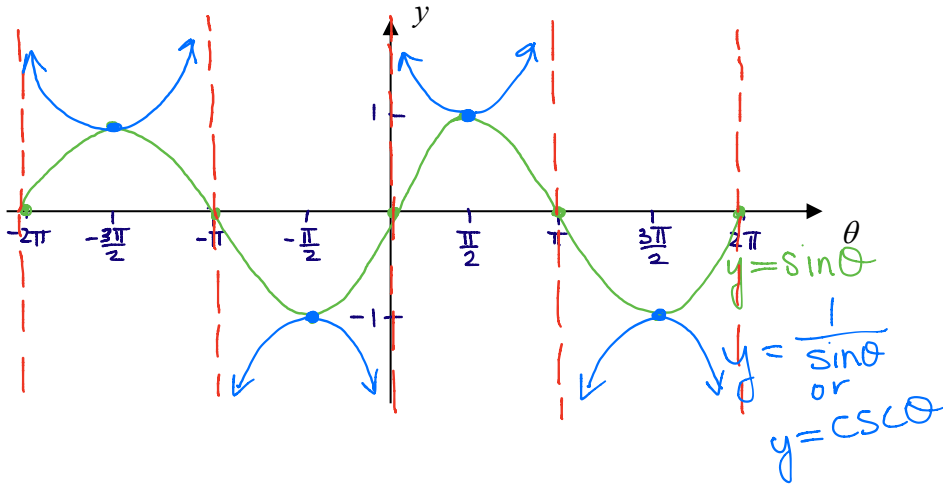


**UNIT 4: GRAPHING TRIGONOMETRIC FUNCTIONS & SOLVING TRIGONOMETRIC EQUATIONS**

**4.1 Graphing the Primary and Reciprocal Trigonometric Functions**

1. Graph  $y = \sin \theta$  and its *reciprocal* function on the same grid for  $-2\pi \leq \theta \leq 2\pi$ .



Complete:

**Properties of the function**  $y = \sin \theta$

i) Length of the period:  $2\pi$

ii) Domain:  $\{\theta \in \mathbb{R}\}$

iii) Range:  $\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$

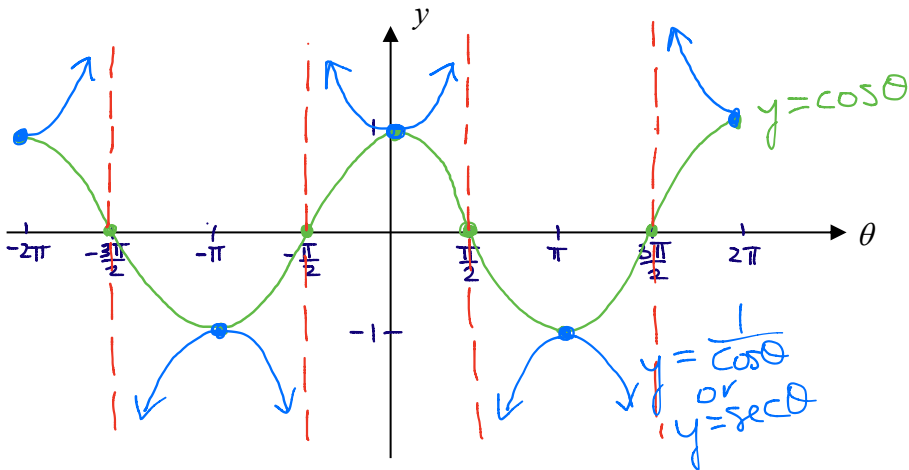
**Properties of the function**  $y = \csc \theta$

i) Length of the period:  $2\pi$

ii) Domain:  $\{\theta \in \mathbb{R} \mid \theta \neq n\pi, n \in \mathbb{I}\}$

iii) Range:  $\{y \in \mathbb{R} \mid y \leq -1 \text{ or } y \geq 1\}$

2. Graph  $y = \cos \theta$  and its *reciprocal* function on the same grid for  $-2\pi \leq \theta \leq 2\pi$ .



Complete:

**Properties of the function**  $y = \cos \theta$

i) Length of the period:  $2\pi$

ii) Domain:  $\{\theta \in \mathbb{R}\}$

iii) Range:  $\{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$

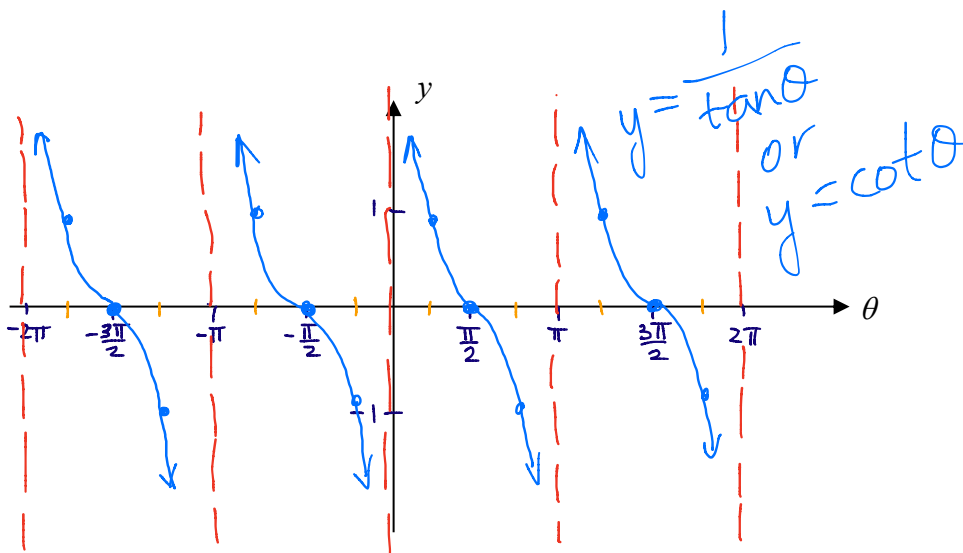
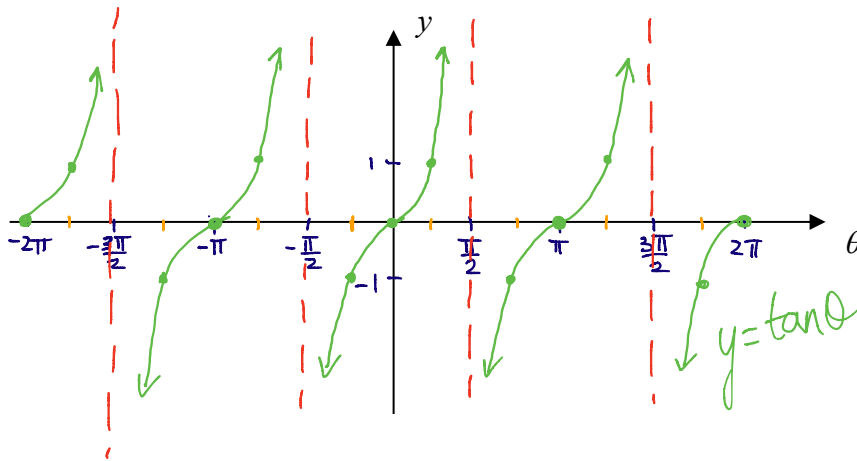
**Properties of the function**  $y = \sec \theta$

i) Length of the period:  $2\pi$

ii) Domain:  $\{\theta \in \mathbb{R} \mid \theta \neq \frac{\pi}{2} + n\pi, n \in \mathbb{I}\}$

iii) Range:  $\{y \in \mathbb{R} \mid y \leq -1 \text{ or } y \geq 1\}$

3. Graph  $y = \tan \theta$  and its *reciprocal* function on separate grids for  $-2\pi \leq \theta \leq 2\pi$ .



Complete:

**Properties of the function**  $y = \tan \theta$

i) Length of the period:  $\pi$

ii) Domain:  $\{\theta \in \mathbb{R} \mid \theta \neq \frac{\pi}{2} + n\pi, n \in \mathbb{I}\}$

iii) Range:  $\{y \in \mathbb{R}\}$

iv) Asymptotes:  $\theta = \frac{\pi}{2} + n\pi, n \in \mathbb{I}$

**Properties of the function**  $y = \cot \theta$

i) Length of the period:  $\pi$

ii) Domain:  $\{\theta \in \mathbb{R} \mid \theta \neq n\pi, n \in \mathbb{I}\}$

iii) Range:  $\{y \in \mathbb{R}\}$

iv) Asymptotes:  $\theta = n\pi, n \in \mathbb{I}$

**HW.** Memorize the graphs of all trigonometric functions for  $-2\pi \leq \theta \leq 2\pi$  and complete Exercise 4.1

**Date:** \_\_\_\_\_ **4.2 Solving Linear & Quadratic Trigonometric Equations**

**Ex. 1.** Determine **exact** solutions for each equation in the interval  $x \in [0, 2\pi]$ .

a)  $2\sin x - \sqrt{3} = 0$   
 $\sin x = \frac{\sqrt{3}}{2}$   
 r.a.a. =  $\frac{\pi}{3}$

<u>In QI:</u> ( $x = \text{raa}$ ) $x = \frac{\pi}{3}$	<u>In QII:</u> ( $x = \pi - \text{raa}$ ) $x = \frac{2\pi}{3}$
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b)  $4\sec^2 x - 8 = 0$   
 $\sec^2 x = \frac{8}{4}$   
 $\sec^2 x = 2$   
 $\cos^2 x = \frac{1}{2}$   
 $\cos x = \pm \frac{1}{\sqrt{2}}$   
 r.a.a. =  $\frac{\pi}{4}$

<u>In QI:</u> ( $x = \text{raa}$ ) $x = \frac{\pi}{4}$	<u>In QII:</u> ( $x = \pi - \text{raa}$ ) $x = \frac{3\pi}{4}$	<u>In QIII:</u> ( $x = \pi + \text{raa}$ ) $x = \frac{5\pi}{4}$	<u>In QIV:</u> ( $x = 2\pi - \text{raa}$ ) $x = \frac{7\pi}{4}$
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**Ex. 2.** Determine **approximate** solutions for each equation in the interval  $x \in [0, 2\pi]$ , to the nearest hundredth of a radian.

a)  $2\tan x + 1 = 0$   
 $\tan x = -\frac{1}{2}$   
 $\boxed{\text{raa} = \tan^{-1}(\pm \frac{1}{2})}$   
 r.a.a.  $\doteq 0.464$  rad

<u>In QII:</u> $x = \pi - 0.464$ $x \doteq 2.68$ rad.	<u>In QIV:</u> $x = 2\pi - 0.464$ $x \doteq 5.82$ rad.
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b)  $\cot^2 x - 0.64 = 0$   
 $\cot^2 x = 0.64$   
 $\cot x = \pm \sqrt{0.64}$   
 $\cot x = \pm 0.8$   
 $\tan x = \frac{\pm 1}{0.8}$   
 r.a.a.  $\doteq 0.896$

<u>In QI:</u> $x \doteq 0.90$	<u>In QII:</u> $x = \pi - 0.896$ $x \doteq 2.25$	<u>In QIII:</u> $x = \pi + 0.896$ $x \doteq 4.04$	<u>In QIV:</u> $x = 2\pi - 0.896$ $x \doteq 5.39$
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**Ex. 3.** Determine **exact** solutions where possible for each equation in the interval  $x \in [0, 2\pi]$ .

Round approximate solutions to the nearest hundredth of a radian.

a)  $\sin x \cos x + \sin x = 0$   
 $\sin x (\cos x + 1) = 0$   
 $\sin x = 0$  or  $\cos x + 1 = 0$   
 $x = 0, \pi, 2\pi$  or  $\cos x = -1$   
 (graph, unit circle)  $x = \pi$  (graph)  
 $\therefore$  the solutions are  $x = 0, \pi, 2\pi$

b)  $2\csc^2 x + \csc x - 1 = 0 \rightarrow \text{Let } a = \csc x$   
 $2a^2 + a - 1 = (2a - 1)(a + 1)$   
 $(2\csc x - 1)(\csc x + 1) = 0$   
 $2\csc x - 1 = 0$  or  $\csc x + 1 = 0$   
 $\csc x = \frac{1}{2}$  or  $\csc x = -1$   
 $\sin x = 2$  or  $\sin x = -1$   
 no solution  $x = \frac{3\pi}{2}$  (graph)  
 $\therefore -1 \leq \sin x \leq 1$

$\therefore$  the solution is  $x = \frac{3\pi}{2}$

c)  $6\cos^2 x = 7\cos x - 2$   
 $6\cos^2 x - 7\cos x + 2 = 0$   
 $(3\cos x - 2)(2\cos x - 1) = 0$   
 $\cos x = \frac{2}{3}$  or  $\cos x = \frac{1}{2}$   
 r.a.a.  $\doteq 0.841$  or r.a.a. =  $\frac{\pi}{3}$

<u>In QI:</u> $x \doteq 0.84$	<u>In QIV:</u> $x \doteq 5.44$	<u>In QI:</u> $x = \frac{\pi}{3}$	<u>In QIV:</u> $x = \frac{5\pi}{3}$
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$\therefore$  the solutions are  $x = 0.84, 5.44, \frac{\pi}{3}, \frac{5\pi}{3}$



Date: \_\_\_\_\_

**4.3 Solving Trigonometric Equations With Compound Angles**Ex. 1. Solve for  $0 \leq \theta \leq 2\pi$  and  $0 \leq A \leq 4\pi$ .

a)  $\cos^2 2\theta + \cos 2\theta = 0 \quad 0 \leq 2\theta \leq 4\pi$

$$\cos 2\theta (\cos 2\theta + 1) = 0$$

$$\cos 2\theta = 0$$

$$2\theta = \frac{\pi}{2} \text{ or } 2\theta = \frac{3\pi}{2} \text{ or } 2\theta = \frac{5\pi}{2} \text{ or } 2\theta = \frac{7\pi}{2}$$

$$\cdot \frac{1}{2}) \theta = \frac{\pi}{4}, \theta = \frac{3\pi}{4}, \theta = \frac{5\pi}{4}, \theta = \frac{7\pi}{4}$$

 $\therefore$  the solutions are:

$$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

b)  $\tan \frac{A}{2} + 1 = 0 \quad 0 \leq A \leq 2\pi$

$$\tan \frac{A}{2} = -1 \quad \text{raa} = \frac{\pi}{4}$$

In QII:

$$\frac{A}{2} = \frac{3\pi}{4}$$

$$\cdot 2) A = \frac{3\pi}{2}$$

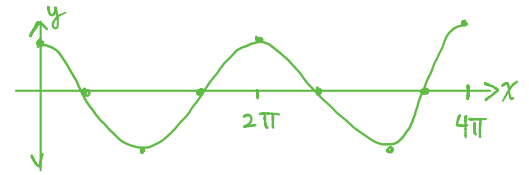
In QIV:

$$\frac{A}{2} = \frac{7\pi}{4}$$

$$A = \frac{7\pi}{2}$$

 $\therefore$  the solutions are

$$A = \frac{3\pi}{2}, \frac{7\pi}{2}$$

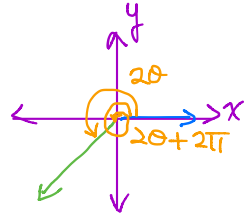


or  $\cos 2\theta = -1$

$$2\theta = \pi \text{ or } 2\theta = 3\pi$$

$$\theta = \frac{\pi}{2}$$

$$\theta = \frac{3\pi}{2}$$



c)  $2 \sin 2\theta + \sqrt{3} = 0 \quad 0 \leq 2\theta \leq 4\pi$

$$\sin 2\theta = -\frac{\sqrt{3}}{2} \quad \text{raa} = \frac{\pi}{3}$$

In QIII:

$$2\theta = \frac{4\pi}{3}$$

$$\cdot \frac{1}{2}) \theta = \frac{2\pi}{3}$$

or  $2\theta = \frac{4\pi}{3} + 2\pi$

$$2\theta = \frac{10\pi}{3}$$

$$\theta = \frac{5\pi}{3}$$

In QIV:

$$2\theta = \frac{5\pi}{3}$$

$$\theta = \frac{5\pi}{6}$$

or  $2\theta = \frac{5\pi}{3} + 2\pi$

$$2\theta = \frac{11\pi}{3}$$

$$\theta = \frac{11\pi}{6}$$

 $\therefore$  the solutions are:

$$\theta = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$$

Ex. 2. Solve for  $\theta$ . Give exact solutions where possible and round approximate solutions to the nearest hundredth of a radian.

a)  $-5\sin\theta = \cos 2\theta + 2, -2\pi \leq \theta \leq 0$

$$-5\sin\theta = 1 - 2\sin^2\theta + 2$$

$$2\sin^2\theta - 5\sin\theta - 3 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 3) = 0$$

$$\sin\theta = -\frac{1}{2} \quad \text{or} \quad \sin\theta = 3$$

no solution

$$\text{raa} = \frac{\pi}{6} \quad \because -1 \leq \sin\theta \leq 1$$

In Q III:  $\theta = -\pi + \frac{\pi}{6}$   
 $= -\frac{5\pi}{6}$

In Q IV:  $\theta = -\frac{\pi}{6}$

$\therefore$  the solutions are

$$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}$$

b)  $-5\cos\theta - \cos 2\theta = 2, 0 \leq \theta \leq 2\pi$

$$-5\cos\theta - (2\cos^2\theta - 1) = 2$$

$$-5\cos\theta - 2\cos^2\theta + 1 = 2$$

$$0 = 2\cos^2\theta + 5\cos\theta + 1$$

$$\cos\theta = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(1)}}{2(2)}$$

$$\cos\theta = \frac{-5 - \sqrt{17}}{4}$$

$$\cos\theta \doteq -2.2808$$

no solution

$$\because -1 \leq \cos\theta \leq 1$$

or  $\cos\theta = \frac{-5 + \sqrt{17}}{4}$

$$\cos\theta = -0.2192$$

$$\text{raa} = \cos^{-1}(-0.2192)$$

$$\text{raa} \doteq 1.350$$

In Q II:

$$\theta \doteq 1.79$$

In Q III:

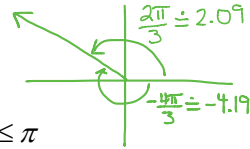
$$\theta \doteq 4.49$$

Date: \_\_\_\_\_

## 4.4 Solving Trigonometric Equations Involving Absolute Value & Solving Trigonometric Inequalities by Graphing

### Warmup

Solve each of the following equations. State exact answers where possible. Otherwise round your answers to two decimal places.



a)  $2\cos\left(x + \frac{5\pi}{6}\right) + 4 = 3, 0 \leq x \leq 2\pi$

$$2\cos\left(x + \frac{5\pi}{6}\right) = -1$$

$$\cos\left(x + \frac{5\pi}{6}\right) = -\frac{1}{2} \quad 0 \leq x \leq 2\pi$$

Let  $\theta = x + \frac{5\pi}{6}$ ,  $0 + \frac{5\pi}{6} \leq x + \frac{5\pi}{6} \leq 2\pi + \frac{5\pi}{6}$

$$\cos \theta = -\frac{1}{2} \quad \frac{5\pi}{6} \leq \theta \leq \frac{17\pi}{6}$$

$$r.a.a. = \frac{\pi}{3}$$

In QII:

$$\theta = \frac{2\pi}{3} + 2\pi$$

$$\theta = \frac{8\pi}{3}$$

$$x + \frac{5\pi}{6} = \frac{16\pi}{6}$$

$$x = \frac{11\pi}{6}$$

In QIII:

$$\theta = \frac{4\pi}{3}$$

$$x + \frac{5\pi}{6} = \frac{8\pi}{6}$$

$$x = \frac{3\pi}{6}$$

$$x = \frac{\pi}{2}$$

∴ the solutions are

$$x = \frac{\pi}{2}, \frac{11\pi}{6}$$

b)  $5\tan\left(x - \frac{\pi}{3}\right) + 3 = 1, -\pi \leq x \leq \pi$

$$\tan\left(x - \frac{\pi}{3}\right) = -\frac{2}{5}$$

Let  $\theta = x - \frac{\pi}{3}$

$$\tan \theta = -\frac{2}{5}$$

$$r.a.a. \doteq 0.38$$

In QII:

$$x - \frac{\pi}{3} = -\pi - 0.38$$

$$x \doteq -2.47$$

In QIV:

$$x - \frac{\pi}{3} = -r.a.a.$$

$$x - \frac{\pi}{3} = -0.38$$

$$x \doteq 0.67$$

∴ the solutions are

$$x \doteq -2.47, 0.67$$

**Ex. 1.** Find all values of  $x$  in the interval  $[0, 2\pi]$  that satisfy the equation  $|\sin x| = 1$ .

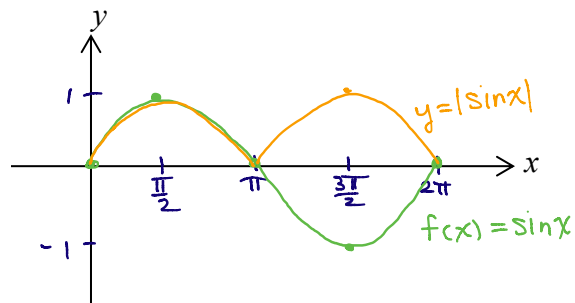
Let  $f(x) = \sin x$ :

First graph  $f(x) = \sin x$

for  $0 \leq x \leq 2\pi$

Then graph  $y = |f(x)|$   
(or  $y = |\sin x|$ )

for  $0 \leq x \leq 2\pi$



$$\therefore \text{S.S.} = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

Ex. 2. Find all values of  $x$  in the interval  $[0, 2\pi]$  that satisfy the inequality  $\sqrt{2} - 2\cos x > 0$ .

$$\sqrt{2} - 2\cos x > 0$$

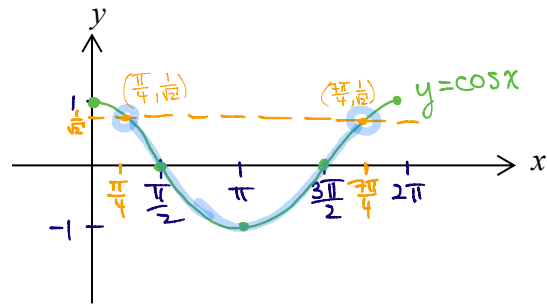
$$-2\cos x > -\sqrt{2}$$

$$\cos x < \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos x < \frac{2}{2\sqrt{2}}$$

$$\cos x < \frac{1}{\sqrt{2}}$$

Let  $f(x) = \cos x$  and graph for  $0 \leq x \leq 2\pi$



Find  $x$  if  $\cos x = \frac{1}{\sqrt{2}}$   
 $\text{raa} = \frac{\pi}{4}$

In Q I:  $x = \frac{\pi}{4}$

In Q IV:  $x = \frac{7\pi}{4}$

$$\therefore \text{S.S.} = \left\{ x \in \mathbb{R} \mid \frac{\pi}{4} < x < \frac{7\pi}{4} \right\} \text{ or } x \in \left( \frac{\pi}{4}, \frac{7\pi}{4} \right)$$

Ex. 3. Find all values of  $x$  in the interval  $[-2\pi, 0]$  that satisfy the inequality  $\sin x - \cos x \leq 0$ .

$$\sin x - \cos x \leq 0$$

$$\sin x \leq \cos x$$

Let  $f(x) = \sin x$ ,  $g(x) = \cos x$  and graph for  $-2\pi \leq x \leq 0$

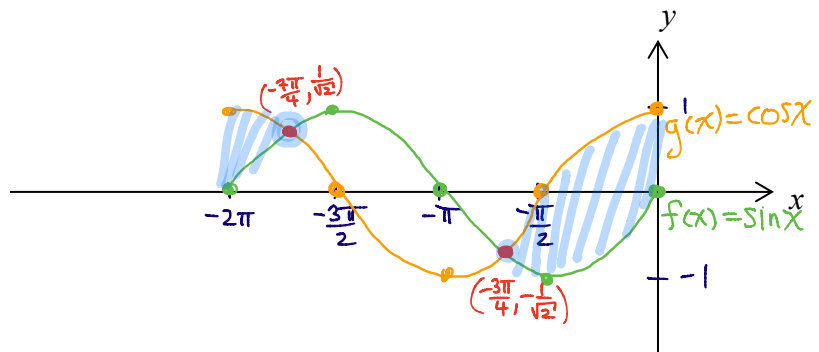
Find  $x$  if  $f(x) = g(x)$

$$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$$

$$\frac{\sin x}{\cos x} = 1$$

$$\tan x = 1$$

$$\text{raa} = \frac{\pi}{4}$$



$$\therefore \text{S.S.} = \left\{ x \in \mathbb{R} \mid -2\pi \leq x \leq -\frac{7\pi}{4} \text{ or } -\frac{3\pi}{4} \leq x \leq 0 \right\}$$

or

$$x \in \left[ -2\pi, -\frac{7\pi}{4} \right] \cup \left[ -\frac{3\pi}{4}, 0 \right]$$

In Q I:

$$x = -2\pi + \frac{\pi}{4}$$

$$x = -\frac{7\pi}{4}$$

$$f\left(-\frac{7\pi}{4}\right) \text{ or } g\left(-\frac{7\pi}{4}\right) \\ = \sin\left(-\frac{7\pi}{4}\right) = \cos\left(-\frac{7\pi}{4}\right) \\ = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

In Q III:

$$x = -\pi + \frac{\pi}{4}$$

$$x = -\frac{3\pi}{4}$$

$$f\left(-\frac{3\pi}{4}\right) = g\left(-\frac{3\pi}{4}\right) \\ = -\frac{1}{\sqrt{2}}$$