Date:_______________

1.7 Solving Polynomial Equations

Note: ***A polynomial equation of the n^{th} degree has n roots.***

Ex. 1. Solve for x in each of the following, $x \in C$.

a) $27x^3 + 8 = 0$

$(3x+2)(9x^2-6x+4)=0$

$3x+2=0$ or $9x^2-6x+4=0$ $\quad a=9,\ b=-6,\ c=4$

$\therefore x=-\frac{2}{3},\ x=\frac{1-i\sqrt{3}}{3},\ x=\frac{1+i\sqrt{3}}{3}$

$x=\frac{6\pm\sqrt{(-6)^2-4(9)(4)}}{2(9)}$

$x=\frac{6\pm\sqrt{-108}}{18}$

$x=\frac{6\pm 6i\sqrt{3}}{18}$

$x=\frac{\cancel{6}^{1}(1\pm i\sqrt{3})}{\cancel{18}_{3}}$

$x=\frac{1\pm i\sqrt{3}}{3}$

b) $6x^3 - 13x^2 + x + 2 = 0$

$(x-2)(6x^2-x-1)=0$

$(x-2)(3x+1)(2x-1)=0$

$\therefore x=-\frac{1}{3},\ x=\frac{1}{2},\ x=2$

$$
\begin{array}{r|l}
 & 6x^2 - x - 1 \\
\hline
x-2 & 6x^3 - 13x^2 + x + 2 \\
 & 6x^3 - 12x^2 \\
 & -x^2 + x \\
 & -x^2 + 2x \\
 & -x + 2 \\
 & -x + 2 \\
 & 0
\end{array}
$$

c) $x^3 - 7x^2 + 8 = 0$

$(x+1)(x^2-8x+8)=0$

$x+1=0$ or $x^2-8x+8=0$ $\quad a=1,\ b=-8,\ c=8$

$\therefore x=-1,\ x=4-2\sqrt{2},\ x=4+2\sqrt{2}$

$$
\begin{array}{r|l}
 & x^2 - 8x + 8 \\
\hline
x+1 & x^3 - 7x^2 + 0x + 8 \\
 & x^3 + x^2 \\
 & -8x^2 + 0x \\
 & -8x^2 - 8x \\
 & 8x + 8 \\
 & 8x + 8 \\
 & 0
\end{array}
$$

$x=\frac{8\pm\sqrt{(-8)^2-4(1)(8)}}{2(1)}$

$x=\frac{8\pm\sqrt{32}}{2}$

$x=\frac{8\pm 4\sqrt{2}}{2}$ $\quad \div 2,\ \div 2$

$x=4\pm 2\sqrt{2}$

d) $3x^3 + x^2 + 24x + 8 = 0$

$x^2(3x+1) + 8(3x+1) = 0$

$(3x+1)(x^2+8) = 0$

$3x+1 = 0$ or $x^2+8=0$

$x = -\frac{1}{3}$ $\quad x^2 = -8$

$x = \pm\sqrt{-8}$

$x = \pm 2i\sqrt{2}$

$\therefore x = -\frac{1}{3}$, $x = -2i\sqrt{2}$, $x = 2i\sqrt{2}$

e) $-4x^4 - 18x^3 + 10x^2 = 0$

$-2x^2(2x^2+9x-5) = 0$

$-2x^2 = 0$ or $2x^2+9x-5=0$

$x \cdot x = 0$ $\quad (2x-1)(x+5) = 0$

$2x-1=0$ or $x+5=0$

$\therefore x=-5$, $x=0$, $x=0$, $x=\frac{1}{2}$

f) $x^4 - 24x^2 = 25$

$x^4 - 24x^2 - 25 = 0$

$(x^2-25)(x^2+1) = 0$

$(x-5)(x+5)(x^2+1) = 0$

$x=5$, $x=-5$, $x^2+1=0$

$x^2 = -1$

$x = \pm i$

$\therefore x=-5$, $x=5$, $x=-i$, $x=i$

g) $(x^2 - 5x - 5)(x^2 - 5x + 3) = 9$

Let $y = x^2 - 5x$

$(y-5)(y+3) = 9$

$y^2 - 2y - 15 = 9$

$y^2 - 2y - 24 = 0$

$(y-6)(y+4) = 0$

$x^2 - 5x - 6 = 0$ or $x^2 - 5x + 4 = 0$

$(x-6)(x+1) = 0$ $\quad (x-4)(x-1) = 0$

$\therefore x=-1$, $x=1$, $x=4$, $x=6$

HW. Exercise 1.7

Date:_______________ **1.8 Determining Polynomial Equations From Roots**

1. Determine the roots of the cubic equation $6x^3 - 19x^2 + 9x + 10 = 0$.

$$6x^3 - 19x^2 + 9x + 10 = 0$$
$$(x-2)(6x^2 - 7x - 5) = 0$$
$$(x-2)(3x-5)(2x+1) = 0$$

$\therefore x = -\frac{1}{2},\ x = \frac{5}{3},\ x = 2$

$$\begin{array}{r|l} & 6x^2 - 7x - 5 \\ x-2 & 6x^3 - 19x^2 + 9x + 10 \\ & 6x^3 - 12x^2 \\ \hline & -7x^2 + 9x \\ & -7x^2 + 14x \\ \hline & -5x + 10 \\ & -5x + 10 \\ \hline & 0 \end{array}$$

2. Write an appropriate equation in expanded form with integral coefficients having the given roots.

a) -3 *and* $\frac{2}{3}$

$x = -3$, $x = \frac{2}{3}$

$x + 3 = 0$ $\quad 3x - 2 = 0$

$$(x+3)(3x-2) = 0$$
$$3x^2 - 2x + 9x - 6 = 0$$

$\therefore 3x^2 + 7x - 6 = 0$ is the required equation.

b) $2 - 3i$ *and* $2 + 3i$

$x = 2 - 3i$, $x = 2 + 3i$

$x - 2 + 3i = 0$ $\quad x - 2 - 3i = 0$

$$(x - 2 + 3i)(x - 2 - 3i) = 0$$
$$x^2 - 2x \cancel{- 3ix} - 2x + 4 + \cancel{6i} \cancel{+ 3ix} \cancel{- 6i} - 9i^2 = 0$$
$$x^2 - 4x + 4 - 9(-1) = 0$$

$\therefore x^2 - 4x + 13 = 0$ is the required equation.

c) $0,\ 0,\ 1 - 2\sqrt{5}$ *and* $1 + 2\sqrt{5}$

$x = 0,\ x = 0,\ x = 1 - 2\sqrt{5},\ x = 1 + 2\sqrt{5}$

$x = 0,\ x = 0,\ x - 1 + 2\sqrt{5} = 0,\ x - 1 - 2\sqrt{5} = 0$

Recall: $(a-b)(a+b) = a^2 - b^2$

$$x \cdot x(\underbrace{x-1}_{a} \underbrace{+}_{+} \underbrace{2\sqrt{5}}_{b})(\underbrace{x-1}_{a} \underbrace{-}_{-} \underbrace{2\sqrt{5}}_{b}) = 0$$
$$x^2[(x-1)^2 - (2\sqrt{5})^2] = 0$$
$$x^2[(x-1)(x-1) - (2\sqrt{5})(2\sqrt{5})] = 0$$
$$x^2[(x^2 - 2x + 1) - 20] = 0$$
$$x^2(x^2 - 2x - 19) = 0$$

$\therefore x^4 - 2x^3 - 19x^2 = 0$ is the required equation.

d) $2,\ -\frac{1}{2}$ *and* $\frac{5}{3}$

$x = 2 \quad x = -\frac{1}{2},\ x = \frac{5}{3}$

$x - 2 = 0,\ 2x + 1 = 0,\ 3x - 5 = 0$

$$(x-2)(2x+1)(3x-5) = 0$$
$$(x-2)(6x^2 - 7x - 5) = 0$$

$\therefore 6x^3 - 19x^2 + 9x + 10 = 0$ is the required equation.

*** See ex. 1 ***

HW. Exercise 1.8

Date:________________ **1.9 Solving Radical Equations**

Warmup

1. Solve, $x \in C$.

a) $x^2 - 18 = 0$

$x^2 = 18$

$x = \pm\sqrt{18}$

$x = \pm 3\sqrt{2}$

b) $4x^2 + 25 = 0$

$4x^2 = -25$

$x^2 = -\frac{25}{4}$

$x = \pm\sqrt{-\frac{25}{4}}$

$x = \pm \frac{5}{2}i$

c) $(x+3)^2 = 16$

$x+3 = \pm 4$

$x = -3 \pm 4$

$x = -3-4, \ x = -3+4$

$x = -7, \ x = 1$

d) $(x-6)^2 + 12 = 0$

$(x-6)^2 = -12$

$x - 6 = \pm\sqrt{-12}$

$x = 6 \pm 2i\sqrt{3}$

$x = 6 - 2i\sqrt{3}, \ x = 6 + 2i\sqrt{3}$

2. Square each of the following:

a) $5\sqrt{x}$

$[5\sqrt{x}]^2$

$= 25x$

b) $3\sqrt{x+1}$

$[3\sqrt{x+1}]^2$

$= 9(x+1)$

$= 9x + 9$

c) $\sqrt{x-2} - 3$

$(\sqrt{x-2} - 3)^2$

$= (\sqrt{x-2} - 3)(\sqrt{x-2} - 3)$

$= (x-2) - 3\sqrt{x-2} - 3\sqrt{x-2} + 9$

$= x + 7 - 6\sqrt{x-2}$

d) $2 - 5\sqrt{x+1}$

$(2 - 5\sqrt{x+1})^2$

$= (2 - 5\sqrt{x+1})(2 - 5\sqrt{x+1})$

$= 4 - 10\sqrt{x+1} - 10\sqrt{x+1} + 25(x+1)$

$= 4 - 20\sqrt{x+1} + 25x + 25$

$= 25x + 29 - 20\sqrt{x+1}$

Solving Radical Equations

1. Isolate the radical on one side of the equation.
2. Square both sides of the equation.
3. Repeat 1. and 2. until no radicals remain.
4. Solve and check answers in the original equation to identify and reject any extraneous roots.

3. Solve.

a) $4\sqrt{2x+7} - 5 = 7$

$4\sqrt{2x+7} = 12$

$\sqrt{2x+7} = 3$

Square both sides:

$2x + 7 = 9$

$2x = 2$

$x = 1$

$LS = 4\sqrt{2x+7} - 5$

$= 4\sqrt{2(1)+7} - 5$

$= 4\sqrt{9} - 5$

$= 4(3) - 5$

$= 7$

$RS = 7$

$\because LS = RS$

$\therefore$ the solution is $x = 1$.

b) $x+\sqrt{x-2}=4$

$\sqrt{x-2}=4-x$

Square both sides

$x-2=16-8x+x^2$

$0=x^2-9x+18$

$0=(x-6)(x-3)$

$x=6$ or $x=3$

Check: $x=6$

$LS=x+\sqrt{x-2}$ $\quad RS=4$

$=6+\sqrt{6-2}$

$=6+2$

$=8$

$\therefore LS\neq RS$

$\therefore x=6$ is not a solution

Check: $x=3$

$LS=x+\sqrt{x-2}$ $\quad RS=4$

$=3+\sqrt{3-2}$

$=3+1$

$=4$

$\therefore LS=RS$

$\therefore x=3$ is a solution

$\therefore$ the solution is $x=3$.

c) $\sqrt{4x+5}-\sqrt{2x-6}=3$

$\sqrt{4x+5}=\sqrt{2x-6}+3$

Square both sides

$4x+5=2x-6+6\sqrt{2x-6}+9$

$2x+2=6\sqrt{2x-6}$

$\div 2)$ $\quad x+1=3\sqrt{2x-6}$

Square both sides

$x^2+2x+1=9(2x-6)$

$x^2+2x+1=18x-54$

$x^2-16x+55=0$

$(x-11)(x-5)=0$

$x=11$ or $x=5$

Check: $x=11$

$LS=\sqrt{4x+5}-\sqrt{2x-6}$ $\quad RS=3$

$=\sqrt{44+5}-\sqrt{22-6}$

$=7-4$

$=3$

$\therefore LS=RS$

$\therefore x=11$ is a solution.

Check: $x=5$

$LS=\sqrt{4x+5}-\sqrt{2x-6}$ $\quad RS=3$

$=\sqrt{20+5}-\sqrt{10-6}$

$=5-2$

$=3$

$\therefore LS=RS$

$\therefore x=5$ is a solution

$\therefore$ the solutions are $x=5$ and $x=11$.

HW. Exercise 1.9

Date:________________

1.10 Solving Rational Equations

1. Solve, $x \in C$. Include restrictions on the variable.
Hint: *Multiply both sides of the equation by the lowest common denominator (**LCD**).*

a) $\frac{2}{3}+\frac{1}{2x}=\frac{2-x}{x}$, $x \neq 0$

$$\cancel{6x}^{2x}\left(\frac{2}{\cancel{3}_1}\right)+\cancel{6x}^{3}\left(\frac{1}{\cancel{2x}_1}\right)=\cancel{6x}^{6}\left(\frac{2-x}{\cancel{x}_1}\right)$$

$$4x+3=6(2-x)$$

$$4x+3=12-6x$$

$$10x=9$$

$$x=\frac{9}{10}$$

b) $\frac{4}{x+1}=\frac{x+1}{4}$, $x \neq -1$

$$4\cancel{(x+1)}^1\left(\frac{4}{\cancel{x+1}_1}\right)=\cancel{4}^1(x+1)\left(\frac{x+1}{\cancel{4}_1}\right)$$

$$16=(x+1)(x+1)$$

$$16=x^2+2x+1$$

$$0=x^2+2x-15$$

$$0=(x+5)(x-3)$$

$$\therefore x=-5,\ x=3.$$

c) $\frac{4}{x-1}-\frac{3}{x+2}=2$, $x \neq -2, 1$

$$\cancel{(x-1)}^1(x+2)\left(\frac{4}{\cancel{x-1}_1}\right)-(x-1)\cancel{(x+2)}^1\left(\frac{3}{\cancel{x+2}_1}\right)=2(x-1)(x+2)$$

$$4(x+2)-3(x-1)=2(x-1)(x+2)$$

$$4x+8-3x+3=2(x^2+x-2)$$

$$x+11=2x^2+2x-4$$

$$0=2x^2+x-15$$

$$0=(2x+5)(x-3)$$

$$\therefore x=-\frac{5}{2}\ ,\ x=3$$

d) $\frac{x^2-2x+1}{x^2-1}-\frac{3x-1}{x+2}=0$

$x \neq -1, 1, -2$

$$\frac{\cancel{(x-1)}^1(x-1)}{\cancel{(x-1)}_1(x+1)}-\frac{3x-1}{x+2}=0$$

$$\frac{x-1}{x+1}=\frac{3x-1}{x+2}$$

$$(x-1)(x+2)=(3x-1)(x+1)$$

$$x^2+x-2=3x^2+2x-1$$

$$0=2x^2+x+1$$

$$x=\frac{-1\pm\sqrt{(1)^2-4(2)(1)}}{2(2)}$$

$$x=\frac{-1\pm\sqrt{-7}}{4}$$

$$x=\frac{-1\pm i\sqrt{7}}{4}$$

$$\therefore x=\frac{-1-i\sqrt{7}}{4}\ ,\ x=\frac{-1+i\sqrt{7}}{4}$$

2. Determine all real roots of the following equation. Include restrictions on the variable.

a) $x^{-2}(8x^{-3}+1)=0$ $x \neq 0$

$x^{-2}=0$

$\frac{1}{x^2}=0$

$\therefore$ no solution

$8x^{-3}+1=0$

$x^3(8x^{-3})+x^3(1)=x^3(0)$

$x^3+8=0$

$(x+2)(x^2-2x+4)=0$

$x+2=0$ or $x^2-2x+4=0$

$x=-2$ $x=\frac{2\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$

$x=\frac{2\pm\sqrt{-12}}{2}$

$x=\frac{(2\pm 2i\sqrt{3})\div 2}{2 \div 2}$

$x=1\pm i\sqrt{3}$

$\therefore x=-2,\ x=1-i\sqrt{3},\ x=1+i\sqrt{3}$

b) $x^2-2x=2-\frac{1}{x^2-2x}$ $x \neq 0, 2$

Let $y=x^2-2x$

$y=2-\frac{1}{y}$

$\cdot y)$ $y^2=2y-1$

$y^2-2y+1=0$

$(y-1)(y-1)=0$

$(x^2-2x-1)^2=0$

$x=\frac{2\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

$x=\frac{2\pm\sqrt{8}}{2}$

$x=\frac{(2\pm 2\sqrt{2})\div 2}{2\div 2}$

$x=1\pm\sqrt{2}$

$\therefore x=1-\sqrt{2}, x=1-\sqrt{2},$
$x=1+\sqrt{2}, x=1+\sqrt{2}$

c) $\sqrt{x-7}+\sqrt{x}=\frac{21}{\sqrt{x-7}}$ $x>7$ and $x>0$

$\boxed{\therefore x>7}$
restriction

$\cdot\sqrt{x-7})$ $x-7+\sqrt{x^2-7x}=21$

$\sqrt{x^2-7x}=28-x$

Square both sides

$x^2-7x=784-56x+x^2$

$49x=784$

$x=16$

$\therefore$ the solution is $x=16$.

<u>Check: $x=16$</u>

$LS=\sqrt{x-7}+\sqrt{x}$

$=\sqrt{16-7}+\sqrt{16}$

$=3+4$

$=7$

$RS=\frac{21}{\sqrt{x-7}}$

$=\frac{21}{\sqrt{16-7}}$

$=\frac{21}{3}$

$=7$

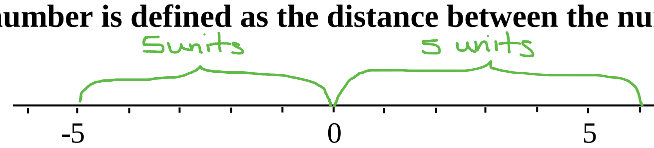
$\therefore LS=RS$

$\therefore x=16$ is a solution

HW. Exercise 1.10

Date:______________ **1.11 Absolute Value Equations and Inequalities**

The *absolute value* of a number is defined as the distance between the number and the origin.



$|-5| = 5$ $|5| = 5$

Ex. 1. Evaluate each of the following:

a) $|-2-8|$

$= |-10|$

$= 10$

b) $2|5| - |-10|$

$= 2(5) - (10)$

$= 10 - 10$

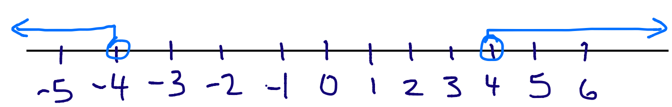
$= 0$

The Absolute Value of $x, x \in R$, **is**

$$|x| = \begin{cases} -x, \text{ if } x < 0. \\ +x, \text{ if } x \geq 0. \end{cases}$$

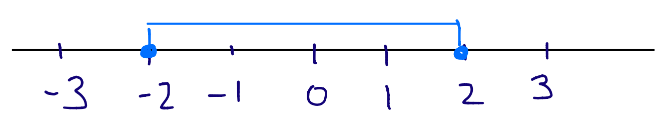
Ex. 2. Graph each of the following on the number line, for $x \in R$. Rewrite each statement without the absolute value bars.

a) $|x| > 4$



$\therefore x < -4$ or $x > 4$

b) $|x| \leq 2$



$\therefore -2 \leq x \leq 2$

For a given function $f(x)$,

$$|f(x)| = \begin{cases} -f(x), \text{ if } f(x) < 0. \\ +f(x), \text{ if } f(x) \geq 0. \end{cases}$$

Ex. 3. Solve for x, $x \in R$.

a) $|x-2| = 3$ by inspection: $x=-1, x=5$

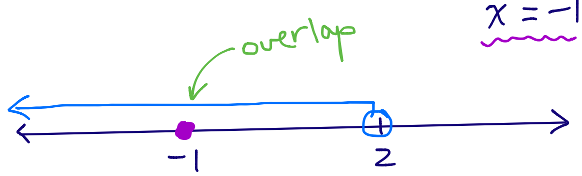
Case ①: If $x-2<0$ then (and) $-(x-2)=3$

$x<2$

$-x+2=3$

$-x=1$

$x=-1$



∴ the solution for case① is $x=-1$

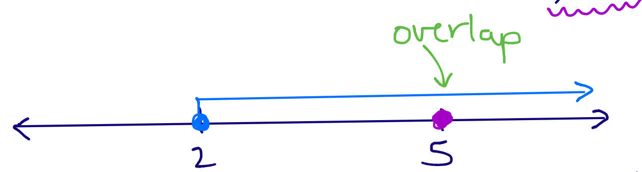
← or →

Case ②: If $x-2\geq 0$ then $+(x-2)=3$

$x\geq 2$

$x-2=3$

$x=5$



∴ the solution for case② is $x=5$

∴ the solution is $x=-1, x=5$.

b) $|x-3| = 2x$

case ①:

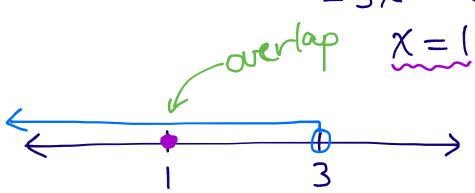
If $x-3<0$ then $-(x-3)=2x$

$x<3$

$-x+3=2x$

$-3x=-3$

$x=1$



∴ the solution for case① is $x=1$

case ②:

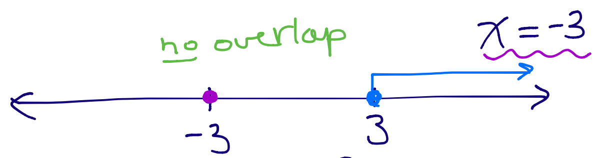
If $x-3\geq 0$ then $+(x-3)=2x$

$x\geq 3$

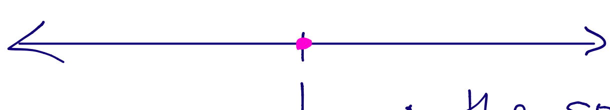
$x-3=2x$

$-x=3$

$x=-3$



∴ no solution for case ②



∴ the solution is $x=1$

c) $|3x-1|<5$

case ①:

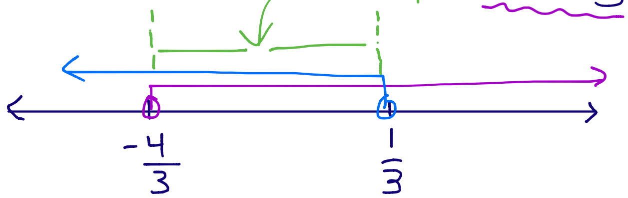
If $3x-1<0$ then $-(3x-1)<5$

$3x<1$ $\qquad -3x+1<5$

$x<\frac{1}{3}$ $\qquad -3x<4$

$x>-\frac{4}{3}$

overlap



$\therefore$ the solution for case ① is

$-\frac{4}{3}<x<\frac{1}{3}$.

case ②

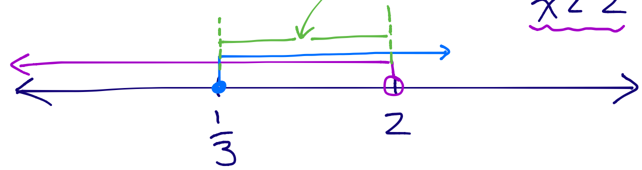
If $3x-1\geq 0$ then $+(3x-1)<5$

$3x\geq 1$ $\qquad 3x-1<5$

$x\geq\frac{1}{3}$ $\qquad 3x<6$

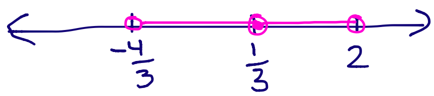
overlap.

$x<2$



$\therefore$ the solution for case ② is

$\frac{1}{3}\leq x<2$



$\therefore$ the solution is $-\frac{4}{3}<x<2$.

d) $|x-2|\geq 2x$

Case ①:

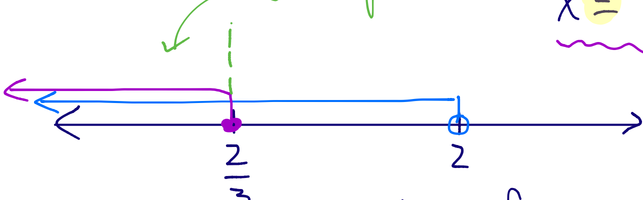
If $x-2<0$ then $-(x-2)\geq 2x$

$x<2$ $\qquad -x+2\geq 2x$

$-3x\geq -2$

overlap

$x\leq\frac{2}{3}$



$\therefore$ the solution for case ①

is $x\leq\frac{2}{3}$

Case ②:

If $x-2\geq 0$ then $+(x-2)\geq 2x$

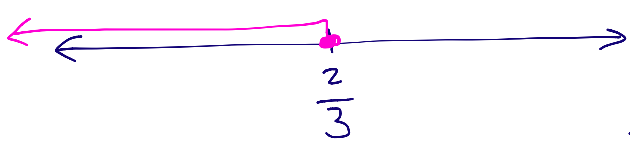
$x\geq 2$ $\qquad x-2\geq 2x$

$-x\geq 2$

$x\leq -2$

no overlap

$\therefore$ no solution for case ②



$\therefore$ the solution is $x\leq\frac{2}{3}$

HW. Exercise 1.11

Date:________________ **Unit 1 Part II Test Review**

Warmup

1. Write a polynomial equation in expanded form with roots $-\frac{2}{3}, 3+2i\sqrt{3}$ *and* $3-2i\sqrt{3}$.

$x=-\frac{2}{3}$, $x=3+2i\sqrt{3}$, $x=3-2i\sqrt{3}$

$3x+2=0$, $x-3-2i\sqrt{3}=0$, $x-3+2i\sqrt{3}=0$

$$(3x+2)(x-3-2i\sqrt{3})(x-3+2i\sqrt{3})=0$$
$$(3x+2)\left[(x-3)^2-(2i\sqrt{3})^2\right]=0$$
$$(3x+2)(x^2-6x+9-4(3)i^2)=0$$
$$(3x+2)(x^2-6x+21)=0$$
$$3x^3-18x^2+63x+2x^2-12x+42=0$$

$\therefore 3x^3-16x^2+51x+42=0$ is the required equation.

2. If one root is 2, find the value of k, and the other root(s) for $25x^4+kx^2+16=0$.

Let $f(x)=25x^4+kx^2+16$

$f(2)=0$

$0=25(2)^4+k(2)^2+16$

$0=400+4k+16$

$-4k=416$

$\therefore k=-104$

$25x^4-104x^2+16=0$

$(25x^2-4)(x^2-4)=0$

$(5x-2)(5x+2)(x-2)(x+2)=0$

$\therefore$ the other roots are

$x=-2$, $x=-\frac{2}{5}$, $x=\frac{2}{5}$

3. Solve, $x \in C$. Include restrictions on the variable.

a) $24x^4+8x^3-3x-1=0$

$8x^3(3x+1)-(3x+1)=0$

$(3x+1)(8x^3-1)=0$

$(3x+1)(2x-1)(4x^2+2x+1)=0$

$3x+1=0$, $2x-1=0$, $4x^2+2x+1=0$

$\therefore x=-\frac{1}{3}$, $x=\frac{1}{2}$, $x=\frac{-1-i\sqrt{3}}{4}$, $x=\frac{-1+i\sqrt{3}}{4}$

$$x=\frac{-2\pm\sqrt{(2)^2-4(4)(1)}}{2(4)}$$
$$x=\frac{-2\pm\sqrt{-12}}{8}$$
$$x=\frac{(-2\pm 2i\sqrt{3})\div 2}{8\div 2}$$
$$x=\frac{-1\pm i\sqrt{3}}{4}$$

b) $\left(x+\frac{1}{x}\right)^2-6\left(x+\frac{1}{x}\right)+8=0$

Let $a=x+\frac{1}{x}$

$a^2-6a+8=0$

$(a-2)(a-4)=0$

$x+\frac{1}{x}-2=0$ $\qquad$ $x+\frac{1}{x}-4=0$

$x^2-2x+1=0$ $\qquad$ $x^2-4x+1=0$

$(x-1)(x-1)=0$

$x=\frac{4\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$

$x=\frac{4\pm\sqrt{12}}{2}$

$x=\frac{(4\pm2\sqrt{3})\div 2}{2\div 2}$

$x=2\pm\sqrt{3}$

$\therefore$ $x=2-\sqrt{3}$, $x=1$, $x=1$, $x=2+\sqrt{3}$ are the solutions.

c) $\frac{3}{x^2}+\frac{2x}{x+2}=\frac{3x}{x+2}+\frac{1}{x^2}$

$\frac{3}{x^2}-\frac{1}{x^2}=\frac{3x}{x+2}-\frac{2x}{x+2}$

$\frac{2}{x^2}=\frac{x}{x+2}$

$2(x+2)=x(x^2)$

$2x+4=x^3$

$0=x^3-2x-4$

$$
\begin{array}{r}
x^2+2x+2 \\
x-2\,\overline{)\,x^3+0x^2-2x-4} \\
\underline{x^3-2x^2} \\
2x^2-2x \\
\underline{2x^2-4x} \\
2x-4 \\
\underline{2x-4} \\
0
\end{array}
$$

$0=(x-2)(x^2+2x+2)$

$x-2=0$ or $x^2+2x+2=0$

$x=\frac{-2\pm\sqrt{(2)^2-4(1)(2)}}{2(1)}$

$x=\frac{-2\pm\sqrt{-4}}{2}$

$x=\frac{-2\pm2i}{2}$

$x=-1\pm i$

$\therefore$ $x=2$, $x=-1-i$, $x=-1+i$ are the solutions.

4. Solve, $x \in R$.

a) $\sqrt{3x+1} - \sqrt{x+1} = 2$

$\sqrt{3x+1} = \sqrt{x+1} + 2$

Square both sides

$3x+1 = x+1 + 4\sqrt{x+1} + 4$

$2x - 4 = 4\sqrt{x+1}$

$\div 2)$ $\quad x - 2 = 2\sqrt{x+1}$

Square both sides

$x^2 - 4x + 4 = 4(x+1)$

$x^2 - 4x + 4 = 4x + 4$

$x^2 - 8x = 0$

$x(x-8) = 0$

$x = 0$ or $x = 8$

Check: $x=0$

$LS = \sqrt{3x+1} - \sqrt{x+1}$

$= \sqrt{1} - \sqrt{1}$

$= 0$

$RS = 2$

$\therefore LS \neq RS$

$\therefore x = 0$ is not a solution

Check: $x = 8$

$LS = \sqrt{3x+1} - \sqrt{x+1}$

$= \sqrt{25} - \sqrt{9}$

$= 5 - 3$

$= 2$

$RS = 2$

$\therefore LS = RS$

$\therefore x = 8$ is a solution.

$\therefore$ the solution is $\underline{x=8}$

b) $|5 - 3x| \leq 3x - 1$

Case ①:

If $5 - 3x < 0$ then $-(5-3x) \leq 3x - 1$

$-3x < -5$

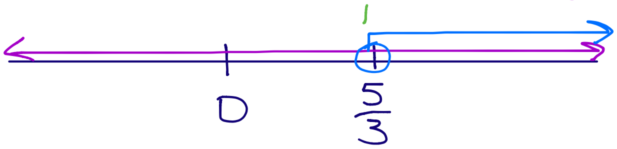
$x > \frac{5}{3}$

$-5 + 3x \leq 3x - 1$

$0 \leq 4$

$\therefore x \in \mathbb{R}$

overlap



$\therefore$ the solution for case ① is $x > \frac{5}{3}$

Case ②

If $5 - 3x \geq 0$ then $+(5-3x) \leq 3x - 1$

$-3x \geq -5$

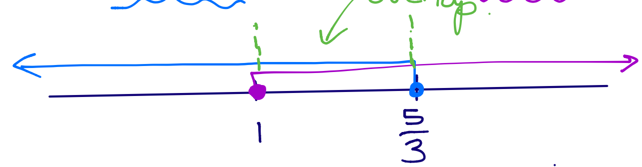
$x \leq \frac{5}{3}$

$5 - 3x \leq 3x - 1$

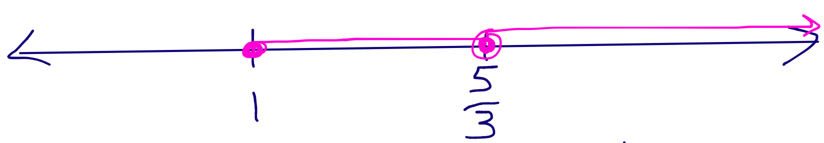
$-6x \leq -6$

$x \geq 1$

overlap



$\therefore$ the solution for case ② is $1 \leq x \leq \frac{5}{3}$



$\therefore$ the solution is $x \geq 1$.

HW. Part II Review 1.7 to 1.11