### 1.7 Solving Polynomial Equations

Note: A polynomial equation of the $n^{\text {th }}$ degree has $n$ roots.
Ex. 1. Solve for $x$ in each of the following, $x \in C$.
a) $27 x^{3}+8=0$
b) $6 x^{3}-13 x^{2}+x+2=0$
c) $x^{3}-7 x^{2}+8=0$
d) $3 x^{3}+x^{2}+24 x+8=0$
e) $-4 x^{4}-18 x^{3}+10 x^{2}=0$
f) $x^{4}-24 x^{2}=25$
g) $\left(x^{2}-5 x-5\right)\left(x^{2}-5 x+3\right)=9$

### 1.8 Determining Polynomial Equations From Roots

1. Determine the roots of the cubic equation $6 x^{3}-19 x^{2}+9 x+10=0$.
2. Write an appropriate equation in expanded form with integral coefficients having the given roots.
a) -3 and $\frac{2}{3}$
b) 2-3i and $2+3 i$
c) $0,0,1-2 \sqrt{5}$ and $1+2 \sqrt{5}$
d) $2,-\frac{1}{2}$ and $\frac{5}{3}$
$\qquad$ 1.9 Solving Radical Equations

## Warmup

1. Solve, $x \in C$.
a) $x^{2}-18=0$
b) $4 x^{2}+25=0$
c) $(x+3)^{2}=16$
d) $(x-6)^{2}+12=0$
2. Square each of the following:
a) $5 \sqrt{x}$
b) $3 \sqrt{x+1}$
c) $\sqrt{x-2}-3$
d) $2-5 \sqrt{x+1}$

Solving Radical Equations

1. Isolate the radical on one side of the equation.
2. Square both sides of the equation.
3. Repeat 1. and 2. until no radicals remain.
4. Solve and check answers in the original equation to identify and reject any extraneous roots.
5. Solve.
a) $4 \sqrt{2 x+7}-5=7$
b) $x+\sqrt{x-2}=4$
c) $\sqrt{4 x+5}-\sqrt{2 x-6}=3$

### 1.10 Solving Rational Equations

1. Solve, $x \in C$. Include restrictions on the variable.

Hint: Multiply both sides of the equation by the lowest common denominator (LCD).
a) $\frac{2}{3}+\frac{1}{2 x}=\frac{2-x}{x}$
b) $\frac{4}{x+1}=\frac{x+1}{4}$
c) $\frac{4}{x-1}-\frac{3}{x+2}=2$
d) $\frac{x^{2}-2 x+1}{x^{2}-1}-\frac{3 x-1}{x+2}=0$
2. Determine all real roots of the following equation. Include restrictions on the variable.
a) $x^{-2}\left(8 x^{-3}+1\right)=0$
b) $x^{2}-2 x=2-\frac{1}{x^{2}-2 x}$
c) $\sqrt{x-7}+\sqrt{x}=\frac{21}{\sqrt{x-7}}$

### 1.11 Absolute Value Equations and Inequalities

The absolute value of a number is defined as the distance between the number and the origin.


Ex. 1. Evaluate each of the following:
a) $|-2-8|$
b) $2|5|-|-10|$

The Absolute Value of $x, x \in R$, is
$|x|= \begin{cases}-x, & \text { if } \\ x<0 . \\ +x, & \text { if } \\ x \geq 0 .\end{cases}$

Ex. 2. Graph each of the following on the number line, for $x \in R$. Rewrite each statement without the absolute value bars.
a) $|x|>4$
b) $|x| \leq 2$

For a given function $f(x)$,
$|f(x)|=\left\{\begin{array}{l}-f(x), \text { if } f(x)<0 . \\ +f(x), \text { if } f(x) \geq 0 .\end{array}\right.$

Ex. 3. Solve for $x, x \in R$.
a) $|x-2|=3$
b) $|x-3|=2 x$
c) $|3 x-1|<5$
d) $|x-2| \geq 2 x$

## Unit 1 Part II Test Review

## Warmup

1. Write a polynomial equation in expanded form with roots $-\frac{2}{3}, 3+2 i \sqrt{3}$ and $3-2 i \sqrt{3}$.
2. If one root is 2 , find the value of $k$, and the other $\operatorname{root}(\mathrm{s})$ for $25 x^{4}+k x^{2}+16=0$.
3. Solve, $x \in C$. Include restrictions on the variable.
a) $24 x^{4}+8 x^{3}-3 x-1=0$
b) $\left(x+\frac{1}{x}\right)^{2}-6\left(x+\frac{1}{x}\right)+8=0$
c) $\frac{3}{x^{2}}+\frac{2 x}{x+2}=\frac{3 x}{x+2}+\frac{1}{x^{2}}$
4. Solve, $x \in R$.
a) $\sqrt{3 x+1}-\sqrt{x+1}=2$
b) $|5-3 x| \leq 3 x-1$
