Date:_____________

UNIT 2: GRAPHING FUNCTIONS

Polynomial, Radical, Absolute Value, Piecewise, Reciprocal & Rational

2.1 Graphing Quadratic, Cubic, Square Root, Absolute Value & Reciprocal Functions Using Transformations

1. Graph each of the following by naming and applying transformations on an appropriate function. Then, state the domain and range. Carefully follow the examples in the note.

a) $f(x)=(x-2)^3+1$ **b)** $y=-\frac{1}{3}x^3$ **c)** $g(x)=2\sqrt{x+5}$

d) $f(x)=-\frac{1}{2}(x-4)^2+4$ **e)** $y=|x+3|-2$ **f)** $g(x)=\sqrt{-(x-6)}+1$

g) $f(x)=\frac{2}{x+4}$ **h)** $y=-1.5|2x-10|-1$ **i)** $g(x)=\left(-\frac{1}{2}x-2\right)^3$

j) $f(x)=\left(-\frac{2}{3}x+2\right)^2+2$ **k)** $y=-\frac{1}{0.5x-1}-3$ **l)** $g(x)=4-2\sqrt{8-4x}$

2. Write an equation for each function that results from the given transformations. Then, state the domain and range.

a) $y=x^3$ has been reflected in the *y*-axis, horizontally stretched by a factor of 4 and translated 2 units to the right.

b) $y=\frac{1}{x}$ has been stretched vertically by a factor of 3, reflected in the *x*-axis and translated 4 units up.

c) $y=|x|$ has been has been both horizontally and vertically compressed by a factor of 0.35 and translated 7 units down and 5 units to the left.

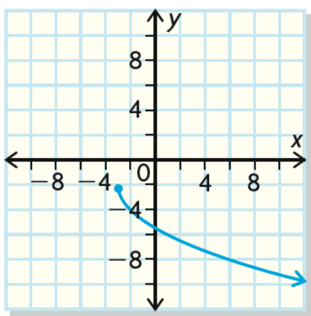
3. Match each equation to its graph. Explain your reasoning.

a) $f(x) = \dfrac{3}{-(x-2)} + 1$ **b)** $y = 2|x-3| - 2$ **c)** $g(x) = -2\sqrt{x+3} - 2$

d) $y = [0.25(x-2)]^2 - 3$ **e)** $g(x) = -\dfrac{4}{x} - 3$ **f)** $f(x) = -\dfrac{1}{3}(x+1)^3 - 1$

g) $g(x) = -0.5|x+4| + 2$ **h)** $y = -0.5(x+4)^2 + 1$ **i)** $f(x) = -\dfrac{1}{2}\sqrt{1-x} + 1$

A



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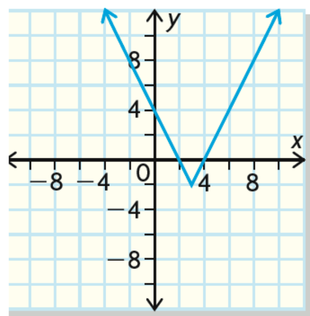

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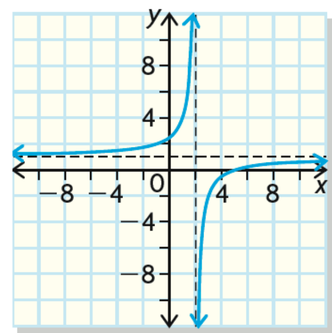

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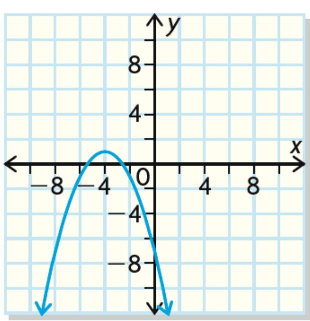

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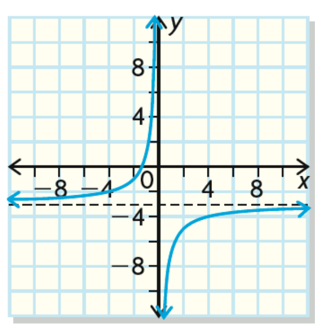

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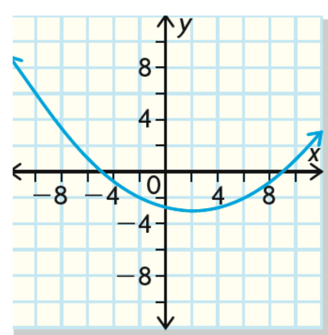

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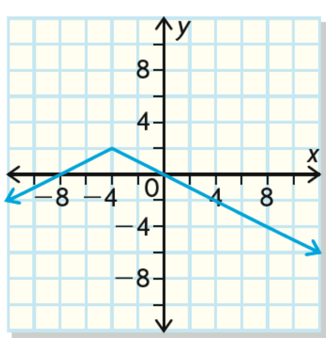

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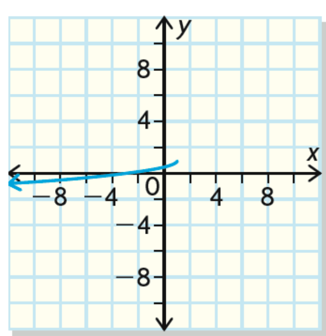

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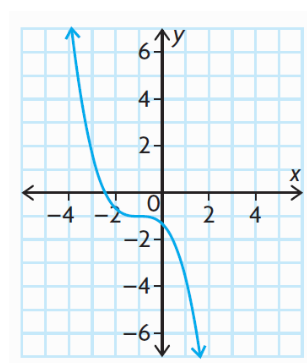

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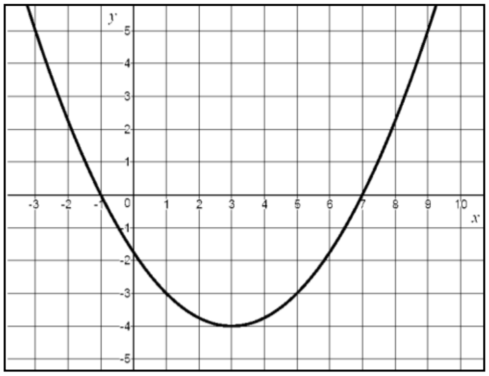
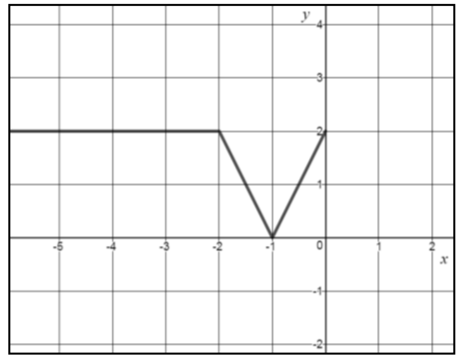
2.2 Graphing Reciprocal and Absolute Value Functions of $y = f(x)$

1. Use the graph of $y = f(x)$ to sketch the graph of $y = \dfrac{1}{f(x)}$ for each of the following:

a)

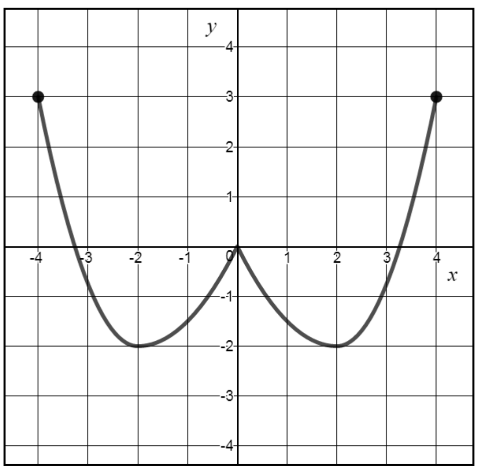


b)



2. Use the graph of $y = f(x)$ to sketch the graph of $y = |f(x)|$ for each of the following:

a)



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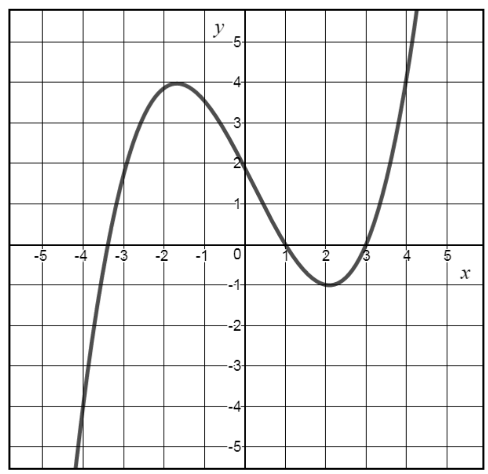


3. Graph each function $y = f(x)$ and its reciprocal function $y = \dfrac{1}{f(x)}$ on the same grid.

a) $f(x) = \dfrac{1}{2}x - 3$ **b)** $f(x) = 4 - x^2$ **c)** $f(x) = x^2 - 3x - 4$

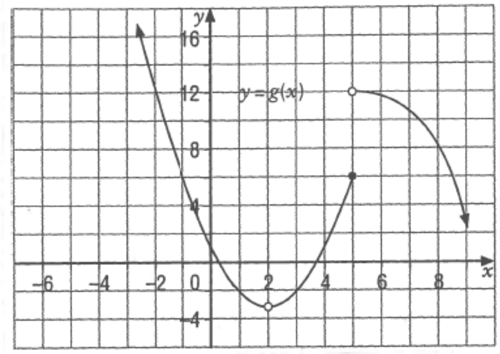
d) $f(x) = \dfrac{1}{3}(x-2)^3$ **e)** $f(x) = |0.5x| + 1$ **f)** $f(x) = 2\sqrt{x+3} - 4$

4. Graph each function $y = f(x)$ and its absolute value function $y = |f(x)|$ on the same grid.

a) $f(x) = -2x - 3$ **b)** $f(x) = \sqrt{-x+6} - 2$ **c)** $f(x) = -\dfrac{1}{x+2}$

d) $f(x) = (x-2)^3 - 1$ **e)** $f(x) = -2|x+4| + 4$ **f)** $f(x) = \dfrac{2}{3}x^2 - 4x$

Date:______________ **2.3 Graphing Piecewise Functions**



1. Use the graph of $g(x)$ to determine the following:

a) $g(-2)$ **b)** $g(2)$ **c)** $g(4)$ **d)** $g(8)$

e) the value(s) of *x* at which the function is discontinuous and type(s) of discontinuity

f) as $x \to 5^-$, $g(x) \to$ ______

g) as $x \to 5^+$, $g(x) \to$ ______

h) the end behaviour of the function

2. For each piecewise function sketch the graph and determine the value(s) of *x* at which the function is discontinuous. Identify the discontinuities as *jump*, *removable* or *infinite*.

a) $f(x)=\begin{cases} -x^2-2 & if\ x \le 2 \\ x^2-3x+4 & if\ x>2 \end{cases}$

b) $g(x)=\begin{cases} -\frac{2}{3}x+5 & if\ x \le 3 \\ x-3 & if\ 3<x<5 \\ -x+5 & if\ x \ge 5 \end{cases}$

c) $f(x)=\begin{cases} -(x+2)^3+1 & if\ x<-1 \\ 2\sqrt{x+1} & if\ x \ge -1 \end{cases}$

d) $g(x)=\begin{cases} -1 & if\ x \in (-\infty,0) \\ 0 & if\ x=0 \\ 1 & if\ x \in (0,\infty) \end{cases}$

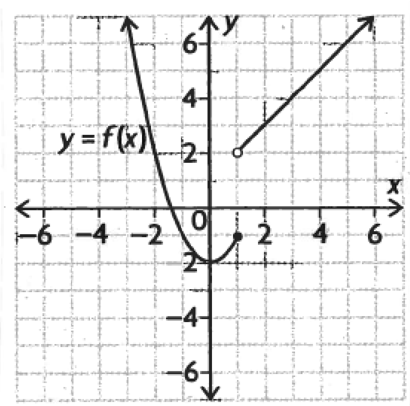
e) $f(x)=\begin{cases} \frac{1}{x+2} & if\ x \ne -2 \\ 0 & if\ x=-2 \end{cases}$

f) $g(x)=\begin{cases} (x-3)^2 & if\ x \in (2,\infty) \\ 2-\frac{1}{2}|x| & if\ x \in (-2,2) \\ (x+3)^2 & if\ x \in (-\infty,-2] \end{cases}$

Date:_____________ **2.4 Piecewise Functions and Continuity Continued**

1. Write the algebraic representation of each piecewise function, using function notation.

a)



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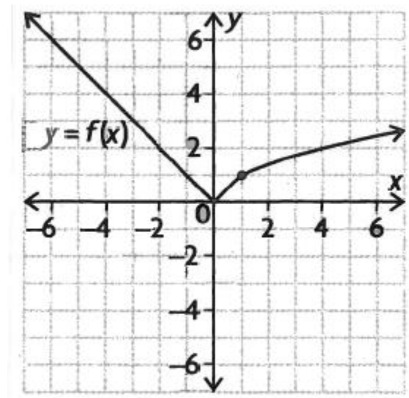


2. The fish population, in thousands, in a lake at any time, t, in years is modelled by the function

$$p(t)=\begin{cases} t^2, & 0\le t\le 6 \\ 4t+8, & t>6 \end{cases}.$$

This function describes a sudden change in the population at time $t=6$, due to a chemical spill.

a) Graph this piecewise function.
b) Describe the continuity of the function.
c) How many fish were killed by the chemical spill?
d) At what time did the population recover to the level it was before the chemical spill?

3. Without graphing determine if each function below is continuous or discontinuous. If it is discontinuous, state where it is discontinuous. Graph to verify your conclusions.

a) $f(x)=\begin{cases} |x+2|, & x\le -1 \\ -x^2+2, & x>-1 \end{cases}$ **b)** $f(x)=\begin{cases} \dfrac{1}{x}, & x<1 \\ -x, & x\ge 1 \end{cases}$

4. Find the value(s) of k that make each function below continuous for all $x\in R$.

a) $g(x)=\begin{cases} x+3, & \text{if } x\ne 3 \\ 2+\sqrt{k}, & \text{if } x=3 \end{cases}$ **b)** $f(x)=\begin{cases} x^2-k, & x<-1 \\ 2x-1, & x\ge -1 \end{cases}$ **c)** $f(x)=\begin{cases} (kx-1)^3 & \text{if } x<2 \\ k^2x^2-1 & \text{if } x\ge 2 \end{cases}$

5. Find the values of a and b that make each function below continuous for all $x\in(-\infty,\infty)$.

a) $g(x)=\begin{cases} -x & \text{if } x\in(-\infty,-2] \\ ax^2+b & \text{if } x\in(-2,0) \\ 6 & \text{if } x\in[0,\infty) \end{cases}$ **b)** $f(x)=\begin{cases} ax+3 & \text{if } x\in(5,\infty) \\ 8 & \text{if } x=5 \\ x^2+bx+a & \text{if } x\in(-\infty,5) \end{cases}$

6. Graph the piecewise function from **5. a)**.

7. Rewrite the following functions involving absolute value as piecewise functions and graph.

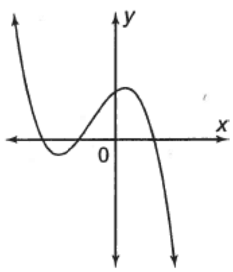
a) $f(x)=|6-2x|+1$ **b)** $f(x)=\dfrac{|x-2|}{x-2}$ **c)** $g(x)=\dfrac{x^2+x}{|x|}$ **d)** $f(x)=\dfrac{x^3-3x+2}{|x+2|}$

Date: ________________ **2.5 Graphing Factored Polynomial Functions**

Complete the "**Summary of Graphs of Polynomial Functions**" lesson in your bound notes by following **Ex. 2, Ex. 3** and **Ex. 4** from the note done in class.

1. Use the graph of each polynomial function to identify the polynomial as cubic or quartic, state the sign of the leading coefficient of its function, state the number of turning points and describe the end behaviour.

a)



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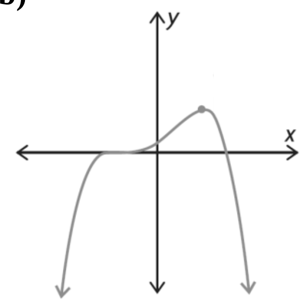

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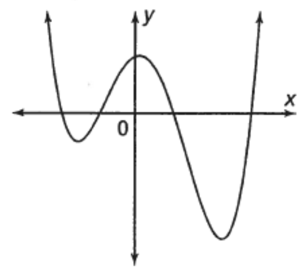
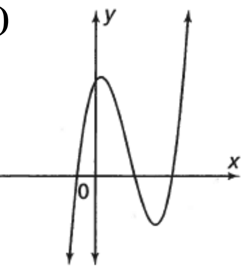


d)



2. State the degree, leading coefficient, and end behaviours of each polynomial function.

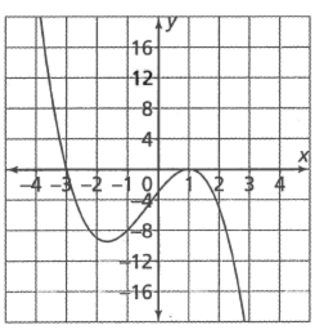
a) $f(x)=\frac{1}{2}(3-2x)(1+3x)$ **b)** $y=(3-x)^3(x+1)^2$

c) $f(x)=(x-2)(x+2)(x-1)(x+1)$ **d)** $g(x)=(2x+1)(2x-1)^2$

e) $y=-(x+2)(x-3)(3x+4)$ **f)** $f(x)=-(3-x)^2(x+1)^2$

3. Use end behaviours, turning points, and zeros (x-intercepts) to match each graph with the appropriate polynomial function. Explain.

a)



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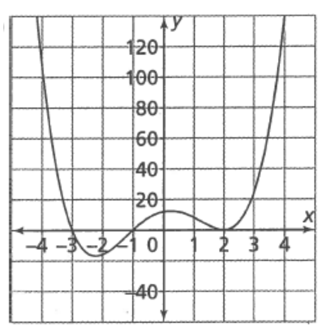
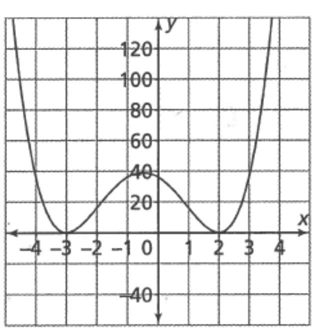
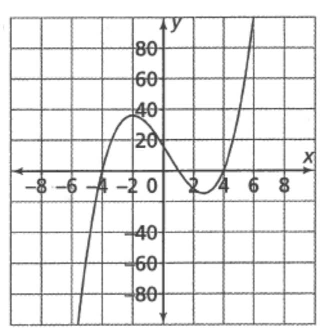


c)



d)



i. $f(x)=(x+3)(x-2)^2(x+1)$ **ii.** $f(x)=(x-1)(x-4)(x+4)$

iii. $f(x)=-(x-1)^2(x+3)$ **iv.** $f(x)=(x-2)^2(x+3)^2$

Date:________________ **2.6 Graphing Expanded Polynomial Functions**

1. For each of the following graphs, state

a) if the function has an even or odd degree

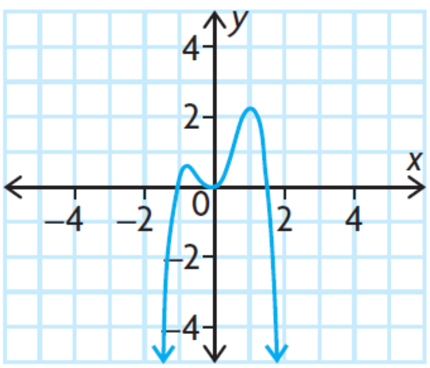
b) if the leading coefficient is positive or negative

c) the degree of the function

d) the number and nature of the roots to the corresponding equation used to find the zeros

e) the number of turning points

i)



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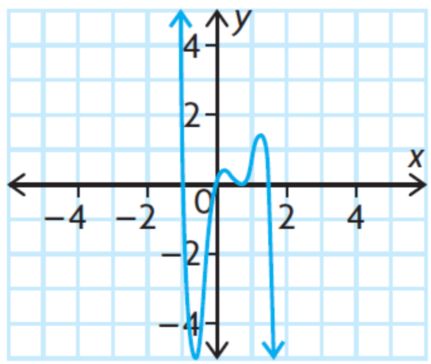

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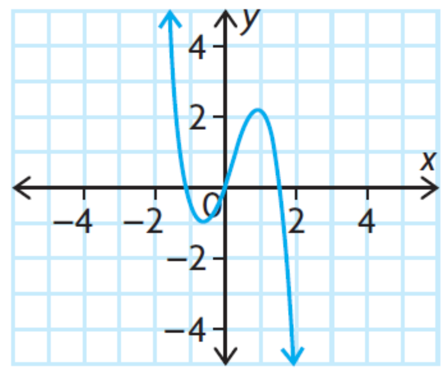

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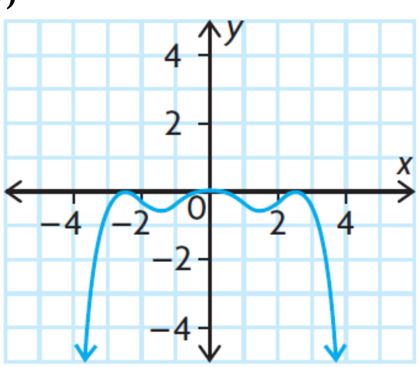
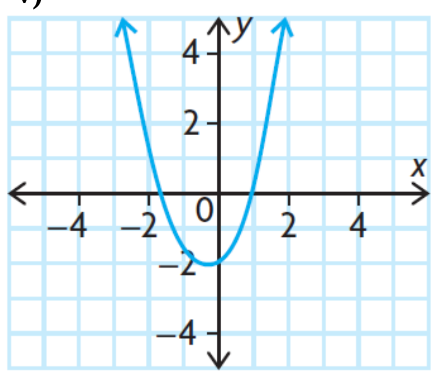


v)



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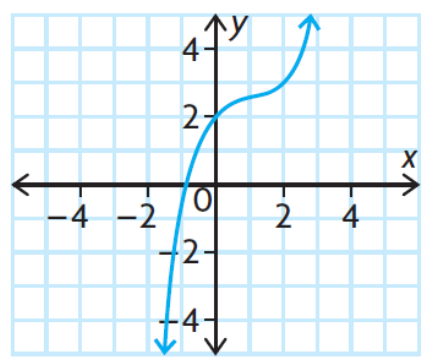


2. Describe the end behavior of each polynomial function using the degree and leading coefficient.

a) $f(x)=5-3x-x^2$ **b)** $f(x)=3x^5+2x^3-4x$

c) $y=0.5x^4+2x^2-6$ **d)** $g(x)=4x^2-3x^3+4-6x$

3. Draw a sketch of the following functions, clearly labeling all x-intercepts.

a) $f(x)=-3x^2+\frac{7}{2}x+\frac{3}{2}$ **b)** $y=-(x-3)^3(x^2+2x+1)$

c) $f(x)=x^4-5x^2+4$ **d)** $g(x)=8x^3-4x^2-2x+1$

e) $y=-3x^3-x^2+22x+24$ **f)** $f(x)=(6x-9-x^2)(x^2+2x+1)$

4. Sketch the function $f(x)=x^3-x^2-7x+3$, by determining ***exact*** values of all x-intercepts and approximating any x-intercepts that are irrational to the nearest tenth.

Date:______________ **2.7 Determining the Equations of Polynomial Functions**

1. Which of the following functions belong to the same families? Explain.

a) $y = 0.3(x-2)(x+4)(x+6)$ **b)** $f(x) = -2(x-5)(2x-1)(1+3x)$

c) $p(x) = 4(x-2)(x+3)^2$ **d)** $y = -(3x+1)(x-5)(1-2x)$

e) $y = -4(x+6)(x-2)(x+4)$ **f)** $g(x) = 0.5(x^2+6x+9)(x-2)$

g) $h(x) = 4(2x-1)(1+3x)(5-x)$ **h)** $y = (5x-10)(x^2+10x+24)$

2. a) Determine an equation for the family of cubic functions whose x-intercepts are -2, 3 and $\frac{2}{5}$.

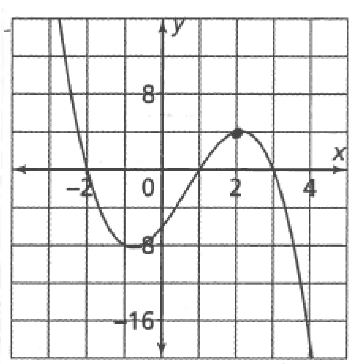
b) Find the particular member of the above family whose graph has a y-intercept of 6 in ***factored*** form and sketch its graph.

3. a) Determine an equation for the family of functions whose zeros are -1 (order 3) and 2 (order 2).

b) Find the particular member of the above family whose graph passes through the point $(3, -16)$ in ***factored*** form and sketch its graph.

4. Determine the equation of the cubic function, in ***standard*** form, with roots $\frac{-2-i\sqrt{3}}{4}$, $\frac{-2+i\sqrt{3}}{4}$ and 1 passing through the point $(-3, -103)$ and sketch its graph using DESMOS.

5. Determine the equation of each polynomial function from its graph in ***factored*** form.

a)


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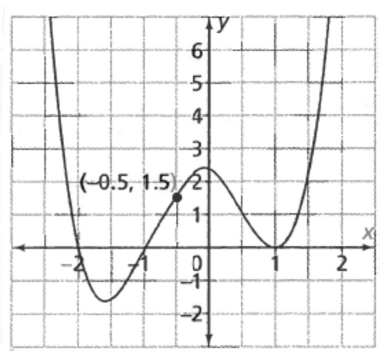

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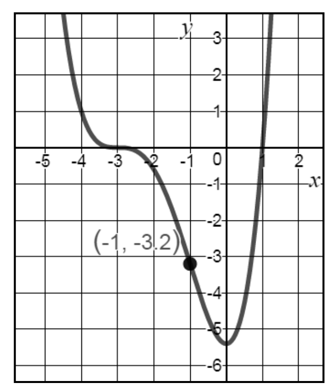
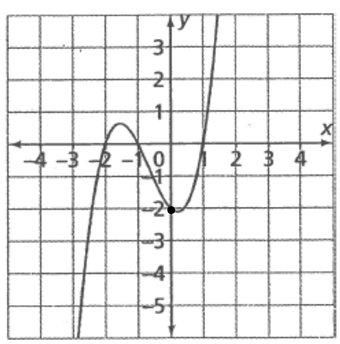


6. Determine the equation of each polynomial function from its graph in ***standard*** form.

a)


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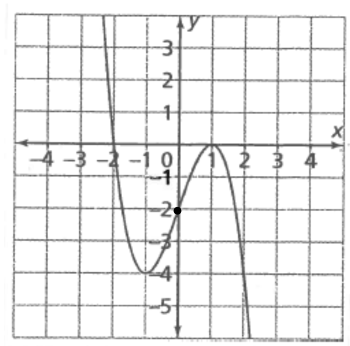

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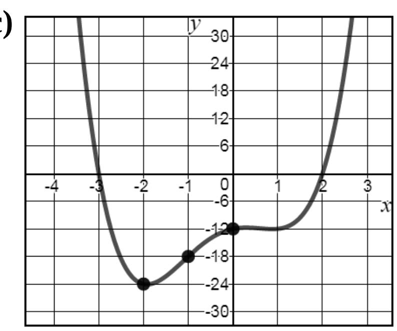


Hint:
Let $f(x) = (ax^2+bx+c)(x+3)(x-2)$

7. In each of the following you are given a set of points that lie on the graph of a function. Determine the equation of the polynomial function.

a) $(1,-34)$, $(2,-42)$, $(3,-38)$, $(4,-16)$, $(5,30)$, $(6,106)$

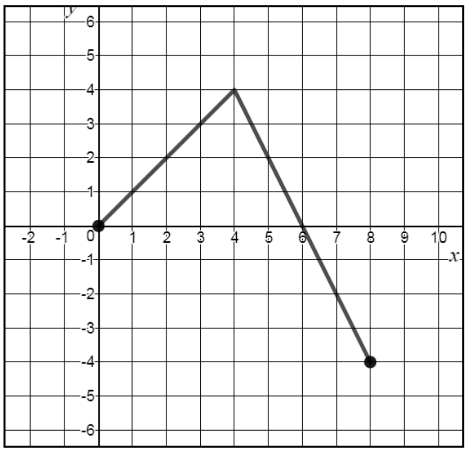
b) $(1,-4)$, $(2,0)$, $(3,30)$, $(4,98)$, $(5,216)$, $(6,396)$

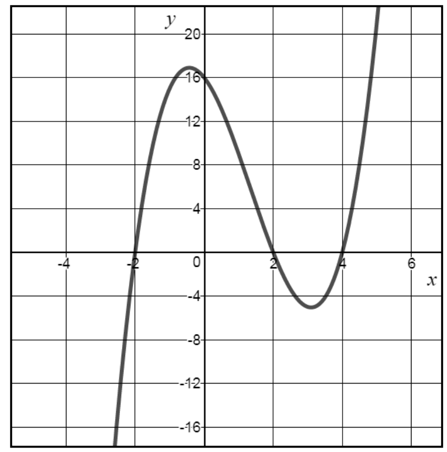
c) $(1,-2)$, $(2,-4)$, $(3,-6)$, $(4,-8)$, $(5,14)$, $(6,108)$, $(7,346)$

Date:_________________ **UNIT 2 REVIEW OF 2.1-2.7**

1. Use the graph of $y = f(x)$ to sketch the graph of :

a) $y = \dfrac{1}{f(x)}$ **b)** $y = |f(x)|$





2. Graph each function $y = f(x)$ and its reciprocal function $y = \dfrac{1}{f(x)}$ on the same grid.

a) $f(x) = -\dfrac{1}{2}(x+2)^2 + 4$ **b)** $f(x) = \sqrt{-\dfrac{1}{2}x + 5} - 2$

3. Graph each function $y = f(x)$ and its absolute value function $y = |f(x)|$ on the same grid.

a) $f(x) = -(x+3)^3 + 5$ **b)** $f(x) = 2|x-1| - 4$

4. The graph of a piecewise function ***f*** is shown. Use the graph to determine the following:

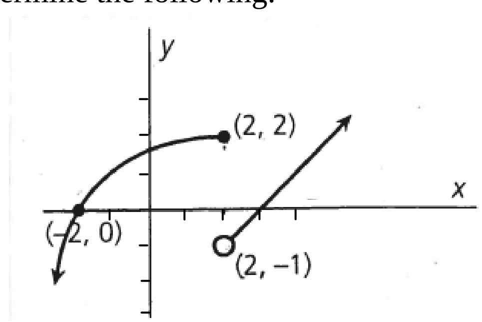
a) $f(2)$ **b)** $f(3)$

c) the value(s) of *x* at which the function is discontinuous and type of discontinuity.

d) as $x \to 2^-$, $f(x) \to$_____ and as $x \to 2^+$, $f(x) \to$_____

e) the end behaviour of the function ***f***

f) the equation of this *quadratic/linear* piecewise function



5. Graph each piecewise function and determine the value(s) of *x* at which the function is discontinuous. Identify the discontinuities as *jump*, *removable* or *infinite*.

a) $f(x) = \begin{cases} -3 + \dfrac{x^2}{3} & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ \dfrac{2x}{3} - 3 & \text{if } x > 0 \end{cases}$ **b)** $g(x) = \begin{cases} \dfrac{2}{x} & \text{if } x \in (-\infty, -1) \\ -x + 1 & \text{if } x \in [-1, 3) \\ x - 2 & \text{if } x \in [3, \infty) \end{cases}$

6. Rewrite the following functions as piecewise functions and graph.

a) $f(x) = \dfrac{x^2 + 2|x+1| - 1}{|x+1|}$ **b)** $g(x) = \dfrac{x^3 - 64}{2|x-4|}$

7. Find the value(s) of k such that the function $g(x)=\begin{cases} -\dfrac{1}{2}x+3 & \text{if } x\in(-\infty,2) \\ -2\sqrt{x+k}-k & \text{if } x\in[2,\infty) \end{cases}$ is continuous for all $x\in(-\infty,\infty)$.

8. Find constants a and b such that the function $f(x)=\begin{cases} x+2 & \text{if } x<2 \\ ax^2-bx+3 & \text{if } 2\le x<3 \\ 2x-a+b & \text{if } x\ge 3 \end{cases}$ is continuous for all $x\in R$.

9. Sketch the following functions, clearly labeling all x-intercepts and identifying these intercepts as single, double or triple roots.

a) $f(x)=0.6(3x-4)(2x+5)(3x-8)$

b) $g(x)=(x+2)^3(3-x)$

c) $y=-\sqrt{3}(x+3)^2(x-2)^2(2x+1)$

d) $f(x)=-\dfrac{1}{5}(4-x)^3(x+2)^2$

e) $g(x)=\left(x^2-x-2\right)^2$

f) $y=-2x^2-16x-14$

g) $f(x)=-\dfrac{1}{4}x^4+\dfrac{5}{4}x^2-1$

h) $g(x)=12x^3-4x^2-27x+9$

i) $y=x^3-6x^2+12x-8$

j) $f(x)=-x^5+3x^3-2x^2$

10. Sketch a possible graph of a polynomial function that satisfies each set of conditions.

a) degree 2, positive leading coefficient and no zeros

b) degree 3, positive leading coefficient, one real root, two imaginary roots and no turning points

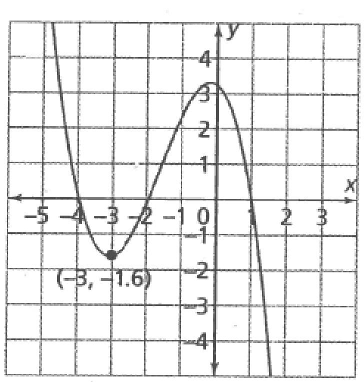
c) degree 4, negative leading coefficient, two distinct real roots and two non-real roots

d) degree 5, negative leading coefficient, one zero and four turning points

11. Find k if -1 is a zero of the function $y=2x^4+4kx^3-8x-k$ and then sketch its graph.

12. Determine the equation of each polynomial function in ***factored*** form.

a)



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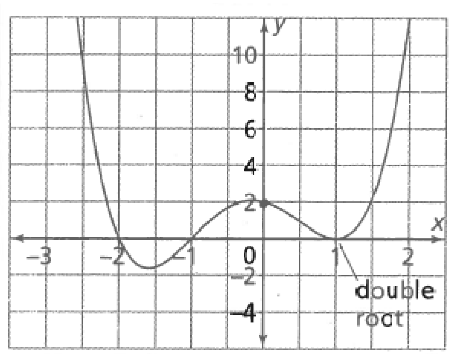

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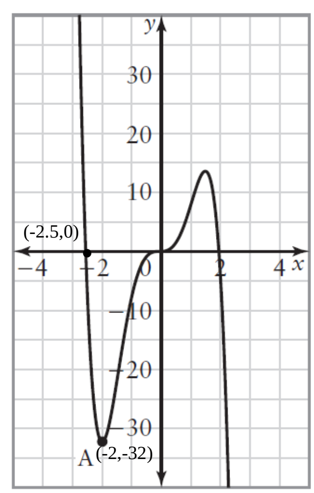


13. The points $(-1,-27)$, $(0,-11)$, $(1,-5)$, $(2,-3)$, $(3,1)$ and $(4,13)$ all lie on the graph of a function. Determine the equation of the polynomial function in ***expanded*** form.

14. An open-top box is to be constructed from a 36 cm by 24 cm piece of cardboard by cutting congruent squares from the corners and then folding up the sides.

a) Express the volume, V, of the box as a function of its height, x, in ***standard*** form.

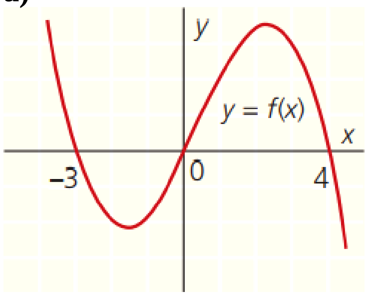
b) Determine possible dimensions of a box with volume 1820 cm^3 to two decimal places if necessary. ***A calculator is required here.***

Date:_______________

2.8 Solving Polynomial Inequalities Graphically

1. Use the graphs of the following functions to state when **i)** $f(x) > 0$ **ii)** $f(x) < 0$
Answer using ***algebraic notation***.

a)



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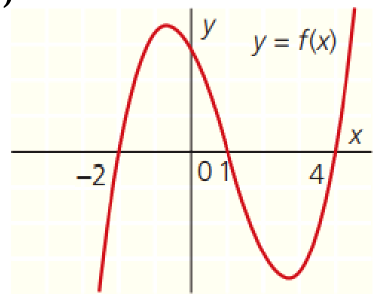

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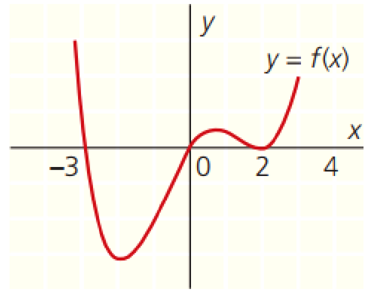


2. Sketch a graph of a cubic polynomial function $y = f(x)$ such that $f(x) < 0$ if $x \in (-4,3) \cup (7,+\infty)$ and $f(x) > 0$ if $x \in (-\infty,-4) \cup (3,7)$. Determine the equation of this function in both factored and expanded forms if the y-intercept is -12.

3. Sketch a graph of a quartic function $y = g(x)$ such that $g(x) > 0$ when $-2 < x < 1$, $g(x) \le 0$ when $x \le -2$ or $x \ge 1$, and $g(x)$ has a double root when $x = 3$. Determine the equation of this function in factored form if $g(-1) = 96$.

4. Solve each of the following graphically where $x \in R$. Answer using a ***solution set***.

a) $(2x+7)(5-2x) < 0$ **b)** $\sqrt{2}x(x+4)^2(x-4)^2 > 0$

c) $2(x-3)^2(1-x) \le 0$ **d)** $(x^2+4x+4)(x^2-25) \ge 0$

5. Solve each of the following graphically where $x \in R$. Answer using ***interval notation***.

a) $\frac{3}{4}x^2 - \frac{5}{2}x - \frac{25}{4} \le 0$ **b)** $2x^2 - 16 > 0$

c) $9x^3 + 18x^2 \le 4x + 8$ **d)** $-x^4 + 3x^2 + 2x \ge 0$

e) $2x^3 - x^2 - 13x - 6 < 0$ **f)** $3x^3 - 11x^2 \ge 2 - 10x$

g) $8x^3 - 12x^2 + 6x - 1 < 0$ **h)** $2x^4 + x^3 - 26x^2 > 37x + 12$

6. Find the solutions that satisfy both $x^3 - x^2 - 20x \le 0$ ***and*** $x^3 - 13x + 12 > 0$.
Hint: *Graph both solutions on the same real number line and look for overlap.*

7. In Canada, hundreds of thousands of cubic metres of wood are harvested each year. The function $V(t) = t^4 - 8t^3 + 19t^2 - 12t + 185000$, $0 \le t \le 4$, models the volume harvested, in cubic metres, where the year 1993 corresponds to $t = 0$. In which years were less than 185000 m^3 harvested ?

8. During a normal five-second respiratory cycle in which a person inhales and then exhales, the volume of air in a person's lungs can be modeled by $V(t) = 0.2t^3 - 1.9t^2 + 5t - 3$, where volume, V, is in litres and time, t, is in seconds, $0 \le t \le 5$. In this cycle, when is the volume of air in the lungs more than 0.3 litres?

Date:______________ **2.9 Solving Polynomial & Rational Inequalities Using a Number Line Strategy**

1. Solve the following polynomial inequalities using a ***number line strategy*** where $x \in R$.
State your final answer using ***set notation***.

a) $(2-x)^3(x+2)^2(1-3x) \le 0$ **b)** $(x-3)(x+1)+(x-3)(x+2) \ge 0$

c) $x^4 - 5x^2 - 36 > 0$ **d)** $12x - x^3 < 0$

e) $12x^3 - 4x^2 - 75x + 25 < 0$ **f)** $x^4 - 8x \le 0$

g) $2x^3 - 4x + 2 > x^3 - 3x - 4$ **h)** $x^4 - 3x^3 + 5x^2 - 27x - 36 \ge 0$

2. Solve each of the following rational inequalities using a ***number line strategy*** where $x \in R$.
State your final answer using ***interval notation***.

a) $\dfrac{x^2 - x - 12}{x-1} \le 0$ **b)** $\dfrac{6x^2 - 5x + 1}{2x+1} > 0$ **c)** $\dfrac{1}{x-5} - 1 \le 0$

d) $\dfrac{3}{x-2} < \dfrac{4}{x}$ **e)** $\dfrac{7}{x-3} \ge \dfrac{2}{x+4}$ **f)** $\dfrac{-2x-10}{x} > x+5$

g) $\dfrac{x}{1-4x} \ge \dfrac{2}{x-9}$ **h)** $\dfrac{x-4}{x+1} < \dfrac{3x-8}{2x-1}$ **i)** $\dfrac{5x-1}{x^2-1} \ge 3$

3. The relationship between the object distance, d, and image distance, I, both in centimetres, for a camera with focal length 2.0 cm is defined by the relation $d = \dfrac{2.0I}{I-2.0}$. For what values of I is d greater than 10.0 cm?

Date:______________ **2.10 Graphing Rational Functions With Horizontal Asymptotes**

1. For each rational function determine,
i) the x-intercept(s)
ii) the y-intercept

a) $f(x)=\dfrac{-2}{3(x^2+2)}$ **b)** $y=\dfrac{4-3x}{x(2x-1)}$ **c)** $g(x)=\dfrac{4x^2+8x}{(2x-1)(x+3)}$ **d)** $f(x)=\dfrac{2+x-x^2}{(x-1)^2}$

2. a) Under what conditions does a rational function have a horizontal asymptote?
b) Explain how to find the equation of the horizontal asymptote.

3. For each rational function determine,
i) the equation(s) of the vertical asymptotes
ii) the equation of the horizontal asymptote
iii) the coordinates of any point where the function crosses the horizontal asymptote

a) $f(x)=\dfrac{-2}{3x^2+6}$ **b)** $y=\dfrac{4-3x}{2x^2-x}$ **c)** $g(x)=\dfrac{4x(x+2)}{2x^2+5x-3}$ **d)** $f(x)=\dfrac{-(x-2)(x+1)}{(x-1)^2}$

4. Let $f(x)=\dfrac{4-x^2}{x^2-2x-3}$, $g(x)=\dfrac{8}{x^2+4}$, $h(x)=\dfrac{3x+4}{2-6x}$ and $p(x)=\dfrac{x^3+8}{x^2+x}$.

a) Which of these rational functions ***has no***:
i) x-intercept?
ii) y-intercept ?
iii) vertical asymptote ?
iv) horizontal asymptote?

b) Which of these rational functions ***has***:
i) an x-intercept of -2?
ii) a y-intercept of 2?
iii) a vertical asymptote of $x=-1$?
iv) a horizontal asymptote of $y=-\dfrac{1}{2}$?

5. Graph the following rational functions by finding and labeling any intercepts, asymptotes and points where the function crosses the horizontal asymptote. Include a table of values for a more accurate graph if appropriate.

a) $f(x)=\dfrac{6-3x}{x+3}$ **b)** $g(x)=\dfrac{x}{(x-1)(x+3)}$ **c)** $y=\dfrac{-2x^2+4x-2}{x^2+1}$

d) $f(x)=\dfrac{-1}{x^2-6x+9}$ **e)** $y=\dfrac{(x+4)(x-3)}{x^2-4}$ **f)** $g(x)=\dfrac{2}{4-x}$

6. Create a function $f(x)$ that has a graph with the given features:
a) a vertical asymptote at $x=1$, a horizontal asymptote at $y=0$ and a y-intercept at 3.
b) vertical asymptotes at $x=-3$ and $x=5$ and a horizontal asymptote at $y=3$.
c) vertical asymptotes at $x=\dfrac{1}{3}$ and $x=-2\dfrac{1}{2}$ and an x-intercept at $-\dfrac{3}{4}$.

7. Find constants a and b that guarantee that the graph of the function defined by $g(x)=\dfrac{ax^2+7}{bx^2-9}$ will have vertical asymptotes at $x=\dfrac{3}{5}$ and $x=-\dfrac{3}{5}$ and a horizontal asymptote at $y=-2$.

Date:______________ 2.11 Graphing Rational Functions With Oblique Asymptotes

1. a) Under what conditions does a rational function have a linear oblique asymptote?
b) Explain how to find the equation of the linear oblique asymptote.

2. Let $f(x)=\dfrac{x^4-1}{x^2+4}$, $g(x)=\dfrac{(2x-1)^2}{x^2-4}$, $h(x)=\dfrac{x^2+4}{1-2x}$ and $p(x)=\dfrac{x^3-1}{x^2}$.

a) Which of these rational functions ***has no***:
- **i)** x-intercept?
- **ii)** y-intercept ?
- **iii)** vertical asymptote ?
- **iv)** linear oblique asymptote?

b) Which of these rational functions ***has***:
- **i)** an x-intercept of 1?
- **ii)** a y-intercept of -0.25?
- **iii)** a vertical asymptote at the y-axis?
- **iv)** a linear oblique asymptote of $y=x$?

3. Graph the following rational functions by finding and labeling any intercepts, asymptotes and points where the function crosses the linear oblique asymptote. Include a table of values for a more accurate graph if appropriate.

a) $f(x)=\dfrac{x^2-4}{1-x}$ **b)** $g(x)=\dfrac{x^2}{x-3}$ **c)** $y=\dfrac{2x^2-4x-8}{x}$

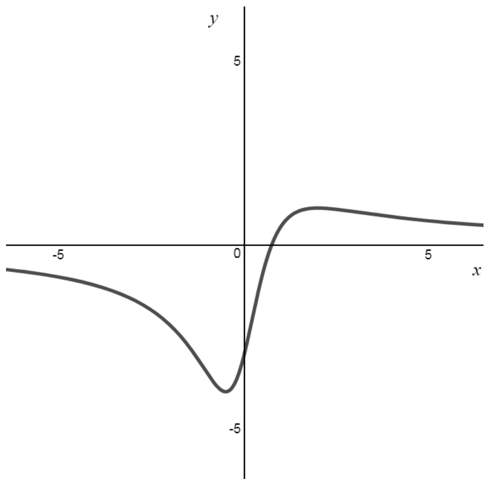
d) $f(x)=\dfrac{-0.5x^2+x}{x+2}$ **e)** $y=\dfrac{x^3+1}{x^2+3x}$ **f)** $g(x)=\dfrac{x^3-3x^2+4}{x^2+3x+4}$

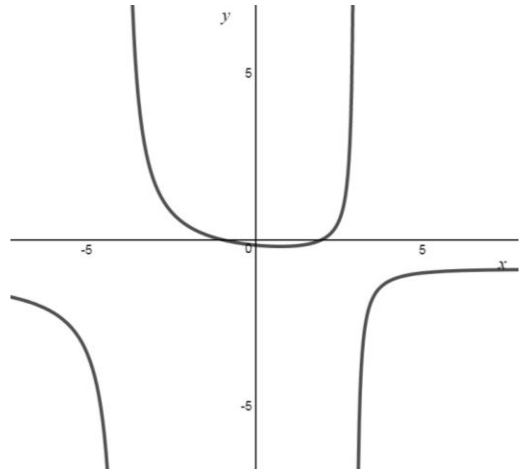
Date:________________ **2.12 Graphing Rational Functions Continued**

1. Find and label on the graphs provided any intercepts and asymptotes and then use the graphs to state, using interval notation, where $f(x) < 0$.

a) $f(x) = \dfrac{4x-3}{x^2+1}$

b) $f(x) = \dfrac{2+x-x^2}{x^2+x-12}$

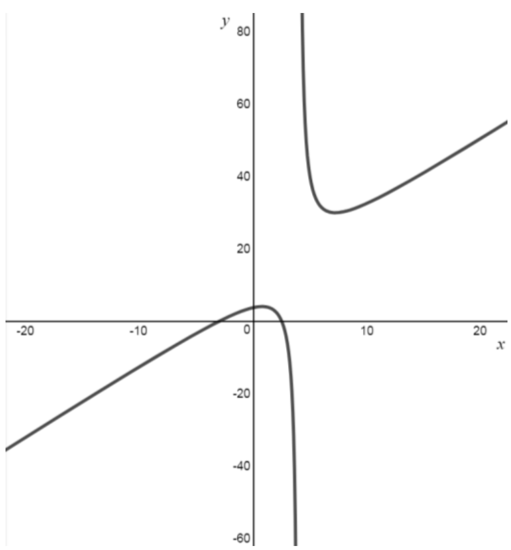


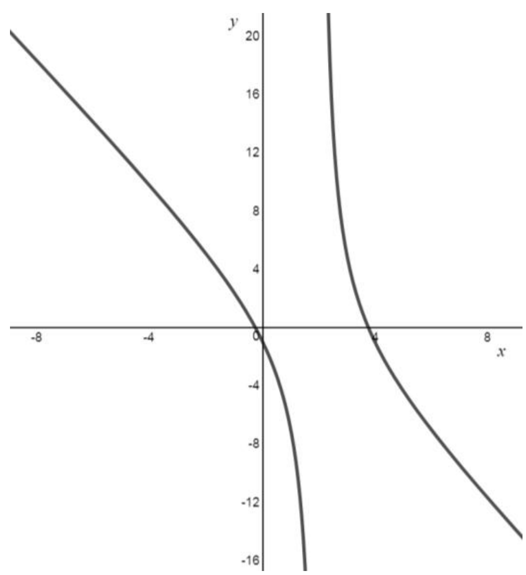


2. Find and label on the graphs provided any intercepts and asymptotes and then use the graphs to state, using interval notation, where $g(x) \geq 0$.

a) $g(x) = \dfrac{2x^2+x-15}{x-4}$

b) $g(x) = \dfrac{2x^2-7x-2}{2-x}$





Date:________________ **2.13 Solving Rational Inequalities Graphically**

1. Solve the following rational inequalities graphically. Answer using set notation.

a) $\dfrac{4-x^2}{x^2+1} \le 0$ **b)** $\dfrac{x+6}{x+3} \ge \dfrac{7}{2}$ **c)** $\dfrac{3}{x+2} < \dfrac{1}{x+4}$

2. Solve the following rational inequalities graphically. Answer using interval notation.

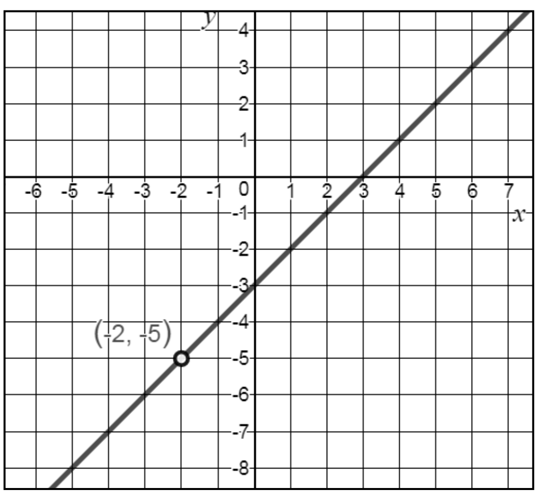
a) $\dfrac{x^2}{x-2} > 8$ **b)** $\dfrac{2x-1}{x+7} \ge -\dfrac{x+1}{x+3}$ **c)** $\dfrac{x^2-4}{x^2-1} \le \dfrac{x}{x+3}$

Date:________________ **2.14 Graphing and Analyzing Polynomial & Rational Functions With Removable and or Infinite Discontinuities**

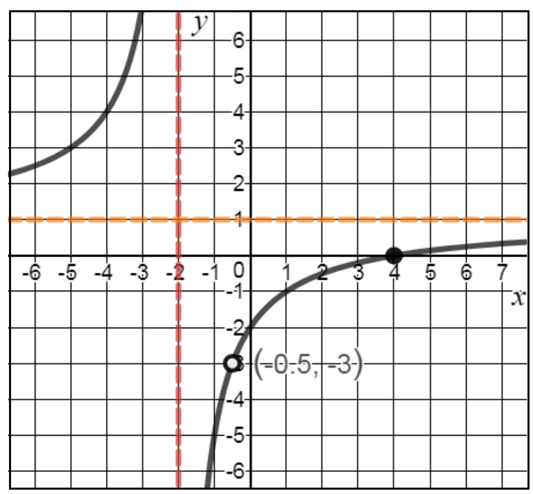
1. For each function $y = f(x)$ graphed below complete the following.

i) State all values of x for which the function is discontinuous and state the type of discontinuity.
ii) Examine how the function behaves near these discontinuities and at the ends of the graph.
iii) Determine the equation of the rational function.

a)



b)



2. Simplify and graph the following rational functions by finding and labeling any holes, intercepts and asymptotes. Include a table of values for a more accurate graph if appropriate.

a) $f(x) = \dfrac{x^3-4x^2+5x-2}{x^2-2x+1}$ **b)** $g(x) = \dfrac{x^2-4}{\frac{1}{x}-\frac{1}{2}}$ **c)** $y = \dfrac{x^3+27}{3x+9}$

d) $f(x) = \dfrac{x^2+3x+2}{x^2+x-2}$ **e)** $y = \dfrac{x+1}{x^3+5x^2+8x+4}$ **f)** $g(x) = \dfrac{2x^4-3x^3+x^2+3x-3}{x^2-x^3}$

Date:________________

UNIT 2 REVIEW OF 2.8-2.14

1. Solve each of the following ***graphically*** where $x \in R$. Answer using ***interval notation***.

a) $2(x+1)^3(1-x) < 0$ **b)** $0.5(2x+1)(5-2x)(x+3) \geq 0$

c) $x^3 + x^2 > 12x + 12$ **d)** $3x^4 + 11x^3 + 8x^2 - 4x \leq 0$

2. Solve the following polynomial inequalities using a ***number line strategy*** where $x \in R$.
State your final answer using ***algebraic notation***.

a) $(3-x)^3(x+1)(4-2x) \leq 0$ **b)** $4x^4 > 5x^2 + 9$

3. Solve each of the following rational inequalities using a ***number line strategy*** where $x \in R$.
State your final answer using a ***solution set***.

a) $\dfrac{(x-2)(x+1)^2}{(x-3)(x+6)} \geq 0$ **b)** $\dfrac{4}{x+5} < 2$ **c)** $\dfrac{x}{x-5} \leq \dfrac{x-1}{x+4}$

4. Without using graphing technology, match each equation with its corresponding graph.
Explain your reasoning.

a) $y = \dfrac{-1}{x-3}$ **b)** $g(x) = \dfrac{x^2-9}{x-3}$ **c)** $y = \dfrac{1}{(x+3)^2}$

d) $f(x) = \dfrac{x}{(x-1)(x+3)}$ **e)** $y = \dfrac{1}{x^2+5}$ **f)** $g(x) = \dfrac{x^2}{x-3}$

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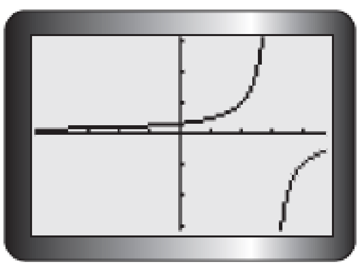

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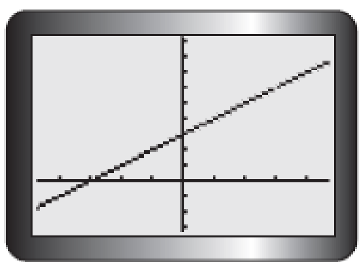

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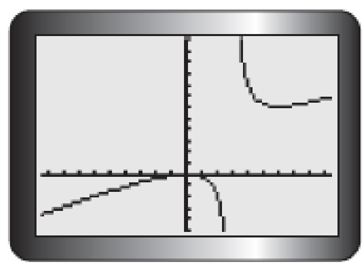

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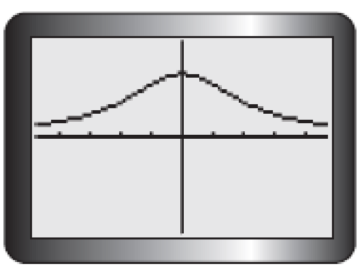

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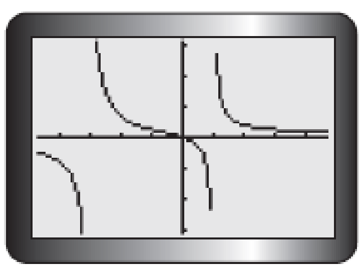

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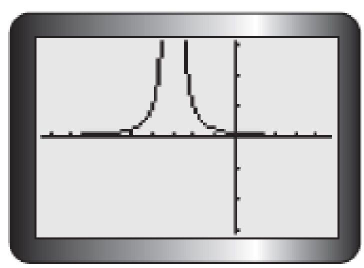


5. Graph the following rational functions by finding and labeling any holes, intercepts, asymptotes and points where the function crosses the horizontal or linear oblique asymptotes. Include a table of values for a more accurate graph if appropriate.

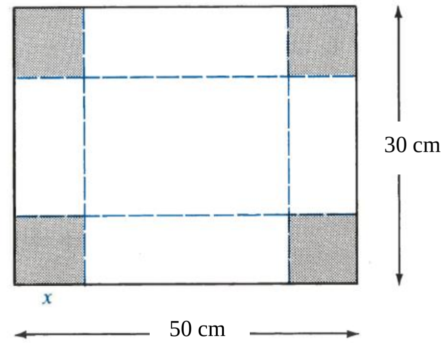
a) $f(x) = \dfrac{6}{x^2-4x-5}$ **b)** $g(x) = \dfrac{4x-2x^2}{x^2+1}$ **c)** $f(x) = \dfrac{x^4-4x^3-x^2+4x}{1-x^2}$ **d)** $y = \dfrac{x^2(x-3)}{x^2-4x+3}$

6. Solve the following rational inequalities ***graphically***. Answer using ***interval notation***.

a) $\dfrac{(2x+1)(x-4)}{1-2x} \geq 0$ **b)** $\dfrac{1}{x+3} > \dfrac{-4}{x-2}$ **c)** $\dfrac{3x-2}{x-2} \leq 4$ **d)** $1 - \dfrac{5}{x} < \dfrac{6}{x^2}$

7. Graph $f(x)=\dfrac{x^4+2x^3-15x^2+4x+20}{x^3-3x^2-25x+75}$ by finding and labeling any holes, intercepts, asymptotes and points where the function crosses the horizontal or linear oblique asymptotes. Use the graph to state, using interval notation, where $f(x)\geq 0$.

8. A rectangular sheet of aluminum measuring 50 cm by 30 cm is made into an open box by cutting squares from the corners and turning up the sides. Determine the range of side lengths that are possible for each square that is cut out and removed that result in a volume greater than 4000 cm^3.
Answer to two decimal places.
A calculator is required here.



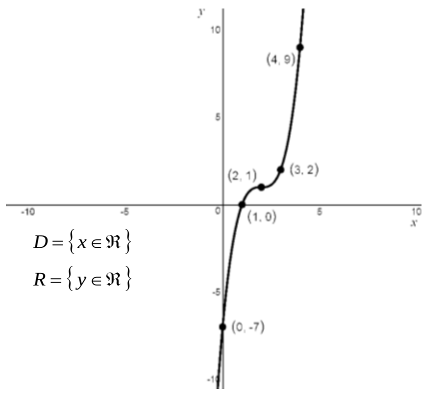
Unit 2 Answers

2.1 Transformations on Quadratic, Cubic, Square Root, Absolute Value & Reciprocal Functions

1. a) *Transformations on* $y = x^3$ *are:*

i) *horizontal translation right* 2 *units*

ii) *vertical translation up* 1 *unit*



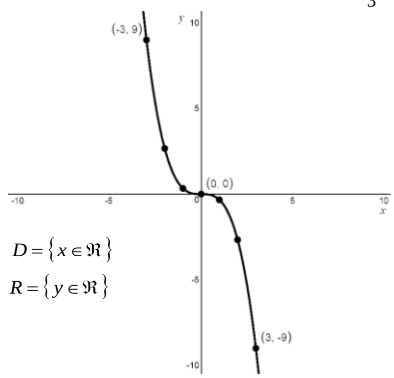
$D = \{x \in \Re\}$

$R = \{y \in \Re\}$

b) *Transformations on* $f(x) = x^3$ *are:*

i) *vertical reflection in the x-axis*

ii) *vertical compression by a factor of* $\frac{1}{3}$



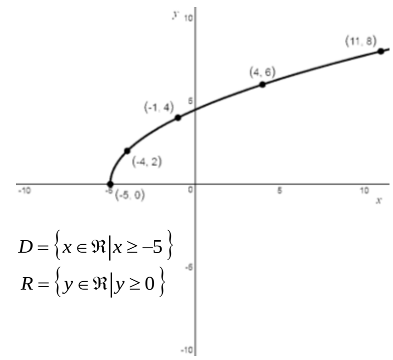
$D = \{x \in \Re\}$

$R = \{y \in \Re\}$

c) *Transformations on* $y = \sqrt{x}$ *are:*

i) *vertical expansion by a factor of* 2

ii) *horizontal translation left* 5 *units*



$D = \{x \in \Re | x \geq -5\}$

$R = \{y \in \Re | y \geq 0\}$

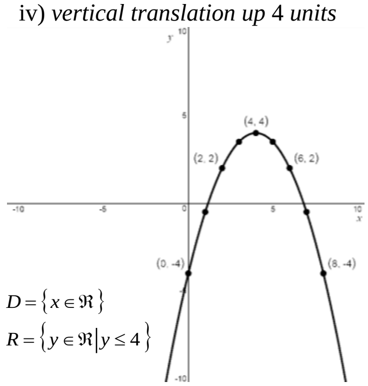
d) *Transformations on* $y = x^2$ *are:*

i) *vertical reflection in the x-axis*

ii) *vertical compression by a factor of* $\frac{1}{2}$

iii) *horizontal translation right* 4 *units*

iv) *vertical translation up* 4 *units*



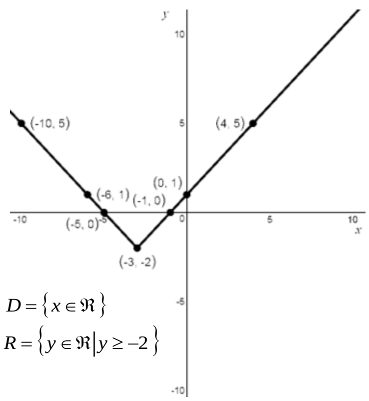
$D = \{x \in \Re\}$

$R = \{y \in \Re | y \leq 4\}$

e) *Transformations on* $f(x) = |x|$ *are:*

i) *horizontal translation left* 3 *units*

ii) *vertical translation down* 2 *units*



$D = \{x \in \Re\}$

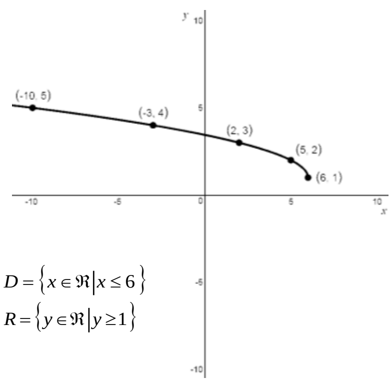
$R = \{y \in \Re | y \geq -2\}$

f) *Transformations on* $y = \sqrt{x}$ *are:*

i) *horizontal reflection in the y-axis*

ii) *horizontal translation right* 6 *units*

iii) *vertical translation up 1 unit*



$D = \{x \in \Re | x \leq 6\}$

$R = \{y \in \Re | y \geq 1\}$

g) *Transformations on* $y = \frac{1}{x}$ *are:*

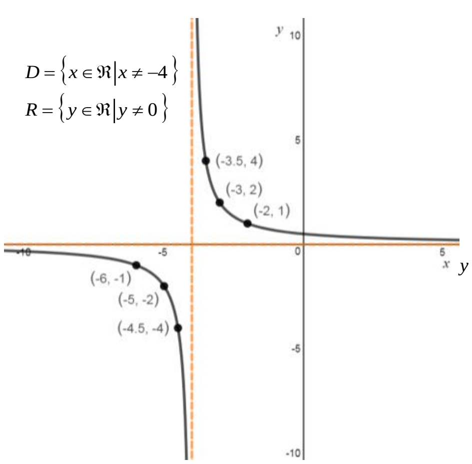
i) *vertical expansion by a factor of* 2

ii) *horizontal translation left* 4 *units*

$x = -4$

$D = \{x \in \Re | x \neq -4\}$

$R = \{y \in \Re | y \neq 0\}$



$y = 0$

h) *Transformations on* $f(x) = |x|$ *are:*

i) *vertical reflection in the x-axis*

ii) *vertical expansion by a factor of* $\frac{3}{2}$

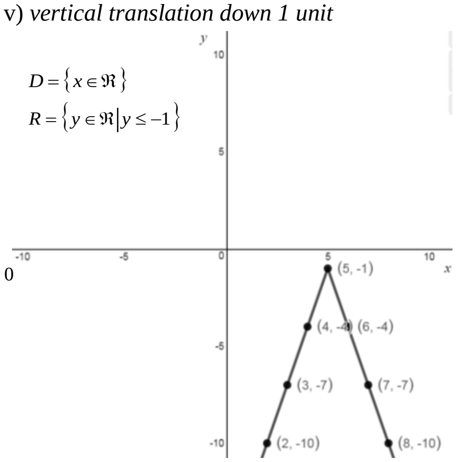
iii) *horizontal compression by a factor of* $\frac{1}{2}$

iv) *horizontal translation right* 5 *units*

v) *vertical translation down 1 unit*

$D = \{x \in \Re\}$

$R = \{y \in \Re | y \leq -1\}$



i) *Transformations on* $y = x^3$ *are:*

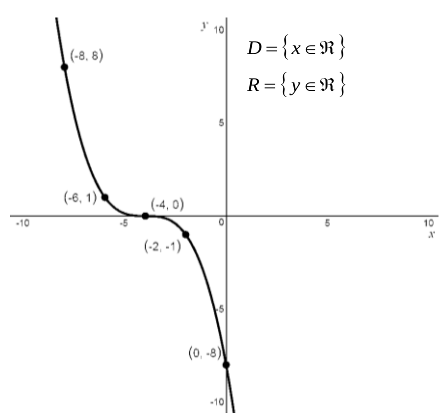
i) *horizontal reflection in the y-axis*

ii) *horizontal expansion by a factor of* 2

iii) *horizontal translation left* 4*units*

$D = \{x \in \Re\}$

$R = \{y \in \Re\}$



j) *Transformations on* $y = x^2$ *are:*

i) *horizontal reflection in the y-axis*
ii) *horizontal expansion by a factor of* $\frac{3}{2}$
iii) *horizontal translation right* 3 *units*
iv) *vertical translation up* 2 *units*

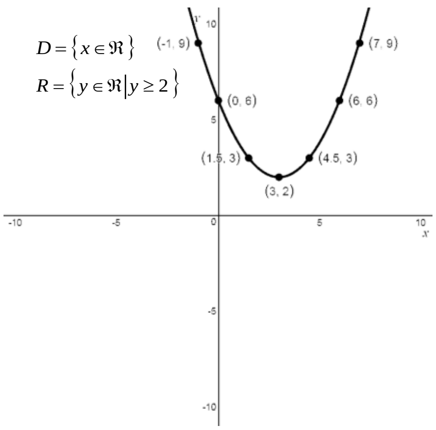
k) *Transformations on* $f(x) = \frac{1}{x}$ *are:*

i) *vertical reflection in the x-axis*
ii) *horizontal expansion by a factor of* 2
iii) *horizontal translation right* 2 *units*
iv) *vertical translation down* 3 *units*

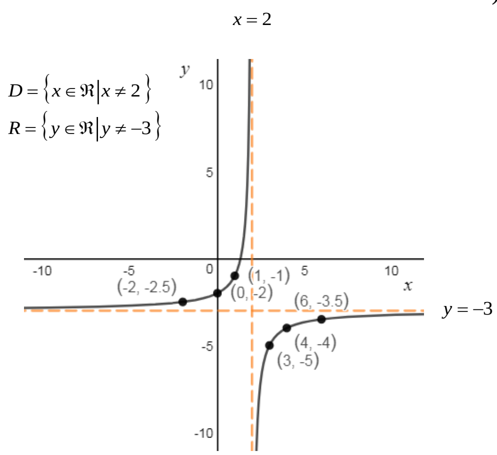
l) *Transformations on* $y = \sqrt{x}$ *are:*

i) *vertical reflection in the x-axis*
ii) *vertical expansion by a factor of* 2
iii) *horizontal reflection in the y-axis*
iv) *horizontal compression by a factor of* $\frac{1}{4}$
v) *horizontal translation right* 2 *units*
vi) *vertical translation up* 4 *units*

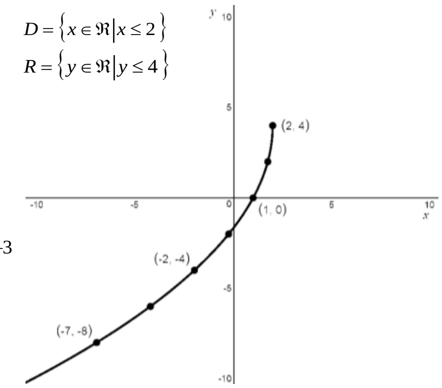
$D = \{x \in \Re\}$
$R = \{y \in \Re | y \geq 2\}$



$D = \{x \in \Re | x \neq 2\}$
$R = \{y \in \Re | y \neq -3\}$



$D = \{x \in \Re | x \leq 2\}$
$R = \{y \in \Re | y \leq 4\}$



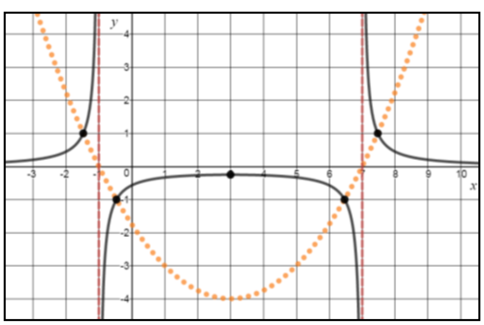
2. a) $f(x) = \left[-\frac{1}{4}(x-2)\right]^3$; $D = \{x \in \Re\}$; $R = \{y \in \Re\}$ **b)** $f(x) = -\frac{3}{x} + 4$; $D = \{x \in \Re | x \neq 0\}$; $R = \{y \in \Re | y \neq 4\}$

c) $f(x) = \frac{7}{20}\left|\frac{20}{7}(x+5)\right| - 7$; $D = \{x \in \Re\}$; $R = \{y \in \Re | y \geq -7\}$

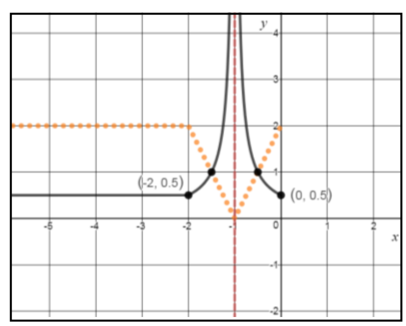
3. a) C **b)** B **c)** A **d)** F **e)** E **f)** I **g)** G **h)** D **i)** H

2.2 Graphing Reciprocal and Absolute Value Functions of $y = f(x)$

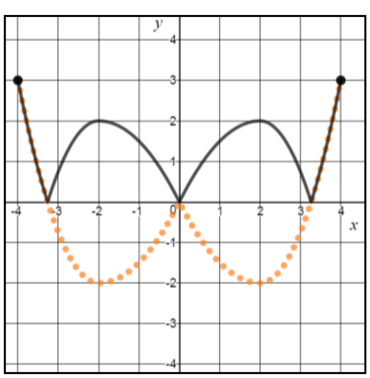
1. a)



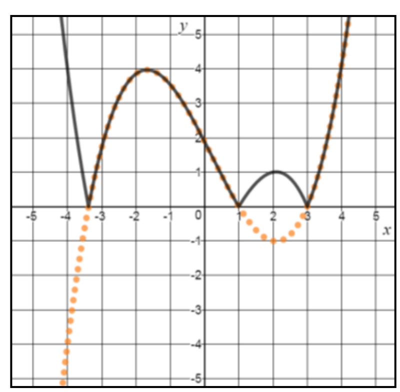
b)



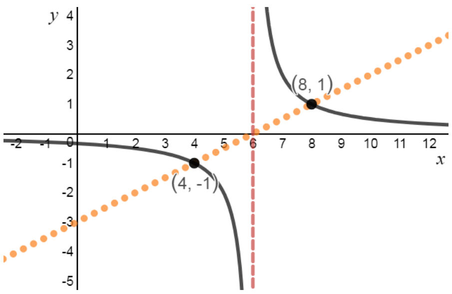
2. a)



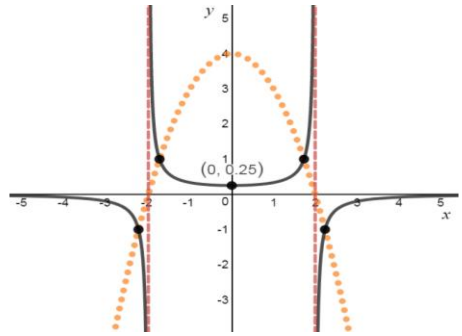
b)



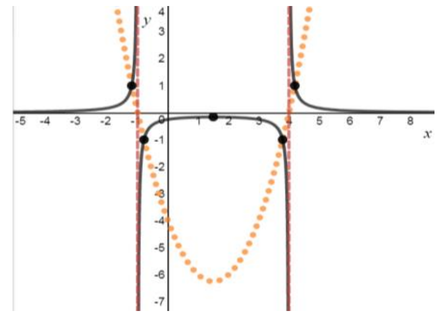
3. a)



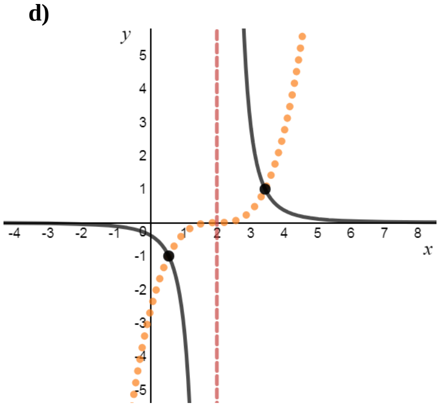
b)



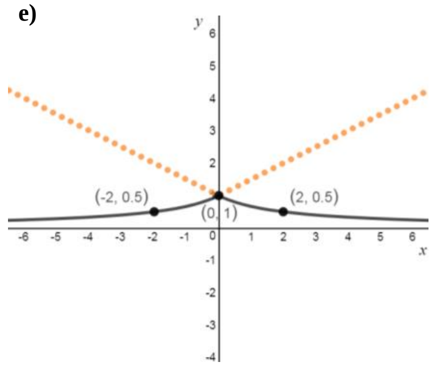
c)



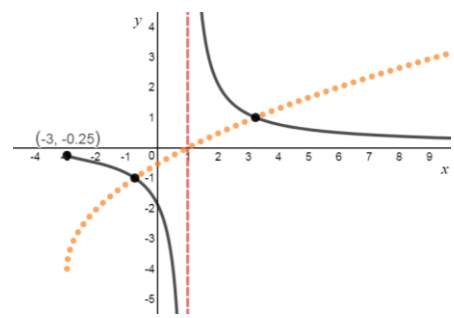
d)



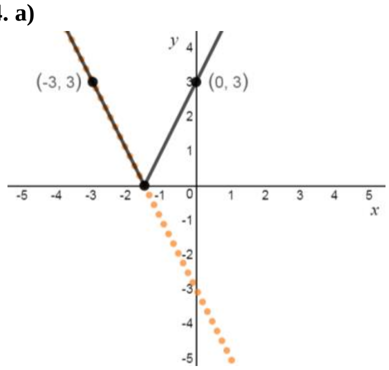
e)



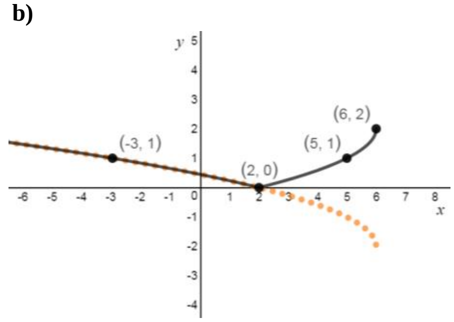
f)



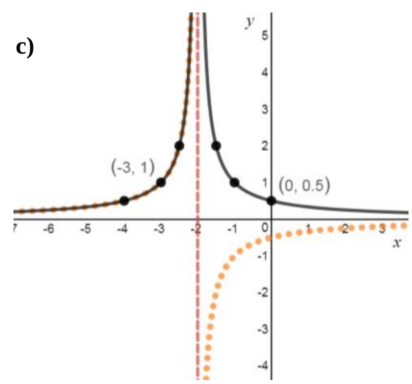
4. a)



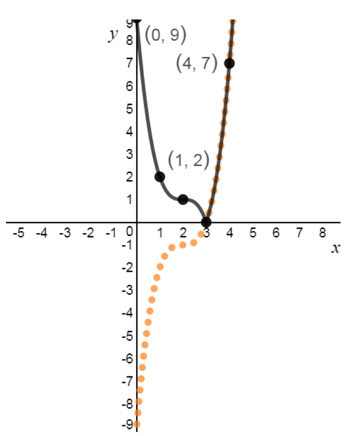
b)



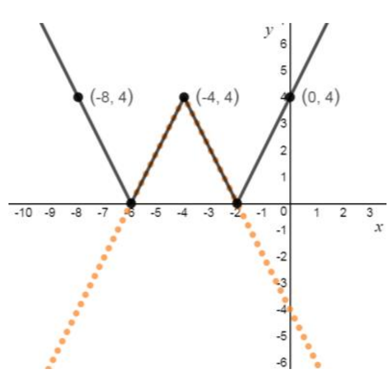
c)



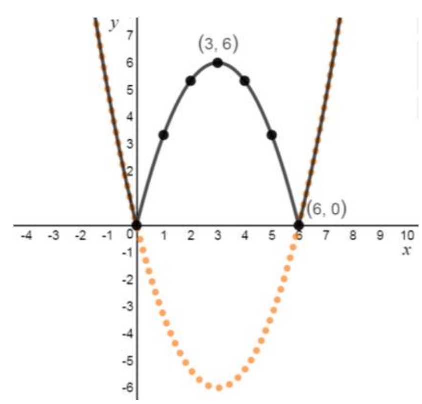
d)



d)



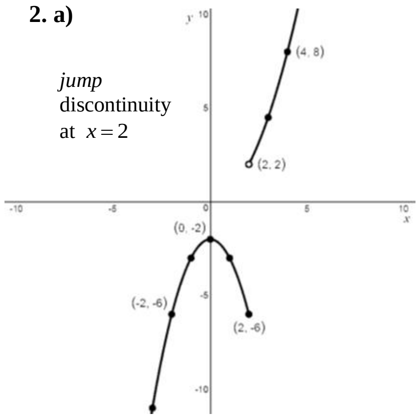
e)



2.3 Graphing Piecewise Functions

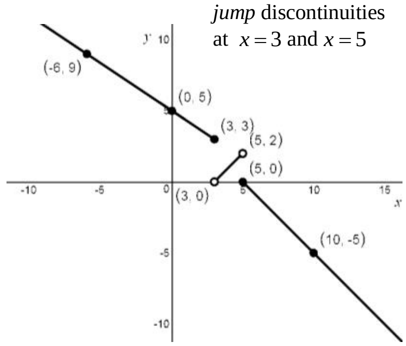
1. a) 12 **b)** *dne* **c)** 1 **d)** 8 **e)** $x = 2$, *removable*; $x = 5$, *jump* **f)** 6 **g)** 12 **h)** *as* $x \to -\infty,\ g(x) \to +\infty$ & *as* $x \to +\infty,\ g(x) \to -\infty$

2. a)



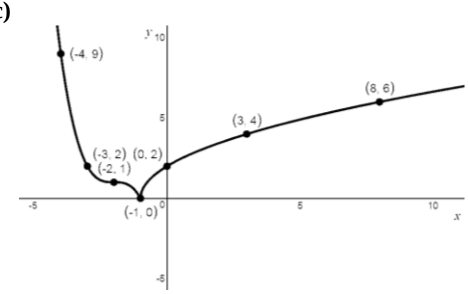
jump discontinuity at $x = 2$

b)

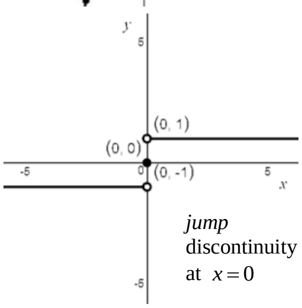


jump discontinuities at $x = 3$ and $x = 5$

c)

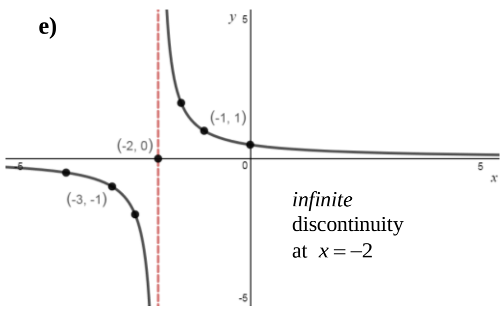


d)



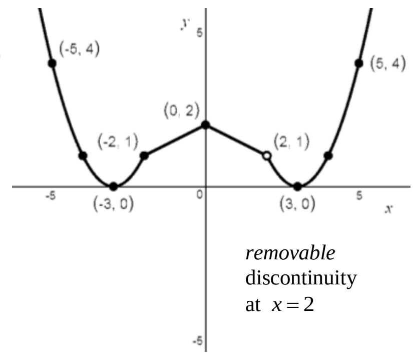
jump discontinuity at $x = 0$

e)



infinite discontinuity at $x = -2$

f)

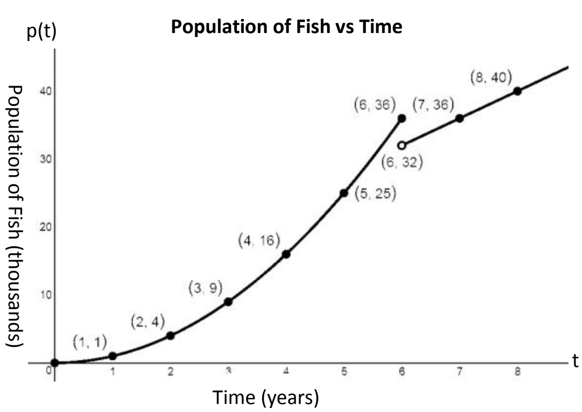


removable discontinuity at $x = 2$

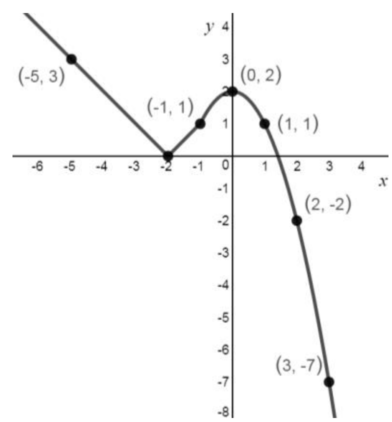
2.4 PiecewiseFunctions and Continuity Continued

1. a) $f(x) = \begin{cases} x^2 - 2, & x \le 1 \\ x + 1, & x > 1 \end{cases}$ **b)** $f(x) = \begin{cases} |x|, & x \le 1 \\ \sqrt{x}, & x > 1 \end{cases}$ *or* $f(x) = \begin{cases} |x|, & x < 1 \\ \sqrt{x}, & x \ge 1 \end{cases}$

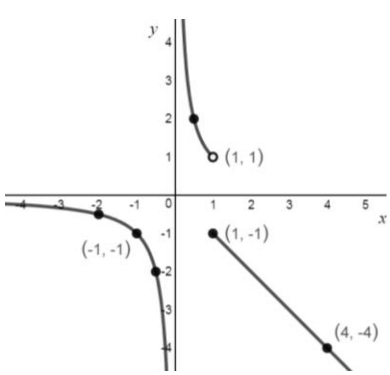
2. a) **b)** *discontinuous at* $t = 6$ **c)** 4000 **d)** $t = 7$



3. a) *continuous at* $x = -1$ & *for all* $x \in \Re$



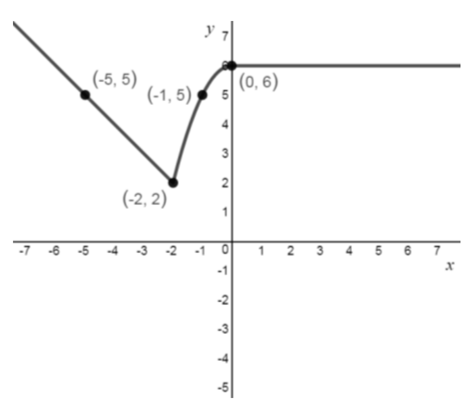
3. b) *discontinuous at x*=0 & *x*=1



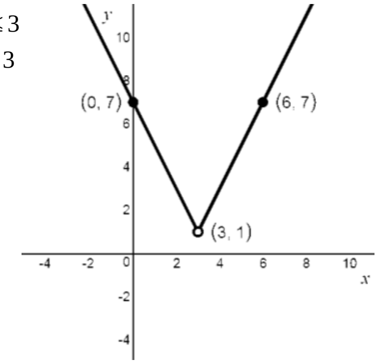
4. a) 16 **b)** 4 **c)** $0, \frac{1}{2}$ *or* $\frac{3}{2}$

5. a) $a = -1, b = 6$ **b)** $a = 1, b = -\frac{18}{5}$

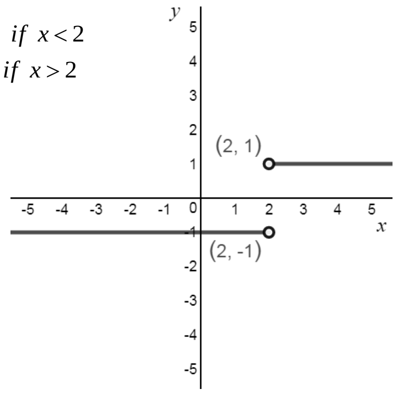
6.



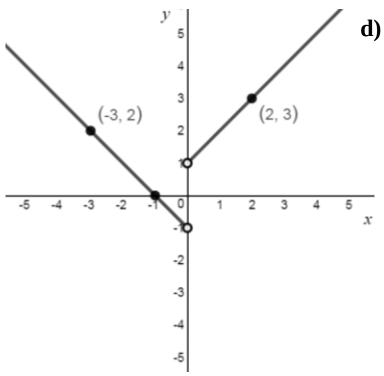
7. a) $f(x)=\begin{cases}-2x+7, & if\ x\le 3\\ 2x-5, & if\ x>3\end{cases}$



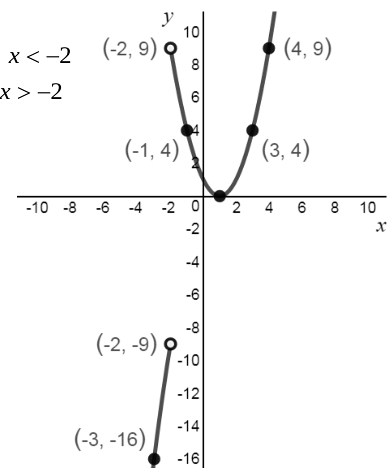
b) $f(x)=\begin{cases}-1, & if\ x<2\\ 1, & if\ x>2\end{cases}$



c) $g(x)=\begin{cases}-x-1, & if\ x<0\\ x+1, & if\ x>0\end{cases}$



d) $f(x)=\begin{cases}-(x-1)^2, & if\ x<-2\\ (x-1)^2, & if\ x>-2\end{cases}$



2.5 Graphing Factored Polynomial Functions

1. a) cubic; negative; 2; $as\ x\to-\infty,\ y\to+\infty\ \&\ as\ x\to+\infty,\ y\to-\infty$ **b)** quartic; negative; 1; $as\ x\to\pm\infty,\ y\to-\infty$
c) quartic; positive; 3; $as\ x\to\pm\infty,\ y\to+\infty$ **d)** cubic; positive; 2; $as\ x\to-\infty,\ y\to-\infty\ \&\ as\ x\to+\infty,\ y\to+\infty$

2. a) degree 2; negative; $as\ x\to\pm\infty,\ f(x)\to-\infty$ **b)** degree 5; negative; $as\ x\to-\infty,\ y\to+\infty\ \&\ as\ x\to+\infty,\ y\to-\infty$
c) degree 4; positive; $as\ x\to\pm\infty,\ f(x)\to+\infty$ **d)** degree 3; positive; $as\ x\to-\infty,\ g(x)\to-\infty\ \&\ as\ x\to+\infty,\ g(x)\to+\infty$
e) degree 3; negative; $as\ x\to-\infty,\ y\to+\infty\ \&\ as\ x\to+\infty,\ y\to-\infty$ **f)** degree 4; negative; $as\ x\to\pm\infty,\ f(x)\to-\infty$

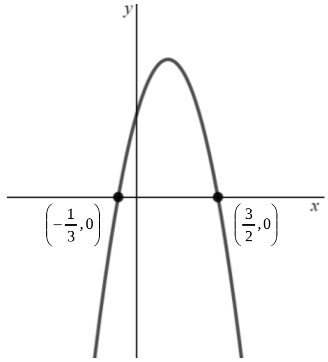
3. a) iii) The function is of degree 3 with a negative leading coefficient and *x*-intercepts of -3, a single root and 1, a double root.
b) i) The function is of degree 4 with a positive leading coefficient and *x*-intercepts of -3, -1, single roots and 2, a double root.
c) iv) The function is of degree 4 with a positive leading coefficient and *x*-intercepts of -3 and 2, both double roots.
d) ii) The function is of degree 3 with a positive leading coefficient and *x*-intercepts of -4, 1 and 4, all single roots.

2.6 Graphing Expanded Polynomial Functions

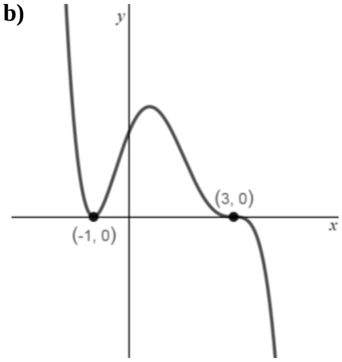
1. i) a) even degree **b)** negative leading coefficient **c)** degree 4 **d)** 4 real roots, 2 distinct & 2 equal **e)** 3 turning points
ii) a) odd degree **b)** negative leading coefficient **c)** degree 5 **d)** 5 real roots, 3 distinct & 2 equal **e)** 4 turning points
iii) a) odd degree **b)** negative leading coefficient **c)** degree 3 **d)** 3 distinct real roots **e)** 2 turning points
iv) a) even degree **b)** negative leading coefficient **c)** degree 6 **d)** 6 real roots, 3 distinct pairs of equal real roots **e)** 5 turning points
v) a) even degree **b)** positive leading coefficient **c)** degree 2 **d)** 2 distinct real roots **e)** 1 turning point
vi) a) odd degree **b)** positive leading coefficient **c)** degree 3 **d)** 3 roots, 1 real & 2 imaginary **e)** no turning points

2. a) $as\ x\to\pm\infty,\ f(x)\to-\infty$ **b)** $as\ x\to-\infty,\ f(x)\to-\infty\ \&\ as\ x\to+\infty,\ f(x)\to+\infty$
c) $as\ x\to\pm\infty,\ y\to+\infty$ **d)** $as\ x\to-\infty,\ g(x)\to+\infty\ \&\ as\ x\to+\infty,\ g(x)\to-\infty$

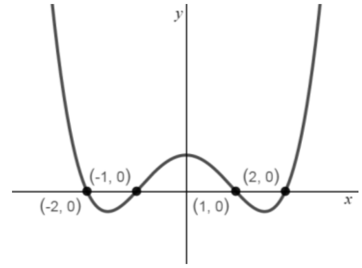
3. a)



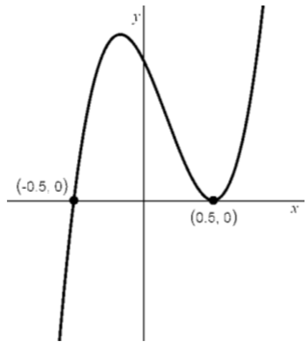
b)



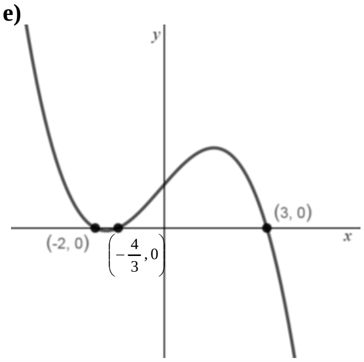
c)



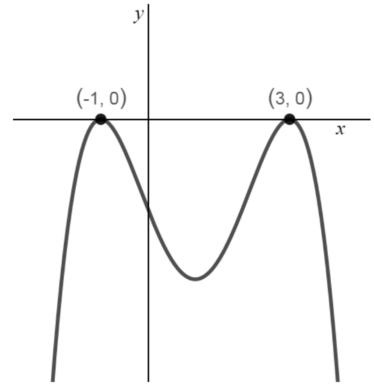
d)



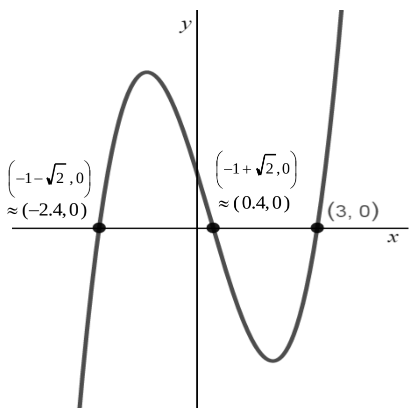
e)



f)



4.



2.7 Determining the Equations of Polynomial Functions

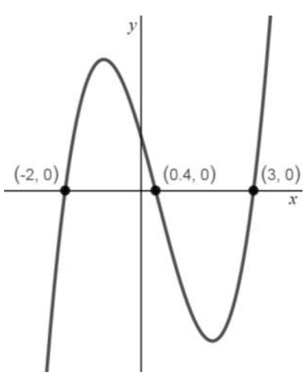
1. The functions of **a)** , **e)** and **h)** belong to the same family since their x-intercepts are $-6, -4$ and 2, all single roots.

The functions of **b)**, **d)** and **g)** belong to the same family since their x-intercepts are $-\frac{1}{3}, \frac{1}{2}$ and 5, all single roots.

The functions of **c)** and **f)** belong to the same family since their x-intercepts are -3, a double root, and 2, a single root.

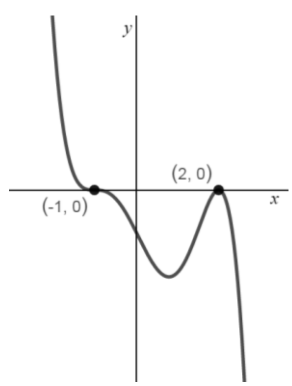
2. a) $f(x) = a(x+2)(5x-2)(x-3)$

b) $f(x) = \frac{1}{2}(x+2)(5x-2)(x-3)$

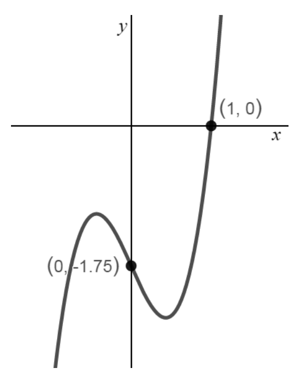


3. a) $f(x) = a(x+1)^3(x-2)^2$

b) $f(x) = -\frac{1}{4}(x+1)^3(x-2)^2$



4. $f(x) = 4x^3 - \frac{9}{4}x - \frac{7}{4}$



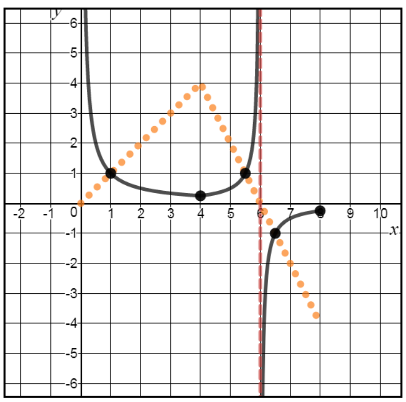
5. a) $f(x) = -(x+2)(x-1)(x-3)$ **b)** $f(x) = \frac{8}{9}(x+2)(x+1)(x-1)^2$ **c)** $f(x) = \frac{1}{5}(x+3)^3(x-1)$

6. a) $f(x) = x^3 + 2x^2 - x - 2$ **b)** $f(x) = -x^3 + 3x - 2$ **c)** $f(x) = (x^2+2)(x+3)(x-2) \rightarrow f(x) = x^4 + x^3 - 4x^2 + 2x - 12$

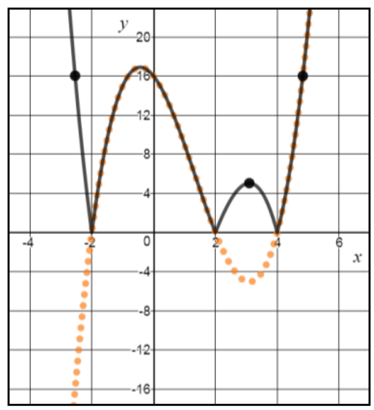
7. a) $f(x) = x^3 - 15x - 20$ **b)** $f(x) = 2x^3 + x^2 - 13x + 6$ **c)** $f(x) = x^4 - 10x^3 + 35x^2 - 52x + 24$

Review Part I of 2.1 to 2.7

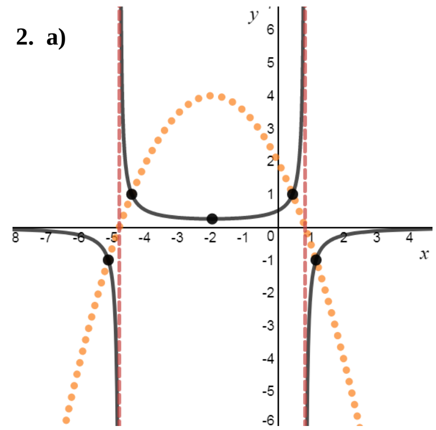
1. a)



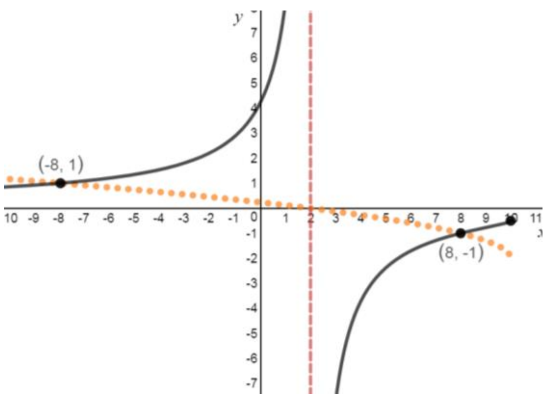
b)



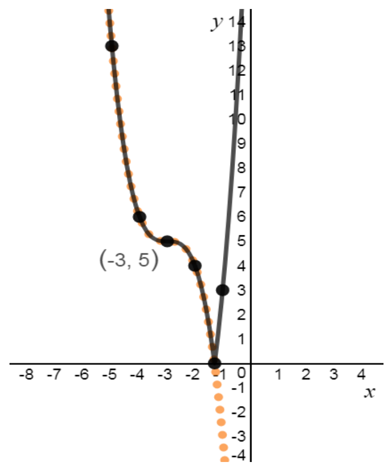
2. a)



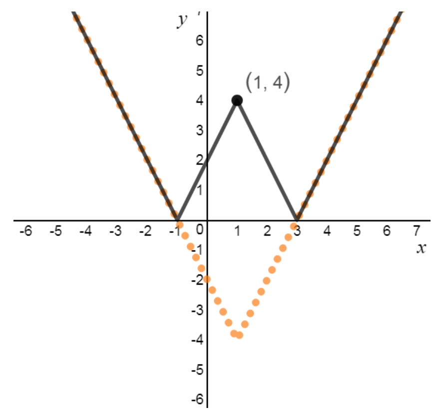
2. b)



3. a)



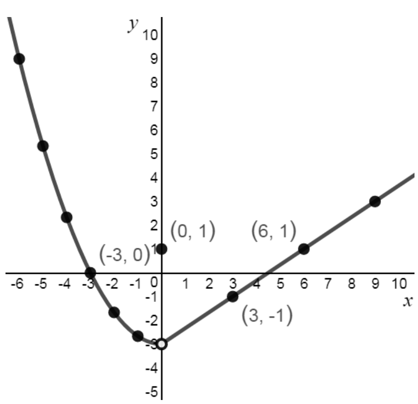
3. b)

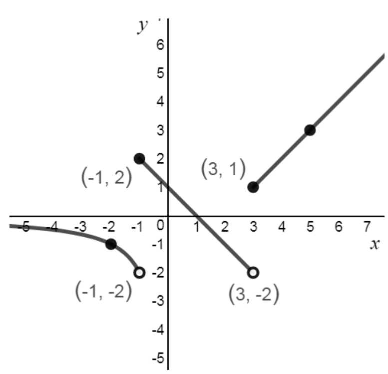


4. a) $f(2) = 2$ **b)** $f(3) = 0$ **c)** *jump* discontinuity at $x = 2$ **d)** $as\ x \rightarrow 2^-,\ f(x) \rightarrow 2$ & $as\ x \rightarrow 2^+,\ f(x) \rightarrow -1$

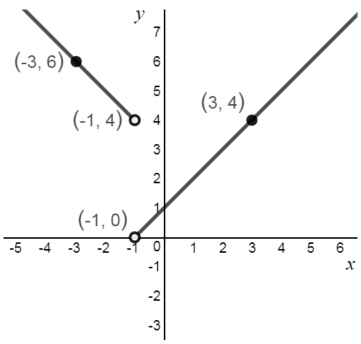
e) $as\ x \rightarrow -\infty,\ f(x) \rightarrow -\infty$ & $as\ x \rightarrow +\infty,\ f(x) \rightarrow +\infty$ **f)** $f(x) = \begin{cases} -\frac{1}{8}(x-2)^2 + 2 & if\ x \le 2 \\ x - 3 & if\ x > 2 \end{cases}$

5. a) *removable* discontinuity at $x = 0$ **5. b)** *jump* discontinuities at $x = -1$ and $x = 3$

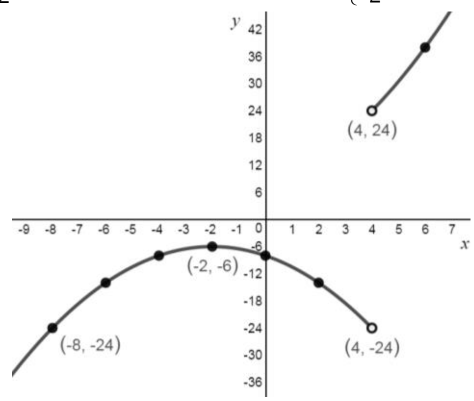




6. a) $f(x)=\begin{cases}-x+3 & if\ x<-1\\ x+1 & if\ x>-1\end{cases}$

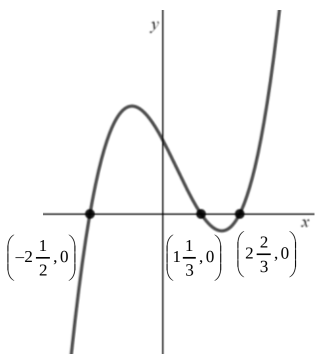


b) $g(x)=\begin{cases}-\dfrac{1}{2}x^2-2x-8 & if\ x<4\\ \dfrac{1}{2}x^2+2x+8 & if\ x>4\end{cases}$ *or* $g(x)=\begin{cases}-\dfrac{1}{2}(x+2)^2-6 & if\ x<4\\ \dfrac{1}{2}(x+2)^2+6 & if\ x>4\end{cases}$

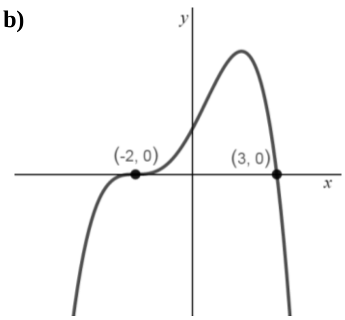


7. $k=-2$ ***Note:*** $k=2$ is an *extraneous* root **8.** $a=\dfrac{1}{2}, b=\dfrac{1}{2}$

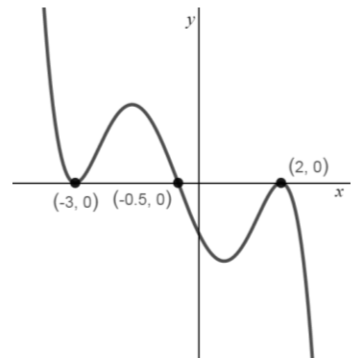
9. a)



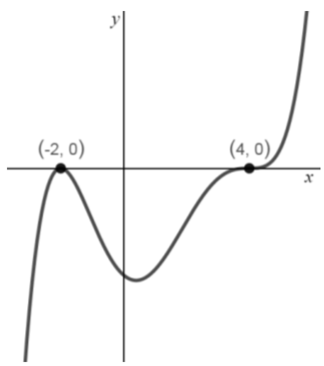
b)



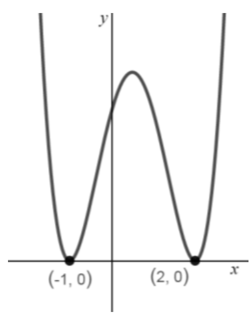
c)



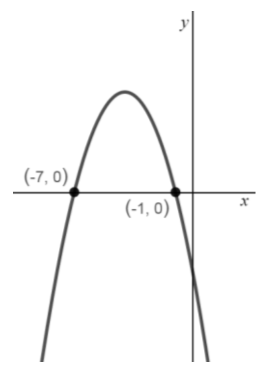
d)



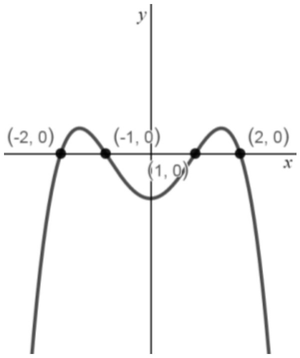
e)



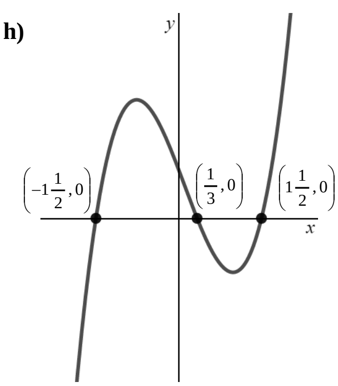
f)



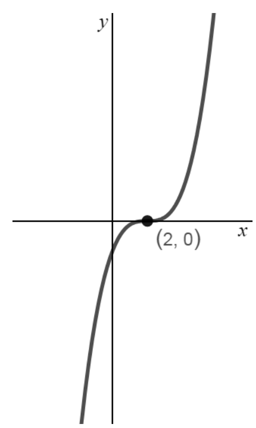
g)



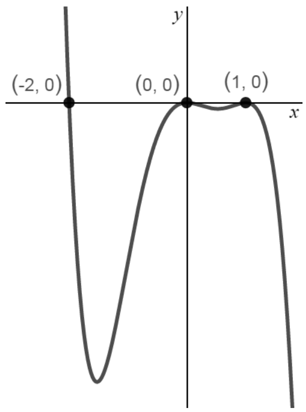
h)



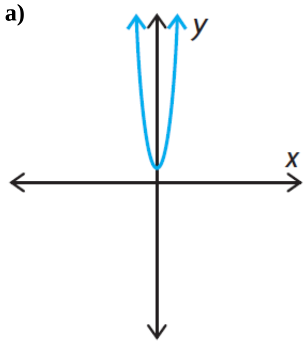
i)



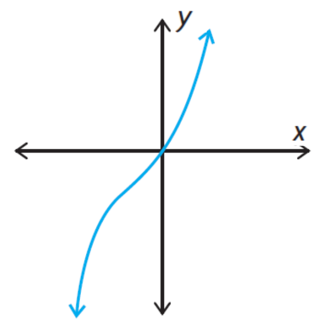
j)



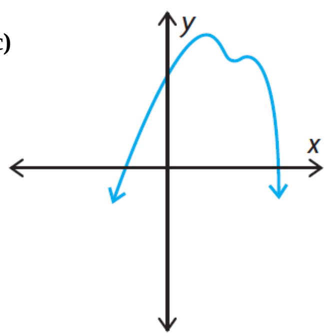
10. a)



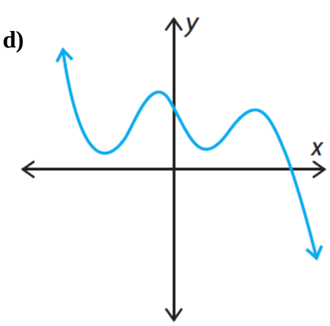
b)



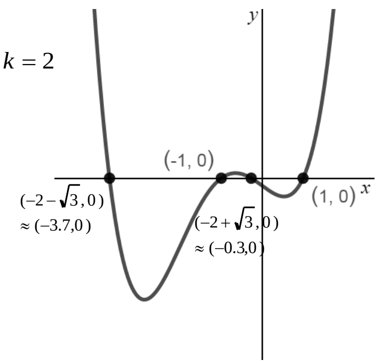
10. c)



d)



11. $k = 2$



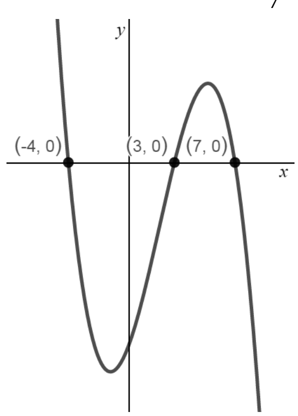
12. a) $f(x) = -\frac{2}{5}(x+4)(x+2)(x-1)$ **b)** $f(x) = (x+2)(x+1)(x-1)^2$ **c)** $f(x) = -x^3(2x+5)(x-2)$

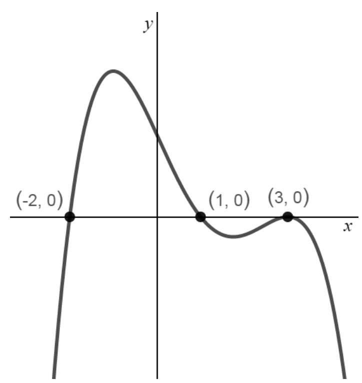
13. $f(x) = x^3 - 5x^2 + 10x - 11$ **14. a)** $V(x) = 4x^3 - 120x^2 + 864x$ **b)** $\approx 27.16 \text{ cm} \times 15.16 \text{ cm} \times 4.42 \text{ cm}$ *or* $26 \text{ cm} \times 14 \text{ cm} \times 5 \text{ cm}$

2.8 Solving Polynomial Inequalities Graphically

1. a) $f(x) > 0$ for $x < -3,\ 0 < x < 4$ **;** $f(x) < 0$ for $-3 < x < 0,\ x > 4$ **b)** $f(x) > 0$ for $-2 < x < 1,\ x > 4$ **;** $f(x) < 0$ for $x < -2,\ 1 < x < 4$
c) $f(x) > 0$ for $x < -3,\ 0 < x < 2,\ x > 2$ **;** $f(x) < 0$ for $-3 < x < 0$

2. $f(x) = -\frac{1}{7}(x+4)(x-3)(x-7)$; $f(x) = -\frac{1}{7}x^3 + \frac{6}{7}x^2 + \frac{19}{7}x - 12$ **3.** $g(x) = -3(x+2)(x-1)(x-3)^2$





4. a) $\left\{x \in \Re \,\middle|\, x < -\frac{7}{2} \textit{ or } x > \frac{5}{2}\right\}$ **b)** $\{x \in \Re \mid 0 < x < 4 \textit{ or } x > 4\}$ **c)** $\{x \in \Re \mid x \geq 1\}$ **d)** $\{x \in \Re \mid x \leq -5 \textit{ or } x = -2 \textit{ or } x \geq 5\}$

5. a) $x \in \left[-\frac{5}{3}, 5\right]$ **b)** $x \in \left(-\infty, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, +\infty\right)$ **c)** $x \in (-\infty, -2] \cup \left[-\frac{2}{3}, \frac{2}{3}\right]$ **d)** $x \in [-1] \cup [0, 2]$

e) $x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 3\right)$ **f)** $x \in \left[\frac{4-\sqrt{10}}{3}, 1\right] \cup \left[\frac{4+\sqrt{10}}{3}, +\infty\right)$ **g)** $x \in \left(-\infty, \frac{1}{2}\right)$ **h)** $x \in (-\infty, -3) \cup \left(-1, -\frac{1}{2}\right) \cup (4, +\infty)$

6. $0 \leq x < 1,\ 3 < x \leq 5$ **7.** between 1993 and 1994 or between 1996 and 1997 **8.** between 1 and 3 seconds

2.9 Solving Polynomial & Rational Inequalities Using a Number Line Strategy

1. a) $\left\{x \in \Re \mid x=-2 \text{ or } \frac{1}{3} \le x \le 2\right\}$ **b)** $\left\{x \in \Re \mid x \le -\frac{3}{2} \text{ or } x \ge 3\right\}$ **c)** $\{x \in \Re \mid x<-3 \text{ or } x>3\}$ **d)** $\{x \in \Re \mid -2\sqrt{3}<x<0 \text{ or } x>2\sqrt{3}\}$

e) $\left\{x \in \Re \mid x<-\frac{5}{2} \text{ or } \frac{1}{3}<x<\frac{5}{2}\right\}$ **f)** $\{x \in \Re \mid 0 \le x \le 2\}$ **g)** $\{x \in \Re \mid x>-2\}$ **h)** $\{x \in \Re \mid x \le -1 \text{ or } x \ge 4\}$

2. a) $x \in (-\infty,-3] \cup (1,4]$ **b)** $x \in \left(-\frac{1}{2},\frac{1}{3}\right) \cup \left(\frac{1}{2},+\infty\right)$ **c)** $x \in (-\infty,5) \cup [6,+\infty)$ **d)** $x \in (0,2) \cup (8,+\infty)$ **e)** $x \in \left[-6\frac{4}{5},-4\right) \cup (3,+\infty)$

f) $x \in (-\infty,-5) \cup (-2,0)$ **g)** $x \in \left[-1,\frac{1}{4}\right) \cup [2,9)$ **h)** $x \in (-\infty,-6) \cup \left(-1,\frac{1}{2}\right) \cup (2,+\infty)$ **i)** $x \in \left(-1,-\frac{1}{3}\right] \cup (1,2]$

3. $2<I<2.5$

2.10 Graphing Rational Functions With Horizontal Asymptotes

1. a) i) *none* **ii)** $-\frac{1}{3}$ **b) i)** $\frac{4}{3}$ **ii)** *none* **c) i)** -2 and 0 **ii)** 0 **d) i)** -1 and 2 **ii)** 2

2. a) *A horizontal asymptote occurs in a simplified rational function when the degree of the numerator is less than or equal to the degree of the denominator.*

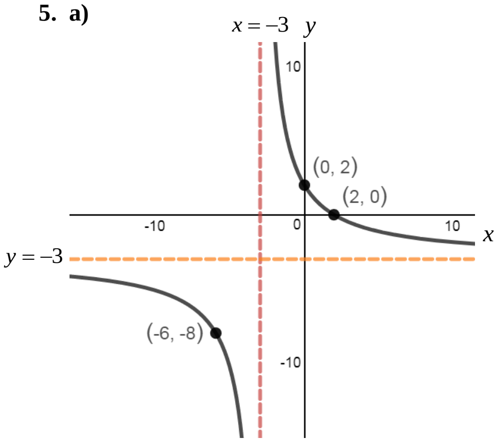
b) *Divide each term in the expanded numerator and denominator by the highest power of x in the denominator and then examine the end behavior of the function.*

3. a) i) *none* **ii)** $y=0$ **iii)** *none* **b) i)** $x=0$, $x=\frac{1}{2}$ **ii)** $y=0$ **iii)** $\left(\frac{4}{3},0\right)$ **c) i)** $x=-3$, $x=\frac{1}{2}$ **ii)** $y=2$ **iii)** $(3,2)$

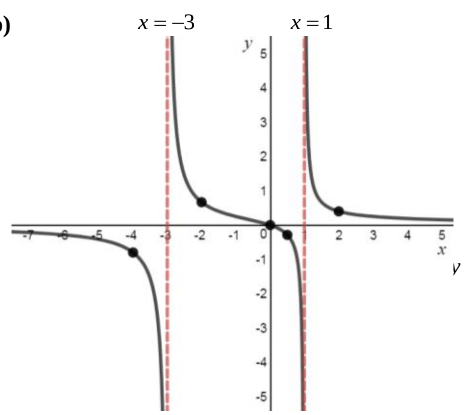
c) i) $x=1$ **ii)** $y=-1$ **iii)** $(3,-1)$

4. a) i) $g(x)$ **ii)** $p(x)$ **iii)** $g(x)$ **iv)** $p(x)$ **b) i)** $f(x)$, $p(x)$ **ii)** $g(x)$, $h(x)$ **iii)** $f(x)$, $p(x)$ **iv)** $h(x)$

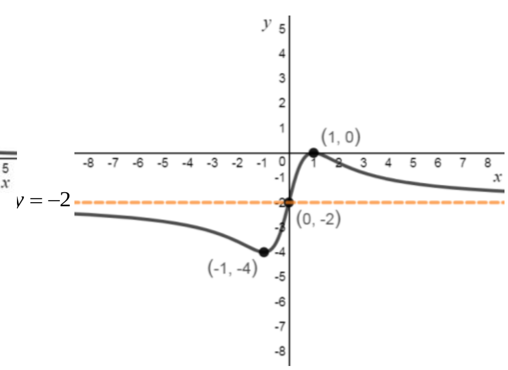
5. a)



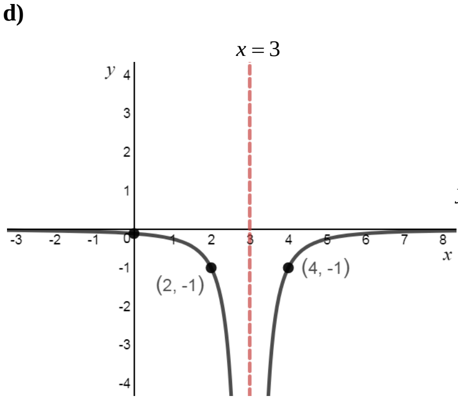
b)



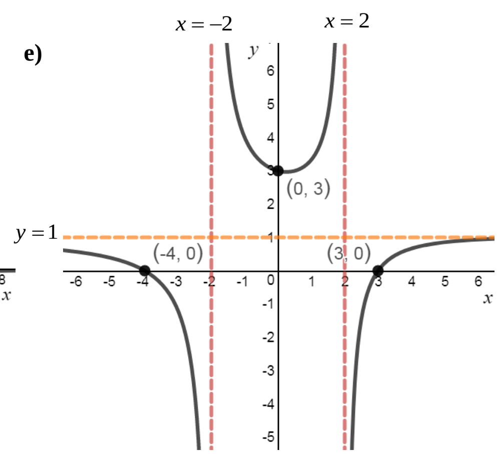
c)



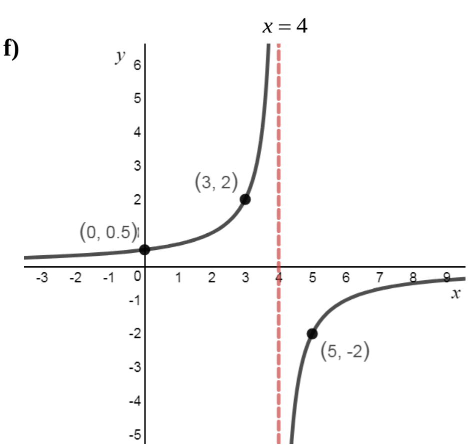
d)



e)



f)



6. a) $y=-\frac{3}{x-1}$ **b)** $y=\frac{3x^2}{(x+3)(x-5)}$ **c)** $y=\frac{4x+3}{(3x-1)(2x+5)}$ **7.** $a=-50$, $b=25$

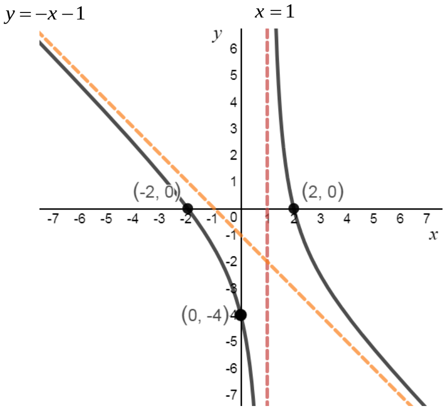
2.11 Graphing Rational Functions With Linear Oblique Asymptotes

1. a) *A linear oblique asymptote occurs in a simplified rational function when the degree of the numerator is exactly one more than the degree of the denominator.*

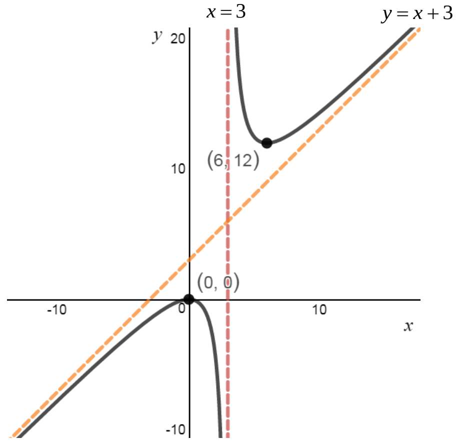
b) *Use long division to rewrite the function in mixed rational form and then examine end behaviour .*
y = quotient of division is the equation of the linear oblique asymptote.

2. a) i) $h(x)$ **ii)** $p(x)$ **iii)** $f(x)$ **iv)** $f(x)$, $g(x)$ **b) i)** $f(x)$, $p(x)$ **ii)** $f(x)$, $g(x)$ **iii)** $p(x)$ **iv)** $p(x)$

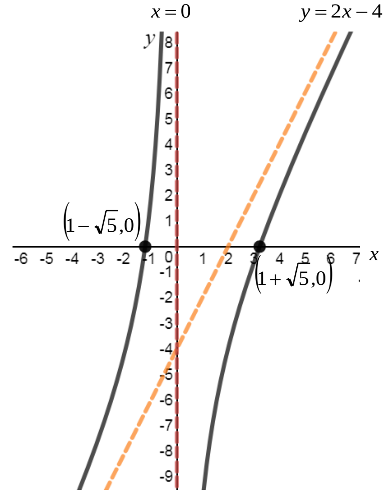
3. a)



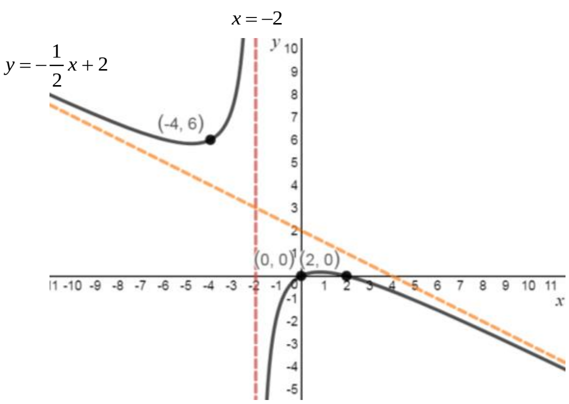
b)



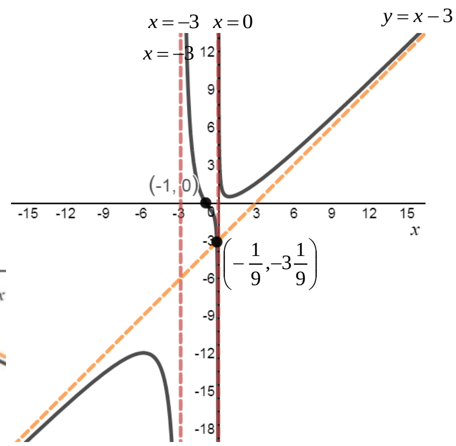
c)



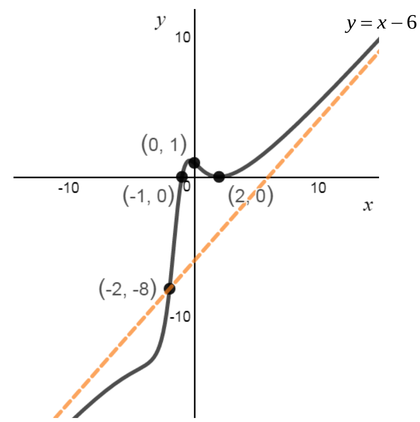
d)



e)



f)



2.12 Graphing Rational Functions Continued

1. a) x-intercept is $\frac{3}{4}$, y-intercept is -3, no vertical asymptote, horizontal asymptote is $y=0$ and $f(x)<0$ for $x\in\left(-\infty,\frac{3}{4}\right)$

b) x-intercepts are -1 and 2, y-intercept is $-\frac{1}{6}$, vertical asymptotes are $x=-4$ and $x=3$, horizontal asymptote is $y=-1$ and $f(x)<0$ for $x\in(-\infty,-4)\cup(-1,2)\cup(3,\infty)$

2. a) x-intercepts are -3 and $2\frac{1}{2}$, y-intercept is $3\frac{3}{4}$, vertical asymptote is $x=4$, linear oblique asymptote is $y=2x+9$ and $g(x)\ge 0$ for $x\in\left[-3,\frac{5}{2}\right]\cup(4,+\infty)$

b) x-intercepts are $\frac{7-\sqrt{65}}{4}$ and $\frac{7+\sqrt{65}}{4}$, y-intercept is -1, vertical asymptote is $x=2$, linear oblique asymptote is $y=-2x+3$ and $g(x)\ge 0$ for $x\in\left(-\infty,\frac{7-\sqrt{65}}{4}\right]\cup\left(2,\frac{7+\sqrt{65}}{4}\right]$

2.13 Solving Rational Inequalities Graphically

1. a) $\{x\in\Re \mid x\le -2 \text{ or } x\ge 2\}$ **b)** $\left\{x\in\Re \mid -3<x\le -\frac{9}{5}\right\}$ **c)** $\{x\in\Re \mid x<-5 \text{ or } -4<x<-2\}$

2. a) $x\in(2,4)\cup(4,+\infty)$ **b)** $x\in(-\infty,-7)\cup[-4,-3)\cup\left[-\frac{1}{3},+\infty\right)$ **c)** $x\in(-\infty,-3)\cup\left[\frac{1-\sqrt{17}}{2},-1\right)\cup\left(1,\frac{1+\sqrt{17}}{2}\right]$

2.14 Graphing and Analyzing Polynomial & Rational Functions With Removable and or Infinite Discontinuities

1. a) i) *removable* discontinuity at $x=-2$

ii) $as\ x\to -2^-,\ f(x)\to -5\ \&\ as\ x\to -2^+,\ f(x)\to -5$

$as\ x\to -\infty,\ f(x)\to -\infty\ \&\ as\ x\to +\infty,\ f(x)\to +\infty$

iii) $f(x)=\frac{x^2-x-6}{x+2}$

b) i) *removable* discontinuity at $x=-\frac{1}{2}$ and *infinite* at $x=-2$

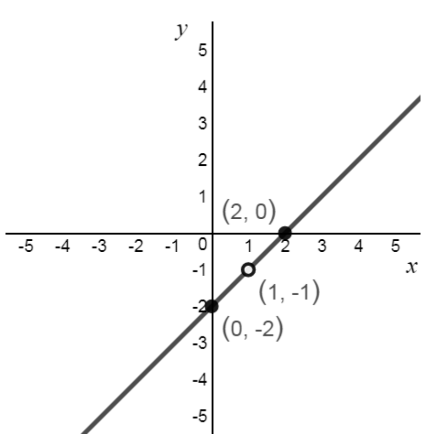
ii) $as\ x\to -\frac{1}{2}^-,\ f(x)\to -3\ \&\ as\ x\to -\frac{1}{2}^+,\ f(x)\to -3$

$as\ x\to -2^-,\ f(x)\to +\infty\ \&\ as\ x\to -2^+,\ f(x)\to -\infty$

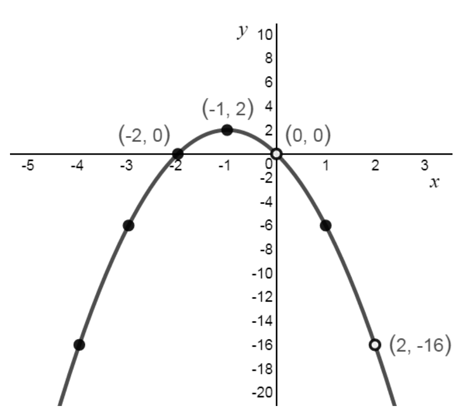
$as\ x\to -\infty,\ f(x)\to 1\ \&\ as\ x\to +\infty,\ f(x)\to 1$

iii) $f(x)=\frac{2x^2-7x-4}{2x^2+5x+2}$

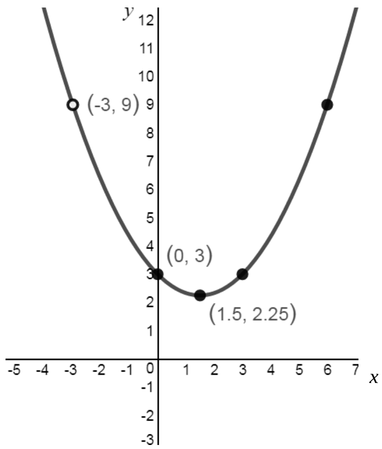
2. a) $f(x)=x-2,\ x\ne 1$



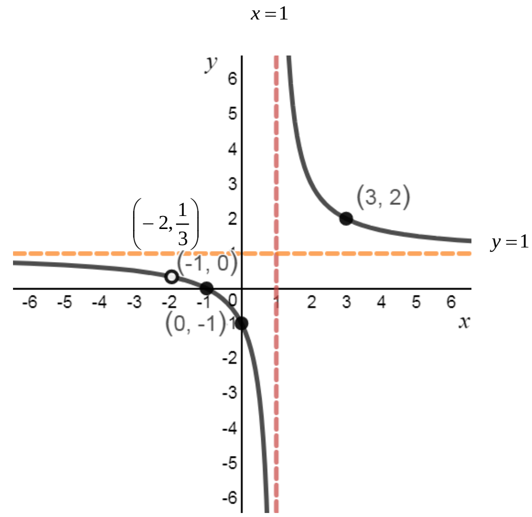
b) $f(x)=-2x^2-4x,\ x\ne 0,2$



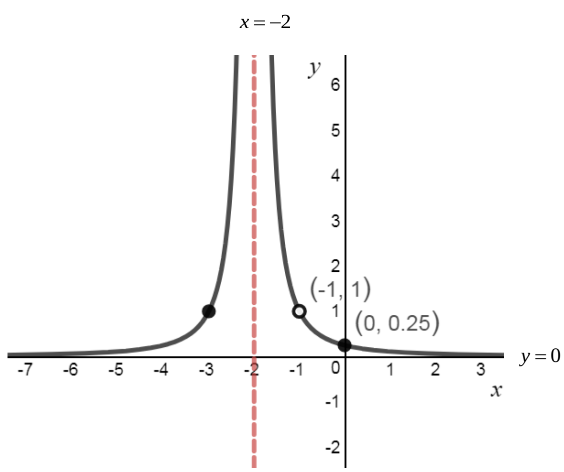
2. c) $f(x)=\dfrac{1}{3}x^2-x+3,\ x\neq -3$



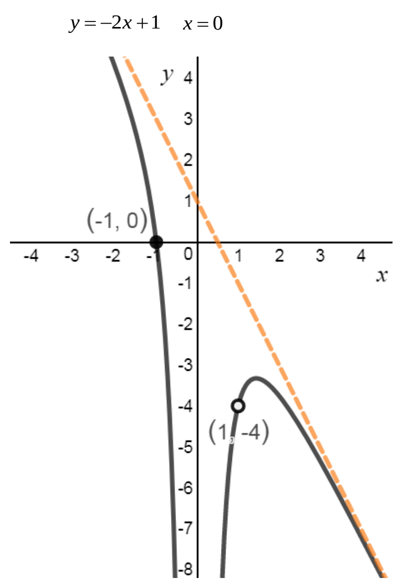
d) $f(x)=\dfrac{x+1}{x-1},\ x\neq -2,1$



2. e) $f(x)=\dfrac{1}{(x+2)^2},\ x\neq -2,-1$



f) $f(x)=\dfrac{-2x^3+x^2-3}{x^2},\ x\neq 0,1$



Review Part II of 2.8 to 2.14

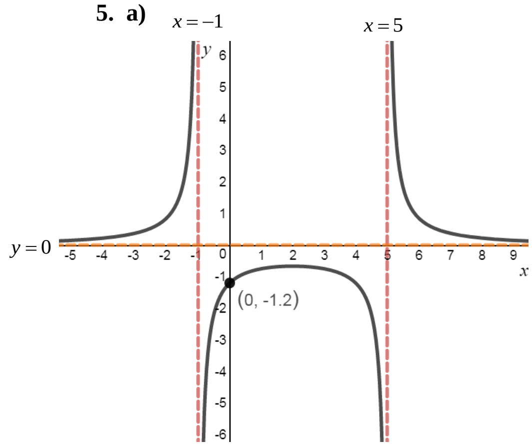
1. a) $x\in(-\infty,-1)\cup(1,+\infty)$ **b)** $x\in(-\infty,-3]\cup\left[-\frac{1}{2},\frac{5}{2}\right]$ **c)** $x\in\left(-2\sqrt{3},-1\right)\cup\left(2\sqrt{3},+\infty\right)$ **d)** $x\in[-2]\cup\left[0,\frac{1}{3}\right]$

2. a) $x\le -1 \; or \; 2\le x\le 3$ **b)** $x<-\frac{3}{2} \; or \; x>\frac{3}{2}$

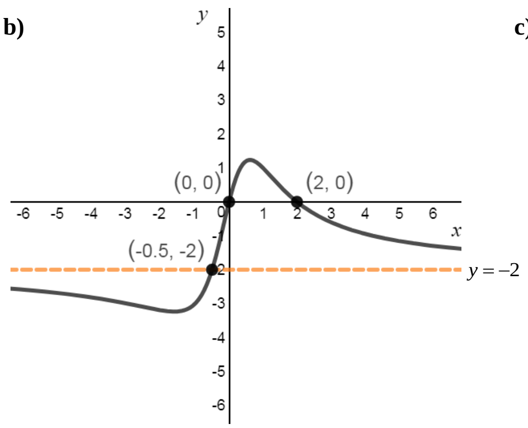
3. a) $\{x\in\Re \mid -6<x\le 2 \; or \; x>3\}$ **b)** $\{x\in\Re \mid x<-5 \; or \; x>-3\}$ **c)** $\left\{x\in\Re \mid x<-4 \; or \; \frac{1}{2}\le x<5\right\}$

4. a) A b) C c) F d) D e) B f) E

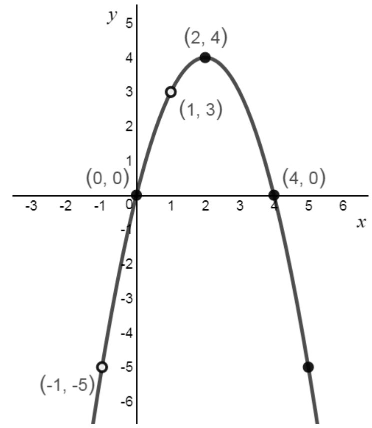
5. a)



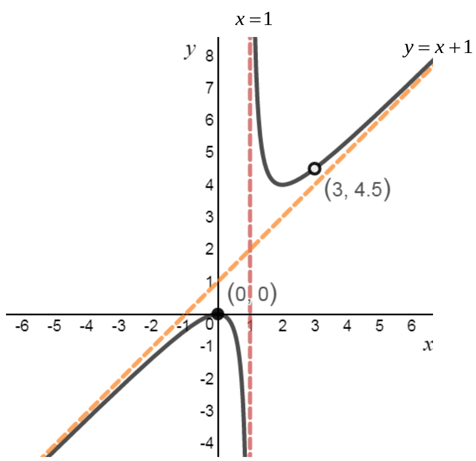
b)



c)

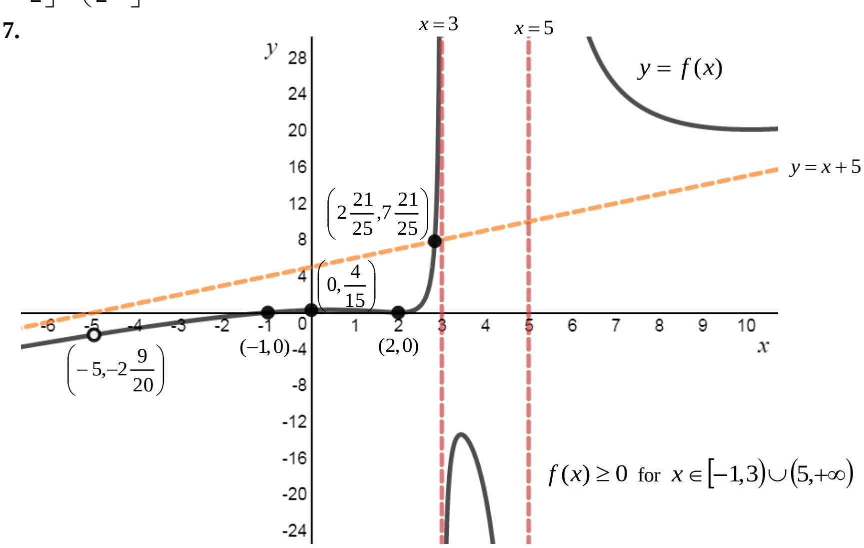


d)



6. a) $x\in\left(-\infty,-\frac{1}{2}\right]\cup\left(\frac{1}{2},4\right]$ **b)** $x\in(-3,-2)\cup(2,+\infty)$ **c)** $x\in(-\infty,2)\cup[6,+\infty)$ **d)** $x\in(-1,0)\cup(0,6)$

7.



8. between 5 cm and 7.19 cm