b) $x^3 + x^2 - 4x - 4 < 0$

Ex. 1. Use the graphs of the following functions to state when i) f(x) > 0 ii) f(x) < 0Answer using *algebraic notation*.



Ex. 2. Solve each of the following graphically where, $x \in R$. Answer using a *solution set*. a) $x^2 - 3x - 10 \ge 0$



Ex. 3. Solve each of the following graphically where, $x \in R$. Answer using *interval notation*. a) $x^4 - 10x^2 + 9 \le 0$

```
b) x^5 - 6x^4 + 8x^3 - 2x^2 - 2 > -4x^3 + 6x^2 - 2
```

2.9 Solving Polynomial & Rational Inequalities Using a Number Line Strategy



Ex. 1. Solve the following polynomial inequalities using a *number line strategy*. State your final answer using *set notation*.

_____ *x*

a) $(x+1)(x-2)(x+3)^2 \le 0$

b) $2x^3 + 3x^2 > 17x - 12$

Ex. 2. Solve the following *rational inequalities* using a *number line strategy*. State your final answer using *interval notation*.

$$a) \quad x-2 < \frac{8}{x}$$

b)
$$\frac{x+3}{x+1} \ge \frac{x-2}{x-3}$$

- A *rational* function is of the form f(x) = p(x)/q(x) and has:
 i) a *vertical asymptote* at x = a if q(a) = 0 and p(a) ≠ 0 For the *vertical asymptote*, set the denominator equal to 0 and solve. &
 ii) a *horizontal asymptote* at y = L if f(x) → L as x → ±∞ and the degree of p(x) is less than or equal to the degree of q(x) For the *horizontal asymptote*, divide each term in the function's *expanded* numerator and denominator by the highest power of x in the denominator and then examine end behaviour.
- **Ex. 1.** Graph the following rational functions by finding and labeling any intercepts, asymptotes and points where the function crosses the horizontal asymptote. Include a table of values for a more accurate graph if appropriate.

a)
$$f(x) = \frac{2(x-2)(x-1)}{x^2 - 2x - 3}$$



b)
$$f(x) = \frac{4}{x^2 + 2}$$

c)
$$f(x) = \frac{x+2}{x-1}$$

d)
$$f(x) = \frac{1}{4 - x^2}$$



A *rational* function of the form $f(x) = \frac{p(x)}{q(x)}$ has:

- i) a *vertical asymptote* at x = a if q(a) = 0 and p(a) ≠ 0 For the *vertical asymptote*, set the denominator equal to 0 and solve. &
 ii) a *linear oblique asymptote* at y = mx+b if f(x) → mx+b as x → ±∞
 - and the degree of p(x) is exactly one more than the degree of q(x)For the *linear oblique asymptote*, rewrite the function in mixed rational form using long division and then examine end behaviour.
- **Ex. 1.** Graph the following rational function by finding and labeling any intercepts, asymptotes and points where the function crosses the linear oblique asymptote. Include a table of values for a more accurate graph if appropriate.

a)
$$f(x) = \frac{9 - x^2}{x + 1}$$



$$\mathbf{c}) \quad f(x) = \frac{x^2 + x + 1}{x}$$



$$\mathbf{d}) \quad f(x) = \frac{x^3}{x^2 - 4}$$

HW. Exercise 2.11

1. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.



2. Using the graphs from the previous question, solve the following inequalities: a) f(x) > 0b) g(x) < 0 **3.** Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.

a)
$$f(x) = \frac{3x-6}{x^2-2x-8}$$

b) $g(x) = \frac{6x^2-5x+1}{2x+1}$

4. Using the graphs from the previous question, solve the following inequalities: Answer using a solution set.

a) $f(x) \ge 0$ **b**) $g(x) \le 0$

5. Graph the following rational functions by finding and labeling any asymptotes and intercepts. Include a table of values for a more accurate graph.

a)
$$f(x) = \frac{2 + x - x^2}{(x - 1)^2}$$

b)
$$g(x) = \frac{x^3 + 8}{x}$$
 ***this graph has a *quadratic* oblique asymptote

6. Using the graphs from the previous question, solve the following inequalities: Answer using interval notation.

a) $f(x) \ge 0$ **b**) $g(x) \le 0$

2.13 Solving Rational Inequalities Graphically

Ex. 1. Solve the following rational inequalities graphically. State your final answer in a solution set.

a)
$$\frac{x^2 - x - 2}{x - 1} \ge 0$$

b) $\frac{x+1}{x-2} < \frac{x+7}{x+1}$

c)
$$-\frac{1}{(2-x)^2} \le -1$$

Date:____

2.14 Graphing and Analyzing Polynomial & Rational Functions With Removable and or Infinite Discontinuities

Examples

For each function given below complete the following.

- **a**) Simplify.
- **c**) Graph.

- **b**) State all values of *x* for which the function is discontinuous.
- **d**) Examine how the function behaves near these discontinuities and at the ends of the graph.

1.
$$f(x) = \frac{9 - x^2}{x + 3}$$

2. $f(x) = \frac{x^3 - 8}{x - 2}$

3.
$$g(x) = \frac{x^2 - x - 2}{x^3 - 4x^2 + x + 6}$$

