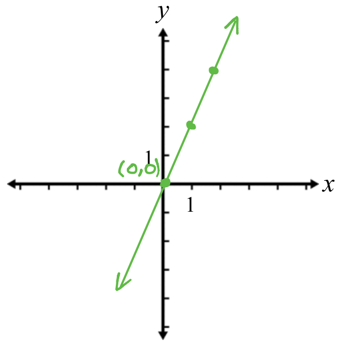
Date:_______________ **2.5 Graphing Factored Polynomial Functions**

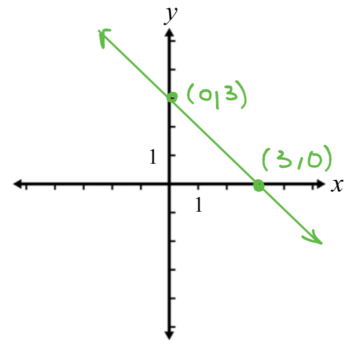
Linear Functions: are *first degree* functions of the form $y = ax + b$ or $f(x) = ax + b$. $(y=mx+b)$

i)



$y = 2x$

ii)



$f(x) = -x + 3$

observations: **i)** x-int: 0

y-int: 0

slope: 2

$\left(\frac{2}{1}\begin{matrix}\leftarrow \text{rise}\\ \leftarrow \text{run}\end{matrix}\right)$

ii) x-int: 3

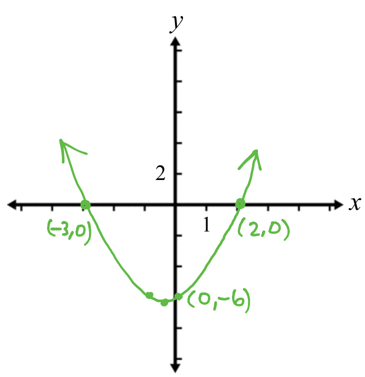
y-int: 3

slope: -1

$\left(\frac{-1}{1}\right)$

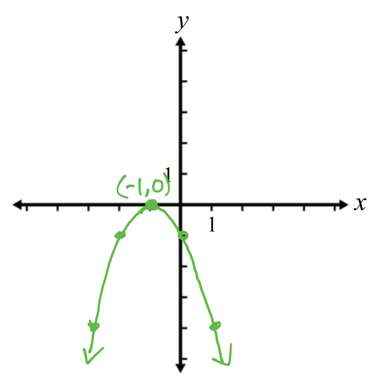
Quadratic Functions: are *second degree* functions of the form $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$.

i)



$y = (x-2)(x+3)$

ii)



$f(x) = -(x+1)^2$

$f(x) = -(x+1)(x+1)$

observations: **i)** x-int(s): $2, -3$

y-int: -6

vertex: $h = \frac{2+(-3)}{2}$

$h = -\frac{1}{2}$

At $x = -\frac{1}{2}$:

$y = (-\frac{1}{2}-2)(-\frac{1}{2}+3)$

$y = -6\frac{1}{4}$

ii) x-int(s): -1 (double)

y-int: -1

vertex: $(-1, 0)$

Cubic Functions: are *third degree* functions of the form $y = ax^3 + bx^2 + cx + d$
or $f(x) = ax^3 + bx^2 + cx + d$.

Ex. 1. Graph the following cubic functions accurately. Find all x-intercepts (zeros) and identify them as single, double or triple roots.

a) $f(x) = x^3$

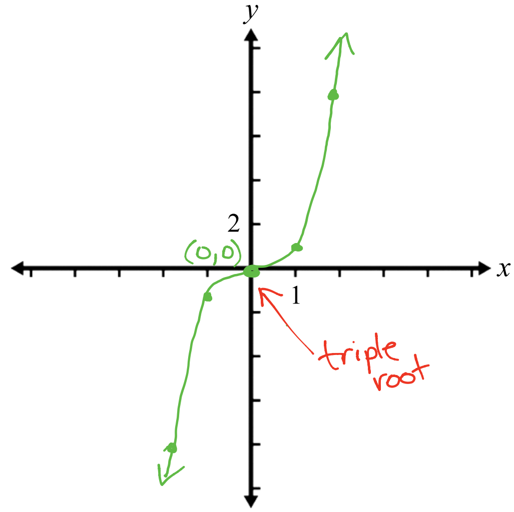
For x-int: $0 = x^3$

$0 = x \cdot x \cdot x$

$x = 0$ or $x = 0$ or $x = 0$

$\therefore$ x-int is 0 (triple root)

x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

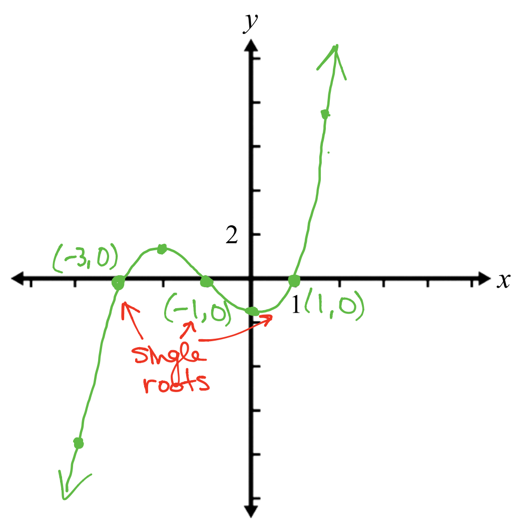


b) $y = \frac{1}{2}(x+3)(x+1)(x-1)$

For x-int: $y = 0$

$\therefore$ x-ints are $\underbrace{-3, -1, 1}_{\text{(single roots)}}$

x	y
-4	-7.5
-3	0
-2	1.5
-1	0
0	-1.5
1	0
2	7.5



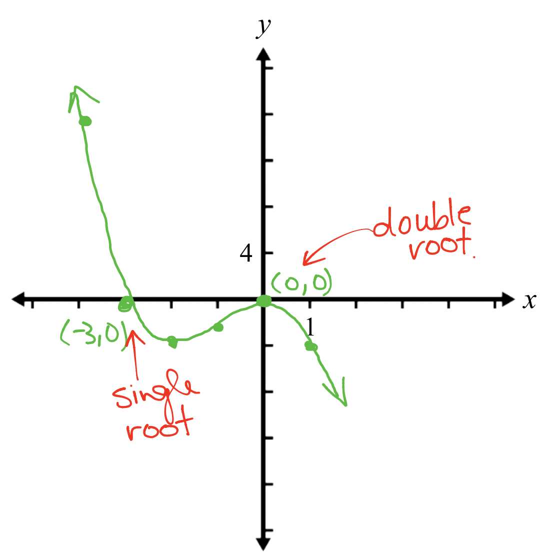
c) $f(x) = -x^2(x+3)$

For x-ints: $f(x) = 0$

$\therefore$ x-ints are 0, -3

double root single root.

x	$f(x)$
-4	16
-3	0
-2	-4
-1	-2
0	0
1	-4

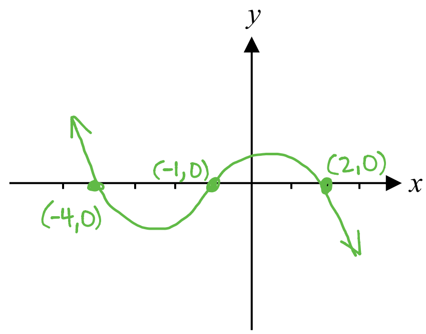


Ex. 2. Sketch the following cubic functions by finding all x-intercepts (zeros) and identifying them as single, double or triple roots.

a) $f(x) = -(x-2)(x+1)(x+4)$

$\therefore$ x-ints are 2, −1, −4 (all single)

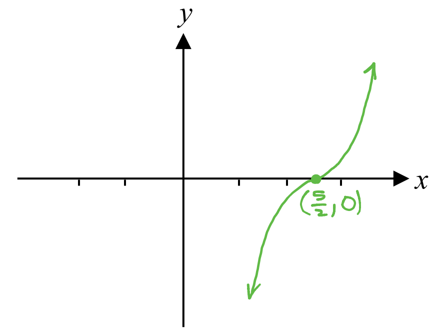
$f(-5) \Rightarrow -(-)(-)(-)$
$\Rightarrow +$



b) $y = (2x-5)^3$

x-ints are $\frac{5}{2}$ (triple)

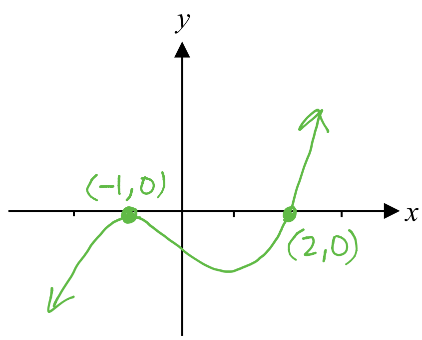
$f(2) \Rightarrow (-)^3$
$\Rightarrow -$



c) $f(x) = (x-2)(x+1)^2$

x-ints are 2, −1
single ↗ ↖ double

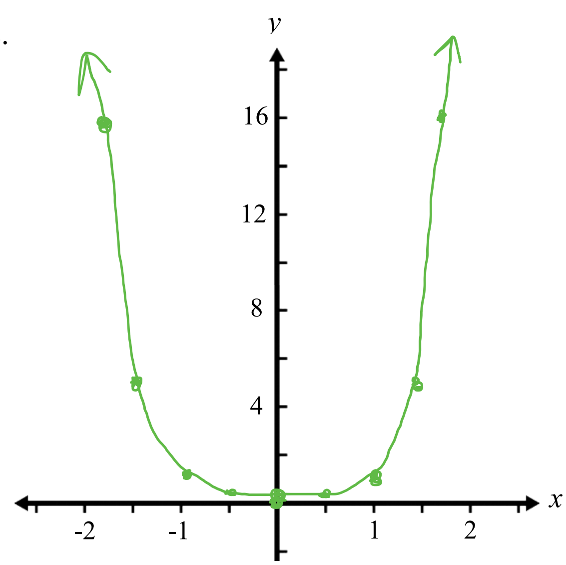
$f(-2) \Rightarrow (-)(-)^2$
$\Rightarrow -$



Quartic Functions: are *fourth degree* functions of the form $y = ax^4 + bx^3 + cx^2 + dx + e$.

Ex. 3. Graph $f(x) = x^4$ accurately by using a table of values.

x	y
-2	16
-3/2	$5\frac{1}{16}$
-1	1
-1/2	$\frac{1}{16}$
x-int → 0	0
1/2	$\frac{1}{16}$
1	1
3/2	$5\frac{1}{16}$
2	16

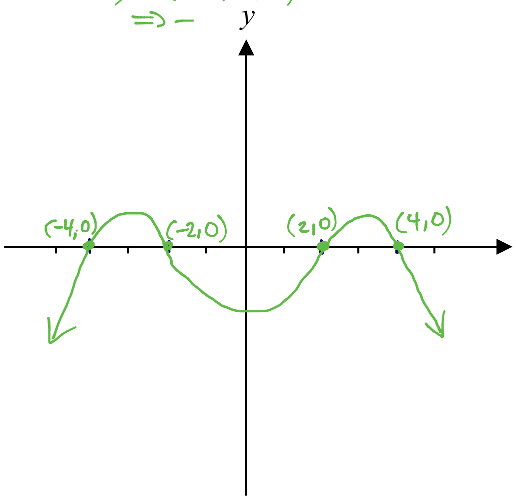


Ex. 4. Sketch the following quartic functions.

a) $f(x) = (2-x)(x+2)(x-4)(x+4)$

x-ints are $\underbrace{2, -2, 4, -4}_{\text{all single}}$

$f(-5) \Rightarrow (+)(-)(-)(-)$
$\Rightarrow -$

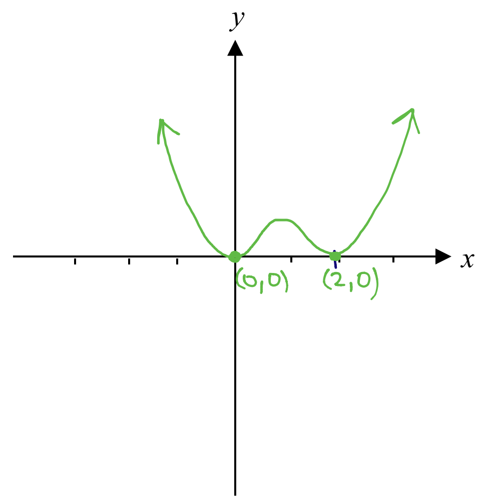


b) $y = \frac{1}{2}x^2(x-2)^2$

x-ints are 0, 2

double double

Test: $x=-1$
$y \rightarrow (-)^2(-)^2$
$\Rightarrow +$

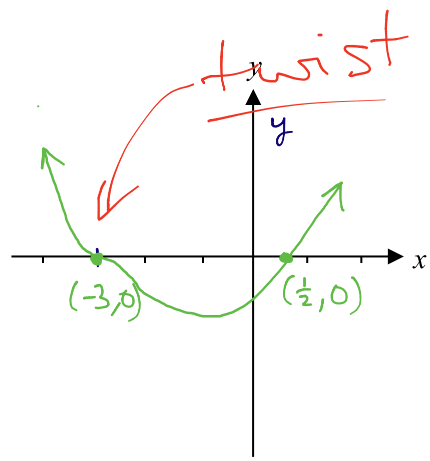


c) $f(x) = (2x-1)(x+3)^3$

x-ints are $\frac{1}{2}$, -3

single triple

Test: $f(-4) \Rightarrow (-)(-)^3$
$\Rightarrow +$

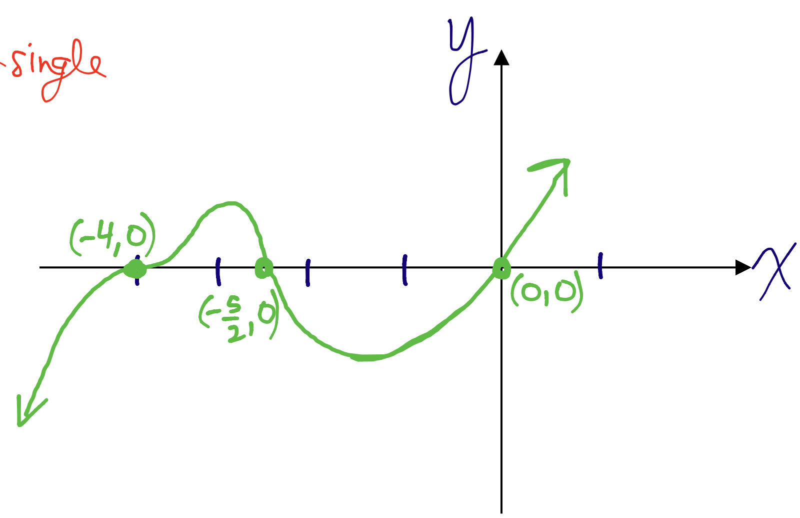


Ex. 5. Sketch the quintic function $y = x(x+4)^3(2x+5)$.

x-ints are 0, -4, $\frac{-5}{2}$

single triple single

Test: $x=-5$
$y \Rightarrow (-)(-)^3(-)$
$\Rightarrow -$



HW. **1.** Complete the **"Summary of Graphs of Polynomial Functions"** lesson in your bound notes by following **Ex. 2, 3, 4 & 5** from the note done in class.

2. Complete **Exercise 2.5**

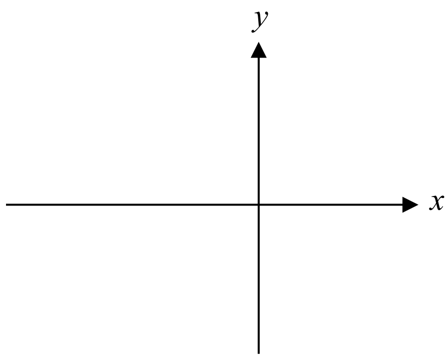
← H.W.

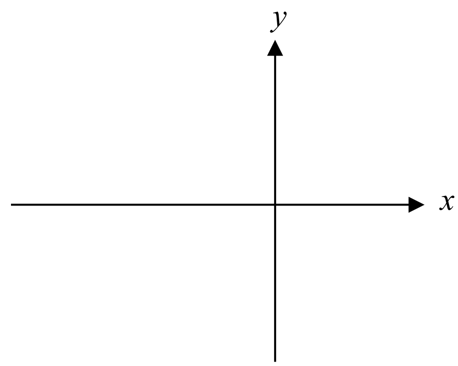
Date:_______________ **2.5 Summary of Graphs of Polynomial Functions**

I Odd Degree Functions (*Linear, Cubic, Quintic*)

1. $y = (x+2)(x-1)(x+3)$

2. $y = (x-1)(x+2)^2$





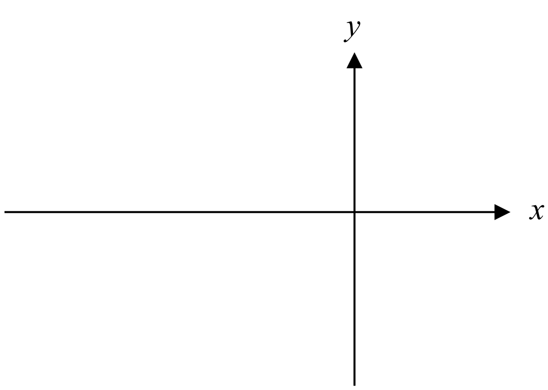
*** **As** $x \to -\infty,\ y \to$ ______ **and as** $x \to +\infty,\ y \to$ ______

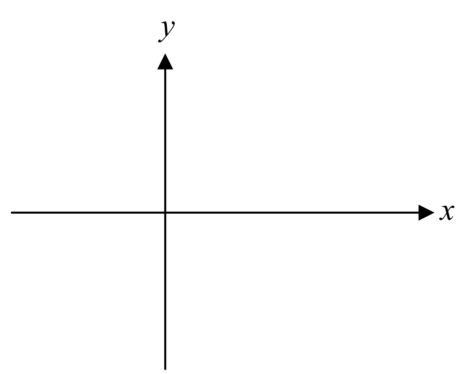
Positive odd degree functions have equations (standard form) with positive odd degree leading terms and graphs that begin with y increasing concave down & end with y increasing concave up.
Note: Concave up/down does not apply to linear functions.

Compare end behaviour to the simplest positive odd degree function: $y = x$

3. $y = 2(1-x)(x+1)(x+4)$

4. $y = -x^2(x-2)^3$





*** **As** $x \to -\infty,\ y \to$ ______ **and as** $x \to +\infty,\ y \to$ ______

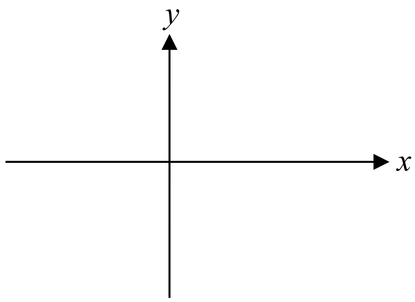
Negative odd degree functions have equations with negative odd degree leading terms and graphs that begin with y decreasing concave up & end with y decreasing concave down.
Note: Concave up/down does not apply to linear functions.

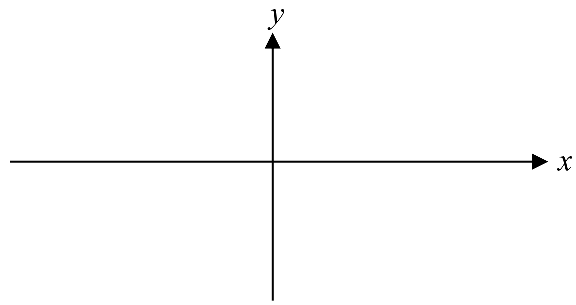
Compare end behaviour to the simplest negative odd degree function: $y = -x$

II <u>Even Degree Functions (*Quadratic, Quartic*)</u>

5. $y=(x+1)(x-2)$

6. $y=(x-2)(x+3)(x+1)(x-4)$



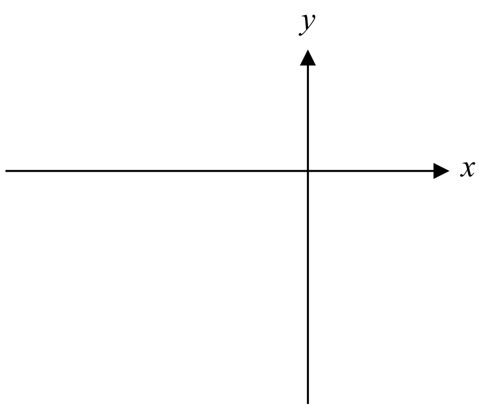


*** **As** $x\to-\infty,\ y\to$ ______ **and as** $x\to+\infty,\ y\to$ ______

Positive even degree functions have equations with positive even degree leading terms and graphs that <u>begin with *y* decreasing concave up</u> & <u>end with *y* increasing concave up</u>.

Compare end behaviour to the simplest positive even degree function: $y=x^2$

7. $y=-x^2(x+3)^2$



*** **As** $x\to-\infty,\ y\to$ ______ **and as** $x\to+\infty,\ y\to$ ______

Negative even degree functions have equations with negative even degree leading terms and graphs that <u>begin with *y* increasing concave down</u> & <u>end with *y* decreasing concave down</u>.

Compare end behaviour to the simplest negative even degree function: $y=-x^2$

Type of Function & Comparison Function	**End Behaviour of *f(x)* as** $x\to-\infty$	**End Behaviour of *f(x)* as** $x\to+\infty$
positive odd degree: $y=x$	$f(x)\to-\infty$	$f(x)\to+\infty$
negative odd degree: $y=-x$	$f(x)\to+\infty$	$f(x)\to-\infty$
positive even degree: $y=x^2$	$f(x)\to+\infty$	$f(x)\to+\infty$
negative even degree: $y=-x^2$	$f(x)\to-\infty$	$f(x)\to-\infty$

Date:_______________

2.6 Graphing Expanded Polynomial Functions

1. Draw a sketch of the following functions, clearly labeling all x-intercepts.

a) $y = 6x^2 + 4x - 16$

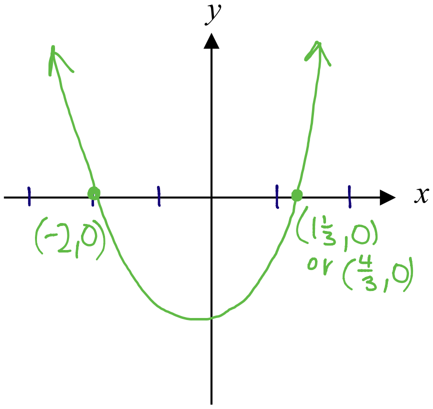
$y = 2(3x^2 + 2x - 8)$

$y = 2(3x - 4)(x + 2)$

$\therefore$ x-ints are $\frac{4}{3}, -2$

(both single)

Compare to $y = x^2$ 



b) $f(x) = -x^3 - x^2 + 9x + 9$

$f(x) = -x^2(x+1) + 9(x+1)$

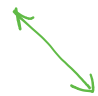
$f(x) = (x+1)(-x^2+9)$

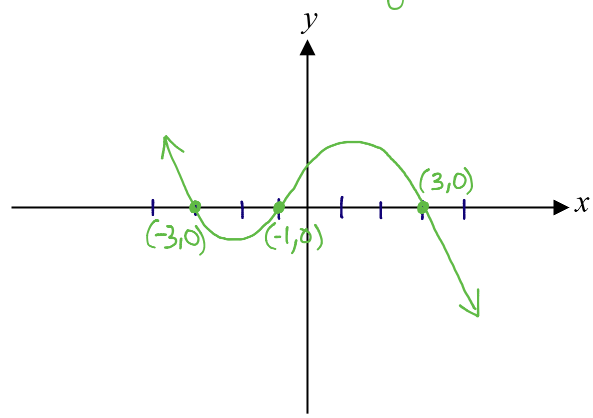
$f(x) = -(x+1)(x^2-9)$

$f(x) = -(x+1)(x-3)(x+3)$

$\therefore$ x-ints are $-3, -1, 3$

(all single)

Compare to $y = -x$ 



c) $y = -x^4 + 9x^2$

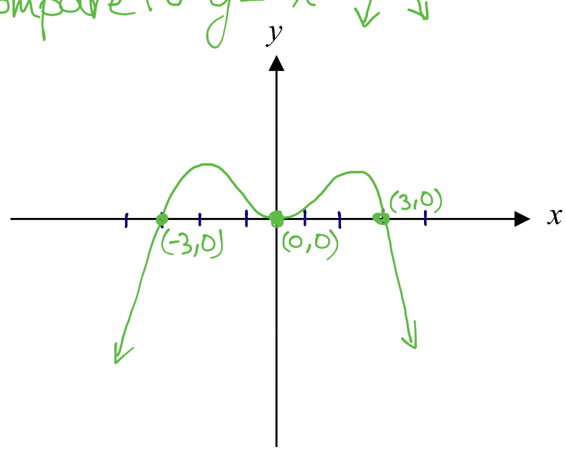
$y = -x^2(x^2 - 9)$

$y = -x^2(x-3)(x+3)$

$\therefore$ x-ints are $0, \underbrace{-3, 3}_{}$

double single

Compare to $y = -x^2$ 



d) $f(x) = 16x^5 + 48x^4 + 36x^3$

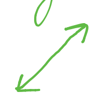
$f(x) = 4x^3(4x^2 + 12x + 9)$

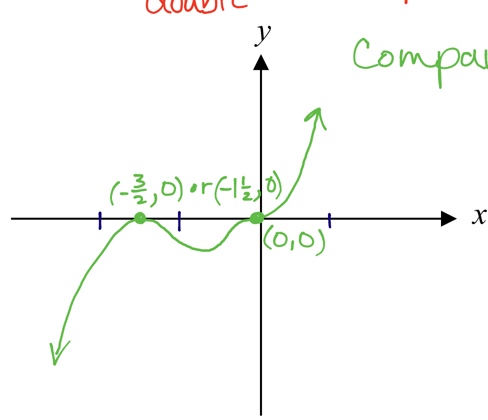
$f(x) = 4x^3(2x+3)(2x+3)$

$f(x) = 4x^3(2x+3)^2$

$\therefore$ x-ints are $-\frac{3}{2}, 0$

double triple

Compare to $y = x$ 



e) $g(x) = x^3 - 9x^2 + 27x - 27$

$g(x) = (x-3)(x^2-6x+9)$

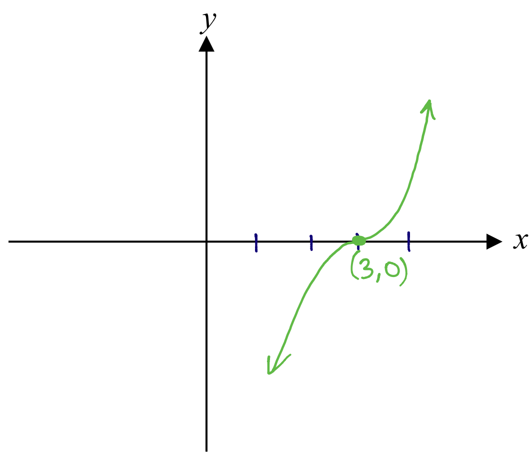
$g(x) = (x-3)(x-3)(x-3)$

$g(x) = (x-3)^3$

$\therefore$ x-int is 3 (triple)

Compare to $y=x$

$$\begin{array}{r} x^2 - 6x + 9 \\ x-3 \overline{\big) x^3 - 9x^2 + 27x - 27} \\ \underline{x^3 - 3x^2} \qquad\qquad \\ -6x^2 + 27x \qquad \\ \underline{-6x^2 + 18x} \qquad \\ 9x - 27 \\ \underline{9x - 27} \\ 0 \end{array}$$



f) $y = -2x^3 - 7x^2 - 2x + 3$

$y = -(2x^3 + 7x^2 + 2x - 3)$

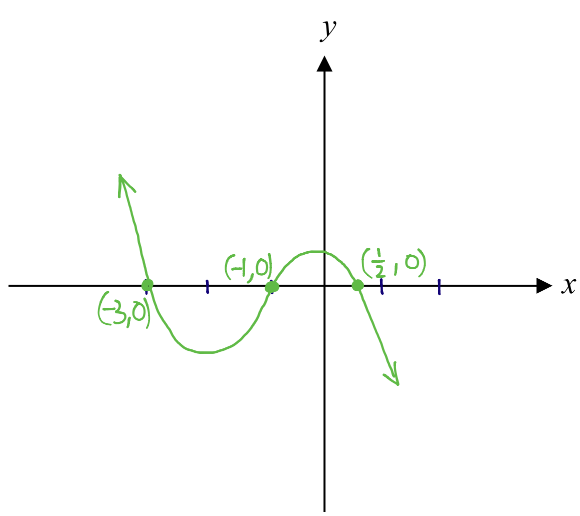
$y = -(x+1)(2x^2+5x-3)$

$y = -(x+1)(2x-1)(x+3)$

$\therefore$ x-ints are $-3, -1, \frac{1}{2}$ (all single)

Compare to $y=-x$

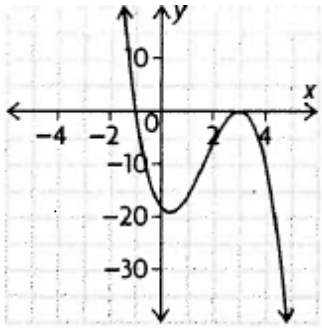
$$\begin{array}{r} 2x^2 + 5x - 3 \\ x+1 \overline{\big) 2x^3 + 7x^2 + 2x - 3} \\ \underline{2x^3 + 2x^2} \qquad\qquad \\ 5x^2 + 2x \qquad \\ \underline{5x^2 + 5x} \qquad \\ -3x - 3 \\ \underline{-3x - 3} \\ 0 \end{array}$$



Ex. 2. Use the graph of each polynomial function to:

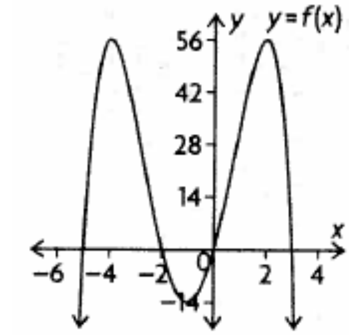
i) identify the polynomial as ***quadratic***, ***cubic***, ***quartic*** or ***quintic***
ii) state the sign of the leading coefficient of its function
iii) state the number & nature of roots to the corresponding equation used to determine the zeros
iv) determine the number of turning points
v) describe the end behavior

a)



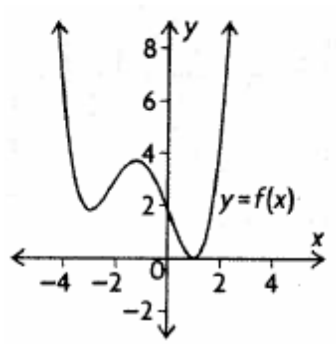
i) cubic
ii) negative
iii) 3 real roots
iv) As $x \to -\infty$, $y \to +\infty$
As $x \to +\infty$, $y \to -\infty$
v) 2 turning pts

b)



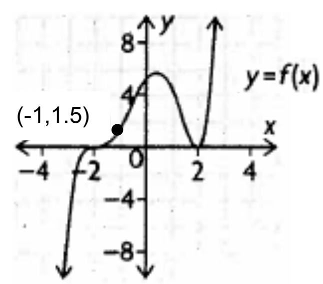
i) quartic
ii) negative
iii) 4 real roots
iv) As $x \to -\infty$, $y \to -\infty$
As $x \to +\infty$, $y \to -\infty$
v) 3 turning pts.

c)



i) quartic
ii) positive
iii) 2 real roots
(2 imaginary)
iii) As $x \to -\infty$, $y \to +\infty$
As $x \to +\infty$, $y \to +\infty$
v) 3 turning points

d)



i) quintic
ii) positive
iii) 5 real root
iv) As $x \to +\infty$, $y \to +\infty$
As $x \to -\infty$, $y \to -\infty$
v) 2 turning points

Day 7: Let $f(x) = a(x+2)^3(x-2)^2$
Find a if $f(-1) = \frac{3}{2}$

$$\frac{3}{2} = a(-1+2)^3(-1-2)^2$$

$$\frac{3}{2} = a(1)^3(-3)^2$$

$$\frac{1}{9}\left(\frac{3}{2}\right) = (9a)\frac{1}{9}$$

$$\therefore a = \frac{1}{6}$$

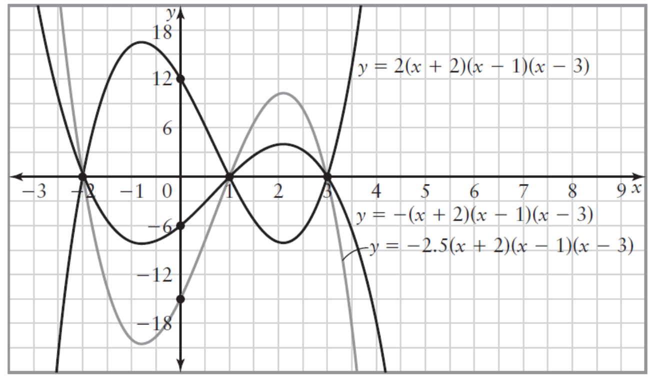
$\therefore$ the equation is
$f(x) = \frac{1}{6}(x+2)^3(x-2)^2$

HW. Exercise 2.6

Date:______________ **2.7 Determining the Equations of Polynomial Functions**

Ex. 1. a) Determine an equation for the family of cubic functions whose x-intercepts are -2, 1 and 3.

Let $f(x) = a(x+2)(x-1)(x-3)$ represent the family of cubic functions.



b) Determine an equation for the particular member of this family, in ***factored*** form, whose y-intercept is 9. Find a if $f(0)=9$

$9 = a(0+2)(0-1)(0-3)$

$9 = a(2)(-1)(-3)$

$9 = 6a$

$\frac{3}{2} = a$

$\therefore$ the required equation is $f(x) = \frac{3}{2}(x+2)(x-1)(x-3)$

Ex. 2. Determine the equation of the quartic function, in ***standard*** form, with zeros $\frac{1-\sqrt{3}}{2}$, $\frac{1+\sqrt{3}}{2}$ and 0 (order 2) passing through the point $(-1,-18)$.

Let $f(x) = a(x)^2(2x-1+\sqrt{3})(2x-1-\sqrt{3})$

$f(-1) = -18$

$-18 = a(-1)^2(-2-1+\sqrt{3})(-2-1-\sqrt{3})$

$(a+b)(a-b) = a^2-b^2$

$-18 = a(1)\underbrace{(-3+\sqrt{3})}_{(a+b)}\underbrace{(-3-\sqrt{3})}_{(a-b)}$

$-18 = a[(-3)^2 - (\sqrt{3})^2]$

$-18 = 6a$

$\therefore a = -3$

$\therefore f(x) = -3x^2(\underbrace{2x-1}_{a}+\underbrace{\sqrt{3}}_{b})(\underbrace{2x-1}_{a}-\underbrace{\sqrt{3}}_{b})$

$f(x) = -3x^2[(2x-1)^2 - (\sqrt{3})^2]$

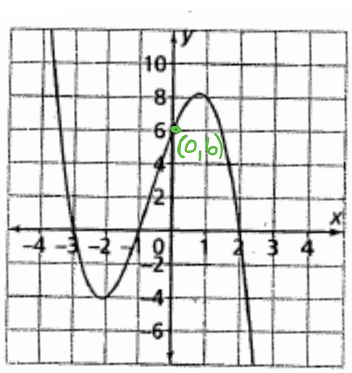
$f(x) = -3x^2(4x^2-4x+1-3)$

$f(x) = -3x^2(4x^2-4x-2)$

$\therefore f(x) = -12x^4+12x^3+6x^2$ is the equation in standard form.

Ex. 3. Determine the equation of each polynomial function in ***factored*** form, from its graph.

a)



x-ints are $-3, -1, 2$ (all single)

Let $f(x) = a(x+3)(x+1)(x-2)$

Find a if $f(0)=6$

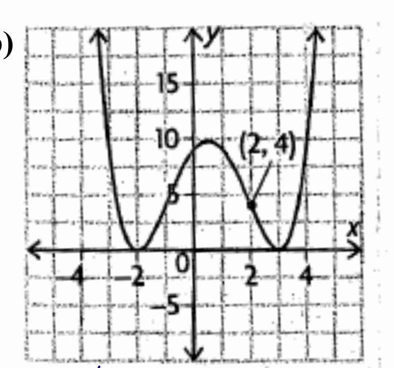
$a(0+3)(0+1)(0-2) = 6$

$-6a = 6$

$a = -1$

$\therefore f(x) = -(x+3)(x+1)(x-2)$ is the required equation.

b)



x-int are -2, 3 (both double)

Let $f(x) = a(x+2)^2(x-3)^2$

Find a if $f(2)=4$

$a(2+2)^2(2-3)^2 = 4$

$16a = 4$

$a = \frac{4}{16}$

$a = \frac{1}{4}$

$\therefore f(x) = \frac{1}{4}(x+2)^2(x-3)^2$ is the required equation.

c) Ex. 2. d) from Unit 2: Day 6

Ex. 4. The points (1,1), (2,–3), (3,5), (4,37), (5,105) and (6,221) lie on the graph of a function. Determine the equation of the polynomial function.

Solution:

Determine if the polynomial function $f(x)$ is ***linear***, ***quadratic***, ***cubic***, ***quartic***, *or* ***quintic*** by calculating the *first differences, second differences, third differences, and so on*.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	1	$-3-1=-4$	$8-(-4)=12$	$24-12=12$
2	-3	$5-(-3)=8$	$32-8=24$	$36-24=12$
3	5	$37-5=32$	$68-32=36$	$48-36=12$
4	37	$105-37=68$	$116-68=48$	
5	105	$221-105=116$		
6	221			

$\because \Delta f(x)$ is not constant $\therefore f(x)$ is not linear

$\because \Delta^2 f(x)$ is not constant $\therefore f(x)$ is not quadratic

$\because \Delta^3 f(x)$ is constant $\therefore f(x)$ is cubic.

$\therefore f(x)$ is cubic

$f(x)=ax^3+bx^2+cx+d$

$f(1)=1$	$a+b+c+d=1$ ①
$f(2)=-3$	$8a+4b+2c+d=-3$ ②
$f(3)=5$	$27a+9b+3c+d=5$ ③
$f(4)=37$	$64a+16b+4c+d=37$ ④

Eliminate d:

② − ①	$7a+3b+c=-4$ ⑤
③ − ②	$19a+5b+c=8$ ⑥
④ − ③	$37a+7b+c=32$ ⑦

Eliminate c:

⑥ − ⑤	$12a+2b=12$ ⑧
⑦ − ⑥	$18a+2b=24$ ⑨

Eliminate b:

⑨ − ⑧ | $6a=12$

$\boxed{\therefore a=2}$

Sub $a=2$ into ⑧:

$12(2)+2b=12$

$24+2b=12$

$2b=-12$

$\boxed{\therefore b=-6}$

Sub $a=2$, $b=-6$ into ⑤

$7(2)+3(-6)+c=-4$

$14-18+c=-4$

$-4+c=-4$

$\boxed{\therefore c=0}$

Sub $a=2$, $b=-6$, $c=0$ into ①

$(2)+(-6)+(0)+d=1$

$-4+d=1$

$\boxed{\therefore d=5}$

$\therefore f(x)=2x^3-6x^2+5$ is the required equation.

HW. Exercise 2.7

HW. for Unit 2 Part I Test: Unit 2 Review of 2.1-2.7