Date:__________________ **2.12 Graphing Rational Functions Continued**

1. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.

a) $f(x)=\dfrac{2x-1}{x+1}$

① x-int. is $\frac{1}{2}$

② y-int is -1

③ V.A. is $x=-1$

④ For H.A. $f(x)=\dfrac{2x-1 \quad \div x}{x+1 \quad \div x}$

$=\dfrac{2+\frac{1}{x}}{1+\frac{1}{x}}$

As $x\to\pm\infty$, $f(x)\to 2$

$\therefore$ H.A is $y=2$

⑤ Does f(x) cross the H.A. ?

$\dfrac{2x-1}{x+1}=\dfrac{2}{1}$

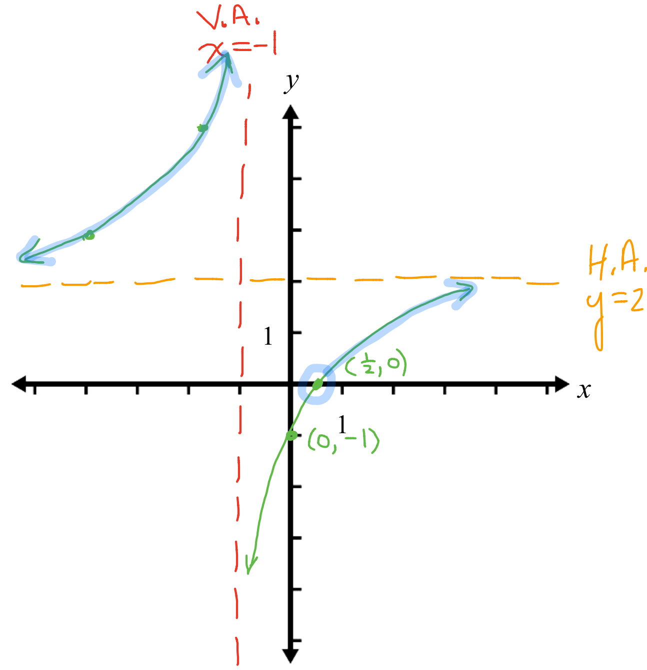
$2x-1=2x+2$

$0=3$

$\therefore$ does not cross

⑥

x	$f(x)$
-2	5
-4	3



b) $g(x)=\dfrac{6}{x^2+2x-3}$

$g(x)=\dfrac{6}{(x+3)(x-1)}$

① x-int: none

② y-int is -2

③ V.A. are $x=-3$, $x=1$

④ For HA: $g(x)=\dfrac{6}{x^2+2x-3}$

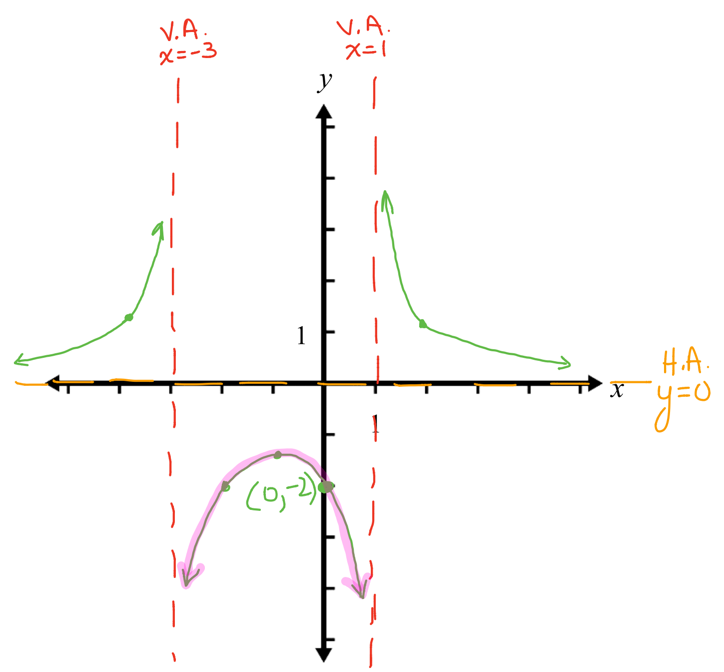
As $x\to\pm\infty$, $g(x)\to 0$

$\therefore$ HA is $y=0$

⑤ g(x) does not cross H.A. since HA is $y=0$ (x-axis) and no x-int.

⑥

x	$g(x)$
-4	$\frac{6}{5}$
-2	-2
-1	$-\frac{3}{2}$
2	$\frac{6}{5}$



2. Using the graphs from the previous question, solve the following inequalities:

a) $f(x)>0$

$\therefore$ $x<-1$ or $x>\frac{1}{2}$

b) $g(x)<0$

$\therefore$ $-3<x<1$

3. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.

a) $f(x)=\dfrac{3x-6}{x^2-2x-8}$

$f(x)=\dfrac{3(x-2)}{(x-4)(x+2)}$

① x-int is 2

② y-int is $\frac{3}{4}$

③ V.A. are $x=-2, x=4$

④ For H.A. $f(x)=\dfrac{3x-6}{x^2-2x-8}$ $\div x^2$ / $\div x^2$

$f(x)=\dfrac{\frac{3}{x}-\frac{6}{x^2}}{1-\frac{2}{x}-\frac{8}{x^2}}$

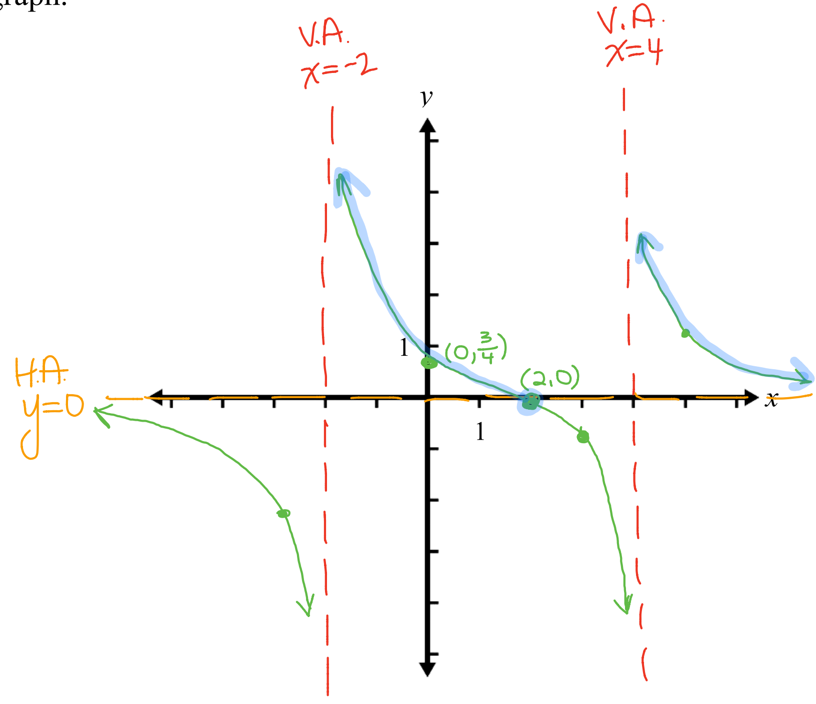
As $x\to\pm\infty$, $f(x)\to 0$

∴ H.A. is $y=0$

⑤ f(x) crosses the H.A. at (2,0) since HA is y=0 (x-axis) and x-int is 2.

⑥

x	$f(x)$
-3	$-\frac{15}{7}$
3	$-\frac{3}{5}$
5	$\frac{9}{7}$



b) $g(x)=\dfrac{6x^2-5x+1}{2x+1}$

$g(x)=\dfrac{(3x-1)(2x-1)}{2x+1}$

① x-ints are $\frac{1}{3}, \frac{1}{2}$

② y-int is 1

③ V.A. is $x=-\frac{1}{2}$

④ For L.O.A: $g(x)=(3x-4)+\dfrac{5}{2x+1}$

$$\begin{array}{r} 3x-4 \\ 2x+1\overline{)6x^2-5x+1} \\ 6x^2+3x \\ \hline -8x+1 \\ -8x-4 \\ \hline 5 \end{array}$$

As $x\to\pm\infty$, $g(x)\to 3x-4$

∴ L.O.A. is $y=3x-4$

⑤ Does g(x) cross the LOA?

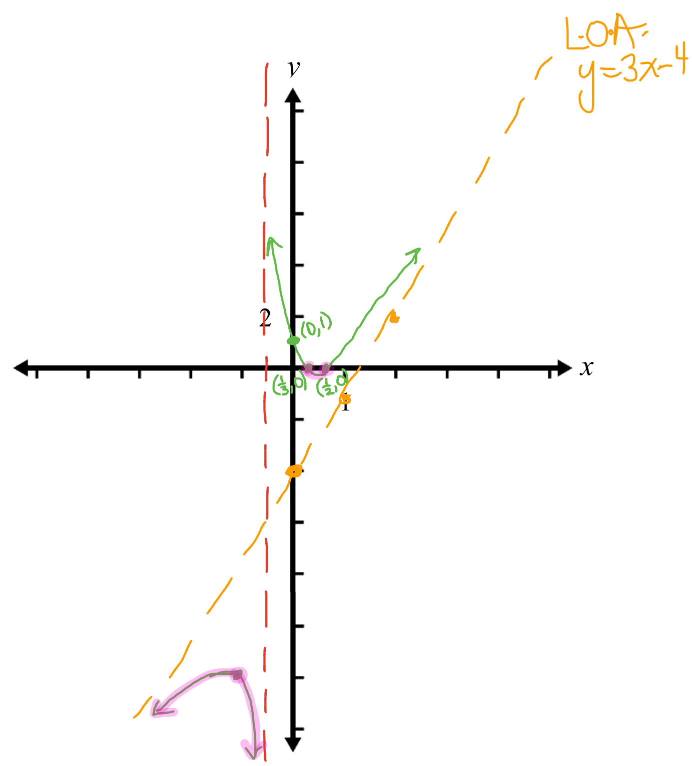
$\dfrac{6x^2-5x+1}{2x+1}=\dfrac{3x-4}{1}$

$6x^2-5x+1=6x^2-5x-4$

$1=-4$ ∴ does not cross.

⑥

x	$g(x)$
-1	-12



4. Using the graphs from the previous question, solve the following inequalities: Answer using a solution set.

a) $f(x)\ge 0$

∴ S.S $=\{x\in\mathbb{R} \mid -2<x\le 2 \text{ or } x>4\}$

b) $g(x)\le 0$

∴ S.S. $=\{x\in\mathbb{R} \mid x<-\frac{1}{2} \text{ or } \frac{1}{3}\le x\le\frac{1}{2}\}$

5. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.

a) $f(x)=\dfrac{2+x-x^2}{(x-1)^2}$

$f(x)=\dfrac{(2-x)(1+x)}{(x-1)^2}$ / $f(x)=\dfrac{2+x-x^2}{x^2-2x+1}$

① x-int are $-1, 2$

② y-int is 2

③ V.A. is $x=1$

④ For H.A.: $f(x)=\dfrac{-x^2+x+2}{x^2-2x+1}\;\begin{matrix}\div x^2\\ \div x^2\end{matrix}$

$f(x)=\dfrac{-1+\frac{1}{x}+\frac{2}{x^2}}{1-\frac{2}{x}+\frac{1}{x^2}}$

As $x\to\pm\infty$, $f(x)\to -1$

$\therefore$ H.A. is $y=-1$

⑤ Does $f(x)$ cross the H.A.?

$\dfrac{2+x-x^2}{x^2-2x+1}=\dfrac{-1}{1}$

$2+x-x^2=-x^2+2x-1$

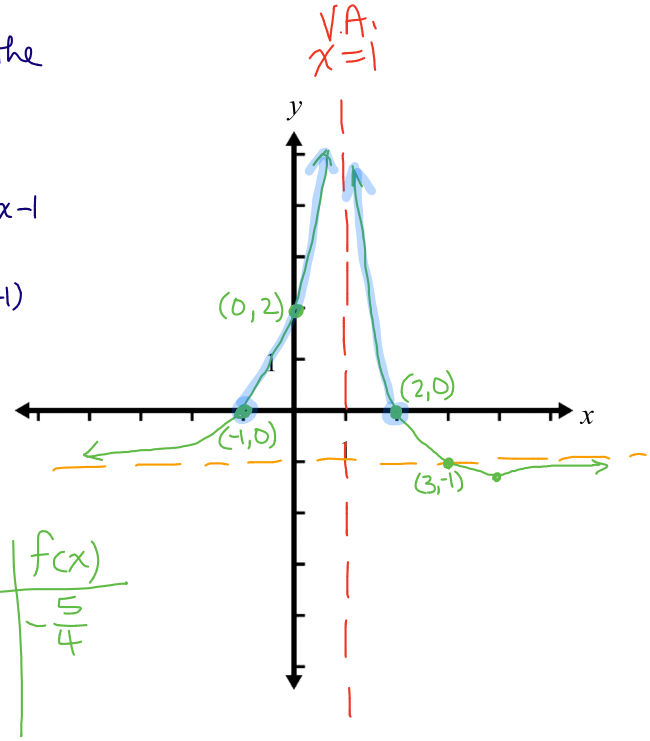
$-x=-3$

$x=3$

$\therefore$ crosses at $(3,-1)$

⑥

x	$f(x)$
4	$-\frac{5}{4}$



b) $g(x)=\dfrac{x^3+8}{x}$ ***this graph has a *quadratic* oblique asymptote

$g(x)=\dfrac{(x+2)(x^2-2x+4)}{x}$

① For x-int: $(x+2)(x^2-2x+4)=0$ ← complex roots

$\therefore x=-2$

$\therefore$ x-int is -2

② y-int: none

③ V.A. is $x=0$

④ For Q.O.A: $g(x)=\dfrac{x^3+8}{x}$

$g(x)=x^2+\dfrac{8}{x}$

As $x\to\pm\infty$, $g(x)\to x^2$

$\therefore$ Q.O.A is $y=x^2$

⑤ Does $g(x)$ cross the Q.O.A?

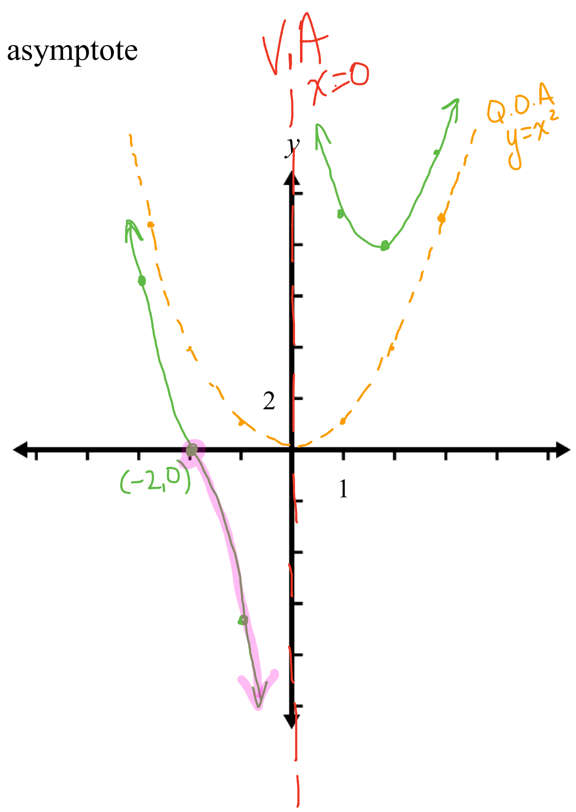
$\dfrac{x^3+8}{x}=x^2$

$x^3+8=x^3$ → $8=0$

$\therefore$ does not cross.

⑥

x	$g(x)$
-3	$\frac{19}{3}$
-1	-7
1	9
2	8
3	$\frac{35}{3}$



6. Using the graphs from the previous question, solve the following inequalities: Answer using interval notation.

a) $f(x)\ge 0$

$x\in[-1,1)\cup(1,2]$

b) $g(x)\le 0$

$x\in[2,0)$

HW. Exercise 2.12

Date:________________

2.13 Solving Rational Inequalities Graphically

Ex. 1. Solve the following rational inequalities graphically. State your final answer in a solution set.

a) $\dfrac{x^2-x-2}{x-1} \geq 0$

Let $f(x) = \dfrac{x^2-x-2}{x-1}$

$f(x) = \dfrac{(x-2)(x+1)}{x-1}$

① x-ints are $-1, 2$

② y-int is 2

③ V.A. is $x=1$

④ For L.O.A.

$$\begin{array}{r} x \\ x-1\overline{)\,x^2-x-2} \\ \underline{x^2-x} \\ 0-2 \end{array}$$

$f(x) = x + \dfrac{-2}{x-1}$

As $x \to \pm\infty$, $f(x) \to x$

$\therefore$ L.O.A is $y=x$

⑤ Does $f(x)$ cross the L.O.A?

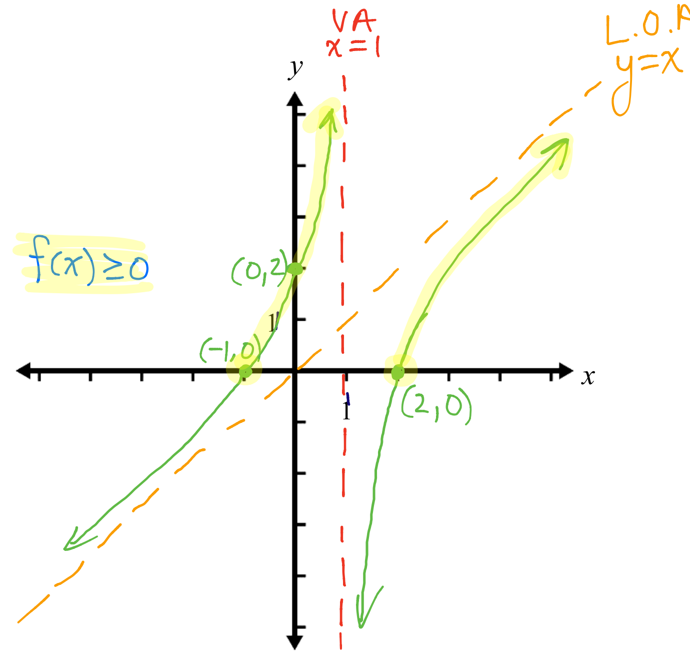
$\dfrac{x^2-x-2}{x-1} = \dfrac{x}{1}$

$x^2-x-2 = x^2-x$

$-2=0$

$\therefore f(x)$ does not cross the L.O.A.

$f(x) \geq 0$



SS. $=\{x \in \mathbb{R} \mid -1 \leq x < 1,\ x \geq 2\}$

b) $\dfrac{x+1}{x-2} < \dfrac{x+7}{x+1}$

$\dfrac{(x+1)(x+1)}{(x+1)(x-2)} - \dfrac{(x+7)(x-2)}{(x+1)(x-2)} < 0$

$\dfrac{(x+1)(x+1)-(x+7)(x-2)}{(x-2)(x+1)} < 0$

$\dfrac{(x^2+2x+1)-(x^2+5x-14)}{(x-2)(x+1)} < 0$

$\dfrac{-3x+15}{(x-2)(x+1)} < 0$

$\dfrac{-3(x-5)}{(x-2)(x+1)} < 0$

Let $f(x) = \dfrac{-3(x-5)}{(x-2)(x+1)}$

$f(x) = \dfrac{-3x+15}{x^2-x-2}$ ← H.A./L.O.A

For L.O.A.

$$\begin{array}{r} x \\ x-1\overline{)\,x^2-x-2} \\ \underline{x^2-x} \\ 0-2 \end{array}$$

$f(x) = x + \dfrac{-2}{x-1}$

As $x \to \pm\infty$, $f(x) \to x$

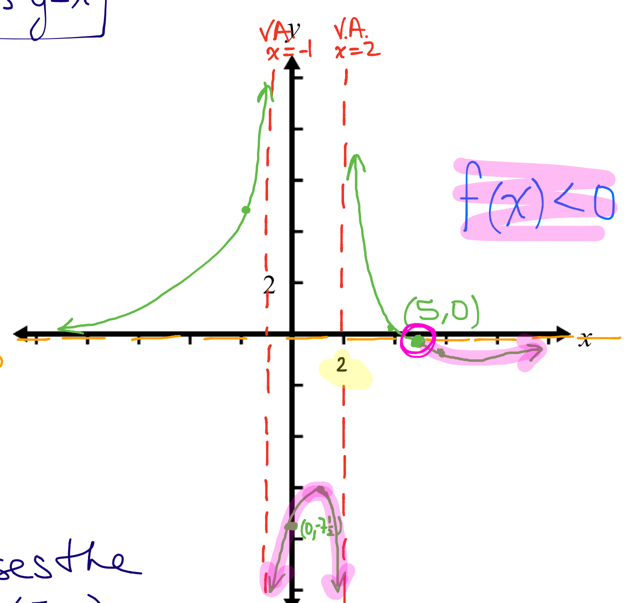
$\boxed{\therefore \text{L.O.A is } y=x}$

⑥

x	$f(x)$
-2	$\frac{21}{4} = 5\frac{1}{4}$
1	-6
4	$\frac{3}{10}$
6	$-\frac{3}{28}$

H.A.
$y=0$

$f(x) < 0$



① x-int is 5

② y-int is $-7\frac{1}{2}$

③ V.A. are $x=2$, $x=-1$

④ For H.A.: $f(x) = \dfrac{-3x+15}{x^2-x-2} \begin{array}{l} \div x^2 \\ \div x^2 \end{array}$

$f(x) = \dfrac{-\frac{3}{x}+\frac{15}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}}$

As $x \to \pm\infty$, $f(x) \to 0$

$\therefore$ the H.A. is $y=0$.

⑤ $f(x)$ crosses the H.A. at $(5,0)$

Since H.A is $y=0$ (x-axis) and x-int at $x=5$

$SS = \{x \in \mathbb{R} \mid -1 < x < 2,\ x > 5\}$

c) $-\frac{1}{(2-x)^2} \le -1$

$$-\frac{1}{(2-x)^2} + 1 \le 0$$

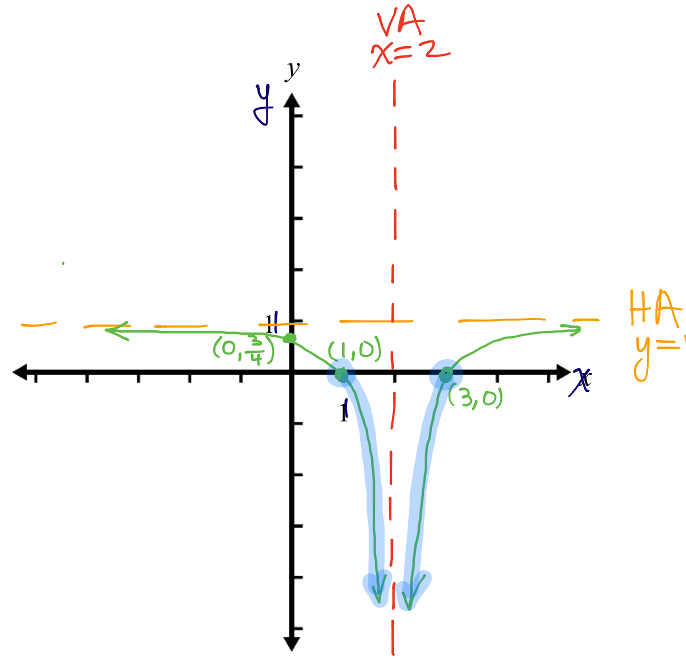
$$\frac{1}{1}\cdot\frac{(2-x)^2}{(2-x)^2} - \frac{1}{(2-x^2)} \le 0$$

$$\frac{4-4x+x^2-1}{4-4x+x^2} \le 0$$

$$\frac{x^2-4x+3}{x^2-4x+4} \le 0$$

Let $f(x) = \frac{x^2-4x+3}{x^2-4x+4}$

$$f(x) = \frac{(x-3)(x-1)}{(x-2)^2}$$



① x-ints are 3 and 1

② y-int is $\frac{3}{4}$

③ V.A. is $x=2$

④ For H.A.: $f(x) = \frac{1-\frac{4}{x}+\frac{3}{x^2}}{1-\frac{4}{x}+\frac{4}{x^2}}$

As $x \to \pm\infty$, $f(x) \to 1$

$\therefore$ H.A is $y=1$

⑤ Does $f(x)$ cross H.A.?

$$\frac{1}{1} = \frac{x^2-4x+3}{x^2-4x+4}$$

$$x^2-4x+4 = x^2-4x+3$$

$$4=3$$

$\therefore$ does not cross

$SS = \{x \in \mathbb{R} \mid 1 \le x < 2$ or $2 < x \le 3\}$

OR

$SS = \{x \in \mathbb{R} \mid 1 \le x \le 3, x \ne 2\}$

HW. Exercise 2.13

Date:_______________ **2.14 Graphing and Analyzing Polynomial & Rational Functions With Removable and or Infinite Discontinuities**

Examples

For each function given below complete the following.

a) Simplify.
b) State all values of x for which the function is discontinuous.
c) Graph.
d) Examine how the function behaves near these discontinuities and at the ends of the graph.

1. $f(x) = \dfrac{9-x^2}{x+3}$

a) $f(x) = \dfrac{(3-x)\cancel{(3+x)}}{\cancel{x+3}}$

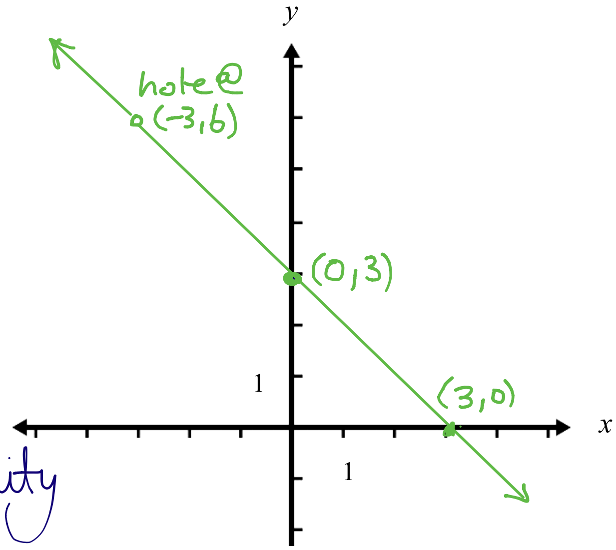
$\boxed{f(x) = -x+3}$, hole @ $x=-3$
$(-3, 6)$

b) $f(x)$ is discontinuous at $x=-3$ (removable)

c) As $x \to -3^-$, $f(x) \to 6$
As $x \to -3^+$, $f(x) \to 6$ } behaviour near the discontinuity

As $x \to -\infty$, $f(x) \to +\infty$
As $x \to +\infty$, $f(x) \to -\infty$ } end behaviour

c)



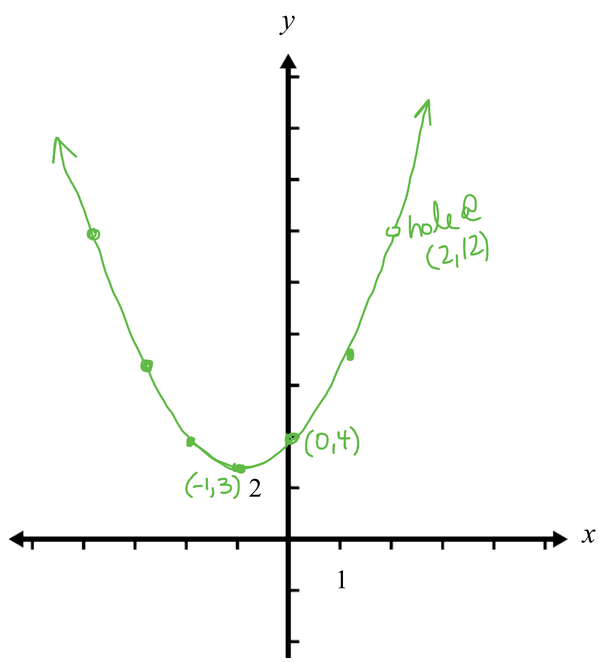
2. $f(x) = \dfrac{x^3-8}{x-2}$

a) $f(x) = \dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$\boxed{f(x) = x^2+2x+4}$, hole @ $x=2$
$(2, 12)$

b) $f(x)$ is discontinuous at $x=2$ (removable)

c) Graph: $x = \dfrac{-b}{2a}$
$x = \dfrac{-2}{2(1)}$
$x = -1$
$f(-1) = 3$
$\therefore$ Vertex @ $(-1, 3)$



d) As $x \to 2^-$, $f(x) \to 12$
As $x \to 2^+$, $f(x) \to 12$
As $x \to -\infty$, $f(x) \to +\infty$
As $x \to +\infty$, $f(x) \to +\infty$

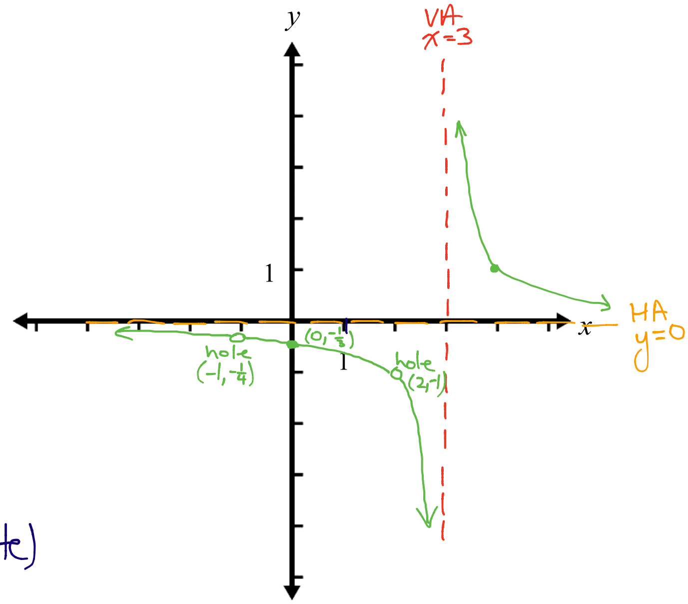
3. $g(x)=\dfrac{x^2-x-2}{x^3-4x^2+x+6}$ ** long division

a) $g(x)=\dfrac{(x-2)(x+1)}{(x+1)(x^2-5x+6)}$

$g(x)=\dfrac{\cancel{(x-2)}\cancel{(x+1)}}{\cancel{(x+1)}(x-3)\cancel{(x-2)}}$

$\boxed{g(x)=\dfrac{1}{x-3}}$, hole @ $x=-1$; $x=2$

$(-1,-\frac{1}{4})$ $(2,-1)$



b) $g(x)$ is discontinuous at $\underbrace{x=-1,\ x=2}_{\text{removable}}$ and $\underbrace{x=3}_{\text{infinite (asymptote)}}$

d) As $x\to-1^-$, $g(x)\to-\frac{1}{4}$

As $x\to-1^+$, $g(x)\to-\frac{1}{4}$

As $x\to2^-$, $g(x)\to-1$

As $x\to2^+$, $g(x)\to-1$

As $x\to3^-$, $g(x)\to-\infty$

As $x\to3^+$, $g(x)\to+\infty$

As $x\to-\infty$, $g(x)\to0$

As $x\to+\infty$, $g(x)\to0$

**

$$\begin{array}{r}
x^2-5x+6 \\
x+1\,\overline{)\,x^3-4x^2+x+6} \\
\underline{x^3+x^2} \\
-5x^2+x \\
\underline{-5x^2-5x} \\
6x+6 \\
\underline{6x+6} \\
0
\end{array}$$

For graph: ① x-int: none

② y-int: $-\frac{1}{3}$

③ V.A: $x=3$ is the V.A.

⑥

x	$g(x)$
4	1

④ For H.A. $y=0$ is the H.A.

As $x\to\pm\infty$, $g(x)=\frac{1}{x-3}\to0$

$\therefore$ H.A. is $y=0$

⑤ $g(x)$ does not cross the H.A.

since H.A is $y=0$ (x-axis) and no x-int

HW. Exercise 2.14

HW. for Unit 2 Part II Test: Unit 2 Review of 2.8-2.14