

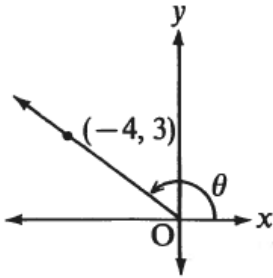
Date: _____ **UNIT 3: TRIGONOMETRIC FUNCTIONS & EQUATIONS**

3.1 The Definitions of Trigonometry

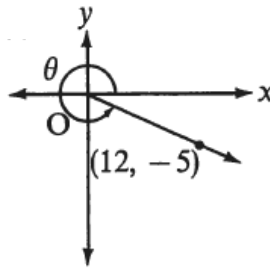
Express answers for trigonometric values in exact simplified fractional or radical form.

1. For each angle, a point on the terminal arm is shown. Calculate the **primary** trigonometric values for **a)** and **b)**. Calculate the **reciprocal** trigonometric values for **c)** and **d)**.

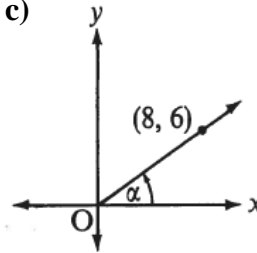
a)



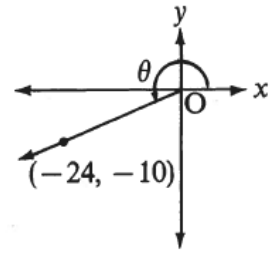
b)



c)



d)



2. The point $P(1,7)$ lies on the terminal arm of θ . For θ , find the
a) sine **b)** cosine **c)** tangent
3. The point $Q(-2,6)$ lies on the terminal arm of α . For α , calculate its
a) cosecant **b)** secant **c)** cotangent
4. **a)** Angle θ is a second quadrant angle, and $\cos \theta = -\frac{3}{4}$. Sketch a detailed diagram.
b) Find the other primary trigonometric values of θ .
5. **a)** Angle θ is in the third quadrant and $\tan \theta = \frac{3}{4}$. Find the reciprocal trigonometric values.
b) Angle α is in the fourth quadrant and $\cos \alpha = \frac{8}{15}$. Find the other trigonometric values.
6. **a)** θ is a first quadrant angle. If $\cos \theta = \frac{1}{\sqrt{5}}$, find $\sin \theta$ and $\sec \theta$.
b) β is a second quadrant angle. If $\tan \beta = -\frac{7}{\sqrt{65}}$, find $\cos \beta$ and $\csc \beta$.
7. Sketch each angle in standard position. Calculate the other trigonometric values of each angle.
a) $\sec \theta = -\frac{13}{12}$, **b)** $\sin \alpha = \frac{7}{25}$, **c)** $\csc \theta = -\frac{17}{8}$,
 θ in the third quadrant α in the first quadrant θ in the fourth quadrant
8. $P(x, y)$ is a point on the terminal arm of α and $OP = r$.
a) Find the possible values of k in each of the following.
b) Find the primary trigonometric values for each value of k .
i) $P(3, k)$, $r = 5$ **ii)** $P(k, 8)$, $r = 10$ **iii)** $P(3, k)$, $r = \sqrt{13}$

9. As in algebra, $\sin^2 \alpha = (\sin \alpha)^2$, α is a third quadrant angle and $\tan \alpha = \frac{5}{12}$. Find values for

a) $\sin^2 \alpha + \cos^2 \alpha$ b) $\sec^2 \alpha - \tan^2 \alpha$

10. β is a second quadrant angle and $\csc \beta = \frac{17}{15}$. Find a value for $2\sin \beta + 3\cos \beta$.

11. For any angle θ , show that

a) $\csc \theta = \frac{1}{\sin \theta}$ b) $\cos \theta = \frac{1}{\sec \theta}$ c) $\cot \theta = \frac{\cos \theta}{\sin \theta}$ d) $\cos^2 \theta = 1 - \sin^2 \theta$

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3.2 Angles and Quadrants

- Sketch each angle in standard position and determine the related acute angle.
 a) 135° b) 315° c) 210° d) -30° e) -225° f) 150° g) -330° h) 120°
- Calculate the next two positive coterminal angles.
 a) 132° b) 275° c) 305° d) 73° e) 270°
- Calculate the next two negative coterminal angles.
 a) -53° b) -138° c) -299° d) -180° e) -192°
- Determine the principal angle.
 a) -187° b) 410° c) -67° d) 905° e) -282° f) -730° g) 135° h) 1249°
- State the principal angle for the given related acute angle and given quadrant.
 a) 24° , quadrant II b) 35° , quadrant III c) 19° , quadrant IV d) 63° , quadrant I
- The point $P(7, -24)$ is on the terminal arm of an angle in standard position.
 a) Sketch the principal angle, θ .
 b) What is the related acute angle to the nearest degree? *Calculator*
 c) What is the measure of θ to the nearest degree?
- β is an angle in standard position and $\sin \beta = \frac{4}{5}$.
 a) In which quadrants is it possible for the terminal arm to lie?
 b) Draw a diagram to show each case in a).
 c) Calculate values for $\cos \beta$ and $\tan \beta$.
 d) Calculate values for β to the nearest degree. *Calculator*
- You know that $\cot \alpha = -\frac{24}{7}$.
 a) In which quadrants is it possible for the terminal arm to lie?
 b) Draw a diagram to show each case in a).
 c) Calculate values for $\sin \alpha$ and $\cos \alpha$.
 d) Calculate values for α to the nearest degree. *Calculator*
- Examine the possibilities of each of the following. *Calculator for angles only.*
 a) If $\sin \theta = \frac{-8}{17}$, find two values of $\cos \theta$ and θ to the nearest degree.
 b) Given that $\cot \alpha = -\frac{12}{5}$, find two values of $\sin \alpha$ and α to the nearest degree.
 c) For $\sec \beta = -\frac{25}{7}$, find $\tan \beta$ and β to the nearest degree.
 d) θ is in standard position. If $\cos \theta = \frac{\sqrt{3}}{2}$, find $\cot \theta$ and θ to the nearest degree.

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3.3 Radian Measure

1. Write each radian measure as a degree measure.

a) $\frac{\pi}{3}$

b) $\frac{\pi}{2}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{6}$

e) $\frac{3\pi}{4}$

f) $-\frac{\pi}{2}$

g) $-\frac{2\pi}{3}$

h) $\frac{5\pi}{6}$

i) $\frac{4}{3}\pi$

j) $-\frac{3}{4}\pi$

k) $-\frac{5}{3}\pi$

l) $\frac{3\pi}{2}$

m) 2π

n) -4π

o) 3π

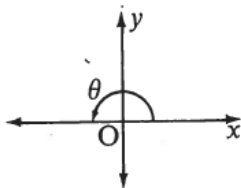
2. Write each degree measure as an exact radian measure.

a) 180° b) 360° c) 90° d) 45° e) -60° f) -150°

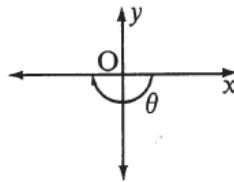
g) 30° h) 240° i) -330° j) 270° k) -90° l) 120°

3. Write the value of θ in exact radian measure.

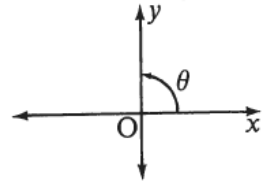
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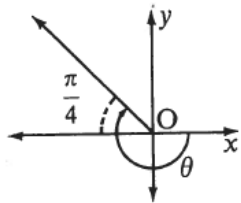
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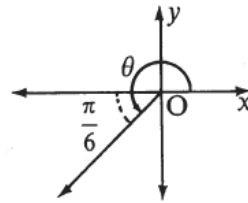
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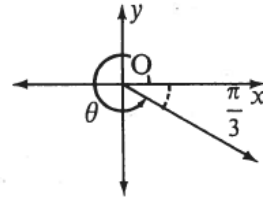
d)



e)



f)



4. In which quadrant does the terminal arm of each angle lie?

a) $\frac{3\pi}{4}$

b) $-\frac{2}{3}\pi$

c) $\frac{7\pi}{6}$

d) $-\frac{7}{4}\pi$

e) $-\frac{\pi}{3}$

f) $\frac{7\pi}{4}$

5. Sketch each angle in standard position.

a) π

b) $\frac{1}{2}\pi$

c) $-\frac{\pi}{4}$

d) $\frac{5}{4}\pi$

e) $\frac{2\pi}{3}$

f) $-\frac{5\pi}{6}$

6. Convert each of the following from degree measure to exact radian measure.

a) $\frac{1}{2}$ revolution

b) $\frac{2}{3}$ revolution

a) 2 revolutions

7. Each radian measure is given to 1 decimal place. Express to the nearest degree. *Calculator*

a) 1.1

b) 0.9

c) -0.3

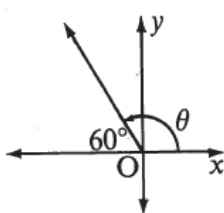
d) 1.5

e) -0.8

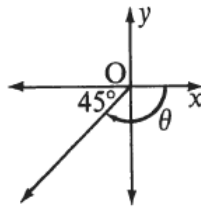
f) 0.4

8. Express the measure of angle θ in exact radian measure.

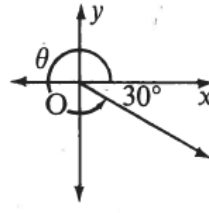
a)



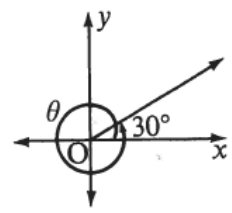
b)



c)



d)



9. Sector angles are drawn in a unit circle. Find the measure of the arc of the circle that subtends an angle measuring. (A unit circle has a radius of 1 unit.)

- a) 90° b) 30° c) 1 rad d) $\frac{\pi}{2}$ rad e) 2.6 rad *Calculator*

10. A point is given on the terminal arm of each angle. Calculate the measure of the principal angle in radians to 1 decimal place. *Calculator*

- a) (2,3) b) (-3,1) c) (-2,-5) d) (3,-5)

11. The radian measures of angles are shown. Write the measure of the coterminal angle θ , $-2\pi \leq \theta \leq 2\pi$.

- a) $\frac{\pi}{3}$ b) $-\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $-\pi$ e) $\frac{3}{4}\pi$ f) $-\frac{3\pi}{2}$

12. Write each degree measure as a radian measure expressed to 4 decimal places. *Calculator*

- a) 1° b) 8° c) 25° d) 65° e) 170° f) -235°

13. Two angles α and $-\frac{3}{4}\pi$ are in standard position and have a terminal arm in common.

Find the measure of α if $-2\pi \leq \alpha \leq 2\pi$.

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3.4 Exact Trigonometric Ratios For Special Angles

1. a) Draw a sketch of $\frac{5}{3}\pi$ in standard position.

b) Calculate exact primary trigonometric values for $\frac{5}{3}\pi$.

2. a) Draw a sketch of -225° in standard position.

b) Calculate exact reciprocal trigonometric values of -225° .

3. Calculate each of the following.

a) $\sin\left(-\frac{\pi}{3}\right)$ b) $\sin 300^\circ$

c) What do you notice about your answers in a) and b)?
Give a reason for your answer.

4. Calculate each of the following.

a) $\cos\frac{5\pi}{4}$ b) $\sin 120^\circ$ c) $\tan\left(-\frac{5\pi}{6}\right)$ d) $\tan 135^\circ$
 e) $\sec\left(-\frac{\pi}{6}\right)$ f) $\cot\left(\frac{11\pi}{6}\right)$ g) $\sec\left(-\frac{\pi}{4}\right)$ h) $\tan(-225^\circ)$
 i) $\csc\left(\frac{4}{3}\pi\right)$ j) $\cos \pi$ k) $\sec\left(-\frac{3\pi}{2}\right)$ l) $\csc(-90^\circ)$

5. Calculate each of the following.

a) $\tan\left(\frac{13\pi}{6}\right)$ b) $\cos\left(-\frac{8}{3}\pi\right)$ c) $\tan 510^\circ$ d) $\csc\left(-\frac{11}{4}\pi\right)$
 e) $\tan 585^\circ$ f) $\cos 780^\circ$ g) $\cot\left(-\frac{7\pi}{2}\right)$ h) $\sec 540^\circ$

6. Calculate each of the following.

a) $\cos 45^\circ \cdot \sin 225^\circ + \cos 330^\circ$ b) $\csc 315^\circ \cdot \sin(-120^\circ) \cdot \cot 225^\circ$
 c) $\tan^2\left(\frac{5\pi}{4}\right) - \sin\frac{\pi}{3} \cdot \cos\frac{\pi}{6}$

7. θ is an angle in the second quadrant and $\cos \theta = -\frac{1}{2}$.

a) Find $\tan \theta$. b) What is the value of θ , $0^\circ \leq \theta \leq 360^\circ$?

8. For each of the following $0^\circ \leq \theta \leq 360^\circ$. Find possible values of θ .

a) $\cos \theta = \frac{\sqrt{3}}{2}$ b) $\sin \theta = -\frac{1}{2}$ c) $\cot \theta = 1$

9. For each of the following $0 \leq \theta \leq 2\pi$. Find possible values of θ .

a) $\csc \theta = -\sqrt{2}$ b) $\sec \theta = -2$ c) $\sin \theta = -\frac{1}{\sqrt{2}}$

Date: _____ **3.6 Using the Addition, Subtraction and Double-Angle Formulas**

1. Use the identity for $\cos(A \pm B)$ to calculate each quantity without using a calculator.

a) $\cos \frac{\pi}{12}$ b) $\cos \frac{5\pi}{12}$ c) $\cos \frac{11\pi}{12}$

2. Use the identity for $\sin(A \pm B)$ to calculate each quantity without using a calculator.

Rationalize the denominator of your answer.

a) $\sin \frac{7\pi}{12}$ b) $\sin \frac{13\pi}{12}$ c) $\sin \frac{19\pi}{12}$

3. Find the exact value of the following, leaving the answer in simplified radical form.

a) $\tan 165^\circ$ b) $\tan \frac{13\pi}{12}$

4. Express each quantity in terms of $\sin x$ and $\cos x$.

a) $\sin\left(\frac{\pi}{3} + x\right)$ b) $\cos\left(x + \frac{3\pi}{4}\right)$ c) $\cos\left(\frac{\pi}{4} - x\right)$ d) $\sin(2\pi - x)$

5. If $\cos A = \frac{1}{3}$, with $0 < A < \frac{\pi}{2}$, and $\sin B = \frac{1}{4}$, with $\frac{\pi}{2} < B < \pi$, calculate each quantity.

a) $\cos(A + B)$ b) $\sin(A + B)$ c) $\cos 2A$
c) $\sin 2B$ d) $\cos 4A$ e) $\sin 4B$

6. If $\sin A = -\frac{1}{\sqrt{5}}$, with $-\frac{\pi}{2} < A < 0$, and $\cos B = -\frac{1}{\sqrt{3}}$, with $\frac{\pi}{2} < B < \pi$, calculate each quantity.

a) $\cos(A - B)$ b) $\sin(A - B)$ c) $\cos(2A - B)$ d) $\sin(A - 2B)$

7. If $\tan A = \frac{1}{3}$ and $\pi < A < \frac{3\pi}{2}$, calculate each quantity. In what quadrant does the angle $2A$ lie?

a) $\sin 2A$ b) $\cos 2A$

8. If $\sec A = -2$ and $\frac{\pi}{2} < A < \pi$, calculate each quantity.

Rationalize the denominator of your answer.

a) $\sin\left(A + \frac{\pi}{4}\right)$ b) $\cos\left(A - \frac{\pi}{4}\right)$

Date: _____ **3.7 Addition, Subtraction and Double-Angle Formulas Continued**

1. Express as a single trigonometric ratio, then evaluate.

a) $\sin 45^\circ \cos 15^\circ - \cos 45^\circ \sin 15^\circ$

b) $\sin \frac{\pi}{4} \cos \frac{\pi}{12} + \cos \frac{\pi}{4} \sin \frac{\pi}{12}$

c) $\cos \frac{\pi}{4} \cos \frac{11\pi}{12} + \sin \frac{\pi}{4} \sin \frac{11\pi}{12}$

d) $\cos 65^\circ \cos 115^\circ - \sin 65^\circ \sin 115^\circ$

e) $\frac{\tan 80^\circ - \tan 320^\circ}{1 + \tan 80^\circ \tan 320^\circ}$

f) $2 \cos^2 \frac{\pi}{12} - 1$

g) $2 \sin\left(-\frac{\pi}{8}\right) \cos\left(-\frac{\pi}{8}\right)$

h) $\frac{\tan \frac{8\pi}{9} + \tan \frac{10\pi}{9}}{1 - \tan \frac{8\pi}{9} \tan \frac{10\pi}{9}}$

2. Express as a single trigonometric expression.

a) $\sin \theta \cos \beta - \cos \theta \sin \beta$

b) $\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$

c) $\cos A \cos 2A - \sin A \sin 2A$

d) $\sin M \sin N + \cos M \cos N$

e) $\sin x \sin y - \cos x \cos y$

f) $\frac{2 \tan(2x)}{1 - \tan^2(2x)}$

g) $\cos^2 x - \sin^2 x$

h) $\cos^2 x + \sin^2 x$

3. Express as a single sine or cosine expression.

a) $10 \sin x \cos x$

b) $5 \sin(2x) \cos(2x)$

c) $\sin(6x) \cos(6x)$

d) $4 \sin \frac{x}{2} \cos \frac{x}{2}$

e) $\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$

f) $2 \cos^2 5\theta - 1$

g) $1 - 2 \sin^2 \frac{2\theta}{3}$

h) $2 \cos^2(3\theta - 2) - 1$

4. Simplify.

a) $\sin\left(\frac{3\pi}{2} + x\right)$

b) $\cos\left(-\frac{\pi}{2} - x\right)$

c) $\frac{\sin(x - 30^\circ) + \cos(60^\circ - x)}{\sin x}$

d) $\frac{\tan\left(\frac{\pi}{4} - x\right) - \tan\left(\frac{\pi}{4} + x\right)}{\tan x}$

5. If θ is in the interval $\left[\frac{\pi}{2}, \pi\right]$, β is in the interval $\left[-\pi, -\frac{\pi}{2}\right]$, $\sec \theta = -\frac{5}{4}$, and $\cot \beta = \frac{5}{12}$,

Determine each value. Include two detailed diagrams.

a) $\sin(\theta + \beta)$

b) $\cos(\theta + \beta)$

c) $\sin 2\theta$

d) $\csc 2\theta$

e) $\cos \frac{\theta}{2}$

f) $\sin \frac{\theta}{2}$

6. Determine formulas for

a) $\sin 3\theta$ in terms of $\sin \theta$

b) $\cos 3\theta$ in terms of $\cos \theta$

c) $\tan 3\theta$ in terms of $\tan \theta$

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3.8 Trigonometric Identities

Prove the following trigonometric identities.

1. $\cos \theta \tan \theta = \sin \theta$
2. $\frac{\cot \theta}{\tan \theta} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}$
3. $\cot^2 \theta = \frac{\cos^2 \theta}{1 - \cos^2 \theta}$
4. $\frac{\csc \theta}{\sec \theta} = \cot \theta$
5. $(\sin t + \cos t)^2 = 1 + 2 \sin t \cos t$
6. $2 \sin^2 t - 1 = \sin^2 t - \cos^2 t$
7. $\frac{1}{\sin^2 t} + \frac{1}{\cos^2 t} = \frac{1}{\sin^2 t \cos^2 t}$
8. $\cos^2 t = \sin^2 t + 2 \cos^2 t - 1$
9. $\tan x = \tan^2 x \cot x$
10. $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$
11. $\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x} = 2 \sec^2 x$
12. $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$
13. $\cot^2 \theta + \sec^2 \theta = \tan^2 \theta + \csc^2 \theta$
14. $\frac{1 + 2 \sin \theta \cos \theta}{\sin \theta + \cos \theta} = \sin \theta + \cos \theta$
15. $\frac{\sec \theta + 1}{\sec \theta - 1} + \frac{\cos \theta + 1}{\cos \theta - 1} = 0$
16. $\frac{\tan \theta \sin \theta}{\tan \theta + \sin \theta} = \frac{\tan \theta - \sin \theta}{\tan \theta \sin \theta}$
17. $\frac{\csc \beta + \cot \beta}{\csc \beta - \cot \beta} = \frac{1 + 2 \cos \beta + \cos^2 \beta}{\sin^2 \beta}$
18. $\sin \beta \cos \beta \tan \beta = 1 - \cos^2 \beta$
19. $\sin \alpha + \tan \alpha = \tan \alpha (1 + \cos \alpha)$
20. $\tan \alpha + \cot \alpha = \sec \alpha \csc \alpha$
21. $\frac{\csc x}{\csc x - 1} + \frac{\csc x}{\csc x + 1} = 2 \sec^2 x$
22. $(1 - \cos^2 x)(1 + \cot^2 x) = 1$
23. $\sec^4 x - \tan^4 x = 1 + 2 \tan^2 x$
24. $\frac{\cos x}{\sec x} - \frac{\sin x}{\cot x} = \frac{\cos x \cot x - \tan x}{\csc x}$
25. $\sin^4 x - \cos^4 x = 1 - 2 \cos^2 x$
26. $\sec^2 x - \sin^2 x = \cos^2 x + \tan^2 x$

Date: _____ **3.9 Trigonometric Identities Involving Compound Angle Formulas**

Prove the following trigonometric identities.

$$1. \frac{\cos 2x + 1}{\sin 2x} = \cot x$$

$$2. \frac{\sin 2x}{1 - \cos 2x} = \cot x$$

$$3. (\sin x + \cos x)^2 = 1 + \sin 2x$$

$$4. \cos^4 x - \sin^4 x = \cos 2x$$

$$5. \cot \theta - \tan \theta = 2 \cot 2\theta$$

$$6. \cot \theta + \tan \theta = 2 \csc 2\theta$$

$$7. \frac{1 + \tan \theta}{1 - \tan \theta} = \tan \left(\theta + \frac{\pi}{4} \right)$$

$$8. \csc 2\theta + \cot 2\theta = \cot \theta$$

$$9. \sec 2t = \frac{\csc t}{\csc t - 2 \sin t}$$

$$10. \csc 2t = \frac{1}{2} (\sec t)(\csc t)$$

$$11. \sec t = \frac{\sin 2t}{\sin t} - \frac{\cos 2t}{\cos t}$$

$$12. \frac{\sin 2t}{1 - \cos 2t} = 2 \csc 2t - \tan t$$

$$13. \tan 2x - 2 \tan 2x \sin^2 x = \sin 2x$$

$$14. \frac{\cos x - \sin 2x}{\cos 2x + \sin x - 1} = \cot x$$

$$15. \sin(x + y) \cdot \sin(x - y) = \sin^2 x - \sin^2 y$$

$$16. \frac{\cos(x - y)}{\cos(x + y)} = \frac{1 + \tan x \tan y}{1 - \tan x \tan y}$$

$$17. \tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$$

$$18. \frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}$$

$$19. \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$20. \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

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Unit 3 Review

1. a) An arc 32 m long subtends a central angle of a circle with a radius of 16 m. Determine the Measure of the central angle in radians.

b) A circle has a radius of 75 cm and a central angle of $\frac{14\pi}{15}$. Determine the arc length.

2. Write each degree measure as an exact radian measure.

a) -50° b) 420°

3. Write each radian measure as a degree measure.

a) $-\frac{5\pi}{4}$ b) $\frac{11\pi}{6}$

4. Draw the angle -420° in standard position and determine each of the following:

a) the principal angle b) a negative coterminal angle
c) the related acute angle d) $\csc(-420^\circ)$

5. Draw the angle $\frac{17\pi}{6}$ in standard position and determine each of the following:

a) the principal angle b) a negative coterminal angle
c) the related acute angle d) $\cot \frac{17\pi}{6}$

6. If the point $P(-4, -4)$ is on the terminal arm of an angle θ in standard position where $0 \leq \theta \leq 2\pi$, draw a detailed diagram and determine each of the following:

a) the related acute angle b) the measure of angle θ c) $\sec \theta$

7. For each of the following values of $\sin \theta$, determine the measure of θ if $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

a) $\frac{1}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{2}}{2}$ d) 1

8. Determine exact values for each of the following.

a) $\cot\left(-\frac{13\pi}{6}\right)$ b) $\sec \frac{4\pi}{3}$ c) $\csc(-\pi)$ d) $\csc \frac{11\pi}{4}$

9. Use an appropriate compound angle formula to express each of the following as a single trigonometric ratio and then evaluate.

a) $\frac{\tan \frac{\pi}{12} + \tan \frac{7\pi}{4}}{1 - \tan \frac{\pi}{12} \tan \frac{7\pi}{4}}$ b) $\cos \frac{\pi}{9} \cos \frac{19\pi}{18} - \sin \frac{\pi}{9} \sin \frac{19\pi}{18}$ c) $\frac{2 \tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}}$
d) $1 - 2 \sin^2\left(-\frac{3\pi}{8}\right)$ e) $\sin \frac{2\pi}{9} \cos \frac{13\pi}{18} - \cos \frac{2\pi}{9} \sin \frac{13\pi}{18}$ f) $\sin \frac{3\pi}{8} \cos \frac{3\pi}{8}$

10. Use appropriate compound angle formulas to simplify each of the following:

a) $\cos(\theta - \pi)$ b) $\sin\left(\theta + \frac{3}{2}\pi\right)$ c) $\cos\left(x + \frac{\pi}{3}\right) - \sin\left(x + \frac{\pi}{6}\right)$

11. Use an appropriate compound angle formula to determine the exact value of each trigonometric ratio. Rationalize the denominator of your final answer.

a) $\sin \frac{\pi}{12}$

b) $\tan\left(-\frac{5\pi}{12}\right)$

c) $\cos 165^\circ$

12. If the point $P(4, 6)$ is on the terminal arm of an angle β in standard position where $-360^\circ \leq \beta \leq 0^\circ$, draw a detailed diagram and determine each of the following:

a) the related acute angle to the nearest degree *Calculator*

b) the measure of angle β to the nearest degree *Calculator*

c) exact values for i) $\tan 2\beta$ ii) $\cot 2\beta$ iii) $\cos 2\beta$ iv) $\cos 4\beta$

13. If $\csc \theta = -\frac{17}{8}$, and θ is on the interval $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$, draw a detailed diagram and

determine each of the following:

a) the related acute angle to the nearest hundredth of a radian *Calculator*

b) the measure of angle θ to the nearest hundredth of a radian *Calculator*

c) exact values for i) $\sin 2\theta$ ii) $\tan\left(\theta + \frac{\pi}{4}\right)$ iii) $\cos \frac{\theta}{2}$ iv) $\sin \frac{\theta}{2}$

14. Prove the following trigonometric identities.

a) $\frac{-\cos^2 \theta}{\sin^2 \theta + 2\sin \theta + 1} = \frac{\sin \theta - 1}{\sin \theta + 1}$

b) $\frac{\cos 2x}{\cos^2 x + \sin x \cos x} = \frac{\cot x - 1}{\cot x}$

c) $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

d) $\sec 2t = \frac{\csc t}{\csc t - 2\sin t}$

e) $\csc 2x + \cot 2x = \cot x$

f) $\sec \theta = \frac{\sin 2\theta}{\sin \theta} - \frac{\cos 2\theta}{\cos \theta}$

g) $\frac{\cos 2x}{1 + \sin 2x} = \frac{\cot x - 1}{\cot x + 1}$

h) $\frac{2\sec^2 \theta - 2\tan^2 \theta}{\csc \theta} = \frac{\sec \theta}{\csc 2\theta}$

Answers 3.1

1. a) $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$ b) $\sin\theta = -\frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = -\frac{5}{12}$
c) $\csc\alpha = \frac{5}{3}$, $\sec\alpha = \frac{5}{4}$, $\cot\alpha = \frac{4}{3}$ d) $\csc\theta = -\frac{13}{5}$, $\sec\theta = -\frac{13}{12}$, $\cot\theta = \frac{12}{5}$
2. a) $\sin\theta = \frac{7}{5\sqrt{2}}$ b) $\cos\theta = \frac{1}{5\sqrt{2}}$ c) $\tan\theta = 7$ 3. a) $\csc\alpha = \frac{\sqrt{10}}{3}$ b) $\sec\alpha = -\sqrt{10}$ c) $\cot\alpha = -\frac{1}{3}$ 4. a) $\sin\theta = \frac{\sqrt{7}}{4}$, $\tan\theta = -\frac{\sqrt{7}}{3}$
5. a) $\csc\theta = -\frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = \frac{4}{3}$ b) $\sin\alpha = -\frac{\sqrt{161}}{15}$, $\tan\alpha = -\frac{\sqrt{161}}{8}$
6. a) $\sin\theta = \frac{2}{\sqrt{5}}$, $\sec\theta = \sqrt{5}$ b) $\cos\beta = -\frac{\sqrt{65}}{\sqrt{114}}$, $\csc\beta = \frac{\sqrt{114}}{7}$
7. a) $\sin\theta = -\frac{5}{13}$, $\cos\theta = -\frac{12}{13}$, $\tan\theta = \frac{5}{12}$, $\csc\theta = -\frac{13}{5}$, $\cot\theta = \frac{12}{5}$
b) $\cos\alpha = \frac{24}{25}$, $\tan\alpha = \frac{7}{24}$, $\csc\alpha = \frac{25}{7}$, $\sec\alpha = \frac{25}{24}$, $\cot\alpha = \frac{24}{7}$
c) $\sin\theta = -\frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $\tan\theta = -\frac{8}{15}$, $\sec\theta = \frac{17}{15}$, $\cot\theta = -\frac{15}{8}$
8. a) i) $k = \pm 4$ ii) $k = \pm 6$ iii) $k = \pm 2$ b) i) $\sin\theta = \pm \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \pm \frac{4}{3}$ ii) $\sin\theta = \frac{4}{5}$, $\cos\theta = \pm \frac{3}{5}$, $\tan\theta = \pm \frac{4}{3}$
iii) $\sin\theta = \pm \frac{2}{\sqrt{13}}$, $\cos\theta = \frac{3}{\sqrt{13}}$, $\tan\theta = \pm \frac{2}{3}$ 9. a) 1 b) 1 10. $\frac{6}{17}$

Answers 3.2

1. a) QII, 45° b) QIV, 45° c) QIII, 30° d) QIV, 30° e) QII, 45° f) QII, 30° g) QI, 30° h) QII, 60°
2. a) $492^\circ, 852^\circ$ b) $635^\circ, 995^\circ$ c) $665^\circ, 1025^\circ$ d) $433^\circ, 793^\circ$ e) $630^\circ, 990^\circ$
3. a) $-413^\circ, -773^\circ$ b) $-498^\circ, -858^\circ$ c) $-659^\circ, -1019^\circ$ d) $-540^\circ, -900^\circ$ e) $-552^\circ, -912^\circ$
4. a) 173° b) 50° c) 293° d) 185° e) 78° f) 350° g) 135° h) 169°
5. a) 156° b) 215° c) 341° d) 63° 6. a) 74° b) 286°
7. a) QI, QII, c) $\cos\beta = \pm \frac{3}{5}$, $\tan\beta = \pm \frac{4}{3}$ d) $53^\circ, 127^\circ$ 8. a) QII, QIV, c) $\sin\alpha = \pm \frac{7}{25}$, $\cos\alpha = \pm \frac{24}{25}$ d) $164^\circ, 344^\circ$
9. a) $\cos\theta = \pm \frac{15}{17}$, $208^\circ, 332^\circ$ b) $\sin\alpha = \pm \frac{5}{13}$, $157^\circ, 337^\circ$ c) $\tan\beta = \pm \frac{24}{7}$, $106^\circ, 254^\circ$ d) $\cot\theta = \pm \sqrt{3}$, $30^\circ, 330^\circ$

Answers 3.3

1. a) 60° b) 90° c) 45° d) 30° e) 135° f) -90° g) -120° h) 150° i) 240° j) -135° k) -300° l) 270° m) 360° n) -720° o) 540°
2. a) π b) 2π c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$ e) $-\frac{\pi}{3}$ f) $-\frac{5}{6}\pi$ g) $\frac{\pi}{6}$ h) $\frac{4\pi}{3}$ i) $-\frac{11}{6}\pi$ j) $\frac{3\pi}{2}$ k) $-\frac{\pi}{2}$ l) $\frac{2}{3}\pi$
3. a) π b) $-\pi$ c) $\frac{\pi}{2}$ d) $-\frac{5\pi}{4}$ e) $\frac{7\pi}{6}$ f) $\frac{5\pi}{3}$ 4. a) QII b) QIII c) QIII d) QI e) QIV f) QIV 6. a) π b) $\frac{4\pi}{3}$ c) 4π
7. a) 63° b) 52° c) -17° d) 86° e) -46° f) 23° 8. a) $\frac{2\pi}{3}$ b) $-\frac{3\pi}{4}$ c) $\frac{11\pi}{6}$ d) $\frac{13\pi}{6}$ 9. a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) 1 d) $\frac{\pi}{2}$ e) 2.6
10. a) 1.0 rad b) 2.8 rad c) 4.3 rad d) 5.3 rad 11. a) $-\frac{5}{3}\pi$ b) $\frac{7\pi}{4}$ c) $-\frac{11}{6}\pi$ d) π e) $-\frac{5\pi}{4}$ f) $\frac{\pi}{2}$
12. a) 0.0175 b) 0.1396 c) 0.4363 d) 1.1345 e) 2.9671 f) -4.1015 13. $\frac{5\pi}{4}$

Answers 3.4

1. b) $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$, $\cos\frac{5\pi}{3} = \frac{1}{2}$, $\tan\frac{5\pi}{3} = -\sqrt{3}$ 2. b) $\csc(-225^\circ) = \sqrt{2}$, $\sec(-225^\circ) = -\sqrt{2}$, $\cot(-225^\circ) = -1$
3. a) $-\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) Answers same because the angles are coterminal.
4. a) $-\frac{1}{\sqrt{2}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{3}}$ d) -1 e) $\frac{2}{\sqrt{3}}$ f) $-\sqrt{3}$ g) $\sqrt{2}$ h) -1 i) $-\frac{2}{\sqrt{3}}$ j) -1 k) ϕ l) -1
5. a) $\frac{1}{\sqrt{3}}$ b) $-\frac{1}{2}$ c) $-\frac{1}{\sqrt{3}}$ d) $-\sqrt{2}$ e) 1 f) $\frac{1}{2}$ g) 0 h) -1 6. a) $\frac{-1+\sqrt{3}}{2}$ b) $\frac{\sqrt{6}}{2}$ c) $\frac{1}{4}$ 7. a) $-\sqrt{3}$ b) 120°
8. a) $30^\circ, 330^\circ$ b) $210^\circ, 330^\circ$ c) $45^\circ, 225^\circ$ 9. a) $\frac{5\pi}{4}, \frac{7\pi}{4}$ b) $\frac{2\pi}{3}, \frac{4\pi}{3}$ c) $\frac{5\pi}{4}, \frac{7\pi}{4}$

Answers 3.6

1. a) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ c) $\frac{-1-\sqrt{3}}{2\sqrt{2}}$ 2. a) $\frac{\sqrt{2}+\sqrt{6}}{4}$ b) $\frac{\sqrt{2}-\sqrt{6}}{4}$ c) $\frac{-\sqrt{2}-\sqrt{6}}{4}$ 3. a) $-2+\sqrt{3}$ b) $2-\sqrt{3}$
4. a) $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$ b) $-\frac{1}{\sqrt{2}}(\cos x + \sin x)$ c) $\frac{1}{\sqrt{2}}(\cos x + \sin x)$ d) $-\sin x$
5. a) $-\frac{\sqrt{15}+2\sqrt{2}}{12}$ b) $\frac{1-2\sqrt{30}}{12}$ c) $-\frac{7}{9}$ d) $-\frac{\sqrt{15}}{8}$ e) $\frac{17}{81}$ f) $-\frac{7\sqrt{15}}{32}$ 6. a) $\frac{-2-\sqrt{2}}{\sqrt{15}}$ b) $\frac{1-2\sqrt{2}}{\sqrt{15}}$ c) $-\frac{3+4\sqrt{2}}{5\sqrt{3}}$ d) $\frac{1+4\sqrt{2}}{3\sqrt{5}}$
7. a) $\frac{3}{5}$ b) $\frac{4}{5}$; first 8. a) $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\frac{\sqrt{6}-\sqrt{2}}{4}$

Answers 3.7

1. a) $\sin 30^\circ = \frac{1}{2}$ b) $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ c) $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$ d) $\cos 180^\circ = -1$ e) $\tan(-240^\circ) = -\sqrt{3}$ f) $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$
- g) $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ h) $\tan 2\pi = 0$
2. a) $\sin(\theta - \beta)$ b) $\sin 3\theta$ c) $\cos 3A$ d) $\cos(M - N)$ e) $-\cos(x + y)$ f) $\tan 4x$ g) $\cos 2x$ h) 1
3. a) $5\sin 2x$ b) $\frac{5}{2}\sin 4x$ c) $\frac{1}{2}\sin 12x$ d) $2\sin x$ e) $\cos \theta$ f) $\cos 10\theta$ g) $\cos \frac{4}{3}\theta$ h) $\cos(6\theta - 4)$
4. a) $-\cos x$ b) $-\sin x$ c) $\sqrt{3}$ d) $\frac{-4}{1-\tan^2 x}$ 5. a) $\frac{33}{65}$ b) $\frac{56}{65}$ c) $-\frac{24}{25}$ d) $-\frac{25}{24}$ e) $\frac{1}{\sqrt{10}}$ f) $\frac{3}{\sqrt{10}}$
6. a) $3\sin \theta - 4\sin^3 \theta$ b) $4\cos^3 \theta - 3\cos \theta$ c) $\frac{3\tan \theta - \tan^3 \theta}{1 - 3\tan^2 \theta}$

Answers Review

1. a) 2 radians b) 70π cm 2. a) $-\frac{5\pi}{18}$ b) $\frac{7\pi}{3}$ 3. a) -225° b) 330° 4. a) 300° b) -780° or -60° c) 60° d) $-\frac{2}{\sqrt{3}}$
5. a) $\frac{5\pi}{6}$ b) $-\frac{7\pi}{6}$ c) $\frac{\pi}{6}$ d) $-\sqrt{3}$ 6. a) $\frac{\pi}{4}$ b) $\frac{5\pi}{4}$ c) $-\sqrt{2}$ 7. a) $\frac{5\pi}{6}$ b) $\frac{4\pi}{3}$ c) $\frac{3\pi}{4}$ d) $\frac{\pi}{2}$
8. a) $-\sqrt{3}$ b) -2 c) *d.n.e.* d) $\sqrt{2}$ 9. a) $\tan \frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$ b) $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ c) $\tan \frac{\pi}{3} = \sqrt{3}$ d) $\cos\left(-\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$
- e) $\sin\left(-\frac{\pi}{2}\right) = -1$ f) $\frac{1}{2}\sin \frac{3\pi}{4} = \frac{1}{2\sqrt{2}}$ 10. a) $-\cos \theta$ b) $-\cos \theta$ c) $-\sqrt{3}\sin x$ 11. a) $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $-2-\sqrt{3}$ c) $\frac{-\sqrt{2}-\sqrt{6}}{4}$
12. a) 56° b) -304° c) i) $-\frac{12}{5}$ ii) $-\frac{5}{12}$ iii) $-\frac{5}{13}$ iv) $-\frac{119}{169}$ 13. a) 0.49 b) -2.65 c) i) $\frac{240}{289}$ ii) $\frac{23}{7}$ iii) $\frac{1}{\sqrt{17}}$ iv) $-\frac{4}{\sqrt{17}}$