**Date:____________**

# UNIT 1: EQUATIONS
## Polynomial, Rational, Radical & Absolute Value

## 1.1 Review of Factoring Techniques

Factor each of the following completely:

**1.** ***Common Factor***

**a)** $-4x^3 + 16x^2$

**b)** $2x^2(2x+1) - 6x(2x+1)$

**2.** ***Trinomial Factoring***

**a)** $x^2 - x - 20$

**b)** $x^4 + 6x^2 + 9$

**c)** $6a^2 - 4ab - 2b^2$

**d)** $-8x^2 + 22x - 12$

**3.** ***Difference of Squares*** ***Recall:*** $a^2 - b^2 = (a-b)(a+b)$

**a)** $25x^2 - 16y^2z^6$

**b)** $81 - x^4$

**c)** $36(x-2)^2 - 25(x+1)^2$

**d)** $-w^4 + 13w^2 - 36$

**4.** ***Factor by Grouping***

**a)** $ax + bx - ay - by$

**b)** $4x^3 + 8x^2 - x - 2$

**c)** $a^2 - b^2 + 9 - 6a$

**d)** $16x^2 - 4y^2 + 12y - 9$

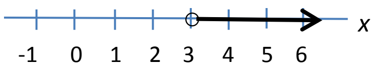
**HW. Exercise 1.1**

**Date:**______________

# 1.2 Solving Linear Inequalities & Quadratic Equations

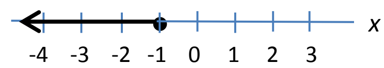
## ***Solving Linear Inequalities***

**1.** Interpret each graphed solution using **i) set** notation and **ii) interval** notation.

**a)** 

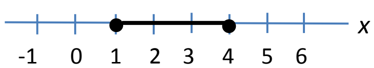

**i)** ______________________________

**ii)** ______________________________

**b)** 

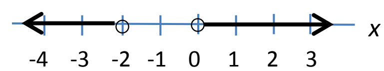

**i)** ______________________________

**ii)** ______________________________

**c)** 


**i)** ______________________________

**ii)** ______________________________

**d)** 


**i)** ______________________________

**ii)** ______________________________

**2.** Solve each of the following inequalities and **graph** the *simplified* solution on a number line.

**a)** $2x-3 \le 6(x+2)+1$

______________________________ $x$

**b)** $-3 < 2x-1 < 5$

______________________________ $x$

**c)** $1-\frac{x}{2} \ge \frac{5}{2}$ *or* $1-\frac{x}{4} > \frac{1}{2}$

______________________________ $x$

*The **simplified** solution of graphed is:*

______________________________ $x$

**d)** $x-2 < 0$ *and* $-(x-2) \ge 2x$

______________________________ $x$

*The **simplified** solution of graphed is:*

______________________________ $x$

***Solving Quadratic Equations by Factoring*** **Recall:** ***A quadratic equation is of the form*** $ax^2 + bx + c = 0$ .

**3.** Solve by factoring.

**a)** $-2x^2 - 6x + 20 = 0$ **b)** $25 = 20t - 4t^2$ **c)** $\frac{x^2}{3} - \frac{x}{6} = 0$

***Solving Quadratic Equations Using the Quadratic Formula***

***Recall:***

***If*** $ax^2 + bx + c = 0$ ***then*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **.**

*A* ***complex number*** *is of the form* $a + bi$ *, where a and b are real numbers and i is the imaginary unit.*

***In Complex Numbers*** $i = \sqrt{-1}$ ***&*** $i^2 = -1$

***so*** $\sqrt{-25} =$ ***and*** $\sqrt{-72} =$

**4.** Solve using the quadratic formula, $y \in C$ . *Answer in simplified radical form if appropriate.*

**a)** $2y - 5 = y^2$ **b)** $6(t-1) = 11 - 3(t-2)^2$

**HW. Exercise 1.2**

**Date:________________** 

# 1.3 Division of Polynomials

## *Long Division and the Division Statement*

**Ex. 1.** For each of the following, perform long division and write the **division statement**, where $dividend = divisor \times quotient + remainder$ **or** $f(x) = d(x)q(x) + r(x)$.

**a)** Divide 579 by 8

**b)** Divide $x^2 - 7x - 10$ by $x - 3$

**Ex. 2.** Divide and express each answer using a **division statement**, where $f(x) = d(x)q(x) + r(x)$. Factor fully if the remainder is 0.

**a)** $(6x^3 - 29x^2 + 43x - 20) \div (2x - 5)$

**b)** $(-9x - 3 + 6x^3 - 4x^2) \div \left(2x^2 - 3\right)$

**Ex. 3.** The volume, *V*, in $cm^3$, of a rectangular box is given by $V = x^3 + 7x^2 + 14x + 8$. If the height, *h*, in *cm*, is given by $x + 1$, determine expressions for the other dimensions.

**Ex. 4.** For $f(x) = (2x+1)(x^2 - 5x + 1) - 8$,

**a)** the linear divisor, $d(x) =$

**b)** the quotient, $q(x) =$

**c)** the remainder, $r(x) =$

**d)** the polynomial function (dividend)

$f(x) =$

***<u>Long Division and Mixed Rational Form</u>***

**Ex. 5.** For each of the following, perform long division and write in **mixed rational form**, where

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor} \quad \textbf{or} \quad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

**a)** $\dfrac{827}{12}$

**b)** $\dfrac{6x^2 - 7x - 10}{3x + 1}$

**Ex. 6.** Divide and express each answer in **mixed rational form**, where $\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

**a)** $\dfrac{x^3}{x-1}$

**b)** $\dfrac{(3x+4)(x-3)}{2-x}$

**HW. Exercise 1.3**

**Date:**________________ **1.4 The Remainder Theorem**

***Recall Long Division with Polynomials***

| |
|---|
| ***When a function $f(x)$ is divided by a divisor $d(x)$, producing a quotient $q(x)$ and a remainder $r(x)$, then $f(x) = d(x)q(x) + r(x)$, where the degree of $r(x)$ is less than the degree of $d(x)$.*** |

**1. a.** For the function $f(x) = x^3 + x^2 - 9$, use long division to divide $(x^3 + x^2 - 9)$ by $(x-2)$.

**b.** What is the remainder?

**c.** What is the value of $f(2)$?

**2. a.** For the function $f(x) = x^3 + 3x^2 - 2x + 1$, use long division to divide $(x^3 + 3x^2 - 2x + 1)$ by $(x+1)$.

**b.** What is the remainder?

**c.** What is the value of $f(-1)$?

***Based on these examples, complete the following statement:***

When $f(x)$ is divided by $(x-2)$, then the remainder $r = f(\quad)$.
When $f(x)$ is divided by $(x+1)$, then the remainder $r = f(\quad)$.
When $f(x)$ is divided by $(x-a)$, then the remainder $r = f(\quad)$.
When $f(x)$ is divided by $(2x-1)$, then the remainder $r = f\left(\quad\right)$.
When $f(x)$ is divided by $(5x+2)$, then the remainder $r = f\left(\quad\right)$.

| |
|---|
| **Remainder Theorem: i)** ***When a polynomial $f(x)$* is divided by $x-a$*, the remainder is $f(a)$.*** |
| **ii)** ***When a polynomial $f(x)$* is divided by $ax-b$*, the remainder is $f\left(\frac{b}{a}\right)$.*** |

**Ex. 1.** Without using long division, determine the remainder when $2x^3 - 4x^2 + 3x - 6$ is divided by $x + 2$.

**Ex. 2.** Find the remainder when $2x^3 + 3x^2 - x - 3$ is divided by $3x - 2$.
Use the ***Remainder Theorem***.

**Ex. 3.** When $x^3 - kx^2 + 17x + 6$ is divided by $x - 3$, the remainder is 12. Find the value of *k*.

**Ex. 4.** When the polynomial $f(x) = 3x^3 + cx^2 + dx - 7$ is divided by $x - 2$, the remainder is $-3$.
When $f(x)$ is divided by $x + 1$, the remainder is $-18$. What are the values of $c$ and *d*?

**HW. Exercise 1.4**

**Date:_______________** **1.5 The Factor Theorem and Sum & Difference of Cubes**

**THE FACTOR THEOREM:** $(x-a)$ **is a factor of** $f(x)$ **if and only if** $f(a)=0$

**ie. If** $(x-a)$ **is a factor of** $f(x)$ **then**

***or***

**If** $f(a)=0$ **then**

**Note:** ***If the leading coefficient of the polynomial is*** $1$ ***then*** $a$ ***is a factor of the constant term.***

**Ex. 1.** **a)** If $(x-2)$ is a factor of $f(x)$, then $f(2)=$

**b)** If $f(-1)=0$, then a factor of $f(x)$ is

**Ex. 2.** Is $(x+3)$ a factor of $x^3+5x^2+2x-9$?

**Ex. 3.** Determine the value(s) of $k$ so that $(x-4)$ is a factor of $x^3-k^2x^2-16x+4k$.

**Ex. 4.** Completely factor the following polynomials.

**a)** $x^3+3x^2-13x-15$

**b)** $x^3 - 3x^2 - 4x + 12$

**c)** $x^3 - 27$

***Factoring the Sum and Difference of Cubes:***

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

**&**

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

**Ex. 5.** Factor the following using the formulas for factoring the sum and difference of cubes.

**a)** $x^3 - 27$

**b)** $16x^3 + 250y^3$

**c)** $x^6 - 64$

**HW. Exercise 1.5**

**Date:_______________**

# 1.6 The Extended Factor Theorem

***Recall:***

| ***Factoring the Sum and Difference of Cubes*** |
| --- |
| $a^3 + b^3 =$ |
| $a^3 - b^3 =$ |

**Ex. 1.** Completely factor each of the following.

**a)** $125x^4 - 216xy^6$

**b)** $(2x+1)^3 + (x-4)^3$

**c)** $\frac{1}{27}x^9 - 64$

**d)** $8x^6 + 15x^3 - 2$

**THE EXTENDED FACTOR THEOREM:**

$(ax-b)$ **is a factor of** $f(x)$ **if and only if** $f\left(\frac{b}{a}\right)=0$

**ie. If** $(ax-b)$ **is a factor of** $f(x)$ **then** $f\left(\frac{b}{a}\right)=0$

***or***

**If** $f\left(\frac{b}{a}\right)=0$ **then** $(ax-b)$ **is a factor of** $f(x)$

- *b* is a factor of the constant term of $f(x)$
- *a* is a factor of the leading coefficient of $f(x)$

**Ex. 2.** Write the binomial factor that corresponds to the polynomial $f(x)$ if:

**a)** $f\left(\frac{1}{2}\right)=0$

**b)** $f\left(-\frac{2}{5}\right)=0$

**Ex. 3.** If $2x^3-kx^2-4x+6$ is divisible by $2x-3$, what is the value of *k*.

**Ex. 4.** Completely factor each of the following.

**a)** $2x^3-5x^2-4x+3$

**b)** $8x^3 + 12x^2 + 6x + 1$

**c)** $9x^4 + 6x^3 + 4x^2 - 5x - 2$

**HW. Exercise 1.6**
**Unit 1 Part I Test covers Days 1 to 6**
**HW. Part I Review 1.1 to 1.6**