Date:_____

UNIT 1: EQUATIONS

Polynomial, Rational, Radical & Absolute Value

1.1 Review of Factoring Techniques

1. Factor fully.

a) $p^2+2pr+r^2$ **b)** $16n^2+8n+1$ **c)** $9u^2-30u+25$ **d)** v^2+4v+3

e) $2w^2+3w+1$ **f)** $3k^2+7k+2$ **g)** $7y^2+15y+2$ **h)** $5x^2-16x+3$

2. Factor fully.

a) $x^2-14x+48$ **b)** y^2-3y+2 **c)** $3x^2-10x+7$ **d)** $3x^3-75x$

e) $6x^2+7x-3$ **f)** x^3+x^2-56x **g)** $4x^2+20x$ **h)** $3x^3-12x$

3. Factor fully.

a) $25x^2-y^2$ **b)** m^2-p^2 **c)** $1-81r^2$ **d)** $49m^2-64$

e) $p^6r^4-100x^2$ **f)** $3-48y^2$ **g)** $(x+n)^2-9$ **h)** $49u^2-(x-y)^2$

4. Factor fully.

a) $kx+px-ky-py$ **b)** $fx-gy+gx-fy$ **c)** h^3+h^2+h+1

d) $x-d+(x-d)^2$ **e)** $4y^2+4yz+z^2-1$ **f)** $x^2-y^2+z^2-2xz$

5. Factor fully.

a) $4x^2+2x-6$ **b)** $28s^2+8st-20t^2$ **c)** $y^2-(r-n)^2$

d) $8+24m-80m^2$ **e)** $6x^2-13x+6$ **f)** y^3+y^2-5y-5

g) $60y^2-10y-120$ **h)** $10x^2+38x+20$ **i)** $27x^2-48$

6. Factor fully.

a) $3v^2-11v+10$ **b)** $6x^2-14x-12$

c) $36(2x-y)^2-25(u-2y)^2$ **d)** $g(1-x)-gx+gx^2$

e) y^3-y^2-y+1 **f)** $n^4+2n^2w^2+w^4$

g) $9(x+2y+z)^2-16(x-2y+z)^2$ **h)** $8u^2(u+1)+2u(u+1)-3(u+1)$

i) $p^2-2p+1-y^2-2yz-z^2$ **j)** $9y^4+12y^2+4$

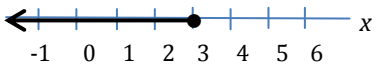
k) x^8-16 **l)** $10x^4-5x^3-6x^2+3x$

m) $(x^2+4x)^2+7(x^2+4x)+12$ **n)** $(2a^2+5a)^2-10(2a^2+5a)-24$

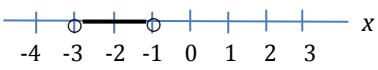
Date:_____ **1.2 Solving Linear Inequalities & Quadratic Equations**

1. Interpret each graphed solution using **i) set** notation and **ii) interval** notation.

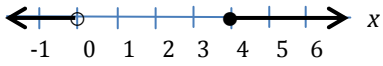
a)



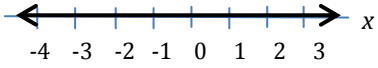
b)



c)



d)



2. Solve each of the following inequalities and graph on a number line.

a) $3x-2>2x+5$

b) $\frac{x-1}{5}+1\geq\frac{x+2}{4}$

c) $4x-5\leq 2(x-7)$

d) $4x+7<9x+17$

3. For each of the following inequalities, determine if $x = 0$ is contained in the solution set.

a) $3x\leq 4x+1$

b) $3x\leq x-1\leq 3x+1$

c) $3(x+2)>x+6$

4. Solve and graph each of the following on a number line.

a) $x+6>2x$ ***and*** $x+6<4x$

b) $2x-3>3-x$ ***or*** $2x-3>x-3$

c) $-5\leq 2x-1\leq 5$

d) $4+x\geq 4-x$ ***and*** $3(4-x)<x-16$

e) $x>\frac{x-2}{3}$ ***or*** $x<\frac{2x-1}{3}$

f) $2(x-1)\leq 3(x-2)$ ***or*** $2(x+1)\geq 3(x+2)$

5. Solve by factoring.

a) $36x+9x^2=0$

b) $16y^2-1=0$

c) $x^2-2x-15=0$

d) $y^2+18y+81=0$

e) $x^2+5x=6$

f) $3x=4-7x^2$

g) $x(3x-13)+4=0$

h) $\frac{x^2}{2}-\frac{4x^2}{3}=-x$

i) $0=9y^2-\frac{3}{2}(y+2)$

6. Solve using the quadratic formula. $x, y\in C$.

a) $x^2+4x-8=0$

b) $x^2-4x+8=0$

c) $3x^2=2-6x$

d) $2=4y-3y^2$

e) $\frac{1}{5}y^2-\frac{1}{6}y=\frac{1}{5}$

f) $(2x+1)^2=26-(2x+3)^2$

Date:_____ **1.3 Division of Polynomials**

1. Perform each of the following divisions and express the result in the form ***dividend = divisor × quotient + remainder***.

a) $753 \div 22$ **b)** $1053 \div 9$

2. **a)** In Question 1 **a**, explain why 22 is not a factor of 753.
b) In Question 1 **b**, explain why 9 is a factor of 1053.

3. For $f(x) = (x-2)(x^2+3x-2)+5$,

a) identify the linear divisor $d(x)$.

b) identify the quotient $q(x)$.

c) identify the remainder $r(x)$.

d) identify the dividend $f(x)$.

4. When a certain polynomial is divided by x^2+x+1, its quotient is x^2-x+1 and its remainder is -1. What is the polynomial in expanded form?

5. Divide and express each answer using a **division statement**, where $f(x) = d(x)q(x)+r(x)$. Factor fully if the remainder is 0.

a) $(2x^2-5x-1) \div (x-3)$ **b)** $(3x^3+7x^2+5x+1) \div (3x+1)$

c) $(3x^3+x^2-x-6) \div (x+1)$ **d)** $(x^3-2x-4) \div (x-2)$

e) $(5x-2x^3+3+x^4) \div (1+2x+x^2)$ **f)** $(3x^4-5x^3-x^2+5x-2) \div (3x-2)$

6. Divide and express each answer in **mixed rational form**, where $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

a) $\frac{385}{4}$ **b)** $\frac{3757}{18}$ **c)** $\frac{3x^2-x-7}{x+1}$

d) $\frac{x^2}{3-x}$ **e)** $\frac{(x-2)(x+1)}{x-1}$ **f)** $\frac{(x^2-2)^2}{(x-1)(x+1)}$

7. The volume of a rectangular box is $(x^3+6x^2+11x+6)\ cm^3$. The box is $(x+3)\ cm$ long and $(x+2)\ cm$ wide. What is the height of the box? $(V = lwh)$

8. The volume of a cylindrical can is $(4\pi x^3+28\pi x^2+65\pi x+50\pi)\ cm^3$. The can is $(x+2)\ cm$ high. What is the radius? $(V = \pi r^2 h)$

Date:_____ **1.4 The Remainder Theorem**

1. a) Use the remainder theorem to determine the remainder when $16x^3 - 100x + 25 - 4x^2$ is divided by $4x - 1$.

b) Verify your answer to **a)** using long division.

c) Express $16x^3 - 4x^2 - 100x + 25$ in fully factored form.

2. Determine the remainder in each of the following using the Remainder Theorem.

a) $(x^3 - 2x^2 + 3x + 4) \div (x + 1)$ **b)** $(x^3 + 3x^2 - 7) \div (x - 2)$

c) $(6x^2 - 10x + 7) \div (3x + 1)$ **d)** $(4x^3 + 9x - 10) \div (2x + 1)$

3. Determine the value(s) of *k* in each of the following:

a) When $x^4 - kx^3 - 2x^2 + x + 4$ is divided by $x - 3$, the remainder is 16.

b) When $2x^3 - 3x^2 + kx - 1$ is divided by $2x - 1$, the remainder is -2.

c) When $6x^2 + 10x - 3$ is divided by $x + k$, the remainder is -3.

d) When $2x^3 + 2kx + k^2$ is divided by $2x + 3$, the remainder is $-2\frac{1}{4}$.

4. If $f(x) = mx^3 + gx^2 - x + 3$ is divided by $x + 1$, the remainder is 3. If $f(x)$ is divided by $x + 2$, the remainder is –7. What are the values of *m* and *g*?

5. If $f(x) = mx^3 + gx^2 - x + 3$ is divided by $x - 1$, the remainder is 3. If $f(x)$ is divided by $x + 3$, the remainder is –1. What are the values of *m* and *g*?

6. Determine the value of *p* in each of the following:

a) When $-2x^3 + px^2 - 5x + 2$ is divided by $x - 2$ and by $x + 1$, the remainder is the same.

b) When $g(x) = 8x^3 + 10x^2 - px - 5$ is divisible by $2x + 1$.

Note: *There is no remainder since* $2x + 1$ *is a factor.*

c) When $x^6 + x^4 - 2x^2 + p$ is divided by $x^2 + 1$, the remainder is 5.

Hint: *Let* $y = x^2$.

7. a) Find the value of *k* if $3x - 2$ is a factor of $6x^3 + kx^2 - 6x - 8$.

b) Verify your answer to **a)** using long division.

c) Express $6x^3 + kx^2 - 6x - 8$ in fully factored form.

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Date:_____ **1.5 The Factor Theorem and Sum & Difference of Cubes**

1. If $x+8$ is a factor of $f(x)$, then what is the value of $f(-8)$?

2. **a)** If $f(5)=0$, state a factor of $f(x)$.
b) Explain how you would find the other factors of $f(x)$.

3. If $f(x)=x^3+2x^2-5x-6$, is equal to 0 when $x=-1$ or 2 or -3, what are the factors of $f(x)$?

4. Use the factor theorem to determine whether the binomial is a factor of $f(x)$ for the following:
a) $x+2;\ f(x)=x^3-3x^2-4x+12$ **b)** $x-3;\ f(x)=x^3+6x^2-2x+3$

5. Completely factor the following:
a) x^3-4x+3 **b)** x^3+2x^2-x-2 **c)** x^3+5x^2+8x+4
d) y^3-y^2-y-2 **e)** $x^3-9x^2+22x-8$ **f)** $x^4-8x^3+3x^2+40x-12$

6. If $x-1$ is a factor of x^3-2kx^2+3x+1, what is the value of *k*?

7. If $k^2x^3+4x^2-5kx-4$ is divisible by $x+2$, determine the value(s) of *k*.

8. Using the formulas for factoring the sum or difference of cubes, factor each of the following:
a) x^3+27 **b)** $-2y^3+16$ **c)** $125x^3+64y^3$ **d)** x^6-1

Date:_____ **1.6 The Extended Factor Theorem**

1. If $2x+5$ is a factor of $f(x)$, then what is the value of $f\left(-\frac{5}{2}\right)$?

2. Write the binomial factor that corresponds to the polynomial $f(x)$.

a) $f\left(\frac{2}{3}\right)=0$ **b)** $f\left(-\frac{1}{4}\right)=0$

3. A cubic polynomial $g(x)$ has the property that $g\left(-\frac{1}{2}\right)=0$, $g(1)=0$ and $g\left(\frac{2}{5}\right)=0$.

Find $g(x)$ in factored form.

4. Determine the value of *k* so that $3x-2$ is a factor of $3x^3-5x^2+kx+2$.

5. Use the factor theorem to determine whether the binomial is a factor of $f(x)$ for the following:

a) $2x-1;\ f(x)=4x^3-6x^2+8x-3$ **b)** $3x+1;\ f(x)=3x^3+x^2-12x-3$

6. Determine the test values that could be zeros of each polynomial.

a) $2x^3-5x^2-4x+3$ **b)** $4x^3-7x-3$

7. Completely factor the following:

a) $2x^3-5x^2-4x+3$ **b)** $4x^3-7x-3$

c) $2x^3-7x^2+7x-5$ **d)** $64x^3+48x^2+12x+1$

e) $27x^3+18x^2-12x-8$ **f)** $5x^4+x^3-22x^2-4x+8$

g) $2x^4+3x^3-32x^2-48x$ **h)** $24x^4+8x^3+3x+1$

8. Using the formulas for factoring the sum or difference of cubes, factor each of the following:

a) $-3x^4y+24xy^4$ **b)** $(x+5)^3-(2x+1)^3$ **c)** $(x-3)^3+(3x-2)^3$

d) $-54x^5-16x^2$ **e)** $\frac{1}{27}x^3-\frac{8}{125}$ **f)** x^9-512

Date:_____ **Review Part I: 1.1 to 1.6**

1. Factor each of the following completely. Long division is not to be used.

a) $-12x^3+26x^2-12x$ **b)** $4x^4+7x^2-36$ **c)** $16x^3-54y^3$

d) x^6-8x^3-9 **e)** $(3x+1)^2-(x-5)^2$ **f)** $(3x+1)^3-(x-5)^3$

g) $(3x+1)^3+(x-5)^3$ **h)** $3x^3+6x^2+3x+6$ **i)** $8x^5-32x^3-x^2+4$

j) $x^2-8xy+16y^2-9z^2$ **k)** $25x^4-y^2-6y-9$

2. When a polynomial $f(x)$ is divided by $2x-3$, its quotient is x^3-2x-7 and its remainder is -4. Find $f(x)$ in expanded form.

3. Divide and express each answer using a **division statement**. Factor fully if the remainder is 0.

a) $(6x^3+31x^2+25x-12)\div(2x+3)$ **b)** $(x^4-4x^3+3x^2-3)\div(x^2+x-2)$

4. Divide and express each answer in **mixed rational form**.

a) $\dfrac{x^3-4x^2}{1-x}$ **b)** $\dfrac{(2x-3)^2}{2x-1}$

5. Without using long division, determine the remainder when

a) (x^3-5x^2-2x-1) is divided by $(x+2)$ **b)** $(3x^3+x+2)$ is divided by $(3x-1)$

6. **a)** When x^3-3kx^2+x+5 is divided by $x-2$, the remainder is 9. Find the value of *k*.

b) When rx^3+gx^2+4x+1 is divided by $x-1$, the remainder is 12. When it is divided by $x+3$, the remainder is -20. Find the values of *r* and *g*.

7. **a)** If $f(-3)=0$, state a factor of $f(x)$. **b)** If $f\left(\dfrac{2}{3}\right)=0$, state a factor of $f(x)$.

8. Use the factor theorem to determine if

a) $x-2$ is a factor of $x^5-4x^3+x^2-3$. **b)** $2x+3$ is a factor of $6x^3+5x^2-16x-15$.

9. **a)** If $x-1$ is a factor of $x^4+2kx^3-k^2x+2$, determine the value(s) of *k*.

b) If $3x+2$ is a factor of $3kx^3+4x^2-2kx-2$, what is the value of *k*?

10. Determine the values of *m* and *n* so that the polynomials $f(x)=2x^3+mx^2+nx-3$ and $g(x)=x^3-3mx^2+2nx+4$ are both divisible by $x-2$.

11. Factor each of the following.

a) x^3-6x^2+6x-5 **b)** $x^3-x^2-14x+24$ **c)** $2x^3-7x^2+9$

d) $8x^3+12x^2+6x+1$ **e)** $x^4-6x^3-15x^2-6x-16$ **f)** $6x^4+x^3-8x^2-x+2$

12. An artist creates a carving from a rectangular block of soapstone whose volume, *V*, in cubic metres, can be modelled by $V(x)=6x^3+25x^2+2x-8$. Determine possible dimensions of the block, in metres, in terms of binomials of *x*.

MHF4UI Unit 1: Day 7

Date:_____ **1.7 Solving Polynomial Equations**

1. Solve for *x* in each of the following without using long division, $x \in C$.

a) $-2x(2x+5)(4x-2)=0$ **b)** $(x-4)^2(x^2-4)=0$

c) $4(3x+6)(12x^2+8x-15)=0$ **d)** $-(3+5x)(3x^2-8x+2)=0$

e) $x^4-1=0$ **f)** $2x^3-54=0$

g) $x^3-3x^2-9x+27=0$ **h)** $x^4=8-7x^2$

i) $x^6-7x^3-8=0$ **j)** $4x^4+8x=2x^3+16x^2$

2. Determine the roots of each of the following using long division, $x \in C$.

a) $x^3-3x-2=0$ **b)** $x^3-9x^2+26x=24$

c) $x^3+8x=7x^2-10$ **d)** $8x^3-12x^2+6x-1=0$

e) $5x^3-8x^2-27x+18=0$ **f)** $(x+1)(x+5)(x+3)=-3$

g) $x^4+9x^3+21x^2-x-30=0$ **h)** $3x^3+2=x(5x-4)$

3. Solve for *x* in each of the following using the change of variable technique, $x \in C$.

a) $(x^2-x)^2-8(x^2-x)+12=0$

b) $(2x^2+5x)^2-10(2x^2+5x)-24=0$

c) $(x^2+6x+6)(x^2+6x+8)=528$

MHF4UI Unit 1: Day 8

Date:_____ **1.8 Determining Polynomial Equations From Roots**

1. Write a polynomial equation in expanded form with integral coefficients having the given roots.

a) $2, -1-i$ *and* $-1+i$

b) $-\frac{2}{3}, -\frac{2}{3}$ *and* 3

c) $0, -1, 4-3\sqrt{2}$ *and* $4+3\sqrt{2}$

2. Determine the roots of the following by using appropriate factoring techniques, $x \in C$

a) $x^3+3x^2-4=0$ **b)** $x^4-19x^2=-18$

c) $4x^4-2x^3-16x^2+8x=0$ **d)** $(x^2+2x)^2-(x^2+2x)-12=0$

e) $3x^3-13x^2+x+1=0$ **f)** $x(1-4x^2)+12x^2=3$

g) $(x^2-4x+5)(x^2-4x+2)=-2$ **h)** $x^6-26x^3-27=0$

i) $2x^4-x^3-14x^2-5x+6=0$ **j)** $8x^5+32x^3+x^2+4=0$

3. If 2 is one root of the equation $2x^3-13x^2+kx+10=0$, find values for *k* and the other two roots.

4. If $\frac{1}{2}$ and -2 are roots of the equation $2x^3+px^2-qx+6=0$, find the values of *p* and *q*.

5. A rectangular box has length $x-1$, width $x-2$, and height $x-4$, with all dimensions in centimetres. Solve a polynomial equation to determine the dimensions of the box if the volume is $12\,cm^3$.

MHF4UI Unit 1: Day 9

Date:_____ **1.9 Solving Radical Equations**

Remember:
*You **must** verify all the values obtained for the variable when solving a radical equation.*

1. Solve.

a) $\sqrt{5x+6}=4$ **b)** $2\sqrt{x}=3$ **c)** $\sqrt{x+2}+3=0$

d) $2\sqrt{x-1}-1=9$ **e)** $\sqrt{x+1}=1+\sqrt{x-4}$ **f)** $\sqrt{x+2}-\sqrt{x+5}=3$

2. Solve.

a) $5\sqrt{x-6}=x$ **b)** $7-x=-\sqrt{x-1}$

c) $\sqrt{14-10x}+3=x$ **d)** $\sqrt{3x-2}-1=\sqrt{2x-3}$

e) $\sqrt{x+1}+\sqrt{3x+1}=2$ **f)** $2\sqrt{x+6}+\sqrt{2x+10}=2$

g) $\sqrt{x^2+4x+4}-\sqrt{x^2+3x}=1$ **h)** $x-\sqrt{2x+1}=0$

3. Solve by isolating the radical and then cubing both sides of the equation.

a) $\sqrt[3]{1-3x}-4=0$ **b)** $\frac{1}{3}\sqrt[3]{2x+1}=-1$

c) $\sqrt[3]{x^2-19}-5=0$ **d)** $\sqrt[3]{9x+9}=x+1$

MHF4UI Unit 1: Day 10

Date:_____ **1.10 Solving Rational Equations**

1. Determine all real roots of the following equations. Include restrictions on the variable.

a) $\frac{5(x-3)}{4}=\frac{6(x-2)}{5}+\frac{x}{2}$ **b)** $\frac{1}{3y}+\frac{3}{4}=\frac{2-y}{y}$

c) $\frac{2a}{6a+5}=\frac{a+3}{3a-1}$ **d)** $\frac{3}{x+1}=\frac{3}{4}-\frac{x-1}{3x+1}$

e) $-\frac{3}{4}=\frac{-12}{(x-4)^2}$ **f)** $\frac{4}{x+1}-\frac{12}{x+3}=\frac{-5}{x+2}$

2. Solve, $x\in C$. Include restrictions on the variable.

a) $\frac{x-2}{3x}-\frac{x-4}{x+2}=0$ **b)** $x^2-\frac{36}{x^2}=5$

c) $\frac{x+1}{x}+\frac{x+2}{x-2}=\frac{x-1}{x-2}+\frac{3}{x}$ **d)** $\frac{x^2+5x+6}{2x^2+3x-2}=\frac{x-1}{x+2}$

e) $0=1-\frac{3}{x^2}+\frac{2}{x^3}$ **f)** $-x^{-2}=\frac{1+x}{x^2-x}$

3. Determine all real roots of the following equations. Include restrictions on the variable.

a) $x^2+x+\frac{12}{x^2+x}=8$ **b)** $\left(x+\frac{4}{x}\right)^2+2\left(x+\frac{4}{x}\right)-8=0$

c) $1+\frac{\sqrt{y+4}}{\sqrt{y-3}}=\frac{7}{\sqrt{y-3}}$ **d)** $\frac{1}{1-x}+\frac{1}{1+\sqrt{x}}=\frac{1}{1-\sqrt{x}}$

Hint: $1-x=\left(1-\sqrt{x}\right)\left(1+\sqrt{x}\right)$

Date:_____ **1.11 Absolute Value Equations and Inequalities**

1. Evaluate each of the following:

a) $|-3-7|$ **b)** $|4|+|-15|$

c) $|3|-|-5|+|3-9|$ **d)** $|9-3|+5|-3|-3|7-12|$

2. Graph each of the following on the number line, for $x \in R$. Rewrite each statement without the absolute value bars in both algebraic and interval notation.

a) $|x| \geq 3$ **b)** $|x| < 1$

3. Graph each of the following on the number line, for $x \in R$. Rewrite each statement using absolute value notation.

a) $x \geq -6$ *and* $x \leq 6$ **b)** $x \in (-\infty, -5) \cup (5, +\infty)$

4. Determine if $x = 0$ is contained in the solution set for each of the following.

a) $|7x+3| \geq 11$ **b)** $|5x-2| = 2-3x$

5. Solve for *x* by *inspection*, $x \in R$.

a) $|x-5| = 1$ **b)** $|3-x| = 8$

c) $|2x-1| = 7$ **d)** $|x| = -5$

6. Solve for *x algebraically with cases*, $x \in R$. Answer using a *solution set* if appropriate.

a) $|3x+2| = 6$ **b)** $|x-3| \leq 9$

c) $|x+4| \geq 5$ **d)** $|2x-3| < 4$

7. Solve for *x algebraically with cases*, $x \in R$. Answer using *interval notation* if appropriate.

a) $|x| = 3x+4$ **b)** $|x-5| = 4x+1$ **c)** $|4x-8| = 2x$

d) $|x-1| < x$ **e)** $|2x+4| \geq 12x$ **f)** $|3x-1| \leq 5|3x-1|-16$

Date:_____ **Review Part II: 1.7 to 1.11**

1. Solve for *x* in each of the following, $x \in C$. Include restrictions on the variable.

a) $\frac{3}{4}(x-1)-\frac{2(2x+1)}{3}=\frac{3}{2}$

b) $\frac{x+3}{4}=\frac{16}{x+3}$

c) $\frac{x}{x+1}-\frac{5}{x+4}=-\frac{1}{6}$

d) $-2x^4+54x=0$

e) $x^3-4x^2+8=0$

f) $(2x^2+x)^2-7(2x^2+x)+6=0$

g) $x^6+7x^3-8=0$

h) $3x^3+2x^2-75x-50=0$

i) $1-\frac{4}{x}-\frac{3}{x^2}+\frac{18}{x^3}=0$

j) $9x^2-4x^{-2}=-5$

k) $6x^4-x^3-25x^2-23x-5=0$

l) $8x^5-64x^3=8-x^2$

m) $6\left(\frac{1}{x-1}\right)^2-\frac{1}{x-1}-1=0$

n) $3x^3-17x^2-3x+1=0$

2. Write a polynomial equation in expanded form with integral coefficients having the given roots.

a) $-2,\ 0,\ \frac{1}{4}$ *and* 1

b) $2-\sqrt{5},\ 2+\sqrt{5},\ -1-2i$ *and* $-1+2i$

3. Solve for *x*, $x \in R$. Remember to verify.

a) $\sqrt{x+3}-3=0$

b) $\sqrt{3x-5}+1=\sqrt{3x}$

c) $\sqrt{x-1}-x=-1$

d) $\sqrt{3x-2}-2\sqrt{x}=1$

e) $\sqrt{x+1}+\frac{2}{\sqrt{x+1}}=\sqrt{x+6}$

f) $\frac{x}{3\sqrt{x^2+1}}-\frac{1}{5}=0$

4. Rearrange each equation and then solve by inspection.

a) $3|y+8|-5=2|y+8|+1$

b) $2|x-3|+|x-3|=10-2|x-3|$

5. Solve for *x*, $x \in R$. Answer using algebraic notation.

a) $|3x-1|=11$

b) $|2x-3|+2=4x$

c) $|2x-3| \geq 5$

d) $3|x-2|-1<10+|x-2|$

e) $|1-3x| \leq x+5$

f) $\frac{1}{3}|x+4|-5x>0$

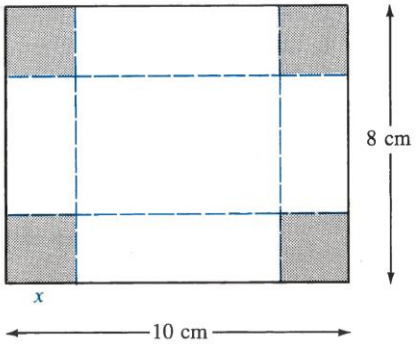
6. Find value(s) of *k* if -1 is a root of each of the following equations.

a) $\sqrt{k-x}+\sqrt{2x+k}=3$

b) $x^3-k^2x^2+x+3k=0$

7. If $-\frac{2}{3}$ and $\frac{1}{2}$ are roots of the equation $6x^3+px^2+q=0$, find values for *p*, *q* and the other root.

8. A rectangular piece of cardboard measuring 10 *cm* by 8 *cm* is made into an open box by cutting squares from the corners and turning up the sides. If the box is to hold a volume of $48\ cm^3$, what are the possible dimensions of the box? $(V=lwh)$



Unit 1 Answers

1.1 Review of Factoring Techniques

1. a) $(p+r)^2$ **b)** $(4n+1)^2$ **c)** $(3u-5)^2$ **d)** $(v+3)(v+1)$ **e)** $(2w+1)(w+1)$ **f)** $(3k+1)(k+2)$ **g)** $(7y+1)(y+2)$
h) $(5x-1)(x-3)$

2. a) $(x-6)(x-8)$ **b)** $(y-2)(y-1)$ **c)** $(3x-7)(x-1)$ **d)** $3x(x-5)(x+5)$ **e)** $(3x-1)(2x+3)$ **f)** $x(x+8)(x-7)$
g) $4x(x+5)$ **h)** $3x(x-2)(x+2)$

3. a) $(5x-y)(5x+y)$ **b)** $(m-p)(m+p)$ **c)** $(1-9r)(1+9r)$ **d)** $(7m-8)(7m+8)$ **e)** $(p^3r^2-10x)(p^3r^2+10x)$
f) $3(1-4y)(1+4y)$ **g)** $(x+n-3)(x+n+3)$ **h)** $(7u-x+y)(7u+x-y)$ **i)** $(x-2)(x+2)(x^2+4)$

4. a) $(k+p)(x-y)$ **b)** $(f+g)(x-y)$ **c)** $(h+1)(h^2+1)$ **d)** $(x-d)(1+x-d)$ **e)** $(2y+z-1)(2y+z+1)$
f) $(x-z-y)(x-z+y)$

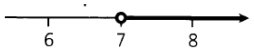
5. a) $2(2x+3)(x-1)$ **b)** $4(7s-5t)(s+t)$ **c)** $(y-r+n)(y+r-n)$ **d)** $8(1+5m)(1-2m)$ *or* $-8(5m+1)(2m-1)$
e) $(3x-2)(2x-3)$ **f)** $(y+1)(y^2-5)$ **g)** $10(3y+4)(2y-3)$ **h)** $2(5x^2+19x+10)$ **i)** $3(3x-4)(3x+4)$

6. a) $(3v-5)(v-2)$ **b)** $2(3x+2)(x-3)$ **c)** $(12x+4y-5u)(12x-16y+5u)$ **d)** $g(1-x)^2$ **e)** $(y+1)(y-1)^2$ **f)** $(n^2+w^2)^2$
g) $-(x-14y+z)(7x-2y+7z)$ **h)** $(u+1)(4u+3)(2u-1)$ **i)** $(p-1+y+z)(p-1-y-z)$ **j)** $(3y^2+2)^2$
k) $(x^2-2)(x^2+2)(x^4+4)$ **l)** $x(2x-1)(5x^2-3)$ **m)** $(x+1)(x+3)(x+2)^2$ **n)** $(2a+1)(a+2)(2a-3)(a+4)$

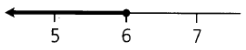
1.2 Review of Solving Linear Inequalities & Quadratic Equations

1. a) $\{x \in R \mid x \le 3\}$; $x \in (-\infty, 3]$ **b)** $\{x \in R \mid -3 < x < -1\}$; $x \in (-3, -1)$
c) $\{x \in R \mid x < 0 \text{ or } x \ge 4\}$; $x \in (-\infty, 0) \cup [4, +\infty)$ **d)** $\{x \in R\}$; $x \in (-\infty, +\infty)$

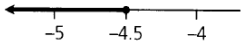
2. a) $x > 7$



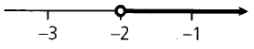
b) $x \le 6$



c) $x \le -4.5$



d) $x > -2$



3. a) Yes **b)** No **c)** No

4. a) $2 < x < 6$

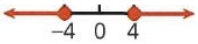
b) $x > 0$

c) $-2 \le x \le 3$

d) $x > 7$

e) $x \ne -1$

f) $x \le -4$ *or* $x \ge 4$



5. a) $-4, 0$ **b)** $-\frac{1}{4}, \frac{1}{4}$ **c)** $5, -3$ **d)** $-9, -9$ **e)** $-6, 1$ **f)** $-1, \frac{4}{7}$ **g)** $\frac{1}{3}, 4$ **h)** $0, \frac{6}{5}$ **i)** $-\frac{1}{2}, \frac{2}{3}$

6. a) $-2-2\sqrt{3}, -2+2\sqrt{3}$ **b)** $2-2i,\ 2+2i$ **c)** $\frac{-3-\sqrt{15}}{3}, \frac{-3+\sqrt{15}}{3}$ **d)** $\frac{2-i\sqrt{2}}{3}, \frac{2+i\sqrt{2}}{3}$ **e)** $-\frac{2}{3}, \frac{3}{2}$ **f)** $-1-\sqrt{3}, -1+\sqrt{3}$

1.3 Division of Polynomials

1. a) $753 = 22(34) + 5$ **b)** $1053 = 9(117)$

2. a) *The remainder is not zero.* **b)** *The remainder is zero.*

3. a) $x - 2$ **b)** $x^2 + 3x - 2$ **c)** 5 **d)** $x^3 + x^2 - 8x + 9$

4. $f(x) = x^4 + x^2$

5. a) $2x^2 - 5x - 1 = (x - 3)(2x + 1) + 2$ **b)** $3x^3 + 7x^2 + 5x + 1 = (3x + 1)(x + 1)^2$

c) $3x^3 + x^2 - x - 6 = (x + 1)(3x^2 - 2x + 1) - 7$ **d)** $x^3 - 2x - 4 = (x - 2)(x^2 + 2x + 2)$

e) $x^4 - 2x^3 + 5x + 3 = (x^2 + 2x + 1)(x^2 - 4x + 7) - 5x - 4$ **f)** $3x^4 - 5x^3 - x^2 + 5x - 2 = (3x - 2)(x + 1)(x - 1)^2$

6. a) $\frac{385}{4} = 96 + \frac{1}{4}$ **b)** $\frac{3757}{18} = 208 + \frac{13}{18}$ **c)** $\frac{3x^2 - x - 7}{x + 1} = 3x - 4 - \frac{3}{x + 1}$

d) $\frac{x^2}{3 - x} = -x - 3 + \frac{9}{3 - x}$ **e)** $\frac{x^2 - x - 2}{x - 1} = x - \frac{2}{x - 1}$ **f)** $\frac{x^4 - 4x^2 + 4}{x^2 - 1} = x^2 - 3 + \frac{1}{x^2 - 1}$

7. $h = (x + 1)$ *cm* **8.** $r = (2x + 5)$ *cm*

1.4 The Remainder Theorem

1. a) 0 **c)** $(4x - 1)(2x - 5)(2x + 5)$

2. a) -2 **b)** 13 **c)** 11 **d)** -15

3. a) 2 **b)** -1 **c)** $0, \frac{5}{3}$ **d)** $\frac{3 - 3\sqrt{3}}{2}, \frac{3 + 3\sqrt{3}}{2}$

4. $m = 2,\ g = 1$

5. $m = \frac{4}{9},\ g = \frac{5}{9}$

6. a) 11 **b)** 7 **c)** 3

7. a) 23 **c)** $(3x - 2)(2x + 1)(x + 4)$

1.5 The Factor Theorem

1. 0

2. a) $x - 5$ **b)** *Divide*

3. $(x + 1), (x - 2), (x + 3)$

4. a) *yes* **b)** *no*

5. a) $(x - 1)(x^2 + x - 3)$ **b)** $(x + 2)(x - 1)(x + 1)$ **c)** $(x + 1)(x + 2)^2$

d) $(y - 2)(y^2 + y + 1)$ **e)** $(x - 4)(x^2 - 5x + 2)$ **f)** $(x + 2)(x - 3)(x^2 - 7x + 2)$

6. $\frac{5}{2}$ **7.** $-\frac{3}{4}, 2$

8 a) $(x + 3)(x^2 - 3x + 9)$ **b)** $-2(y - 2)(y^2 + 2y + 4)$ **c)** $(5x + 4y)(25x^2 - 20xy + 16y^2)$ **d)** $(x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$

1.6 The Extended Factor Theorem

1. 0

2. a) $3x - 2$ **b)** $4x + 1$

3. $g(x) = (2x + 1)(x - 1)(5x - 2)$

4. -1

5. a) *yes* **b)** *no*

6. a) $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$ **b)** $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

7. a) $(x + 1)(x - 3)(2x - 1)$ **b)** $(x + 1)(2x - 3)(2x + 1)$ **c)** $(2x - 5)(x^2 - x + 1)$ **d)** $(4x + 1)^3$

e) $(3x - 2)(3x + 2)^2$ **f)** $(x - 2)(x + 2)(5x^2 + x - 2)$ **g)** $x(x - 4)(x + 4)(2x + 3)$ **h)** $(3x + 1)(2x + 1)(4x^2 - 2x + 1)$

8. a) $-3xy(x - 2y)(x^2 + 2xy + 4y^2)$ **b)** $-(x - 4)(7x^2 + 25x + 31)$ **c)** $7(4x - 5)(x^2 - x + 1)$

d) $-2x^2(3x + 2)(9x^2 - 6x + 4)$ **e)** $\left(\frac{1}{3}x - \frac{2}{5}\right)\left(\frac{1}{9}x^2 + \frac{2}{15}x + \frac{4}{25}\right)$ **f)** $(x - 2)(x^2 + 2x + 4)(x^6 + 8x^3 + 64)$

Part I Review 1.1-1.6

1. a) $-2x(3x-2)(2x-3)$ **b)** $(2x-3)(2x+3)(x^2+4)$ **c)** $2(2x-3y)(4x^2+6xy+9y^2)$ **d)** $(x^3-9)(x+1)(x^2-x+1)$
e) $8(x+3)(x-1)$ **f)** $2(x+3)(13x^2-18x+21)$ **g)** $4(x-1)(7x^2+10x+31)$ **h)** $3(x+2)(x^2+1)$
i) $(x-2)(x+2)(2x-1)(4x^2+2x+1)$ **j)** $(x-4y-3z)(x-4y+3z)$ **k)** $(5x^2-y-3)(5x^2+y+3)$

2. $f(x)=2x^4-3x^3-4x^2-8x+17$

3. a) $6x^3+31x^2+25x-12=(2x+3)(3x-1)(x+4)$ **b)** $x^4-4x^3+3x^2-3=(x^2+x-2)(x^2-5x+10)-20x+17$

4. a) $\dfrac{x^3-4x^2}{1-x}=-x^2+3x+3-\dfrac{3}{1-x}$ **b)** $\dfrac{4x^2-12x+9}{2x-1}=2x-5+\dfrac{4}{2x-1}$

5. a) -25 **b)** $\dfrac{22}{9}$

6. a) $k=\dfrac{1}{2}$ **b)** $r=2,\ g=5$ **7. a)** $(x+3)$ **b)** $(3x-2)$ **8. a)** *No* **b)** *Yes* **9. a)** $k=-1\ or\ k=3$ **b)** $k=\dfrac{1}{2}$

10. $m=-\dfrac{7}{10},\ n=-5\dfrac{1}{10}$

11. a) $(x-5)(x^2-x+1)$ **b)** $(x-2)(x-3)(x+4)$ **c)** $(x+1)(2x-3)(x-3)$ **d)** $(2x+1)^3$ **e)** $(x+2)(x-8)(x^2+1)$
f) $(x-1)(x+1)(2x-1)(3x+2)$

12. $(x+4)m,\ (2x-1)m,\ (3x+2)m$

1.7 Solving Polynomial Equations

1. a) $-\dfrac{5}{2},\ 0,\ \dfrac{1}{2}$ **b)** $-2,\ 2,\ 4,\ 4$ **c)** $-2,\ -\dfrac{3}{2},\ \dfrac{5}{6}$ **d)** $-\dfrac{3}{5},\ \dfrac{4-\sqrt{10}}{3},\ \dfrac{4+\sqrt{10}}{3}$ **e)** $-1,\ 1,\ -i,\ i$ **f)** $3,\ \dfrac{-3-3i\sqrt{3}}{2},\ \dfrac{-3+3i\sqrt{3}}{2}$

g) $-3,\ 3,\ 3$ **h)** $-1,\ 1,\ -2i\sqrt{2},\ 2i\sqrt{2}$ **i)** $-1,\ 2,\ -1-i\sqrt{3},\ -1+i\sqrt{3},\ \dfrac{1-i\sqrt{3}}{2},\ \dfrac{1+i\sqrt{3}}{2}$ **j)** $-2,\ 0,\ \dfrac{1}{2},\ 2$

2. a) $-1,\ -1,\ 2$ **b)** $2,\ 3,\ 4$ **c)** $1-\sqrt{3},\ 1+\sqrt{3},\ 5$ **d)** $\dfrac{1}{2},\ \dfrac{1}{2},\ \dfrac{1}{2}$ **e)** $-2,\ \dfrac{3}{5},\ 3$ **f)** $\dfrac{-7-\sqrt{13}}{2},\ -2,\ \dfrac{-7+\sqrt{13}}{2}$

g) $-5,\ -3,\ -2,\ 1$ **h)** $-\dfrac{1}{3},\ 1-i,\ 1+i$

3. a) $-2,\ -1,\ 2,\ 3$ **b)** $-4,\ -2,\ -\dfrac{1}{2},\ \dfrac{3}{2}$ **c)** $-8,\ 2,\ -3-i\sqrt{21},\ -3+i\sqrt{21}$

1.8 Determining Polynomial Equations From Roots

1. a) $x^3-2x-4=0$ **b)** $9x^3-15x^2-32x-12=0$ **c)** $x^4-7x^3-10x^2-2x=0$

2. a) $-2,\ -2,\ 1$ **b)** $-3\sqrt{2},\ -1,\ 1,\ 3\sqrt{2}$ **c)** $-2,\ 0,\ \dfrac{1}{2},\ 2$ **d)** $-1-\sqrt{5},\ -1+\sqrt{5},\ -1-i\sqrt{2},\ -1+i\sqrt{2}$ **e)** $2-\sqrt{5},\ \dfrac{1}{3},\ 2+\sqrt{5}$

f) $-\dfrac{1}{2},\ \dfrac{1}{2},\ 3$ **g)** $1,\ 2,\ 2,\ 3$ **h)** $-1,\ 3,\ \dfrac{-3-3i\sqrt{3}}{2},\ \dfrac{-3+3i\sqrt{3}}{2},\ \dfrac{1-i\sqrt{3}}{2},\ \dfrac{1+i\sqrt{3}}{2}$ **i)** $-2,\ -1,\ \dfrac{1}{2},\ 3$

j) $-\dfrac{1}{2},\ -2i,\ 2i,\ \dfrac{1-i\sqrt{3}}{4},\ \dfrac{1+i\sqrt{3}}{4}$ **3.** $k=13;\ -\dfrac{1}{2},\ 5$ **4.** $p=-3,\ q=11$ **5.** $4\ cm\times 3\ cm\times 1\ cm$

1.9 Solving Radical Equations

1. a) 2 **b)** $2\dfrac{1}{4}$ **c)** *no solution* **d)** 26 **e)** 8 **f)** *no solution*

2. a) $10,\ 15$ **b)** 10 **c)** *no solution* **d)** $2,\ 6$ **e)** 0 **f)** -5 **g)** $-3,\ 1$ **h)** $1+\sqrt{2}$

3. a) -21 **b)** -14 **c)** $-12,\ 12$ **d)** $-4,\ -1,\ 2$

1.10 Solving Rational Equations

1. a) -3 **b)** $\frac{20}{21},\ y \neq 0$ **c)** $-\frac{3}{5},\ a \neq -\frac{5}{6}, \frac{1}{3}$ **d)** $-\frac{1}{5}, 5,\ x \neq -1, -\frac{1}{3}$ **e)** $0, 8,\ x \neq 4$ **f)** $-\frac{5}{3}, 3,\ x \neq -3, -2, -1$

2. a) $3-\sqrt{7}, 3+\sqrt{7},\ x \neq -2, 0$ **b)** $-3, 3, -2i, 2i,\ x \neq 0$ **c)** $\frac{1-i\sqrt{15}}{2}, \frac{1+i\sqrt{15}}{2},\ x \neq 0, 2$ **d)** $4-\sqrt{21}, 4+\sqrt{21},\ x \neq -2, \frac{1}{2}$

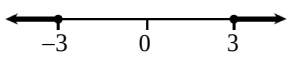
e) $-2, 1, 1,\ x \neq 0$ **f)** $-1-\sqrt{2}, -1+\sqrt{2},\ x \neq 0, 1$

3. a) $-3, -2, 1, 2,\ x \neq -1, 0$ **b)** $-2, -2,\ x \neq 0$ **c)** $12,\ y > 3$ **d)** $\frac{1}{4},\ x \geq 0,\ x \neq 1\ or\ 0 \leq x < 1,\ x > 1$

1.11 Absolute Value Equations and Inequalities

1. a) 10 **b)** 19 **c)** 4 **d)** 6

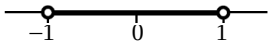
2. a)



$x \leq -3\ or\ x \geq 3$

$x \in (-\infty, -3] \cup [3, +\infty)$

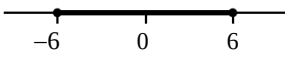
b)



$-1 < x < 1$

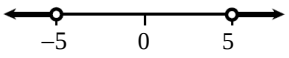
$x \in (-1, 1)$

3. a)



$|x| \leq 6$

b)



$|x| > 5$

4. a) *No* **b)** *Yes* **5. a)** 4, 6 **b)** $-5, 11$ **c)** $-3, 4$ **d)** *no solution*

6. a) $-\frac{8}{3}, \frac{4}{3}$ **b)** $\{x \in R \mid -6 \leq x \leq 12\}$ **c)** $\{x \in R \mid x \leq -9\ or\ x \geq 1\}$ **d)** $\left\{x \in R \,\middle|\, -\frac{1}{2} < x < \frac{7}{2}\right\}$

7. a) -1 **b)** $\frac{4}{5}$ **c)** $\frac{4}{3}, 4$ **d)** $x \in \left(\frac{1}{2}, +\infty\right)$ **e)** $x \in \left(-\infty, \frac{2}{5}\right]$ **f)** $x \in (-\infty, -1] \cup \left[\frac{5}{3}, +\infty\right)$

Part II Review 1.7-1.11

1. a) -5 **b)** $-11, 5,\ x \neq -3$ **c)** $-\frac{13}{7}, 2,\ x \neq -4, -1$ **d)** $0, 3, \frac{-3-3i\sqrt{3}}{2}, \frac{-3+3i\sqrt{3}}{2}$ **e)** $1-\sqrt{5}, 2, 1+\sqrt{5}$

f) $-2, -1, \frac{1}{2}, \frac{3}{2}$ **g)** $-2, 1, \frac{-1-i\sqrt{3}}{2}, \frac{-1+i\sqrt{3}}{2}, 1-i\sqrt{3}, 1+i\sqrt{3}$ **h)** $-5, -\frac{2}{3}, 5$ **i)** $-2, 3, 3,\ x \neq 0$ **j)** $-\frac{2}{3}, \frac{2}{3}, -i, i,\ x \neq 0$

k) $-1, -1, -\frac{1}{3}, \frac{5}{2}$ **l)** $-2\sqrt{2}, -\frac{1}{2}, 2\sqrt{2}, \frac{1-i\sqrt{3}}{4}, \frac{-1+i\sqrt{3}}{4}$ **m)** $-2, 3,\ x \neq 1$ **n)** $-\frac{1}{3}, 3-2\sqrt{2}, 3+2\sqrt{2}$

2. a) $4x^4+3x^3-9x^2+2x=0$ **b)** $x^4-2x^3-4x^2-22x-5=0$

3. a) 6 **b)** 3 **c)** 1, 2 **d)** *no solution* **e)** 3 **f)** $\frac{3}{4}$

4. a) $-14, -2$ **b)** 1, 5

5. a) $-\frac{10}{3}, 4$ **b)** $\frac{5}{6}$ **c)** $x \leq -1\ or\ x \geq 4$ **d)** $-\frac{7}{2} < x < \frac{15}{2}$ **e)** $-1 \leq x \leq 3$ **f)** $x < \frac{2}{7}$

6. a) $k=3$ **b)** $k=1\ or\ k=2$

7. $p=13,\ q=-4;\ -2$

8. $8cm\ by\ 6cm\ by\ 1cm\ \ or\ \ 6cm\ by\ 4cm\ by\ 2cm$