Date: Oct 29/14

TRIGONOMETRIC IDENTITIES

In terms of $\sin\theta$ and $\cos\theta$:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\therefore \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$, etc...

$\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$

Ex. 1. Prove the following identities.

a) $\frac{1+\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$

$LS = \frac{1+\cot\theta}{\csc\theta}$

$= \frac{1+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} \cdot \frac{\sin\theta}{\sin\theta}$

$= \frac{\sin\theta + \cos\theta}{1}$

$= \sin\theta + \cos\theta$

$RS = \sin\theta + \cos\theta$

$\because LS = RS$

$\therefore \frac{1+\cot\theta}{\csc\theta} = \sin\theta + \cos\theta \quad \square$

b) $\cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta$

$LS = \cot^2\theta + \sec^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta} + \frac{1}{\cos^2\theta}$

$= \frac{\cos^2\theta \cdot \cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}$

$= \frac{(1-\sin^2\theta)(1-\sin^2\theta) + \sin^2\theta}{\sin^2\theta\cos^2\theta}$

$= \frac{1 - 2\sin^2\theta + \sin^4\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}$

$= \frac{\sin^4\theta - \sin^2\theta + 1}{\sin^2\theta\cos^2\theta}$

$RS = \tan^2\theta + \csc^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \frac{\sin^2\theta \cdot \sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$

$= \frac{\sin^4\theta + (1-\sin^2\theta)}{\sin^2\theta\cos^2\theta}$

$= \frac{\sin^4\theta - \sin^2\theta + 1}{\sin^2\theta\cos^2\theta}$

$\because LS = RS$

$\therefore \cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta \quad \square$

c) $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

$$LS = \cos^4 x - \sin^4 x$$
$$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$$
$$= \cos^2 x - \sin^2 x$$
$$= (1 - \sin^2 x) - \sin^2 x$$
$$= 1 - 2\sin^2 x$$

$$RS = 1 - 2\sin^2 x$$

$\because LS = RS$

$\therefore \cos^4 x - \sin^4 x = 1 - 2\sin^2 x \; \square$

$$= \cos^2 x \cdot \cos^2 x - \sin^4 x$$
$$= (1 - \sin^2 x)(1 - \sin^2 x) - \sin^4 x$$
$$= 1 - 2\sin^2 x + \sin^4 x - \sin^4 x$$
$$= 1 - 2\sin^2 x$$

d) $\dfrac{\sin t - \cos t}{\cos t} + \dfrac{\sin t + \cos t}{\sin t} = \sec t \csc t$

Don't like t? Let $t = \theta$

$$LS = \frac{\sin t - \cos t}{\cos t} + \frac{\sin t + \cos t}{\sin t}$$

$$RS = \sec t \csc t$$

$$= \frac{\sin^2 t - \sin t\cos t + \sin t \cos t + \cos^2 t}{\sin t \cos t}$$

$$= \frac{\sin^2 t + \cos^2 t}{\sin t \cos t}$$

$$= \frac{1}{\sin t \cos t}$$

$$= \frac{1}{\sin t} \cdot \frac{1}{\cos t} \rightarrow = \sec t \csc t$$

$\because LS = RS$

$\therefore$ I'm lazy

Q.E.D.

e) $\dfrac{1}{1 + \sin\phi} = \sec^2\phi - \dfrac{\tan\phi}{\cos\phi}$

$\phi = $ phi

$$LS = \frac{1}{1 + \sin\phi}$$

$$RS = \sec^2\phi - \frac{\tan\phi}{\cos\phi}$$

$$= \frac{1}{\cos^2\phi} - \frac{\frac{\sin\phi}{\cos\phi}}{\cos\phi} \cdot \frac{\cos\phi}{\cos\phi}$$

$$= \frac{1}{\cos^2\phi} - \frac{\sin\phi}{\cos^2\phi}$$

$$= \frac{1 - \sin\phi}{\cos^2\phi}$$

$$= \frac{1 - \sin\phi}{1 - \sin^2\phi}$$

$$= \frac{\cancel{1 - \sin\phi}\;1}{\cancel{(1 - \sin\phi)}\,1\,(1 + \sin\phi)}$$

$$= \frac{1}{1 + \sin\phi}$$

$\because LS = RS$

$\therefore$ Q.E.D.

HW. Worksheet on Proving Trigonometric Identities #3 to 27 odds

Date: Oct 30/14 **Trigonometric Identities Involving Compound Angle Formulas**

Ex. 1. Prove the following trigonometric identities.

a) $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

$LS = \dfrac{\sin 2x}{1+\cos 2x}$

$RS = \tan x$

$= \dfrac{2\sin x\cos x}{1+(2\cos^2 x-1)}$

$= \dfrac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x}$

$\therefore LS = RS$

$= \dfrac{\sin x}{\cos x}$

$\therefore$ QED.

$= \tan x$

b) $\tan x + \tan y = \dfrac{\sin(x+y)}{\cos x\cos y}$

$LS = \tan x + \tan y$

$RS = \dfrac{\sin(x+y)}{\cos x\cos y}$

$= \dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y}$

$= \dfrac{\sin x\cos y + \sin y\cos x}{\cos x\cos y}$

$= \dfrac{\sin(x+y)}{\cos x\cos y}$

c) $\sin 3\theta + \sin\theta = 2\sin 2\theta\cos\theta$

$LS = \sin 3\theta + \sin\theta$
$= \sin(2\theta+\theta) + \sin\theta$
$= \sin 2\theta\cos\theta + \sin\theta\cos 2\theta + \sin\theta$
$= 2\sin\theta\cos\theta\cos\theta + \sin\theta(1-2\sin^2\theta) + \sin\theta$
$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta + \sin\theta$
$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta + \sin\theta$
$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta + \sin\theta$
$= 4\sin\theta - 4\sin^3\theta$

$RS = 2\sin 2\theta\cos\theta$
$= 2(2\sin\theta\cos\theta)\cos\theta$
$= 4\sin\theta\cos^2\theta$
$= 4\sin\theta(1-\sin^2\theta)$
$= 4\sin\theta - 4\sin^3\theta$

$\because LS = RS$

$\therefore$ Q.E.D.

d) $\tan\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\csc\theta$ Let $\beta = \frac{\theta}{2}$, so $2\beta = \theta$

$\tan\beta + \cot\beta = 2\csc 2\beta$.

$LS = \tan\beta + \cot\beta$
$= \frac{\sin\beta}{\cos\beta} + \frac{\cos\beta}{\sin\beta}$
$= \frac{\sin^2\beta + \cos^2\beta}{\sin\beta\cos\beta}$
$= \frac{1}{\sin\beta\cos\beta}$

$RS = 2\csc 2\beta$
$= \frac{2}{\sin 2\beta}$
$= \frac{\cancel{2}\,1}{\cancel{2}\,1\sin\beta\cos\beta}$
$= \frac{1}{\sin\beta\cos\beta}$

$\because LS = RS$

$\therefore$ QED.

HW. Trigonometric Identities Involving Compound Angle Formulas Worksheet

For Unit 3 Test: do Day 10 Unit 3 Review Sheet