundamental Trigonometric Limit

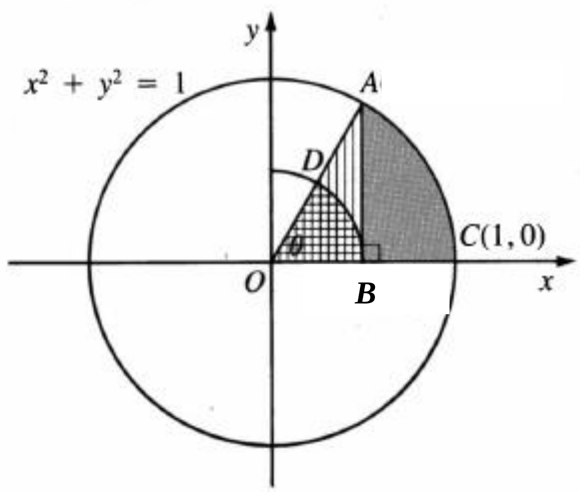
Before we can find the derivatives of the trigonometric functions, we must find the ***fundamental trigonometric limit***, $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$, where θ is in radians.

θ *in radians*	$\frac{\sin\theta}{\theta}$
0.5	
0.2	
0.01	
0.001	

The trend of the values of $\frac{\sin\theta}{\theta}$ in the table suggests

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$. A proof follows.

In the diagram, point A is on the unit circle $x^2 + y^2 = 1$. Point A determines angle θ, $0 < \theta < \frac{\pi}{2}$. The perpendicular drawn from point A meets the x-axis at B. The circle, radius OB, meets line segment OA at D.



Now Area sector $OBD \leqslant$ Area triangle $OAB \leqslant$ Area sector OAC

Therefore $\frac{1}{2}(OB)^2(\theta) \leqslant \frac{1}{2}(OB)(BA) \leqslant \frac{1}{2}(OC)^2(\theta)$

In ΔOAB, ***OA***=____, find OB and BA.

Find OB	***Find BA***

Sector Area

$= \frac{1}{2}r^2\theta$

Area of Triangle

$= \frac{1}{2}bh$

The fundamental trigonometric limit,

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \quad or \quad \lim_{h \to 0} \frac{\sin h}{h} = 1$$

Ex. 1. Evaluate the following limits using the ***fundamental trig limit***, if the limit is ***indeterminate***.

a) $\lim_{x \to 0} \frac{\tan(\sin x)}{\sec x}$

b) $\lim_{5x \to 0} \frac{\sin 5x}{5x}$

c) $\lim_{x \to 0} \frac{\sin 2x}{x}$

or

$\lim_{x \to 0} \frac{\sin 2x}{x}$

d) $\lim_{\theta \to 0} \frac{\tan\theta}{\theta}$

e) $\lim_{x \to 0} \frac{\sin 6x}{x}$

f) $\lim_{h \to 0} \frac{\cos h - 1}{h}$

g) $\lim_{\theta \to 0} \frac{1 - \cos 4\theta}{\theta^2}$

h) $\lim_{x \to \pi} \frac{\sin x}{x - \pi}$

HW: Worksheet on Trigonometric Limits

Date:_______________

WORKSHEET on Trigonometric Limits

1. Use the fundamental trigonometric limit to evaluate $\lim_{h\to 0}\frac{\sin 2h}{h}$ in two ways.

a) First use the double-angle identity for sin 2*h*.

b) Then make the change of variable $u = 2h$.

2. Use the fundamental trigonometric limit to evaluate each limit.

a) $\lim_{h\to 0}\frac{\sin 3h}{h}$

b) $\lim_{x\to 0}\frac{\sin \pi x}{x}$

c) $\lim_{u\to 0}\frac{\tan u}{u}$

d) $\lim_{x\to 0}\frac{\tan 10x}{x}$

e) $\lim_{t\to 0}\frac{1-\cos t}{t^2}$

f) $\lim_{x\to 0}\frac{1-\cos 2x}{x\sin x}$

g) $\lim_{x\to 0}\frac{\sin 3x}{4x}$

h) $\lim_{x\to 0}\frac{\sin 3x}{\sin 4x}$

i) $\lim_{h\to 0}\frac{\sin^2 h}{h}$

j) $\lim_{x\to 0}\frac{\sin(\sin x)}{\sin x}$

k) $\lim_{x\to \pi}\frac{\sin x}{\pi - x}$

l) $\lim_{x\to \frac{\pi}{2}}\frac{\cot x}{\frac{\pi}{2} - x}$

Answers: **1. a) 2 b) 2 2. a) 3 b) π c) 1 d) 10 e) 1/2 f) 2 g) 3/4 h) 3/4 i) 0 j) 1 k) 1 l) 1**

Date:_______________

The Derivatives of sin*x* and cos*x*

Recall:

Fundamental Trig Limits

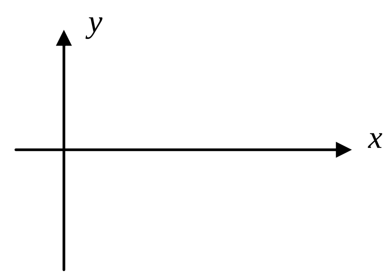
1. $\lim_{h\to 0}\frac{\sin h}{h}$ **2.** $\lim_{h\to 0}\frac{\cos h-1}{h}$

Compound Angle Formulas

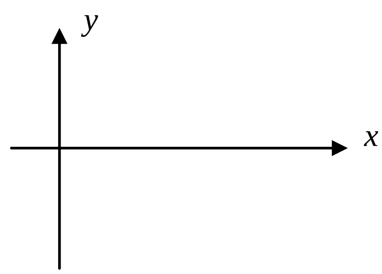
1. $\sin(A+B)=$

2. $\cos(A+B)=$

Ex. 1. Find the derivative of $f(x)=\sin x$ from ***first principles***.



Ex. 2. Find the derivative of $f(x) = \cos x$ from ***first principles***.



SUMMARY

If $y = \sin x$ then $\dfrac{dy}{dx} =$

If $y = \cos x$ then $\dfrac{dy}{dx} =$

If $y = \sin f(x)$ then $\dfrac{dy}{dx} =$

If $y = \cos f(x)$ then $\dfrac{dy}{dx} =$

If $y = \left[\sin f(x)\right]^n$ then

$\dfrac{dy}{dx} =$

If $y = \left[\cos f(x)\right]^n$ then

$\dfrac{dy}{dx} =$

Ex. 3. Find $\frac{dy}{dx}$.

a) $y = \cos x^2$

b) $y = \cos^2 x$

c) $y = \sin \frac{1}{x}$

d) $y = \sin^3(3\pi - x)$

e) $y = \sin 4x \cdot \cos 2x$

f) $y = \frac{\sin x}{1 - 2\cos x}$

g) $y = \sin(\cos 4x)$

h) $\sin xy + 3y = 0$

Ex. 4. Find the equation of the tangent to the curve $y + \cos 2x = y \sin 2x$ at $\left(\frac{\pi}{4}, 1\right)$.

HW: Worksheet on Derivatives of sin *x* and cos *x*: #1, 3, 4ac, 6, 8, 9 (equation of tangent only)

Date:_______________ **WORKSHEET on the Derivatives of sin*x* and cos*x***

1. Find $\frac{dy}{dx}$.

a. $y = 3\cos 4x$
b. $y = \cos\left(3x + \frac{\pi}{2}\right)$
c. $y = \cos(2x^3)$
d. $y = \cos^3 2x$
e. $y = \cos(x^2 + x)$
f. $y = (x + \cos x)^2$
g. $y = \cos\left(\pi x + \frac{1}{\pi}\right)$
h. $y = \cos \pi x + \frac{1}{\pi}$
i. $y = 2\sin \pi x + x^2$
j. $y = 3\sin(x^2 - 1)$
k. $y = 3\sin^2 x$
l. $y = (\sin 2x + \cos x)^3$
m. $x = \sin y$

3. Differentiate each function.

a. $f(x) = x\cos x$
b. $g(u) = u^3 \sin 2u$
c. $f(y) = y^2\cos(3y^3)$
d. $h(u) = \sin(\cos \pi u)$
e. $f(x) = \sin x \cos x$
f. $h(t) = \frac{\sin 2t}{\cos 2t}$
g. $v(t) = \sin^2(\sqrt{t})$
h. $v(t) = \sqrt{1 + \cos t + \sin^2 t}$
i. $g(y) = \sin^2 \pi y \cos \pi y$
j. $h(x) = \sin x \sin 2x \sin 3x$
k. $f(t) = \sqrt{2 + \sin^2 5t}$
l. $g(u) = (u + \sin 3u)^2$
m. $h(x) = \frac{x^2}{2 - \cos \pi x}$
n. $h(t) = \sin\frac{1}{t}$
o. $h(y) = \frac{\sin 2y}{1 + \sin 2y}$
p. $m(x) = (x^2 + \cos^2 x)^3$

4. Find $\frac{dy}{dx}$ in each case, where A, B, m, and n are constants.

a. $y = \cos(Ax + B)$
b. $y = A\cos^n Bx$
c. $y = \sin^m(x^n)$
d. $y = Ax^n \sin^m Bx$

6. Find the equation of the tangent line to the curve at each of the given points.

a. $\cos(x + 2y) = 2x + 2y - \pi$, $\left(\frac{\pi}{2}, 0\right)$

b. $x\sin(xy - y^3) = \frac{x^2 - y}{y}$, $(1, 1)$

8. If $y = A\cos kt + B\sin kt$, where A, B, and k are constants, show that

$$y'' + k^2 y = 0$$

9. Find the equation of the tangent line to the graph of the function at the indicated point. Use the second derivative to determine whether the graph lies above or below the tangent. Sketch the graph near the indicated point.

a. $f(x) = x\sin 2x$, $x = \frac{\pi}{4}$

b. $f(x) = \sin^2 x$, $x = \frac{\pi}{3}$

c. $f(x) = \cos(\pi x^2 + \pi)$, $x = \frac{1}{2}$

ANSWERS

1. a. $-12\sin 4x$ **b.** $-3\sin\left(3x+\frac{\pi}{2}\right)$

c. $-6x^2\sin(2x^3)$ **d.** $-6\cos^2(2x)\sin(2x)$

e. $-(2x+1)\sin(x^2+x)$

f. $2(x+\cos x)(1-\sin x)$

g. $-\pi\sin\left(\pi x+\frac{1}{\pi}\right)$

h. $-\pi\sin(\pi x)$ **i.** $2\pi\cos(\pi x)+2x$

j. $6x\cos(x^2-1)$ **k.** $3\sin 2x$

l. $3(\sin 2x+\cos x)^2(2\cos 2x-\sin x)$ **m.** $\sec y$

3. a. $\cos x - x\sin x$

b. $3u^2\sin(2u)+2u^3\cos(2u)$

c. $2y\cos(3y^3)-9y^4\sin(3y^3)$

d. $-\pi\sin(\pi u)\cos(\cos(\pi u))$ **e.** $\cos 2x$

f. $2\sec^2(2t)$ **g.** $\frac{1}{2\sqrt{t}}\sin(2\sqrt{t})$

h. $\frac{1}{2}(1+\cos t+\sin^2 t)^{-\frac{1}{2}}(2\sin t\cos t-\sin t)$

i. $2\pi\sin(\pi y)\cos^2(\pi y)-\pi\sin^3(\pi y)$

j. $\cos x\sin 2x\sin 3x+2\sin x\cos 2x\sin 3x+$
$3\sin x\sin 2x\cos 3x$ **k.** $\frac{5}{2}\sin(10t)(2+\sin^2 5t)^{-\frac{1}{2}}$

l. $2(u+\sin(3u))(1+3\cos(3u))$

m. $\frac{2x(2-\cos\pi x)-\pi x^2\sin\pi x}{(2-\cos\pi x)^2}$ **n.** $-\frac{1}{t^2}\cos\left(\frac{1}{t}\right)$

o. $2\cos(2y)(1+\sin(2y))^{-2}$

p. $3(x^2+\cos^2 x)(2x-\sin 2x)$

4. a. $-A\sin(Ax+B)$

b. $-nAB\cos^{n-1}(Bx)\sin(Bx)$

c. $mnx^{n-1}\sin^{m-1}(x^n)\cos(x^n)$

d. $Anx^{n-1}\sin^m Bx+mABx^n\sin^{m-1}(Bx)\cos(Bx)$

6. a. $6x+8y-3\pi=0$ **b.** $x+y-2=0$

9. a. $y=x$, below

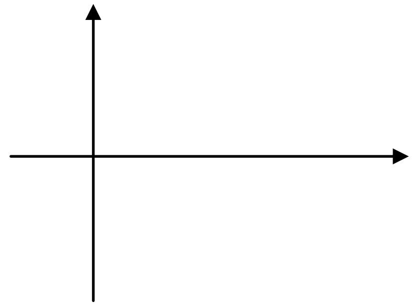
b. $6\sqrt{3}x-12y=2\sqrt{3}\pi-9$, below

c. $\pi x-\sqrt{2}y-\frac{\pi}{2}-1=0$, above

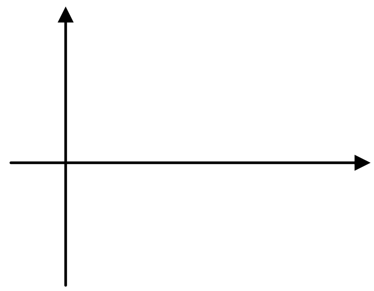
Date:________________ **Further Trigonometric Derivatives**

Ex. 1. Find $\frac{dy}{dx}$ for each of the following reciprocal trig functions and illustrate graphically.

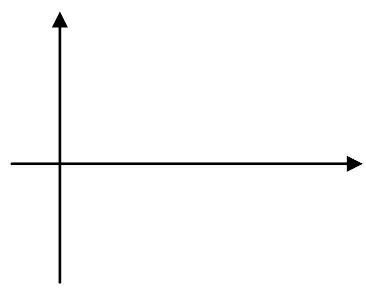
a) $y = \csc x$



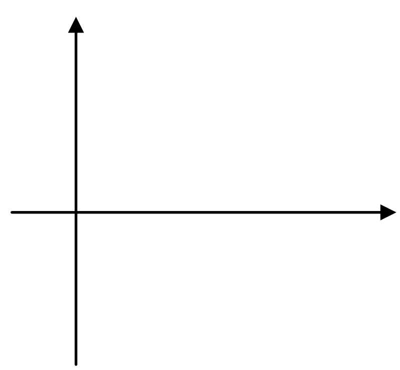
b) $y = \sec x$



c) $y = \tan x$



d) $y = \cot x$



SUMMARY OF TRIGONOMETRIC DERIVATIVES

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Ex. 2. Find $\frac{dy}{dx}$ for the following:

a) $y = \tan(x^3 + 4x)$

b) $y = 2x^3 \cdot \cot\left(\frac{1}{x}\right)$

c) $y = \csc(\tan x)$

d) $y = \sec^3 \pi x$

e) $\cot(x + y) = 1 - y$

HW: Worksheet on Further Trigonometric Derivatives #1, 4, 5

Date:______________ **WORKSHEET on Further Trigonometric Derivatives**

1. Find $\frac{dy}{dx}$ in each case.

a. $y = 2\tan x - \tan 2x$

b. $y = 3\sec(2x^2 + 1)$

c. $y = 3\sec 5x \tan^2 5x$

d. $y = \sqrt{x^2 + \sec^2 x}$

e. $y = \dfrac{x^2}{\tan \pi x}$

f. $y = \tan(x^2) - \tan^2 x$

g. $y = \sqrt{x}\sec\sqrt{x}$

h. $y = x^2 \tan\left(\dfrac{1}{x}\right)$

i. $y = \tan(xy)$

j. $x\tan y = y\tan x$

k. $y = \tan(\sin x)$

l. $y = \sin(\tan x)$

2. Use the identity

$$\sec^2 x = 1 + \tan^2 x$$

and the derivative of $\tan x$ to find the derivative of $\sec x$.

3. Use the derivatives of $\sin x$ and $\cos x$ to find the derivatives of $\cot x$ and $\csc x$, in the form given in the text.

4. Find $\frac{dy}{dx}$ in each case.

a. $y = \cot 2x + \csc 2x$

b. $y = 2x^3 \cot x$

c. $y = (x + \csc x)^2$

d. $y = \sqrt{\pi^2 + \csc^2 x}$

e. $y = 3\cot\sqrt{x^2 + 1}$

f. $y = \dfrac{\cot x}{1 + \csc^2 x}$

g. $y = \sqrt{x}\csc\left(\dfrac{1}{\sqrt{x}}\right)$

h. $y = \cot^2 2x \csc 2x$

i. $\cot(x + y) = 2y$

j. $\cot(xy^2) - 4y + 3 = 0$

k. $y = \dfrac{\tan x - 1}{\tan x + 1}$

l. $y = (\cot x + \sin x)^2$

5. Find the equation of the tangent line and the equation of the normal line to the curve $y + 2x\tan(\pi xy) = x - 1$ at the point $(1, 0)$.

ANSWERS

1. a. $2\sec^2 x - 2\sec^2(2x)$

b. $12x\sec(2x^2+1)\tan(2x^2+1)$

c. $15\sec(5x)\tan(5x)[\tan^2(5x) + 2\sec^2(5x)]$

d. $\dfrac{x + \sec^2 x \tan x}{\sqrt{x^2 + \sec^2 x}}$ **e.** $\dfrac{2x\tan(\pi x) - \pi x^2 \sec^2(\pi x)}{\tan^2(\pi x)}$

f. $2x\sec^2(x^2) - 2\tan x \sec^2 x$

g. $\frac{1}{2}\sec\sqrt{x}\left(\tan\sqrt{x} + x^{-\frac{1}{2}}\right)$

h. $2x\tan\left(\frac{1}{x}\right) - \sec^2\left(\frac{1}{x}\right)$ **i.** $\dfrac{y\sec^2(xy)}{1 - x\sec^2(xy)}$

j. $\dfrac{y\sec^2 x - \tan y}{x\sec^2 y - \tan x}$ **k.** $(\sec^2(\sin x))(\cos x)$

l. $(\cos(\tan x))(\sec^2 x)$ **2.** $\sec x \tan x$

4. a. $-2\csc(2x)[\csc(2x) + \cot(2x)]$

b. $6x^2\cot x - 2x^3\csc^2 x$

c. $2(x + \csc x)(1 - \csc x \cot x)$

d. $-\csc^2 x \cot x(\pi^2 + \csc^2 x)^{-\frac{1}{2}}$

e. $-3x(\csc^2\sqrt{x^2+1})(x^2+1)^{-\frac{1}{2}}$

f. $\dfrac{(1 + \csc^2 x)(-\csc^2 x) + 2\csc^2 x \cot^2 x}{(1 + \csc^2 x)^2}$

g. $\frac{1}{2x}\csc\left(\frac{1}{\sqrt{x}}\right)\left[\sqrt{x} + \cot\left(\frac{1}{\sqrt{x}}\right)\right]$

h. $-2\cot(2x)\csc(2x)[2\csc^2(2x) + \cot^2(2x)]$

i. $\dfrac{-\csc^2(x+y)}{2 + \csc^2(x+y)}$ **j.** $\dfrac{-y^2\csc^2(xy^2)}{4 + 2xy\csc^2(xy^2)}$

k. $\dfrac{2\sec^2 x}{(\tan x + 1)^2}$

l. $2(\cot x + \sin x)(-\csc^2 x + \cos x)$

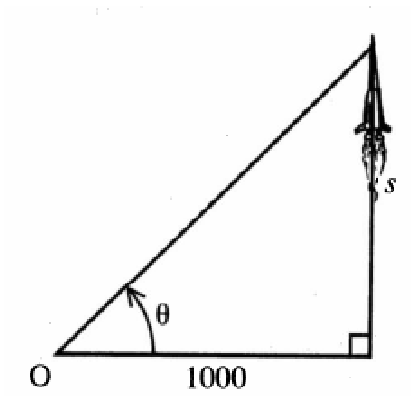
Date:_______________

Applications of Trigonometric Functions
Related Rates

Ex. 1. The motion of a particle is described by the function $s(t) = 10\cos\left(5t - \frac{\pi}{4}\right)$. Find the velocity and acceleration functions and determine the maximum and minimum values of each.

Ex. 2. A ladder of length 8 m leaning against a wall starts to slide. If its upper end slides down the wall at a rate of 0.25 m/s, at what rate is the angle between the ladder and the ground changing when the foot of the ladder is 5 m from the wall?

Ex. 3. A TV camera, located 1000 m from the site of a rocket launch, records the event. The rocket is launched vertically, and its elevation s (in metres) t seconds after lift-off is $s = 200t^2$. How rapidly must the camera angle be increased in order to maintain a view of the rocket, 5 s after lift-off?



HW: Worksheet on Applications of Trigonometric Functions –Related Rates

Date:______________

WORKSHEET on Applications of Trigonometric Functions
Related Rates

1. An object is suspended from the end of a spring. Its displacement from the equilibrium position is

$$s = 8\sin(10\pi t)$$

Calculate the velocity and acceleration of the object at any time t and show that

$$\frac{d^2s}{dt^2} + 100\pi^2 s = 0$$

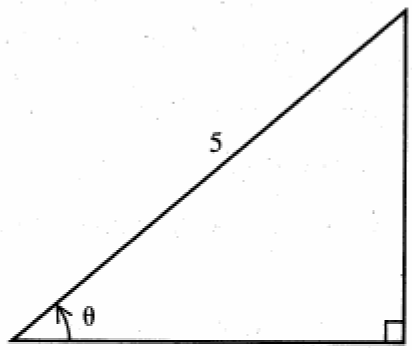
2. The displacement s (in centimetres) of the midpoint of a vibrating violin string (the A string) at time t is

$$s = 0.05\sin(880\pi t)$$

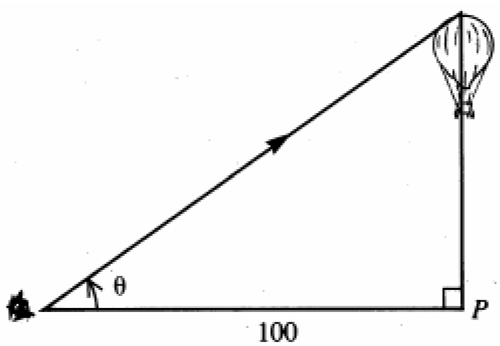
where t is measured in seconds. Find the velocity v and acceleration a of the midpoint of the string, and show that

$$a = -(880\pi)^2 s$$

3. One end of a ladder of length 5 m slides down a vertical wall. When the upper end of the ladder is 3 m above the ground, it has a downward velocity of 0.5 m/s. Find the rate at which the angle of elevation θ is changing at that time.



4. A balloon rises into the air, starting at a point P. An observer 100 m from P looks at the balloon, and the angle θ between her line of sight and the ground increases at a rate of $\frac{1}{20}$ radians per second. Find the velocity of the balloon when $\theta = \frac{\pi}{4}$.



11. A particle moving along a straight line has the position function

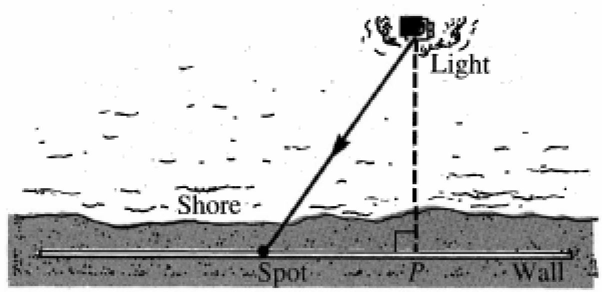
$$s(t) = t\sin(\pi t)$$

a. Find the velocity and acceleration of the particle when $t = \frac{3}{2}$.

b. Is the particle moving towards or away from the origin when $t = \frac{3}{2}$? Is the particle slowing down or speeding up when $t = \frac{3}{2}$?

12. A line L through the point (0, 1) is rotating about the point (0, 1) at the rate of one revolution per minute in a clockwise direction. At what rate is the point of intersection of L and the x-axis moving along the x-axis when $x = 5$?

13. A lighthouse searchlight 1 km from shore makes one revolution every 45 s. How fast is the spot of light moving along a wall on the shore when the spot is 500 m from the point P?



ANSWERS

1. $80\pi\cos(10\pi t)$, $-800\pi^2\sin(10\pi t)$

2. $44\pi\cos(880\pi t)$, $-38\ 720\pi^2\sin(880\pi t)$

3. -0.125 rad/s **4.** 10 m/s

11. a. $-1, \frac{3\pi^2}{2}$ **b.** away, slowing down

12. 52π units/min

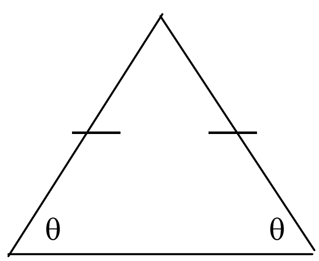
13. $\frac{\pi}{18}$ km/s

Date:______________

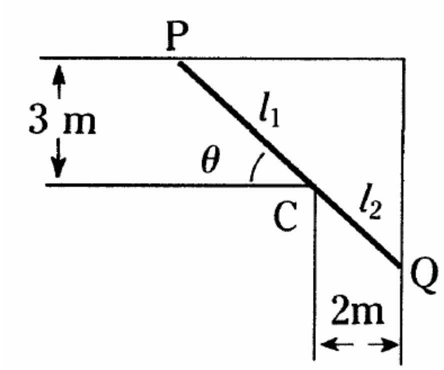
Applications of Trigonometric Functions
Optimization

Ex. 1. Find the maximum and minimum values of $f(x) = \cos 2x + 2\sin x$ for $0 \le x \le \frac{3\pi}{4}$.

Ex. 2. An isosceles triangle has two equal sides of length 15 cm. What is the maximum possible area of the triangle?



Ex. 3. Two corridors, 3 m and 2 m wide respectively, meet at right angles. Find the length of the longest thin straight rod that can pass horizontally around the corner. Neglect the thickness of the rod. Answer to the nearest cm.



HW: Worksheet on Applications of Trigonometric Functions – Optimization
REVIEW for TEST: Review Worksheets on Days 1 to 3 and Days 4 & 5

Date:_______________

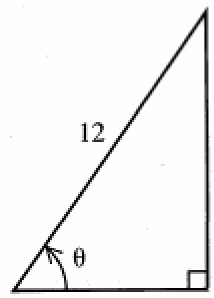
WORKSHEET on Applications of Trigonometric Functions Optimization

4. Find the maximum and minimum value of each function on the given interval.

a. $f(x) = 2\cos x + x, \quad -\pi \leq x \leq \pi$
b. $f(x) = 2\sin x - x, \quad 0 \leq x \leq 2\pi$
c. $f(x) = \sin x + 2x, \quad 0 \leq x \leq 2\pi$
d. $f(x) = 2\cos x - \cos 2x, \quad \frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$
e. $f(x) = \sin^2 x - \sin x, \quad -\pi \leq x \leq \pi$

5. A V-shaped trough 5 m long is made from a rectangular sheet of aluminum 100 cm wide by bending it down the middle. What angle between the sides of the trough will maximize its capacity?

6. The hypotenuse of a right-angled triangle is 12 cm in length. Find the measure of the angles in the triangle that maximize its perimeter.

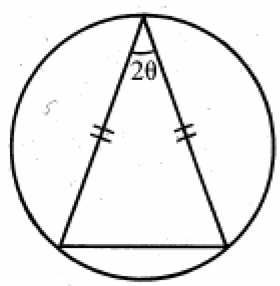


10. The voltage V being supplied to an electrical circuit at time t is

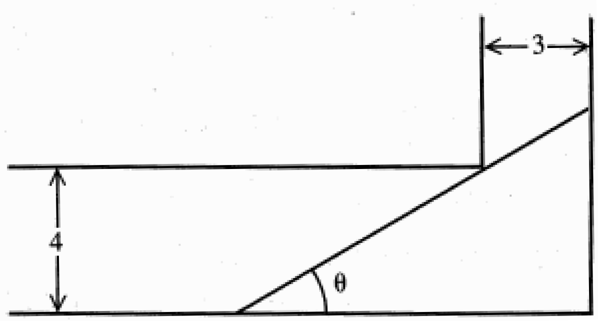
$$V = 100\sin 50t + 50\sin 100t$$

Find the maximum and minimum values of V over one period.

14. An isosceles triangle is inscribed in a circle of radius R. Find the value of θ that maximizes the area of the triangle.



15. Two corridors, 3 m and 4 m wide respectively, meet at right angles. Find the length of the longest thin straight rod that can pass horizontally around the corner.



ANSWERS

4.

	Maximum	Minimum
a.	$\sqrt{3}+\frac{\pi}{6}$	$-2-\pi$
b.	$\sqrt{3}-\frac{\pi}{3}$	$-\sqrt{3}-\frac{5\pi}{3}$
c.	4π	0
d.	1	-3
e.	2	$-\frac{1}{4}$

5. $\frac{\pi}{2}$ **6.** $\frac{\pi}{4}$

10. $75\sqrt{3},\ -75\sqrt{3}$

14. $\frac{\pi}{6}$

15. $\left(4^{\frac{2}{3}}+3^{\frac{2}{3}}\right)^{\frac{3}{2}}$ or approximately 9.87 m

Date: _______________ **REVIEW WORKSHEET: Days 1 to 3**

1. Use the ***fundamental trig limit*** to determine exact values for the following limits.

a. $\lim_{\theta\to 0}\frac{\cos\theta-1}{\sin\theta}$ **b.** $\lim_{x\to 0}\frac{\sin x^2}{x}$ **c.** $\lim_{x\to 0}\frac{\sin 3x\cdot\sin 5x}{x^2}$

d. $\lim_{x\to 0}\frac{\sin 3x}{5x^3-4x}$ **e.** $\lim_{\theta\to\frac{\pi}{4}}\frac{1-\tan\theta}{\sin\theta-\cos\theta}$ **f.** $\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2}$

3. Find y' and y'' in each case.

a. $y = 3\sin 2x - 4\cos 2x$ **b.** $y = \tan 3x$
c. $y = \pi x\cos(\pi x) - \sin(\pi x)$ **d.** $y = \sin^2 2x + \cos^2 2x$
e. $y = \cot^3 x$ **f.** $y = x\tan 2x$
g. $y = \tan^2(3x^2) - \sec^2(3x^2)$ **h.** $y = \cos^2(3x)$
i. $y = \sin(x^3)$ **j.** $y = \frac{1}{2-\cos x}$

4. a. Find the equation of the tangent line to the curve $y = x\sin x$ at the point where $x = \frac{\pi}{2}$.

b. Use the second derivative to determine whether the curve lies above or below the tangent line near $x = \frac{\pi}{2}$.

7. Consider the function $y = \sin^2(\pi x)$. Show that

a. $y' = \pi\sin(2\pi x)$ **b.** $y'' + 4\pi^2 y = 2\pi^2$

16. If $\tan(xy) = x$, find $\frac{dy}{dx}$ at the point $\left(1, \frac{\pi}{4}\right)$.

ANSWERS

1. a. 0 **b.** 0 **c.** 15 **d.** -3/4 **e.** $-\sqrt{2}$ **f.** 1/3

3. a. $6\cos(2x) + 8\sin(2x)$, $-12\sin(2x) + 16\cos(2x)$
b. $3\sec^2(3x)$, $18\sec^2(3x)\tan(3x)$
c. $-\pi^2 x\sin(\pi x)$, $-\pi^2\sin(\pi x) - \pi^3 x\cos(\pi x)$
d. 0, 0 **e.** $-3\cot^2 x\csc^2 x$,
$6\cot x\csc^4 x + 6\cot^3 x\csc^2 x$
f. $\tan(2x) + 2x\sec^2(2x)$,
$4\sec^2(2x) + 8x\sec^2(2x)\tan(2x)$ **g.** 0, 0
h. $-3\sin(6x)$, $-18\cos(6x)$ **i.** $3x^2\cos(x^3)$,
$6x\cos(x^3) - 9x^4\sin(x^3)$ **j.** $\frac{-\sin x}{(2-\cos x)^2}$,
$\frac{-2\cos x + \cos^2 x + 2\sin^2 x}{(2-\cos x)^3}$ **4. a.** $y = x$
b. below

16. $\frac{2-\pi}{4}$

Date: ________________ **REVIEW WORKSHEET: Days 4 & 5**

4. Find the maximum and minimum values of each function on the given interval.

a. $f(x) = \dfrac{13}{5\sin x + 12\cos x}, \quad 0 \leq x \leq \dfrac{\pi}{2}$

b. $f(x) = \sec x - x, \quad 0 \leq x \leq \dfrac{5\pi}{12}$

c. $f(x) = \sin x \cos^2 x, \quad 0 \leq x \leq \dfrac{\pi}{2}$

5. The position function of a particle that moves in a straight line is $x(t) = 2\pi t + \cos(2\pi t)$.

a. Find the velocity of the particle at time t.
b. Find the acceleration of the particle at time t.
c. For what values of t in the interval $0 \leq t \leq 3$ is the particle at rest?
d. What is the maximum velocity of the particle?

6. A particle moves along the x-axis so that at time t its position function is $x(t) = \sin(\pi t^2)$ for $-1 \leq t \leq 1$.

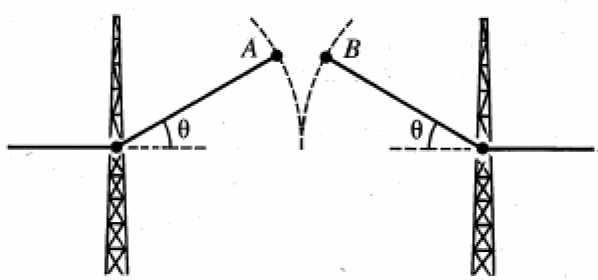
a. Find the velocity of the particle at time t.
b. Find the acceleration of the particle at time t.
c. For what values of t does the particle change direction?
d. Find all values of t for which the particle is moving to the left.

8. Suppose that a person's blood pressure at time t (in seconds) is

$$p = 100 + 18\sin 7t$$

a. Find the maximum value of p (the systolic pressure) and the minimum value of p (the diastolic pressure).
b. How many heartbeats per minute are predicted by the formula for p?

9. A drawbridge with arms of length 10 m is constructed as shown in the diagram. If the arms rotate at $\dfrac{\pi}{60}$ radians/s as the bridge is raised, how fast is the distance AB changing when the angle of inclination is $\theta = \dfrac{\pi}{6}$ radians?



11. A helicopter leaves the ground at a point 80 m from an observer and rises vertically at a rate of 2.5 m/s. How fast is the angle of elevation of the observer's line of sight to the helicopter increasing when the helicopter is at an altitude of 60 m?

12. The height of an object thrown downward from an initial altitude of 200 m is

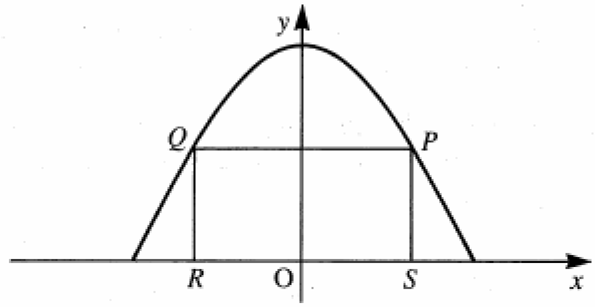
$$h(t) = 200 - 12t - 5t^2$$

The object is being tracked by a searchlight 100 m from where the object will hit the ground. How fast is the angle of elevation of the searchlight changing after 3 s?

19. If $\sin^2 x + \sin^2 y = \frac{5}{4}$, and $\frac{dx}{dt} = -1$, find $\frac{dy}{dt}$ at the point $(x, y) = \left(\frac{2\pi}{3}, \frac{3\pi}{4}\right)$.

20. Find the length of the shortest ladder that will reach from ground level to a high vertical wall if it must clear a 2 m vertical fence that is 3 m from the wall.
(Hint: The use of an angle as a variable can simplify this problem.)

24. A rectangle $PQRS$ is inscribed, as sketched, in the region between the x-axis and the part of the graph of $y = \cos 4x$ specified by $-\frac{\pi}{8} \leq x \leq \frac{\pi}{8}$. Determine the coordinates of P for which the perimeter of $PQRS$ is a maximum.



ANSWERS

4. **a.** $\frac{13}{5}$, 1 **b.** 2.55, 0.6058 **c.** $\frac{2}{3\sqrt{3}}$, 0

5. a. $2\pi - 2\pi \sin(2\pi t)$

b. $-4\pi^2 \cos(2\pi t)$ **c.** $\frac{1}{4}, \frac{5}{4}, \frac{9}{4}$ **d.** 4π

6. a. $2\pi t \cos(\pi t^2)$

b. $2\pi \cos(\pi t^2) - 4\pi^2 t^2 \sin(\pi t^2)$ **c.** $\pm\frac{1}{\sqrt{2}}$, 0

d. $-\frac{1}{\sqrt{2}} < t < 0$, $\frac{1}{\sqrt{2}} < x < 1$

8. a. 118, 82

9. $\frac{\pi}{6}$

11. $\frac{1}{50}$ rad/s

12. -0.174 rad/s

19. $\frac{\sqrt{3}}{2}$

20. $\left(2^{\frac{2}{3}} + 3^{\frac{2}{3}}\right)^{\frac{3}{2}} \approx 7.02$

24. $\left(\frac{\pi}{24}, \frac{\sqrt{3}}{2}\right)$