### 2.8 Solving Polynomial Inequalities Graphically

Ex. 1. Use the graphs of the following functions to state when i) $f(x)>0$ ii) $f(x)<0$ Answer using algebraic notation.
a)

b)

i) $\qquad$
i) $\qquad$
ii) $\qquad$
ii) $\qquad$

Ex. 2. Solve each of the following graphically where, $x \in R$. Answer using a solution set.
a) $x^{2}-3 x-10 \geq 0$
b) $x^{3}+x^{2}-4 x-4<0$

Ex. 3. Solve each of the following graphically where, $x \in R$. Answer using interval notation.
a) $x^{4}-10 x^{2}+9 \leq 0$
b) $x^{5}-6 x^{4}+8 x^{3}-2 x^{2}-2>-4 x^{3}+6 x^{2}-2$

### 2.9 Solving Polynomial \& Rational Inequalities Using a Number Line Strategy

Warmup: Solve the following polynomial inequality graphically.

$$
4 x^{3}+12 x^{2}-3 x-9 \geq 0
$$

Ex. 1. Solve the following polynomial inequalities using a number line strategy. State your final answer using set notation.
a) $(x+1)(x-2)(x+3)^{2} \leq 0$
$\qquad$
b) $2 x^{3}+3 x^{2}>17 x-12$

Ex. 2. Solve the following rational inequalities using a number line strategy. State your final answer using interval notation.
a) $x-2<\frac{8}{x}$
b) $\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$

A rational function is of the form $f(x)=\frac{p(x)}{q(x)}$ and has:
i) a vertical asymptote at $x=a$ if $q(a)=0$ and $p(a) \neq 0$

For the vertical asymptote, set the denominator equal to 0 and solve.
\&
ii) a horizontal asymptote at $y=L$ if $f(x) \rightarrow L$ as $x \rightarrow \pm \infty$ and the degree of $p(x)$ is less than or equal to the degree of $q(x)$ For the horizontal asymptote, divide each term in the function's expanded numerator and denominator by the highest power of $x$ in the denominator and then examine end behaviour.

Ex. 1. Graph the following rational functions by finding and labeling any intercepts, asymptotes and points where the function crosses the horizontal asymptote. Include a table of values for a more accurate graph if appropriate.
a) $f(x)=\frac{2(x-2)(x-1)}{x^{2}-2 x-3}$

b) $f(x)=\frac{4}{x^{2}+2}$

c) $f(x)=\frac{x+2}{x-1}$

d) $f(x)=\frac{1}{4-x^{2}}$


A rational function of the form $f(x)=\frac{p(x)}{q(x)}$ has:
i) a vertical asymptote at $x=a$ if $q(a)=0$ and $p(a) \neq 0$

For the vertical asymptote, set the denominator equal to 0 and solve.
\&
ii) a linear oblique asymptote at $y=m x+b$ if $f(x) \rightarrow m x+b$ as $x \rightarrow \pm \infty$ and the degree of $p(x)$ is exactly one more than the degree of $q(x)$ For the linear oblique asymptote, rewrite the function in mixed rational form using long division and then examine end behaviour.

Ex. 1. Graph the following rational function by finding and labeling any intercepts, asymptotes and points where the function crosses the linear oblique asymptote. Include a table of values for a more accurate graph if appropriate.
a) $f(x)=\frac{9-x^{2}}{x+1}$

c) $f(x)=\frac{x^{2}+x+1}{x}$

d) $f(x)=\frac{x^{3}}{x^{2}-4}$

$\qquad$

### 2.12 Graphing Rational Functions Continued

1. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.
a) $f(x)=\frac{2 x-1}{x+1}$

b) $g(x)=\frac{6}{x^{2}+2 x-3}$

2. Using the graphs from the previous question, solve the following inequalities:
a) $f(x)>0$
b) $g(x)<0$
3. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.
a) $f(x)=\frac{3 x-6}{x^{2}-2 x-8}$

b) $g(x)=\frac{6 x^{2}-5 x+1}{2 x+1}$

4. Using the graphs from the previous question, solve the following inequalities:

Answer using a solution set.
a) $f(x) \geq 0$
b) $g(x) \leq 0$
5. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.
a) $f(x)=\frac{2+x-x^{2}}{(x-1)^{2}}$

b) $g(x)=\frac{x^{3}+8}{x} \quad * * *$ this graph has a quadratic oblique asymptote

6. Using the graphs from the previous question, solve the following inequalities:

Answer using interval notation.
a) $f(x) \geq 0$
b) $g(x) \leq 0$

### 2.13 Solving Rational Inequalities Graphically

Ex. 1. Solve the following rational inequalities graphically. State your final answer in a solution set.
a) $\frac{x^{2}-x-2}{x-1} \geq 0$

b) $\frac{x+1}{x-2}<\frac{x+7}{x+1}$


$$
\text { c) }-\frac{1}{(2-x)^{2}} \leq-1
$$


$\qquad$

### 2.14 Graphing and Analyzing Polynomial \& Rational Functions With Removable and or Infinite Discontinuities

## Examples

For each function given below complete the following.
a) Simplify.
b) State all values of $x$ for which the function is discontinuous.
c) Graph.
d) Examine how the function behaves near these discontinuities and at the ends of the graph.

1. $f(x)=\frac{9-x^{2}}{x+3}$

2. $f(x)=\frac{x^{3}-8}{x-2}$

3. $g(x)=\frac{x^{2}-x-2}{x^{3}-4 x^{2}+x+6}$

