Date:______________ **UNIT 2: GRAPHING FUNCTIONS**

2.1 Graphing Quadratic, Cubic, Square Root, Absolute Value and Reciprocal Functions Using Transformations

Given $y = a\,f[k(x-d)]+c$, the **transformations** on the graph of $y = f(x)$ are as follows:

i) ***vertical reflection*** in the x-axis if $a < 0$

ii) ***vertical stretch*** by a factor of $|a|$

Note: A stretch is an ***expansion*** *if the stretch factor is more than* 1 *or a* ***compression*** *if the stretch factor is between* 0 *and* 1.

iii) ***horizontal reflection*** in the y-axis if $k < 0$

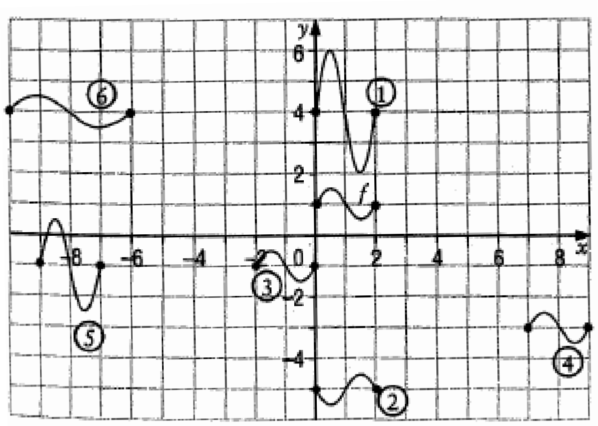
iv) ***horizontal stretch*** by a factor of $\dfrac{1}{|k|}$

v) ***horizontal translation*** **right** $|d|$ units if $d > 0$ or **left** $|d|$ units if $d < 0$

vi) ***vertical translation*** **up** $|c|$ units if $c > 0$ or **down** $|c|$ units if $c < 0$

$$(x,y) \rightarrow \left(\frac{1}{k}x+d, ay+c\right)$$

1. The graph of $y = f(x)$ is shown. Match each equation with its graph by describing the transformations applied to the graph of $y = f(x)$.



a) $y = f(x-7)-4$

b) $y = -f(x)-4$

c) $y = 4f(x)$

d) $y = -f(-x)$

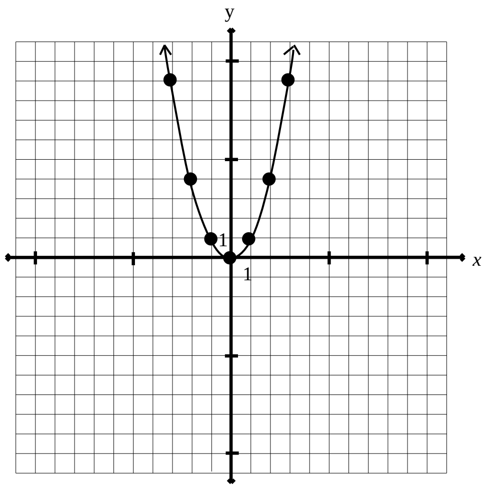
e) $y = 3f(x+9)-4$

f) $y = f\left(\frac{1}{2}x+5\right)+3$

2. Memorize each of the following base functions.

a) ***quadratic function*** $f(x) = x^2$

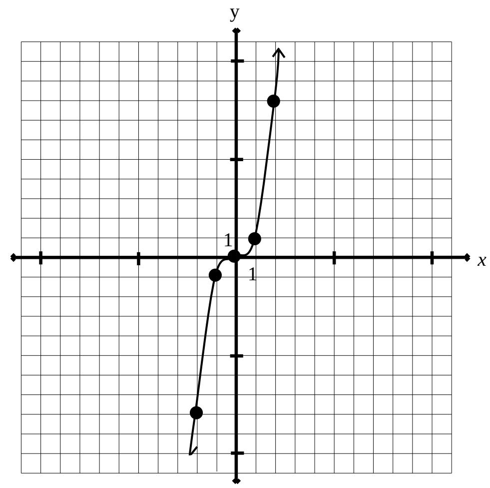
x	y
–4	16
–3	9
–2	4
–1	1
0	0
1	1
2	4
3	9
4	16



Domain: $\{x \in \Re\}$, Range: $\{y \in \Re | y \geq 0\}$

b) ***cubic function*** $f(x) = x^3$

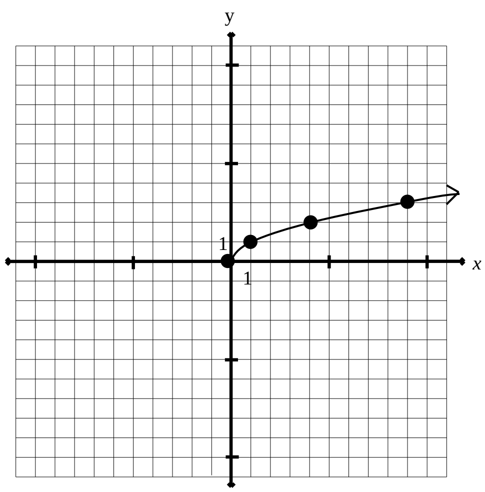
x	y
–3	–27
–2	–8
–1	–1
0	0
1	1
2	8
3	27



Domain: $\{x \in \Re\}$, Range: $\{y \in \Re\}$

c) ***square root function*** $f(x) = \sqrt{x}$

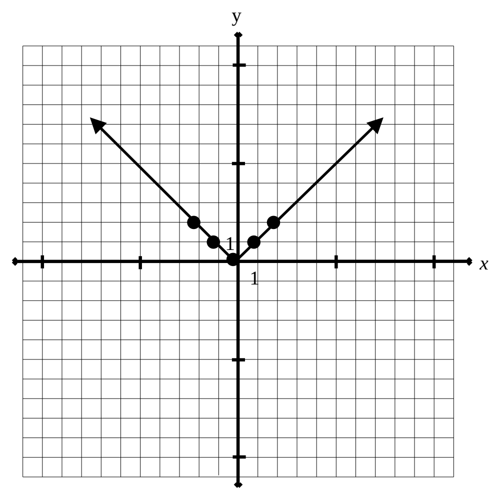
x	y
0	0
1	1
4	2
9	3
16	4
25	5



Domain: $\{x \in \Re | x \geq 0\}$, Range: $\{y \in \Re | y \geq 0\}$

d) ***absolute value function*** $f(x) = |x|$

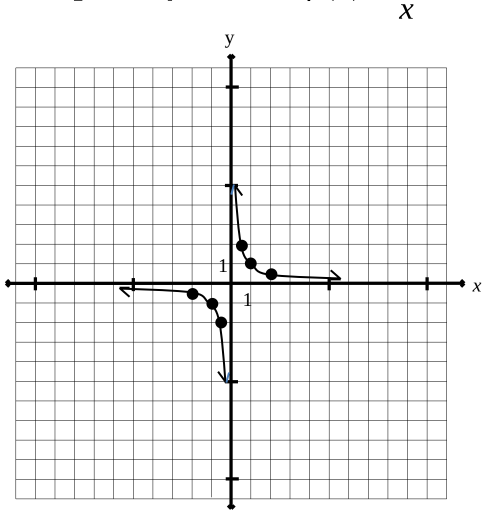
x	y
–4	4
–3	3
–2	2
–1	1
0	0
1	1
2	2
3	3
4	4



Domain: $\{x \in \Re\}$, Range: $\{y \in \Re | y \geq 0\}$

e) ***reciprocal function*** $f(x) = \dfrac{1}{x}$

x	y
–2	$-\frac{1}{2}$
–1	–1
$-\frac{1}{2}$	–2
0	*dne*
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$



Domain: $\{x \in \Re | x \neq 0\}$, Range: $\{y \in \Re | y \neq 0\}$

The ***vertical asymptote*** of $f(x) = \dfrac{1}{x}$ is $x = 0$.

As $x \to 0^-$, $f(x) \to -\infty$ *and* as $x \to 0^+$, $f(x) \to +\infty$.

"As x approaches 0 from the left, $f(x)$ approaches negative infinity" "as x approaches 0 from the right, $f(x)$ approaches positive infinity."

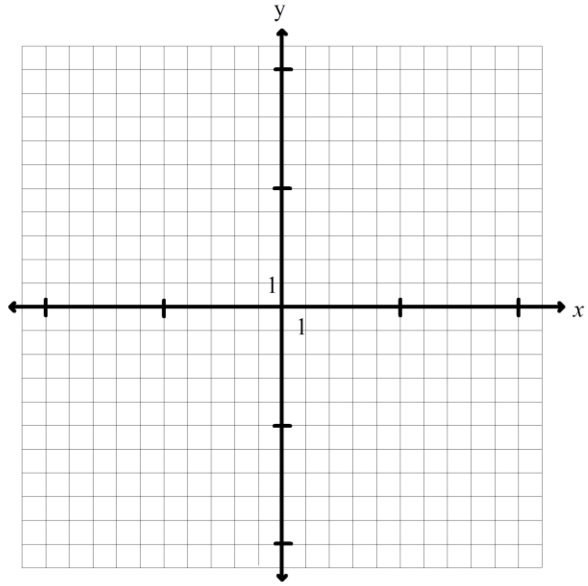
The ***horizontal asymptote*** of $f(x) = \dfrac{1}{x}$ is $y = 0$.

As $x \to -\infty$, $f(x) \to 0$ *and* as $x \to +\infty$, $f(x) \to 0$.

Ex. 1. Graph each of the following by naming and applying transformations on an appropriate function.

a) $y = -2\sqrt{-3(x-5)} + 4 \leftrightarrow y = a\sqrt{k(x-d)} + c$

Transformations on *are:*



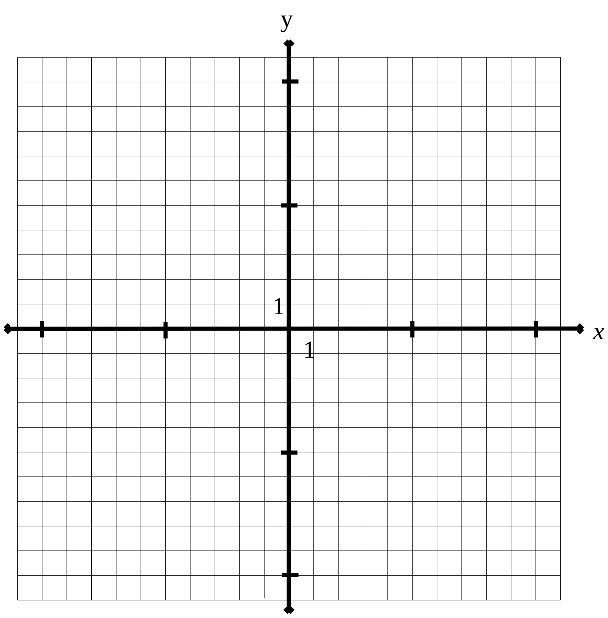
i) Domain: ______________________ **ii)** Range: ______________________

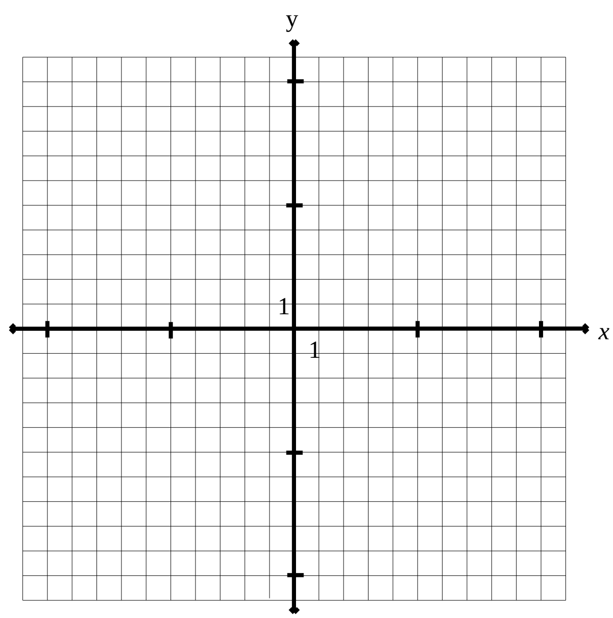
b) $f(x) = -|2(x+1)| - 3 \leftrightarrow f(x) = a|k(x-d)| + c$

Transformations on *are:*

c) $g(x) = \frac{1}{3}(x-2)^3 + 1 \leftrightarrow g(x) = a[k(x-d)]^3 + c$

Transformations on *are:*





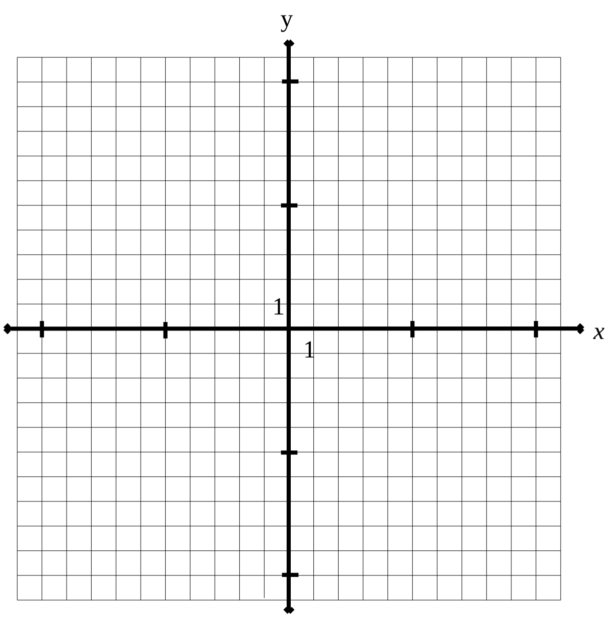
Ex. 2. Graph each of the following by naming and applying transformations on an appropriate function. Then, state the domain and range.

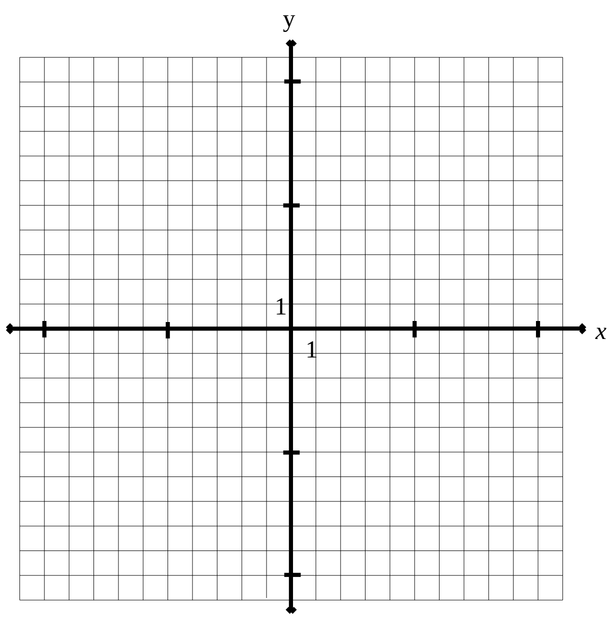
a) $y = \left(0.4x + \frac{4}{5}\right)^2 \leftrightarrow y = a[k(x-d)]^2 + c$

b) $f(x) = \frac{1}{3-x} - 2 \leftrightarrow f(x) = a\left[\frac{1}{k(x-d)}\right] + c$

Transformations on *are:*

Transformations on *are:*





i) Domain: ______________________

ii) Range: ______________________

i) Domain: ______________________

ii) Range: ______________________

HW. Exercise 2.1

Date:_______________

2.2 Graphing Reciprocal and Absolute Value Functions of $y = f(x)$

PART I: ***Using the graph of*** $y = f(x)$ ***to graph its reciprocal function*** $y = \dfrac{1}{f(x)}$

1. *If* $f(x) = 0$, $\dfrac{1}{f(x)}$ *dne. Draw vertical asymptotes at the zeros.*

2. *If* $f(x) = \pm 1$, $\dfrac{1}{f(x)} = \pm 1$. *Mark these invariant points.*

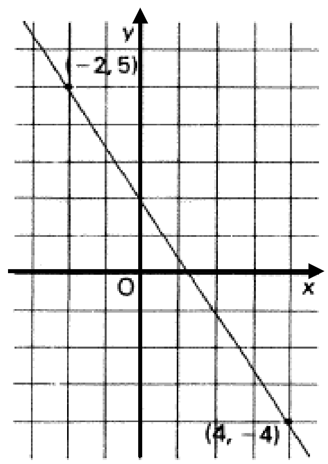
3. i) *If* $f(x)$ *increases,* $\dfrac{1}{f(x)}$ *decreases.* ***ii)*** *If* $f(x)$ *decreases,* $\dfrac{1}{f(x)}$ *increases.*

iii) *If* $f(x)$ *is constant,* $\dfrac{1}{f(x)}$ *is also constant. Graph accordingly.*

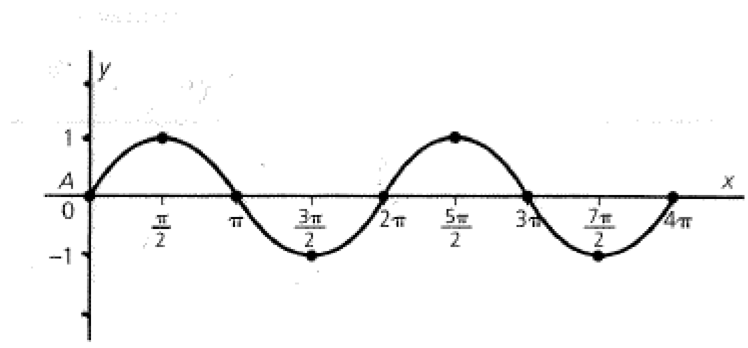
4. *If* $f(x) \to \pm\infty$, $\dfrac{1}{f(x)} \to 0$. *Draw a horizontal asymptote at* $y = 0$.

Ex. 1. Use the graph of $y = f(x)$ to sketch the graph of $y = \dfrac{1}{f(x)}$ for each of the following:

a)



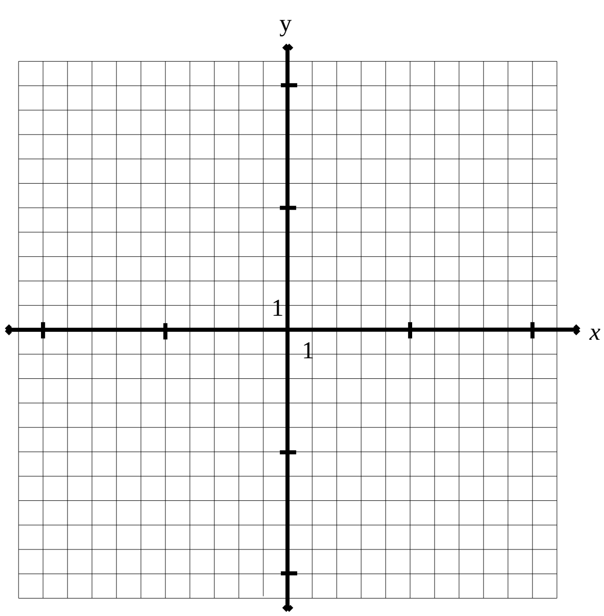
b)

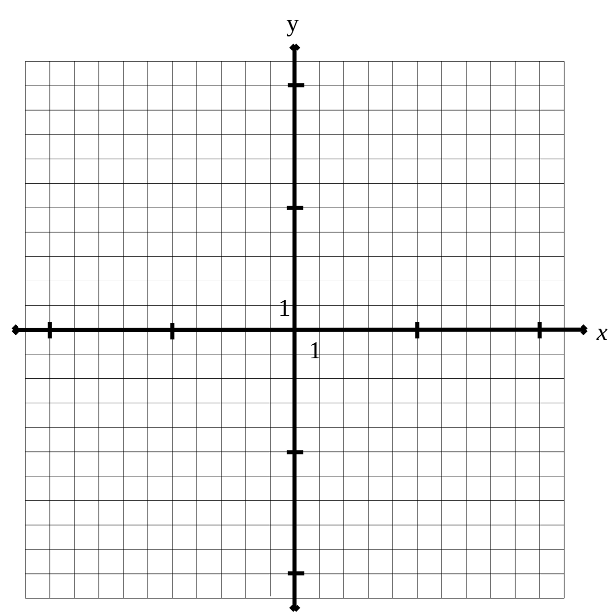


Ex. 2. Graph each function $y = f(x)$ and its reciprocal function $y = \dfrac{1}{f(x)}$ on the same grid.

a) $f(x) = \sqrt{x+5} - 2$

b) $f(x) = -|x-2| + 3$





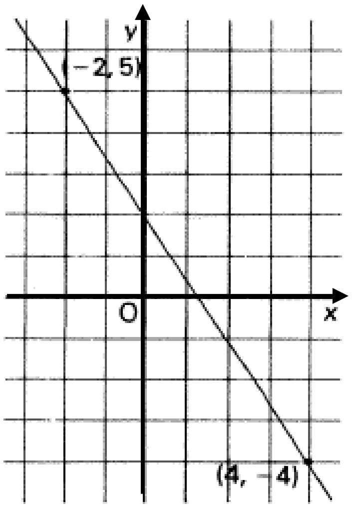
PART II: ***Using the graph of*** $y = f(x)$ ***to graph its absolute value function*** $y = |f(x)|$

1. *All points on the graph of* $y = f(x)$ *where* $f(x) \geq 0$ *are also on the graph of* $y = |f(x)|$.
Graph over these ***invariant*** *points with a different colour.*

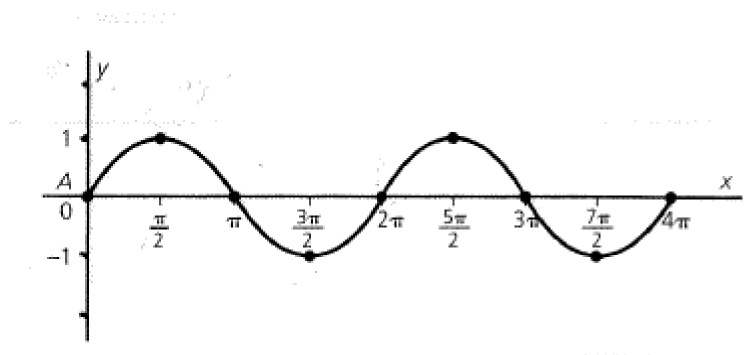
2. *All points on the graph of* $y = f(x)$ *where* $f(x) < 0$ *are vertically reflected in the x-axis.*
Use the same colour to reflect these points in order to complete the graph.

Ex. 1. Use the graph of $y = f(x)$ to sketch the graph of $y = |f(x)|$ for each of the following:

a)



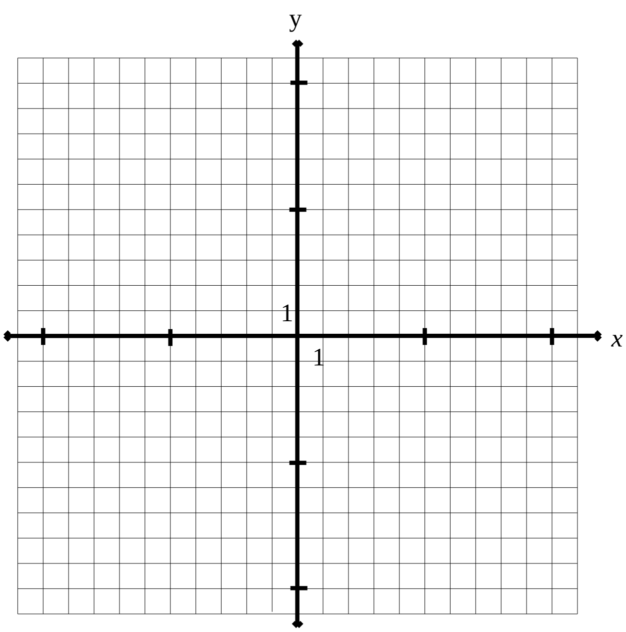
b)

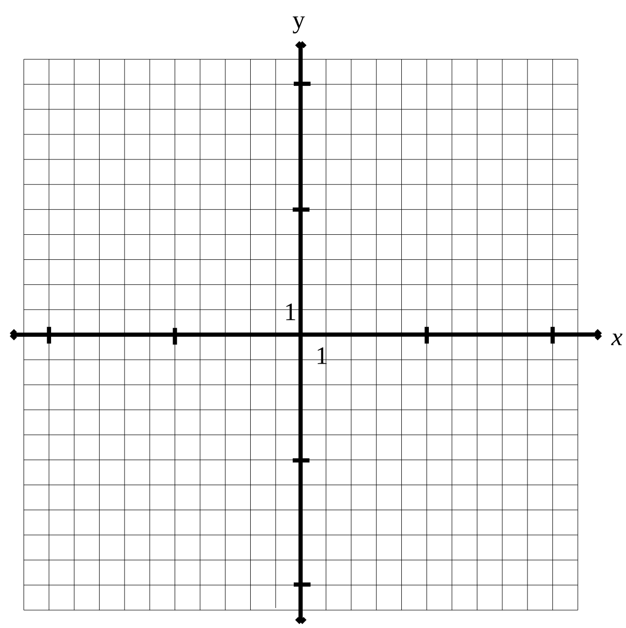


Ex. 2. Graph each function $y = f(x)$ and its absolute value function $y = |f(x)|$ on the same grid.

a) $f(x) = -0.5x^2 - 4x - 5$

b) $f(x) = \left(-\frac{1}{3}x + 1\right)^3 - 1$

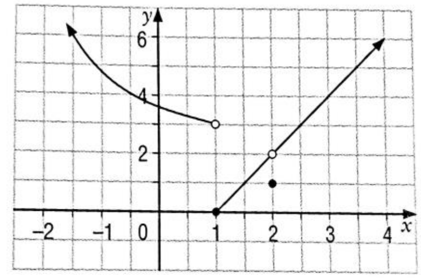




HW. Exercise 2.2

Date:________________ **2.3 Graphing Piecewise Functions**

Ex. 1. The graph of a ***piecewise*** function ***f*** is shown. Use the graph to determine the following:



a) $f\left(-\frac{1}{2}\right)$ **b)** $f(3)$ **c)** $f(1)$ **d)** $f(2)$

e) the value(s) of x at which the function is ***discontinuous***

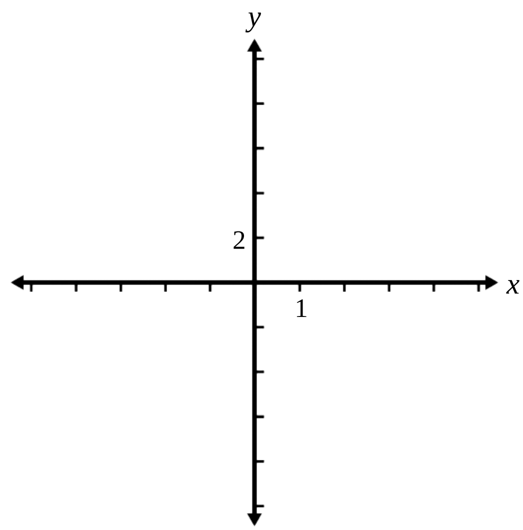
f) as $x \to 1^-$, $f(x) \to$______ and as $x \to 1^+$, $f(x) \to$______

"as *x* approaches 1 from the left" "as *x* approaches 1 from the right"

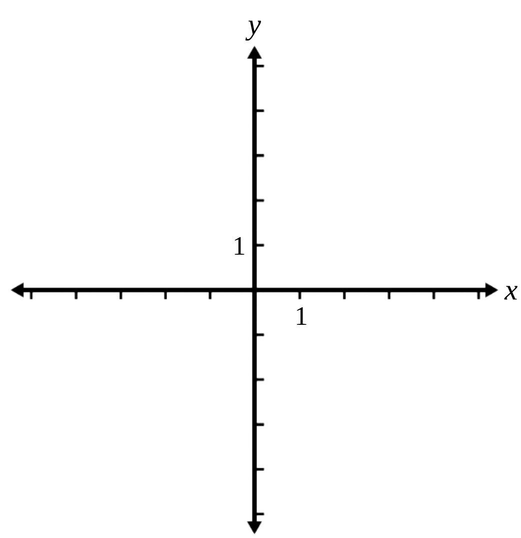
g) the end behaviour of the function ***f***

Ex. 2. For each piecewise function sketch the graph and determine the value(s) of x at which the function is discontinuous. Identify the discontinuities as *jump*, *removable* or *infinite*.

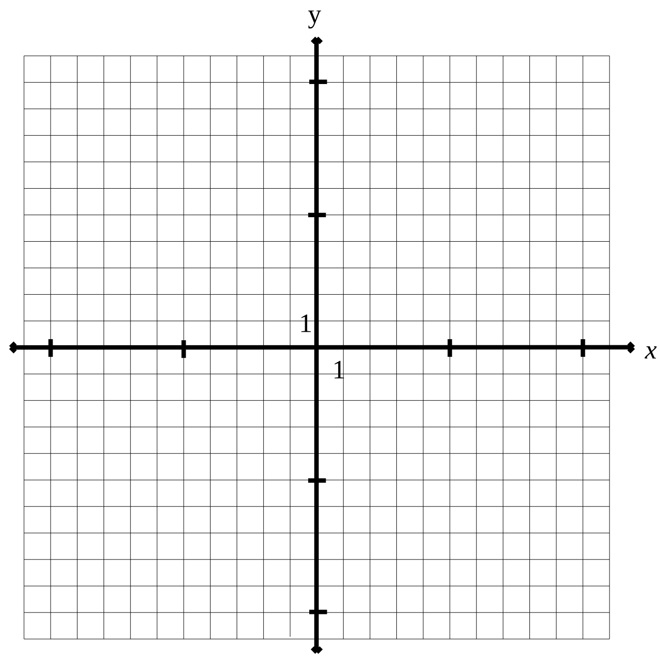
a) $f(x) = \begin{cases} 2 - 2x & \text{if } x < 2 \\ (x-2)^2 & \text{if } x \geq 2 \end{cases}$



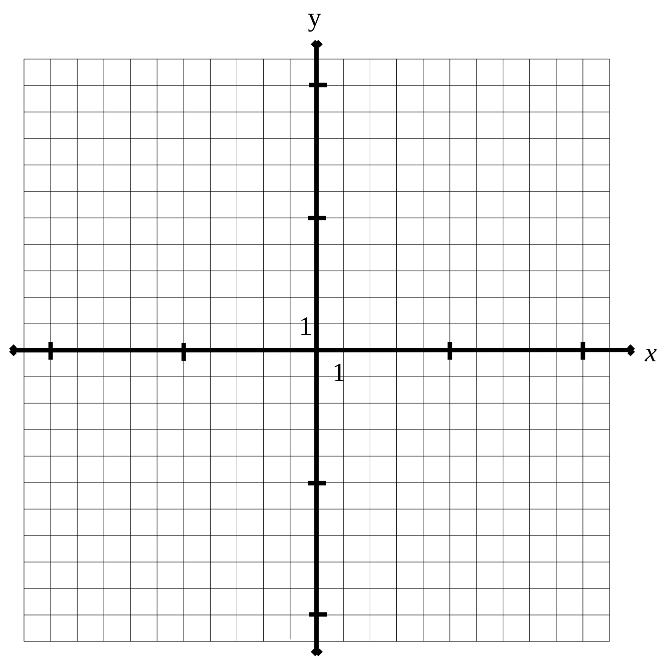
b) $f(x) = \begin{cases} -2 & \text{if } x \in (-\infty, 1) \\ -x - 1 & \text{if } x \in [1, \infty) \end{cases}$



c) $g(x)=\begin{cases} -x^3 & if \;\; x\in(-\infty,0) \\ \dfrac{2}{x-2} & if \;\; x\in(0,\infty) \end{cases}$



d) $f(x)=\begin{cases} \sqrt{3-x} & if \;\; x<-1 \\ 5 & if \;\; x=-1 \\ -2|x|+4 & if \;\; x>-1 \end{cases}$

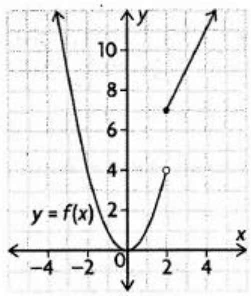


HW. Exercise 2.3

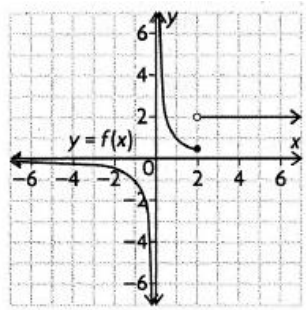
Date:______________ **2.4 Piecewise Functions and Continuity Continued**

Ex. 1. Write an algebraic representation of each piecewise function, using function notation.

a)



b)



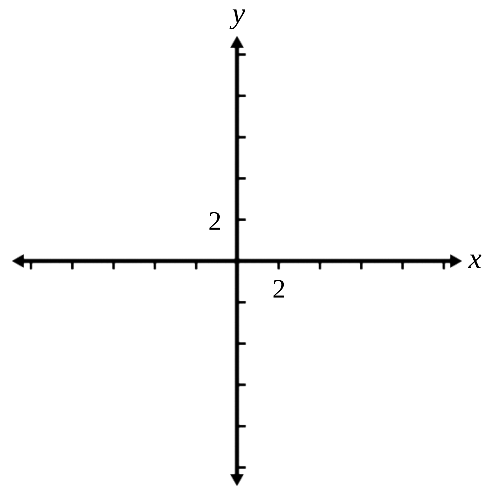
Ex. 2. Without graphing determine if the function below is continuous or discontinuous. If it is discontinuous, state where it is discontinuous.

$$g(x)=\begin{cases} x+1 & \text{if } x\le 0 \\ 2x+1 & \text{if } 0<x<3 \\ 4-x^2 & \text{if } x\ge 3 \end{cases}$$

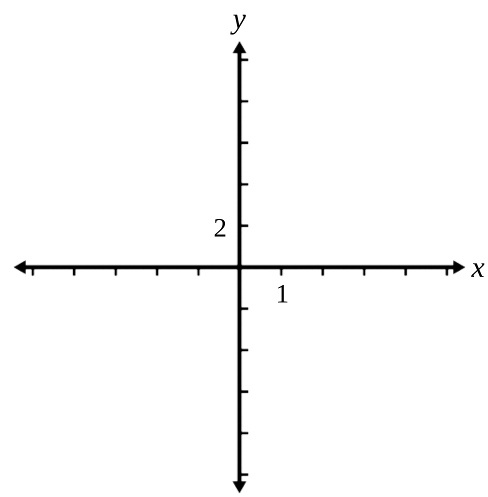
Ex. 3. Given $f(x)=\begin{cases} 5-x^2 & \text{if } x\in(-\infty,-1) \\ ax+b & \text{if } x\in[-1,1) \\ 2x^2 & \text{if } x\in[1,\infty) \end{cases}$, determine the values of *a* and *b* so that the function is continuous for all $x\in(-\infty,\infty)$.

2. Rewrite the following functions involving absolute value as piecewise functions and then graph.

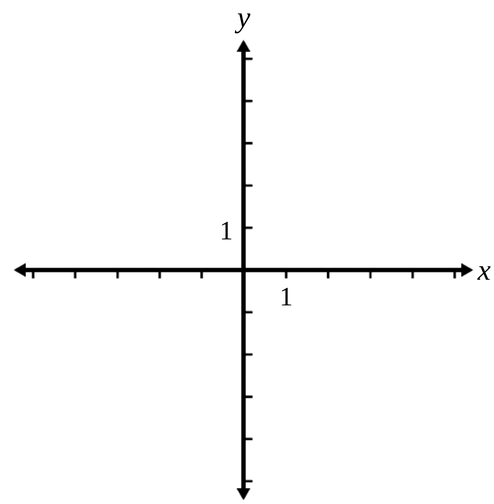
a) $f(x) = |4 - x|$



b) $f(x) = \dfrac{x^2|x+2|}{x+2}$



c) $f(x) = \dfrac{x^2 + |x-1| - 1}{|x-1|}$



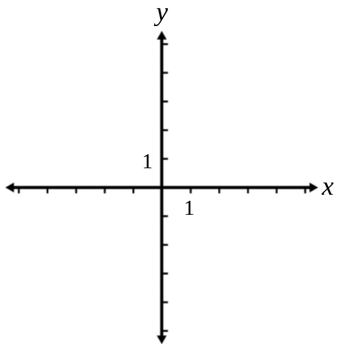
HW. Exercise 2.4

Date:_______________

2.5 Graphing Factored Polynomial Functions

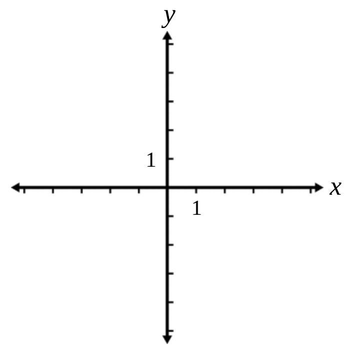
Linear Functions: are *first degree* functions of the form $y = ax + b$ or $f(x) = ax + b$.

i)



$y = 2x$

ii)



$f(x) = -x + 3$

observations: **i)** x-int:

y-int:

slope:

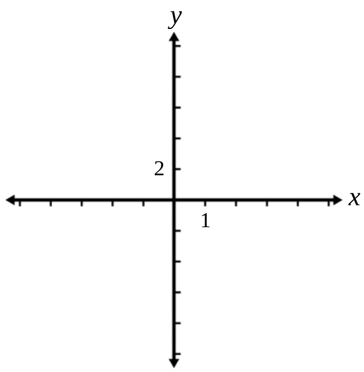
ii) x-int:

y-int:

slope:

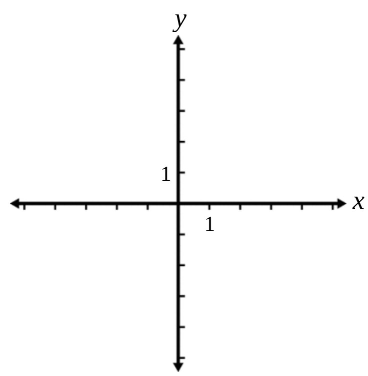
Quadratic Functions: are *second degree* functions of the form $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$.

i)



$y = (x-2)(x+3)$

ii)



$f(x) = -(x+1)^2$

observations: **i)** x-int(s):

y-int:

vertex:

ii) x-int(s):

y-int:

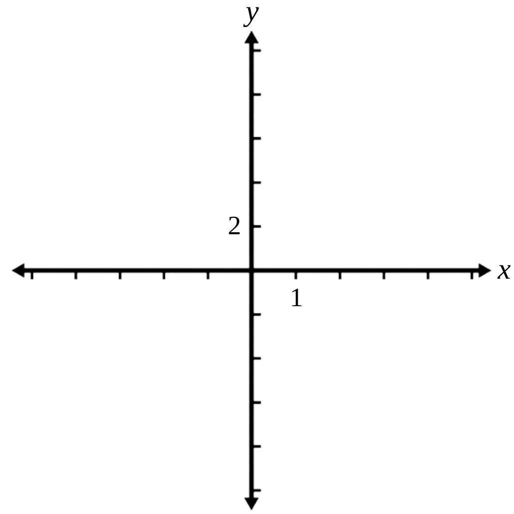
vertex:

<u>Cubic Functions</u>: are *third degree* functions of the form $y = ax^3 + bx^2 + cx + d$
or $f(x) = ax^3 + bx^2 + cx + d$.

Ex. 1. Graph the following cubic functions accurately. Find all *x*-intercepts (zeros) and identify them as single, double or triple roots.

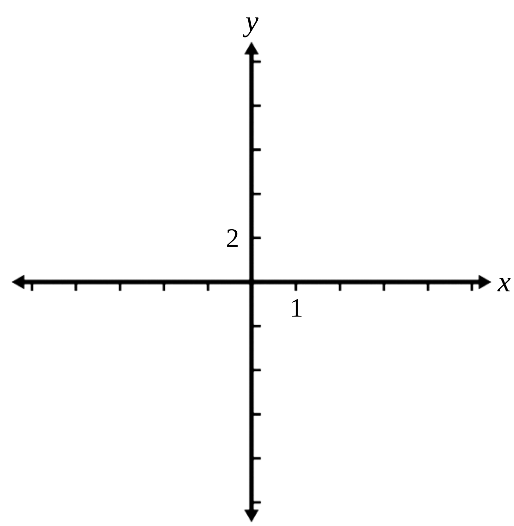
a) $f(x) = x^3$

x	$f(x)$
-2	
-1	
0	
1	
2	



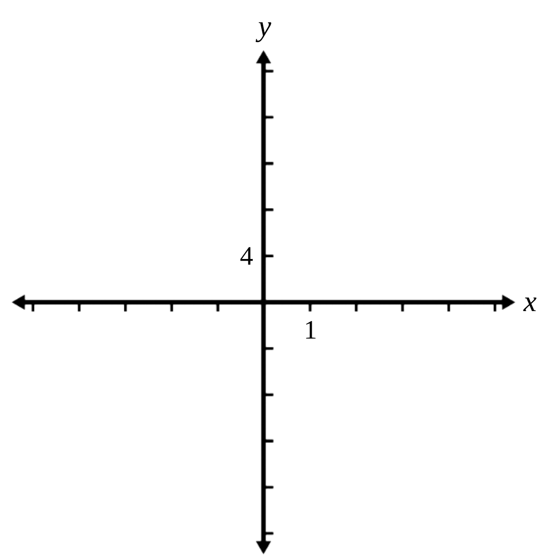
b) $y = \frac{1}{2}(x+3)(x+1)(x-1)$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	



c) $f(x) = -x^2(x+3)$

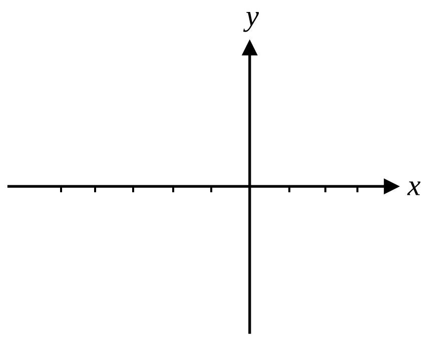
x	$f(x)$
-4	
-3	
-2	
-1	
0	
1	

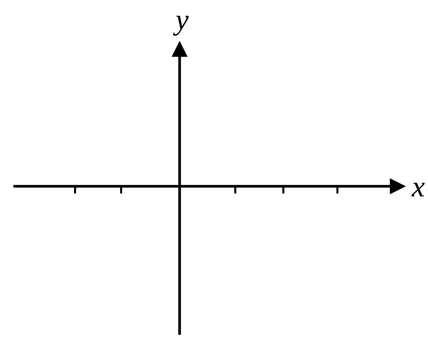


Ex. 2. Sketch the following cubic functions by finding all x-intercepts (zeros) and identifying them as single, double or triple roots.

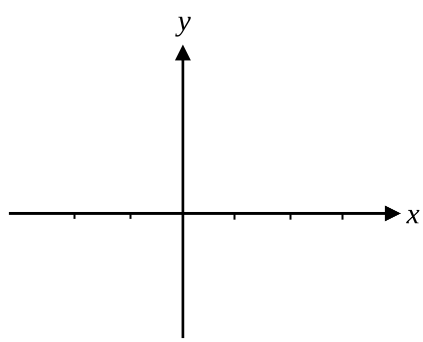
a) $f(x) = -(x-2)(x+1)(x+4)$

b) $y = (2x-5)^3$





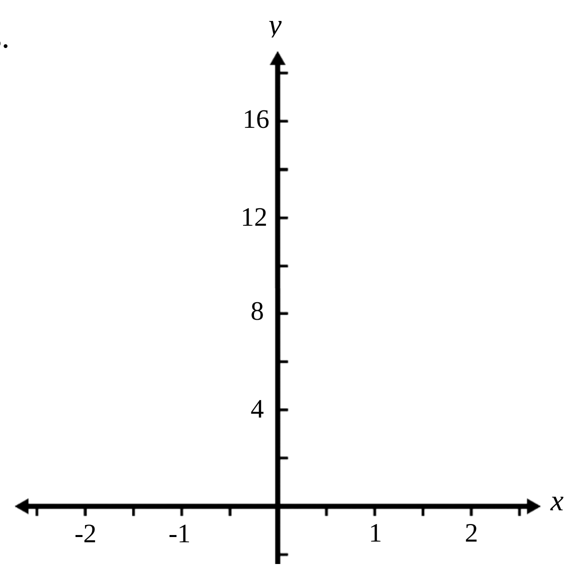
c) $f(x) = (x-2)(x+1)^2$



<u>Quartic Functions</u>: are *fourth degree* functions of the form $y = ax^4 + bx^3 + cx^2 + dx + e$.

Ex. 3. Graph $f(x) = x^4$ accurately by using a table of values.

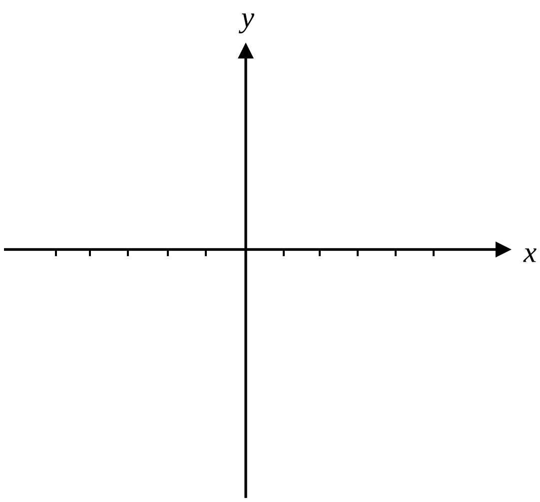
x	y
-2	
-3/2	
-1	
-1/2	
0	
1/2	
1	
3/2	
2	

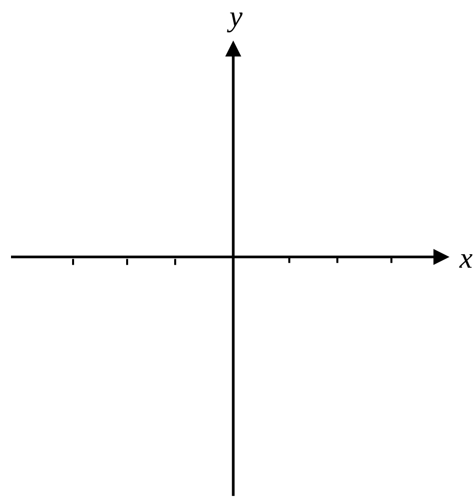


Ex. 4. Sketch the following quartic functions.

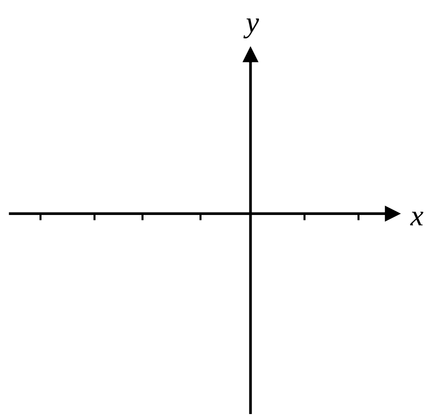
a) $f(x) = (2-x)(x+2)(x-4)(x+4)$

b) $y = \frac{1}{2}x^2(x-2)^2$

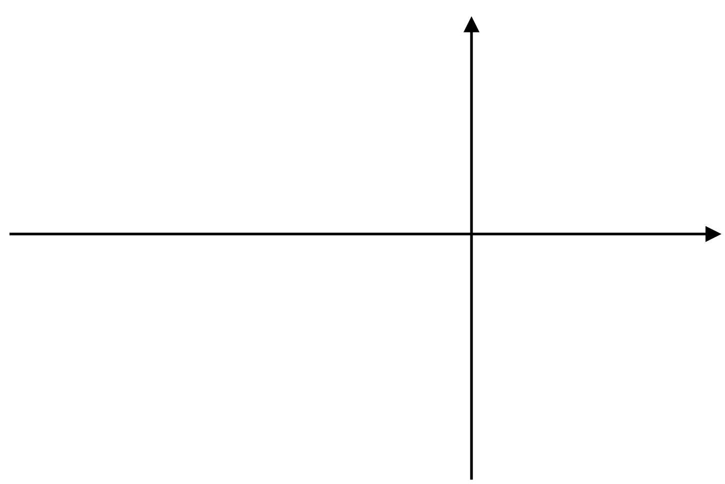




c) $f(x) = (2x-1)(x+3)^3$



Ex. 5. Sketch the quintic function $y = x(x+4)^3(2x+5)$.



HW. **1.** Complete the "**<u>Summary of Graphs of Polynomial Functions</u>**" lesson in your bound notes by following **Ex. 2, 3, 4 & 5** from the note done in class.

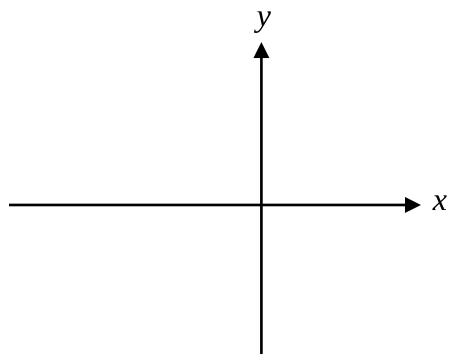
2. Complete **Exercise 2.5**

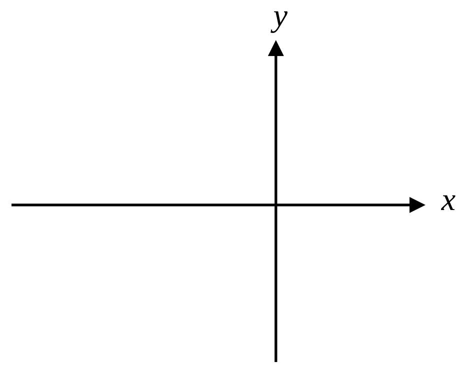
Date:______________ **2.5 Summary of Graphs of Polynomial Functions**

I Odd Degree Functions (*Linear, Cubic, Quintic*)

1. $y = (x+2)(x-1)(x+3)$

2. $y = (x-1)(x+2)^2$





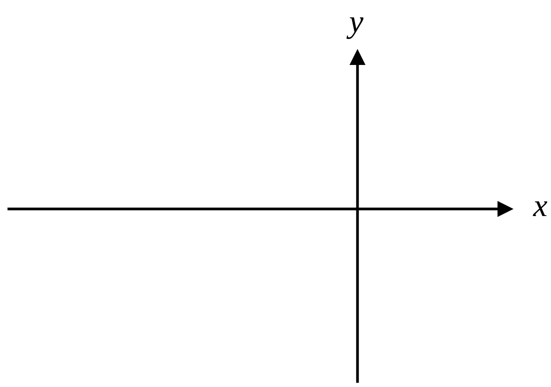
***** As** $x \to -\infty,\ y \to$ ______ **and as** $x \to +\infty,\ y \to$ ______

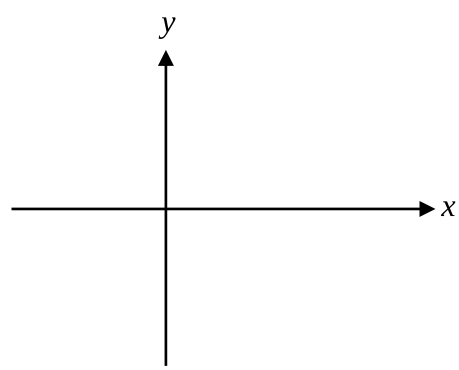
Positive odd degree functions have equations (standard form) with positive odd degree leading terms and graphs that begin with *y* increasing concave down & end with *y* increasing concave up.
Note: Concave up/down does not apply to linear functions.

Compare end behaviour to the simplest positive odd degree function: $y = x$

3. $y = 2(1-x)(x+1)(x+4)$

4. $y = -x^2(x-2)^3$





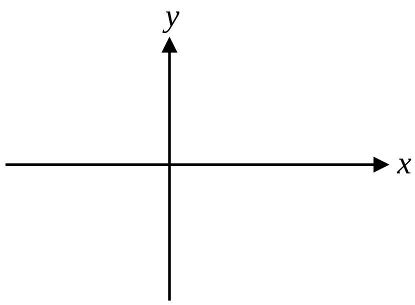
***** As** $x \to -\infty,\ y \to$ ______ **and as** $x \to +\infty,\ y \to$ ______

Negative odd degree functions have equations with negative odd degree leading terms and graphs that begin with *y* decreasing concave up & end with *y* decreasing concave down.
Note: Concave up/down does not apply to linear functions.

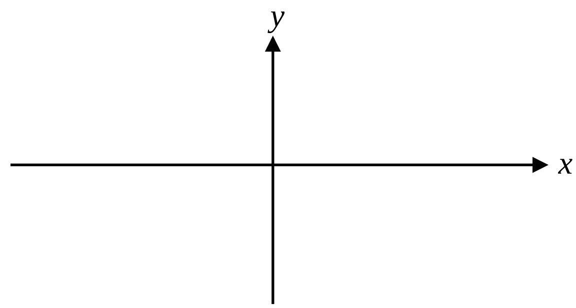
Compare end behaviour to the simplest negative odd degree function: $y = -x$

II Even Degree Functions (*Quadratic, Quartic*)

5. $y=(x+1)(x-2)$



6. $y=(x-2)(x+3)(x+1)(x-4)$

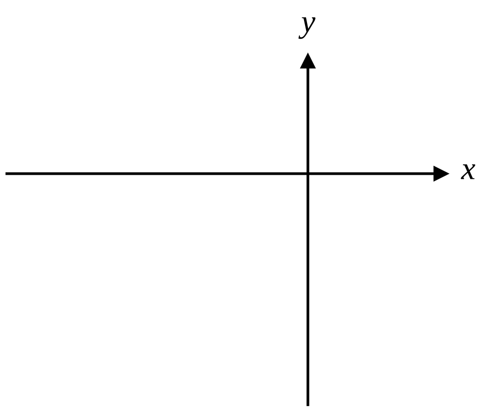


***** As** $x\to-\infty,\ y\to$ ______ **and as** $x\to+\infty,\ y\to$ ______

Positive even degree functions have equations with positive even degree leading terms and graphs that begin with *y* decreasing concave up & end with *y* increasing concave up.

Compare end behaviour to the simplest positive even degree function: $y=x^2$

7. $y=-x^2(x+3)^2$



***** As** $x\to-\infty,\ y\to$ ______ **and as** $x\to+\infty,\ y\to$ ______

Negative even degree functions have equations with negative even degree leading terms and graphs that begin with *y* increasing concave down & end with *y* decreasing concave down.

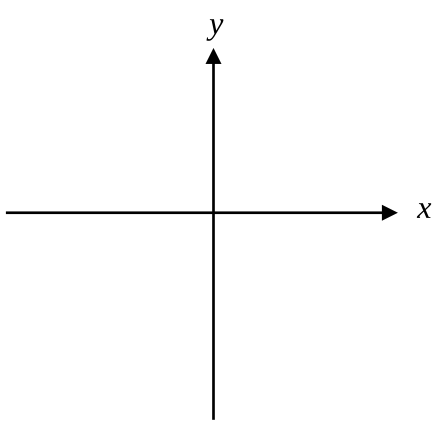
Compare end behaviour to the simplest negative even degree function: $y=-x^2$

Type of Function & Comparison Function	**End Behaviour of *f(x)* as** $x\to-\infty$	**End Behaviour of *f(x)* as** $x\to+\infty$
positive odd degree: $y=x$	$f(x)\to-\infty$	$f(x)\to+\infty$
negative odd degree: $y=-x$	$f(x)\to+\infty$	$f(x)\to-\infty$
positive even degree: $y=x^2$	$f(x)\to+\infty$	$f(x)\to+\infty$
negative even degree: $y=-x^2$	$f(x)\to-\infty$	$f(x)\to-\infty$

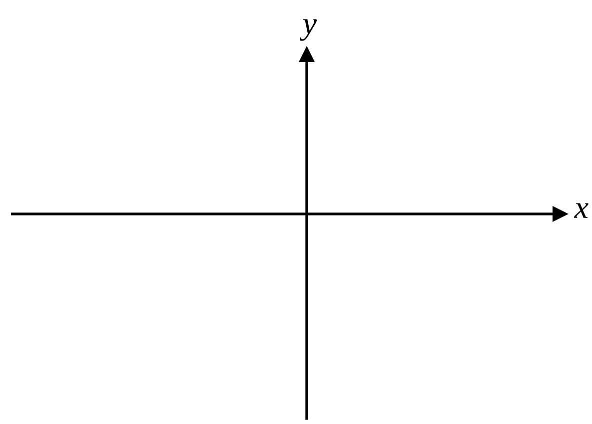
Date:_______________ **2.6 Graphing Expanded Polynomial Functions**

1. Draw a sketch of the following functions, clearly labeling all *x*-intercepts.

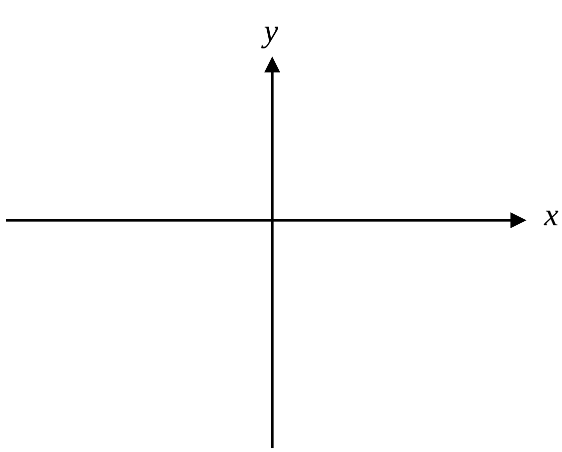
a) $y = 6x^2 + 4x - 16$



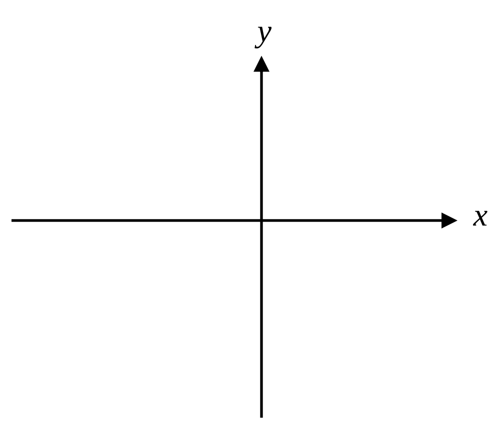
b) $f(x) = -x^3 - x^2 + 9x + 9$



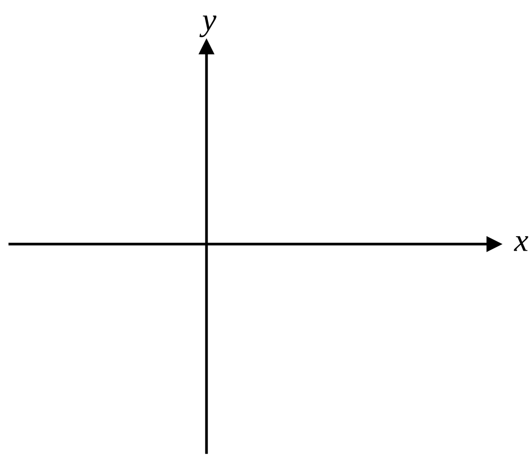
c) $y = -x^4 + 9x^2$



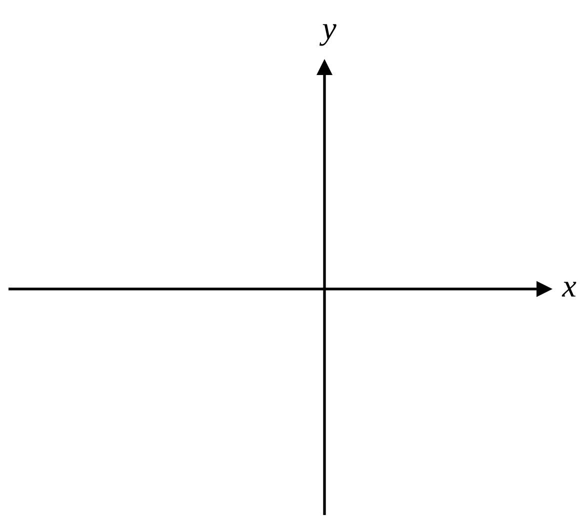
d) $f(x) = 16x^5 + 48x^4 + 36x^3$



e) $g(x) = x^3 - 9x^2 + 27x - 27$



f) $y = -2x^3 - 7x^2 - 2x + 3$



Ex. 2. Use the graph of each polynomial function to:

i) identify the polynomial as ***quadratic***, ***cubic***, ***quartic*** or ***quintic***

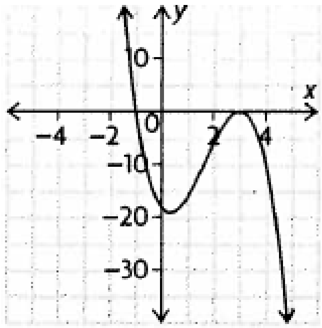
ii) state the sign of the leading coefficient of its function

iii) state the number & nature of roots to the corresponding equation used to determine the zeros

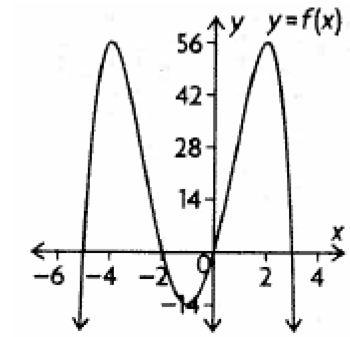
iv) determine the number of turning points

v) describe the end behavior

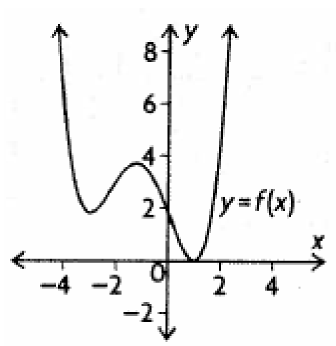
a)



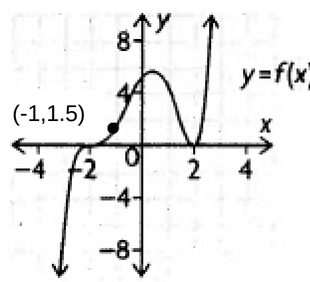
b)



c)



d)

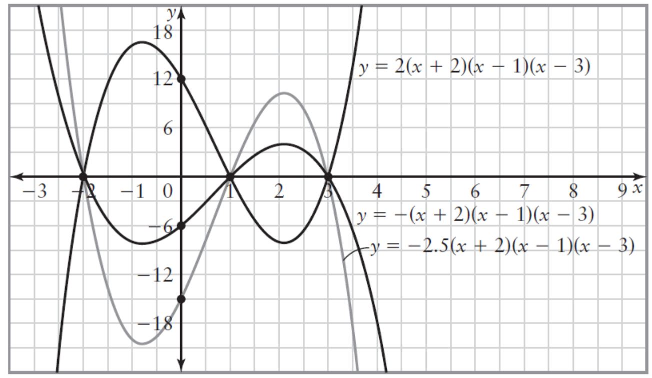


HW. Exercise 2.6

Date:______________ **2.7 Determining the Equations of Polynomial Functions**

Ex. 1. a) Determine an equation for the family of cubic functions whose x-intercepts are -2, 1 and 3.

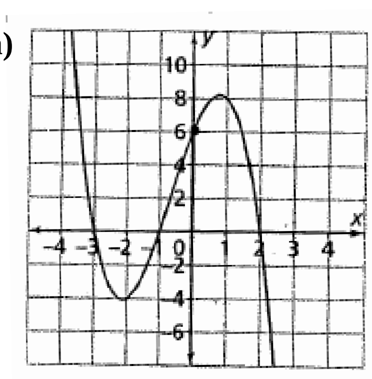
b) Determine an equation for the particular member of this family, in ***factored*** form, whose y-intercept is 9.



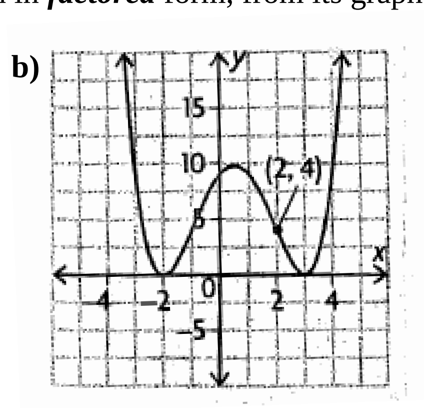
Ex. 2. Determine the equation of the quartic function, in ***standard*** form, with zeros $\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}$ and 0 (order 2) passing through the point $(-1,-18)$.

Ex. 3. Determine the equation of each polynomial function in ***factored*** form, from its graph.

a)



b)



c) Ex. 2. d) from Unit 2: Day 6

Ex. 4. The points $(1,1), (2,-3), (3,5), (4,37), (5,105)$ and $(6,221)$ lie on the graph of a function. Determine the equation of the polynomial function.

Solution:

Determine if the polynomial function $f(x)$ is ***linear***, ***quadratic***, ***cubic***, ***quartic***, *or* ***quintic*** by calculating the *first differences, second differences, third differences, and so on*.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	1			
2	-3			
3	5			
4	37			
5	105			
6	221			

HW. Exercise 2.7

HW. for Unit 2 Part I Test: Unit 2 Review of 2.1-2.7