Date:_____________

UNIT 1: EQUATIONS
Polynomial, Rational, Radical & Absolute Value

1.1 Review of Factoring Techniques

Factor each of the following completely:

1. ***Common Factor***

a) $-4x^3+16x^2$

$=-4x^2(x-4)$

b) $2x^2(2x+1)-6x(2x+1)$

$=\underbrace{2x(2x+1)}_{\text{GCF}}(x-3)$

or let $a=2x+1$

$2x^2a-6xa$

$=2xa(x-3)$

$=2x(2x+1)(x-3)$

2. ***Trinomial Factoring***

a) x^2-x-20

$=(x-5)(x+4)$

b) x^4+6x^2+9

$=(x^2+3)(x^2+3)$

$=(x^2+3)^2$

c) $6a^2-4ab-2b^2$

$=2(3a^2-2ab-b^2)$

$=2(3a+b)(a-b)$

d) $-8x^2+22x-12$

$=-2(4x^2-11x+6)$

$=-2(4x-3)(x-2)$

3. ***Difference of Squares*** ***Recall:*** $a^2-b^2=(a-b)(a+b)$

a) $25x^2-16y^2z^6$

$=(5x-4yz^3)(5x+4yz^3)$

b) $81-x^4$

$=(9-x^2)(9+x^2)$

$=(3-x)(3+x)(9+x^2)$

c) $36(x-2)^2-25(x+1)^2$

$=[6(x-2)-5(x+1)][6(x-2)+5(x+1)]$

$=(6x-12-5x-5)(6x-12+5x+5)$

$=(x-17)(11x-7)$

d) $-w^4+13w^2-36$

$=-(w^4-13w^2+36)$

$=-(w^2-9)(w^2-4)$

$=-(w-3)(w+3)(w-2)(w+2)$

4. ***Factor by Grouping***

a) $\underbrace{ax+bx}\underbrace{-ay-by}$

$=x(a+b)-y(a+b)$

$=(a+b)(x-y)$

b) $\underbrace{4x^3+8x^2}\underbrace{-x-2}$

$=4x^2(x+2)-(x+2)$

$=(x+2)(4x^2-1)$

$=(x+2)(2x-1)(2x+1)$

c) a^2-b^2+9-6a

$=\underbrace{a^2-6a+9}\underbrace{-b^2}$

$=(a-3)^2-b^2$

$=[(a-3)-b][(a-3)+b]$

$=(a-3-b)(a-3+b)$

d) $16x^2\underbrace{-4y^2+12y-9}$

$=16x^2-(4y^2-12y+9)$

$=16x^2-(2y-3)^2$

$=[4x-(2y-3)][4x+(2y-3)]$

$=(4x-2y+3)(4x+2y-3)$

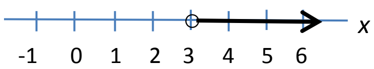
HW. Exercise 1.1

Date:______________ **1.2 Solving Linear Inequalities & Quadratic Equations**

Solving Linear Inequalities

1. Interpret each graphed solution using **i) set** notation and **ii) interval** notation.

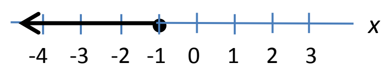
a)



i) $\{x \in \mathbb{R} \mid x > 3\}$

ii) $x \in (3, +\infty)$

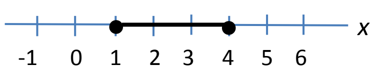
b)



i) $\{x \in \mathbb{R} \mid x \le -1\}$

ii) $x \in (-\infty, -1]$

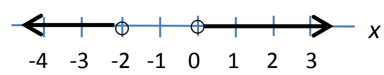
c)



i) $\{x \in \mathbb{R} \mid 1 \le x \le 4\}$

ii) $x \in [1, 4]$

d)



i) $\{x \in \mathbb{R} \mid x < -2, x > 0\}$

ii) $x \in (-\infty, -2) \cup x \in (0, +\infty)$

2. Solve each of the following inequalities and **graph** on a number line.

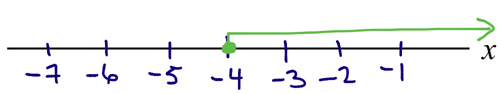
a) $2x - 3 \le 6(x+2) + 1$

$2x - 3 \le 6x + 12 + 1$

$2x - 3 \le 6x + 13$

$-4x \le 16$

$x \ge -4$

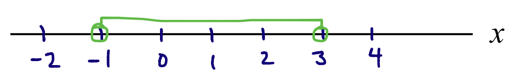


b) $-3 < 2x - 1 < 5$

$+1)$ $-3 + 1 < 2x < 5 + 1$

$\div 2)$ $-2 < 2x < 6$

$-1 < x < 3$



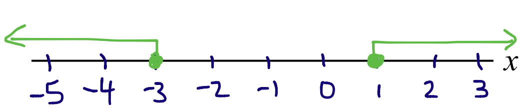
c) $1 - \frac{x}{2} \ge \frac{5}{2}$ *or* $1 - \frac{x}{2} \le \frac{1}{2}$

$\cup$ (union)

$\cdot 2)$ $2 - x \ge 5$ $\quad$ $2 - x \le 1$

$-x \ge 3$ $\quad$ $-x \le -1$

$x \le -3$ $\quad$ $x \ge 1$

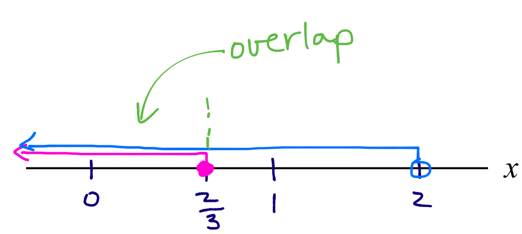


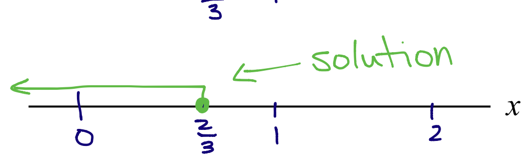
d) $x - 2 < 0$ *and* $-(x-2) \ge 2x$

$x < 2$ $\quad$ $\cap$ (intersection) $\quad$ $-x + 2 \ge 2x$

$2 \ge 3x$

$\frac{2}{3} \ge x$





Solving Quadratic Equations by Factoring ***Recall: A quadratic equation is of the form*** $ax^2+bx+c=0$.

3. Solve by factoring.

a) $-2x^2-6x+20=0$

$-2(x^2+3x-10)=0$

$-2(x+5)(x-2)=0$

$\therefore x=-5$ or $x=2$

b) $25=20t-4t^2$

$4t^2-20t+25=0$

$(2t-5)(2t-5)=0$

$\therefore t=\frac{5}{2}$ or $t=\frac{5}{2}$

c) $\frac{x^2}{3}-\frac{x}{6}=0$

$\frac{\cancel{6}^{2}}{1}\left(\frac{x^2}{\cancel{3}_{1}}\right)-\frac{\cancel{6}^{1}}{1}\left(\frac{x}{\cancel{6}_{1}}\right)=6(0)$

$2x^2-x=0$

$x(2x-1)=0$

$\therefore x=0$ or $x=\frac{1}{2}$

Solving Quadratic Equations Using the Quadratic Formula

Recall:

If $ax^2+bx+c=0$ ***then*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

*A **complex number** is of the form* $a+bi$, *where a and b are real numbers and i is the imaginary unit.*

In Complex Numbers $i=\sqrt{-1}$ **&** $i^2=-1$

so $\sqrt{-25}=\sqrt{25}\sqrt{-1}$
$=5i$

and $\sqrt{-72}=\sqrt{36}\sqrt{-1}\sqrt{2}$
$=6i\sqrt{2}$

4. Solve using the quadratic formula, $y\in C$. *Answer in simplified radical form if appropriate.*

a) $2y-5=y^2$

$y^2-2y+5=0$

$a=1,\ b=-2,\ c=5$

$y=\frac{2\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)}$

$y=\frac{2\pm\sqrt{-16}}{2}$

$y=\frac{2\pm 4i}{2}$

$y=\frac{\cancel{2}^{1}(1\pm 2i)}{\cancel{2}_{1}}$

$\therefore y=1-2i$ or $y=1+2i$

b) $6(t-1)=11-3(t-2)^2$

$6(t-1)=11-3(t-2)(t-2)$

$6t-6=11-3(t^2-4t+4)$

$6t-6=11-3t^2+12t-12$

$3t^2-6t-5=0$

$a=3,\ b=-6,\ c=-5$

$t=\frac{6\pm\sqrt{(-6)^2-4(3)(-5)}}{2(3)}$

$t=\frac{6\pm\sqrt{96}}{6}$

$t=\frac{6\pm 4\sqrt{6}}{6}$ $\frac{\div 2}{\div 2}$

$t=\frac{3\pm 2\sqrt{6}}{3}$

$\therefore t=\frac{3-2\sqrt{6}}{3}$ or $t=\frac{3+2\sqrt{6}}{3}$

HW. Exercise 1.2

Date:________________ **1.3 Division of Polynomials**

Long Division and the Division Statement

Ex. 1. For each of the following, perform long division and write the **division statement**, where *dividend* = *divisor* × *quotient* + *remainder* **or** $f(x) = d(x)q(x) + r(x)$.

a) Divide 579 by 8

$$\begin{array}{r} 72 \\ 8\overline{)579} \\ -56\downarrow \\ 19 \\ -16 \\ 3 \end{array}$$

$\therefore 579 = 8 \times 72 + 3$

b) Divide $x^2 - 7x - 10$ by $x - 3$

$$\begin{array}{r} x - 4 \\ x-3\overline{)x^2 - 7x - 10} \\ -(x^2 - 3x)\downarrow \\ -4x - 10 \\ -4x + 12 \\ -22 \end{array}$$

$\therefore x^2 - 7x - 10 = (x-3)(x-4) - 22$

Ex. 2. Divide and express each answer using a **division statement**, where $f(x) = d(x)q(x) + r(x)$. Factor fully if the remainder is 0.

a) $(6x^3 - 29x^2 + 43x - 20) \div (2x - 5)$

$$\begin{array}{r} 3x^2 - 7x + 4 \\ 2x-5\overline{)6x^3 - 29x^2 + 43x - 20} \\ 6x^3 - 15x^2 \downarrow \\ -14x^2 + 43x \\ -14x^2 + 35x \\ 8x - 20 \\ 8x - 20 \\ 0 \end{array}$$

$\therefore 6x^3 - 29x^2 + 43x - 20 = (2x-5)(3x^2 - 7x + 4)$
$= (2x-5)(3x-4)(x-1)$

b) $(-9x - 3 + 6x^3 - 4x^2) \div (2x^2 - 3)$

$$\begin{array}{r} 3x - 2 \\ 2x^2+0x-3\overline{)6x^3 - 4x^2 - 9x - 3} \\ 6x^3 + 0x^2 - 9x \downarrow \\ -4x^2 + 0x - 3 \\ -4x^2 + 0x + 6 \\ -9 \end{array}$$

$\therefore 6x^3 - 4x^2 - 9x - 3$
$= (2x^2 - 3)(3x - 2) - 9$

Ex. 3. The volume, V, in cm^3, of a rectangular box is given by $V = x^3 + 7x^2 + 14x + 8$. If the height, h, in cm, is given by $x + 1$, determine expressions for the other dimensions.

$\because V = lwh$
$\therefore l \cdot w = \frac{V}{h}$

$$\begin{array}{r} x^2 + 6x + 8 \\ x+1\overline{)x^3 + 7x^2 + 14x + 8} \\ x^3 + x^2 \downarrow \\ 6x^2 + 14x \\ 6x^2 + 6x \\ 8x + 8 \\ 8x + 8 \\ 0 \end{array}$$

$\therefore l \cdot w = x^2 + 6x + 8$
$= (x+4)(x+2)$

$\therefore$ the other dimensions are $(x+2)$ cm and $(x+4)$ cm.

Ex. 4. For $f(x) = (2x+1)(x^2 - 5x + 1) - 8$,

a) the linear divisor, $d(x) = 2x+1$

b) the quotient, $q(x) = x^2 - 5x + 1$

c) the remainder, $r(x) = -8$

d) the polynomial function (dividend)

$$f(x) = (2x+1)(x^2-5x+1) - 8$$
$$= 2x^3 - 10x^2 + 2x + x^2 - 5x + 1 - 8$$
$$\therefore f(x) = 2x^3 - 9x^2 - 3x - 7$$

Long Division and Mixed Rational Form

Ex. 5. For each of the following, perform long division and write in **mixed rational form**, where

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor} \quad \textbf{or} \quad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

a) $\dfrac{827}{12}$

$$\begin{array}{r} 68 \\ 12\overline{)827} \\ -72\downarrow \\ 107 \\ -96 \\ \hline 11 \end{array}$$

$$\therefore \frac{827}{12} = 68 + \frac{11}{12}$$

b) $\dfrac{9x^2 + 6x - 10}{3x+1}$

$$\begin{array}{r} 3x + 1 \\ 3x+1\overline{)9x^2 + 6x - 10} \\ 9x^2 + 3x \quad \downarrow \\ \hline 3x - 10 \\ 3x + 1 \\ \hline -11 \end{array}$$

$$\therefore \frac{9x^2+6x-10}{3x+1} = (3x+1) - \frac{11}{3x+1}$$

Ex. 6. Divide and express each answer in **mixed rational form**, where $\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

a) $\dfrac{x^3}{x-1}$

$$\begin{array}{r} x^2 + x + 1 \\ x-1\overline{)x^3 + 0x^2 + 0x + 0} \\ x^3 - x^2 \quad \downarrow \\ \hline x^2 + 0x \\ x^2 - x \\ \hline x + 0 \\ x - 1 \\ \hline 1 \end{array}$$

$$\therefore \frac{x^3}{x-1} = (x^2 + x + 1) + \frac{1}{x-1}$$

b) $\dfrac{(3x+4)(x-3)}{2-x} = \dfrac{3x^2 - 5x - 12}{-x+2}$

$$\begin{array}{r} -3x - 1 \\ -x+2\overline{)3x^2 - 5x - 12} \\ 3x^2 - 6x \quad \downarrow \\ \hline x - 12 \\ x - 2 \\ \hline -10 \end{array}$$

$$\therefore \frac{3x^3 - 5x - 12}{2-x} = (-3x-1) - \frac{10}{2-x}$$

HW. Exercise 1.3

Date:________________ **1.4 The Remainder Theorem**

Recall Long Division with Polynomials

> ***When a function*** $f(x)$ ***is divided by a divisor*** $d(x)$***, producing a quotient*** $q(x)$ ***and a remainder*** $r(x)$***, then*** $f(x)=d(x)q(x)+r(x)$***, where the degree of*** $r(x)$ ***is less than the degree of*** $d(x)$***.***

1. a. For the function $f(x)=x^3+x^2-9$, use long division to divide (x^3+x^2-9) by $(x-2)$.

$$\begin{array}{r} x^2+3x+6 \\ x-2\,\overline{)\,x^3+x^2+0x-9} \\ \underline{x^3-2x^2} \\ 3x^2+0x \\ \underline{3x^2-6x} \\ 6x-9 \\ \underline{6x-12} \\ 3 \end{array}$$

b. What is the remainder? 3

c. What is the value of $f(2)$?

$f(2)=(2)^3+(2)^2-9$
$=8+4-9$
$=3$

2. a. For the function $f(x)=x^3+3x^2-2x+1$, use long division to divide (x^3+3x^2-2x+1) by $(x+1)$.

$$\begin{array}{r} x^2+2x-4 \\ x+1\,\overline{)\,x^3+3x^2-2x+1} \\ \underline{x^3+x^2} \\ 2x^2-2x \\ \underline{2x^2+2x} \\ -4x+1 \\ \underline{-4x-4} \\ 5 \end{array}$$

b. What is the remainder? 5

c. What is the value of $f(-1)$?

$f(-1)=(-1)^3+3(-1)^2-2(-1)+1$
$=-1+3+2+1$
$=5$

Based on these examples, complete the following statement:

When $f(x)$ is divided by $(x-2)$, then the remainder $r=f(2)$.
When $f(x)$ is divided by $(x+1)$, then the remainder $r=f(-1)$.
When $f(x)$ is divided by $(x-a)$, then the remainder $r=f(a)$.
When $f(x)$ is divided by $(2x-1)$, then the remainder $r=f(\frac{1}{2})$.
When $f(x)$ is divided by $(5x+2)$, then the remainder $r=f(-\frac{2}{5})$.

> **Remainder Theorem: i)** ***When a polynomial*** $f(x)$ **is divided by** $x-a$***, the remainder is*** $f(a)$***.***
>
> **ii)** ***When a polynomial*** $f(x)$ **is divided by** $ax-b$***, the remainder is*** $f\left(\frac{b}{a}\right)$***.***

Ex. 1. Without using long division, determine the remainder when $2x^3 - 4x^2 + 3x - 6$ is divided by $x+2$.

Let $f(x) = 2x^3 - 4x^2 + 3x - 6$; $r = f(-2)$

$$f(-2) = 2(-2)^3 - 4(-2)^2 + 3(-2) - 6$$
$$= -16 - 16 - 6 - 6$$
$$= -44$$

∴ the remainder is -44.

Ex. 2. Find the remainder when $2x^3 + 3x^2 - x - 3$ is divided by $3x - 2$.
Use the ***Remainder Theorem***.

Let $f(x) = 2x^3 + 3x^2 - x - 3$; $r = f\left(\frac{2}{3}\right)$

$$f\left(\tfrac{2}{3}\right) = 2\left(\tfrac{2}{3}\right)^3 + 3\left(\tfrac{2}{3}\right)^2 - \tfrac{2}{3} - 3$$
$$= \frac{16}{27} + \frac{4}{3} - \frac{2}{3} - \frac{3}{1}$$
$$= \frac{16}{27} + \frac{2}{3} - \frac{3}{1}$$
$$= \frac{16}{27} + \frac{18}{27} - \frac{81}{27}$$
$$= -\frac{47}{27}$$

∴ the remainder is $-\frac{47}{27}$.

Ex. 3. When $x^3 - kx^2 + 17x + 6$ is divided by $x - 3$, the remainder is 12. Find the value of *k*.

Let $f(x) = x^3 - kx^2 + 17x + 6$; $f(3) = 12$

$$(3)^3 - k(3)^2 + 17(3) + 6 = 12$$
$$27 - 9k + 51 + 6 = 12$$
$$-9k + 84 = 12$$
$$-9k = -72$$
$$\therefore k = 8$$

Ex. 4. When the polynomial $f(x) = 3x^3 + cx^2 + dx - 7$ is divided by $x - 2$, the remainder is -3. When $f(x)$ is divided by $x + 1$, the remainder is -18. What are the values of c and d?

① $f(2) = -3$

$$3(2)^3 + c(2)^2 + d(2) - 7 = -3$$
$$24 + 4c + 2d - 7 = -3$$
$$4c + 2d = -20$$
$$\div 2) \quad \boxed{2c + d = -10}$$

② $f(-1) = -18$

$$3(-1)^2 + c(-1)^2 + d(-1) - 7 = -18$$
$$-3 + c - d - 7 = -18$$
$$\boxed{c - d = -8}$$

③ Solve:

$$2c + d = -10$$
$$c - d = -8$$

Add: $3c = -18$

$$\boxed{c = -6}$$

Sub $c = -6$ into ②:

$$(-6) - d = -8$$
$$-6 + 8 = d$$
$$\boxed{d = 2}$$

∴ the values of c & d are -6 and 2, respectively.

HW. Exercise 1.4

Date:_______________ **1.5 The Factor Theorem and Sum & Difference of Cubes**

THE FACTOR THEOREM: $(x-a)$ **is a factor of** $f(x)$ **if and only if** $f(a)=0$

ie. If $(x-a)$ **is a factor of** $f(x)$ **then** $f(a)=0$

or

If $f(a)=0$ **then** $(x-a)$ is a factor of $f(x)$

Note: ***If the leading coefficient of the polynomial is*** 1 ***then*** a ***is a factor of the constant term.***

Ex. 1. a) If $(x-2)$ is a factor of $f(x)$, then $f(2)=0$

b) If $f(-1)=0$, then a factor of $f(x)$ is $x+1$

Ex. 2. Is $(x+3)$ a factor of x^3+5x^2+2x-9?

Let $f(x)=x^3+5x^2+2x-9$

$f(-3)=(-3)^3+5(-3)^2+2(-3)-9$

$=-27+45-6-9$

$=3$

$\because f(-3)\neq 0$, the remainder is not 0

$\therefore x+3$ is not a factor.

Ex. 3. Determine the value(s) of k so that $(x-4)$ is a factor of $x^3-k^2x^2-16x+4k$.

Let $f(x)=x^3-k^2x^2-16x+4k$;

$\because x-4$ is a factor

$\therefore f(4)=0$

$f(4)=0$

$(4)^3-k^2(4)^2-16(4)+4k=0$

$-16k^2+4k=0$

$-4k(k-4)=0$

$\therefore k=0$ or $k=4$.

Ex. 4. Completely factor the following polynomials.

a) $x^3+3x^2-13x-15$

$=(x+1)(x^2+2x-15)$

$=(x+1)(x+5)(x-3)$

Let $f(x)=x^3+3x^2-13x-15$

Test values: $\pm1, \pm3, \pm5, \pm15$

x	$f(x)$
1	$\neq 0$
-1	0

$\because f(-1)=0$

$\therefore x+1$ is a factor

$$\begin{array}{r|l} & x^2+2x-15 \\ x+1 & x^3+3x^2-13x-15 \\ & x^3+x^2 \\ \hline & 2x^2-13x \\ & 2x^2+2x \\ \hline & -15x-15 \\ & -15x-15 \\ \hline & 0 \end{array}$$

b) $x^3 - 3x^2 - 4x + 12$ Grouping!

$= x^2(x-3) - 4(x-3)$

$= (x-3)(x^2-4)$

$= (x-3)(x-2)(x+2)$

c) $x^3 - 27$

$= (x-3)(x^2+3x+9)$

does not factor.

Let $f(x) = x^3 - 27$

Test values: $\pm 1, \pm 3, \pm 9, \pm 27$

x	$f(x)$
3	0

$\therefore f(3) = 0$

$\therefore x-3$ is a factor

$$\begin{array}{r} x^2 + 3x + 9 \\ x-3 \overline{\big) x^3 + 0x^2 + 0x - 27} \\ \underline{x^3 - 3x^2 \qquad\qquad} \\ 3x^2 + 0x \qquad \\ \underline{3x^2 - 9x \qquad} \\ 9x - 27 \\ \underline{9x - 27} \\ 0 \end{array}$$

Factoring the Sum and Difference of Cubes:

$$\boxed{\begin{array}{c} a^3 + b^3 = (a+b)(a^2 - ab + b^2) \\ \textbf{\&} \\ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \end{array}}$$

Ex. 5. Factor the following using the formulas for factoring the sum and difference of cubes.

a) $x^3 - 27 = (x)^3 - (3)^3$

$= [(x) - (3)][(x)^2 + (x)(3) + (3)^2]$

or

$= (x-3)(x^2+3x+9)$

b) $16x^3 + 250y^3 = 2(8x^3 + 125y^3)$

$= 2[(2x)^3 + (5y)^3]$

$= 2[(2x) + (5y)][(2x)^2 - (2x)(5y) + (5y)^2]$

$= 2(2x+5y)(4x^2 - 10xy + 25y^2)$

c) $x^6 - 64 = (x^3 - 8)(x^3 + 8)$ *always difference of squares first

$= (x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

HW. Exercise 1.5

Date:_______________

1.6 The Extended Factor Theorem

Recall:

Factoring the Sum and Difference of Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Ex. 1. Completely factor each of the following.

a) $125x^4 - 216xy^6$

$= x(125x^3 - 216y^6)$

$= x[(5x)^3 - (6y^2)^3]$

$= x(5x - 6y^2)(25x^2 + 30xy^2 + 36y^4)$

b) $(2x+1)^3 + (x-4)^3$

* Recall: $(x+y)^2 = (x+y)(x+y)$

$= [(2x+1) + (x-4)][(2x+1)^2 - (2x+1)(x-4) + (x-4)^2]$

$= (2x+1+x-4)[(2x+1)(2x+1) - (2x+1)(x-4) + (x-4)(x-4)]$

$= (3x-3)[(4x^2+4x-1) - (2x^2-7x-4) + (x^2-8x+16)]$

$= 3(x-1)(3x^2+3x+21)$

$= 9(x-1)(x^2+x+7)$

c) $\frac{1}{27}x^9 - 64$

$= (\frac{1}{3}x^3)^3 - (4)^3$

$= [\frac{1}{3}x^3 - 4][(\frac{1}{3}x^3)^2 + (\frac{1}{3}x^3)(4) + (4)^2]$

$= (\frac{1}{3}x^3 - 4)(\frac{1}{9}x^6 + \frac{4}{3}x^3 + 16)$

d) $8x^6 + 15x^3 - 2 = (x^3+2)(8x^3-1)$

$= (x^3+2)(2x-1)(4x^2+2x+1)$

THE EXTENDED FACTOR THEOREM:

$(ax-b)$ **is a factor of** $f(x)$ **if and only if** $f\left(\frac{b}{a}\right)=0$

ie. If $(ax-b)$ **is a factor of** $f(x)$ **then** $f\left(\frac{b}{a}\right)=0$

or

If $f\left(\frac{b}{a}\right)=0$ **then** $(ax-b)$ **is a factor of** $f(x)$

- b is a factor of the constant term of $f(x)$
- a is a factor of the leading coefficient of $f(x)$

Ex. 2. Write the binomial factor that corresponds to the polynomial $f(x)$ if:

a) $f\left(\frac{1}{2}\right)=0$

$\therefore 2x-1$ is a factor

b) $f\left(-\frac{2}{5}\right)=0$

$\therefore 5x+2$ is a factor

Ex. 3. If $2x^3-kx^2-4x+6$ is divisible by $2x-3$, what is the value of k.

Let $f(x)=2x^3-kx^2-4x+6$; $f\left(\frac{3}{2}\right)=0$

$$2\left(\frac{3}{2}\right)^3-k\left(\frac{3}{2}\right)^2-4\left(\frac{3}{2}\right)+6=0$$

$$\frac{27}{4}-\frac{9}{4}k-6+6=0$$

$$\frac{27}{4}=\frac{9}{4}k$$

$$\therefore k=3$$

Ex. 4. Completely factor each of the following.

a) $2x^3-5x^2-4x+3$

$=(2x-1)(x^2-2x-3)$

$=(2x-1)(x-3)(x+1)$

$\pm1, \pm2$ $\qquad$ $\pm1, \pm3$

Let $f(x)=2x^3-5x^2-4x+3$

Test Values: ±1, ±3, $\pm\frac{3}{2}$, $\pm\frac{1}{2}$

x	$f(x)$
1	$\neq 0$
-1	$\neq 0$
3	$\neq 0$
-3	$\neq 0$
$\frac{1}{2}$	0

$\because f\left(\frac{1}{2}\right)=0$

$\therefore 2x-1$ is a factor

$$\begin{array}{r|l} & x^2-2x-3 \\ 2x-1 & 2x^3-5x^2-4x+3 \\ & \underline{2x^3-x^2} \\ & -4x^2-4x \\ & \underline{-4x^2+2x} \\ & -6x+3 \\ & \underline{-6x+3} \\ & 0 \end{array}$$

b) $8x^3+12x^2+6x+1$

$=(2x+1)(4x^2+4x+1)$

$=(2x+1)(2x+1)(2x+1)$

$=(2x+1)^3$

Let $f(x)=8x^3+12x^2+6x+1$

Test Values: $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{1}{8}$

x	$f(x)$
-1	$\neq 0$
1	$\neq 0$
$\frac{1}{2}$	$\neq 0$
$-\frac{1}{2}$	0

$\because f(-\frac{1}{2})=0$

$\therefore 2x+1$ is a factor

$$\begin{array}{r|l} & 4x^2+4x+1 \\ \hline 2x+1 & 8x^3+12x^2+6x+1 \\ & 8x^3+4x^2 \\ \hline & 8x^2+6x \\ & 8x^2+4x \\ \hline & 2x+1 \\ & 2x+1 \\ \hline & 0 \end{array}$$

c) $9x^4+6x^3+4x^2-5x-2$

$=(3x+1)(3x^3+x^2+x-2)$

$=(3x+1)(3x-2)(x^2+x+1)$

Let $f(x)=9x^4+6x^3+4x^2-5x-2$

Test Values: $\pm 1, \pm 2, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}$

x	$f(x)$
$-\frac{1}{3}$	0

$\because f(-\frac{1}{3})=0$

$\therefore 3x+1$ is a factor

$$\begin{array}{r|l} & 3x^3+x^2+x-2 \\ \hline 3x+1 & 9x^4+6x^3+4x^2-5x-2 \\ & 9x^4+3x^3 \\ \hline & 3x^3+4x^2 \\ & 3x^3+x^2 \\ \hline & 3x^2-5x \\ & 3x^2+x \\ \hline & -6x-2 \\ & -6x-2 \\ \hline & 0 \end{array}$$

Let $g(x)=3x^3+x^2+x-2$

Test Values: $\pm 1, \pm 2, \pm\frac{1}{3}, \pm\frac{2}{3}$

x	$g(x)$
$-\frac{1}{3}$	$\neq 0$
$\frac{2}{3}$	0

$\because g(\frac{2}{3})=0$

$\therefore 3x-2$ is a factor

$$\begin{array}{r|l} & x^2+x+1 \\ \hline 3x-2 & 3x^3+x^2+x-2 \\ & 3x^3-2x^2 \\ \hline & 3x^2+x \\ & 3x^2-2x \\ \hline & 3x-2 \\ & 3x-2 \\ \hline & 0 \end{array}$$

HW. Exercise 1.6
Unit 1 Part I Test covers Days 1 to 6
HW. Part I Review 1.1 to 1.6