Date:__________________ **4.5 Transformations of Sine and Cosine Graphs**

Given $y = a\,f[k(x-d)]+c$, the **transformations** on the graphs of $y=f(x)$ where $f(x)=\sin x$ or $f(x)=\cos x$ are as follows:

i) ***vertical reflection*** in the x-axis if $a<0$

ii) ***vertical stretch*** by a factor of $|a|$

Note: A stretch is an ***expansion*** *if the stretch factor is more than* 1 *or a* ***compression*** *if the stretch factor is between* 0 *and* 1.

iii) ***horizontal reflection*** in the y-axis if $k<0$

iv) ***horizontal stretch*** by a factor of $\frac{1}{|k|}$

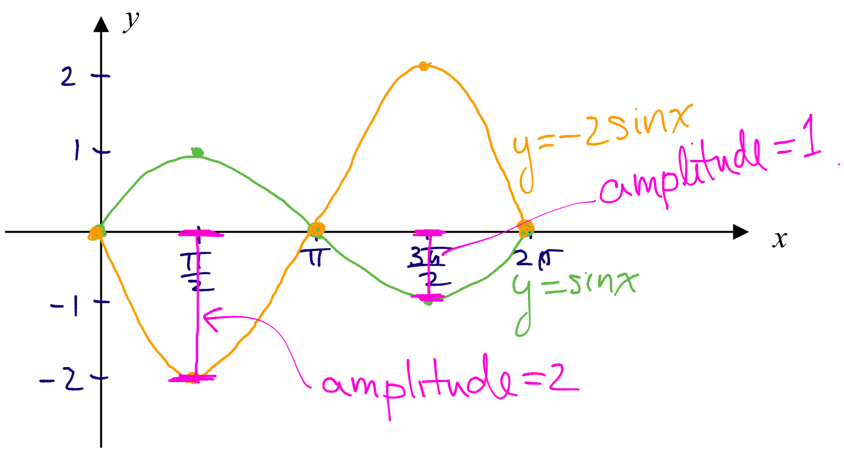
v) ***horizontal translation*** **right** $|d|$ units if $d>0$ or **left** $|d|$ units if $d<0$

vi) ***vertical translation*** **up** $|c|$ units if $c>0$ or **down** $|c|$ units if $c<0$

$$(x,y)\rightarrow\left(\frac{1}{k}x+d,ay+c\right)$$

Ex. 1. Graph each of the following functions by naming and using transformations on $y=\sin x$.

a) $y=-2\sin x$, $0\le x\le 2\pi$



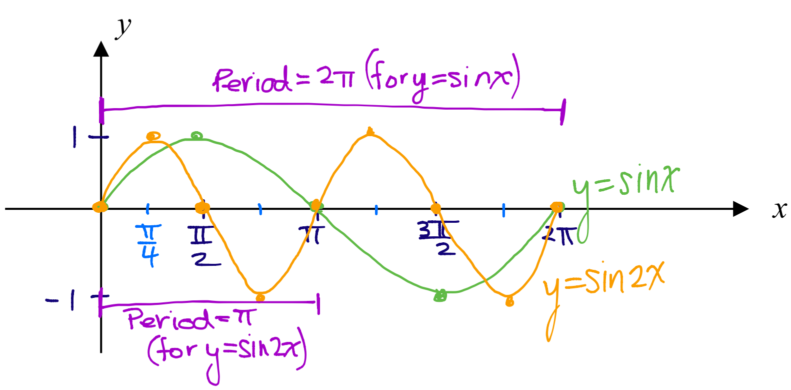
Transformations on $y=\sin x$ are:

i) V.R. across x-axis

ii) V.E. by a factor of 2

$(x,y)\rightarrow(x,-2y)$

b) $y=\sin 2x$, $0\le x\le 2\pi$



Transformations on $y=\sin x$ are:

i) H.C. by a factor of $\frac{1}{2}$

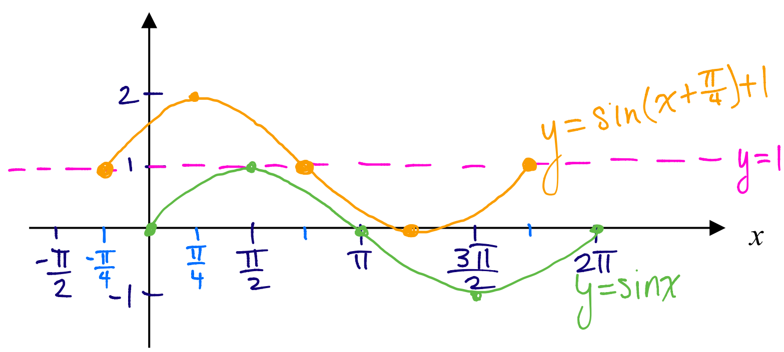
$(x,y)\rightarrow(\frac{1}{2}x,y)$

c) $y=\sin\left(x+\frac{\pi}{4}\right)+1,\ 0\le x\le 2\pi$

Transformations on $y=\sin x$ are:

i) H.T. $\frac{\pi}{4}$ units left

ii) V.T. 1 unit up



Summary of Transformations on the Periodic Functions $y=\sin\theta$ ***and*** $y=\cos\theta$

For $y=a\sin k(\theta-d)+c$ and $y=a\cos k(\theta-d)+c$,

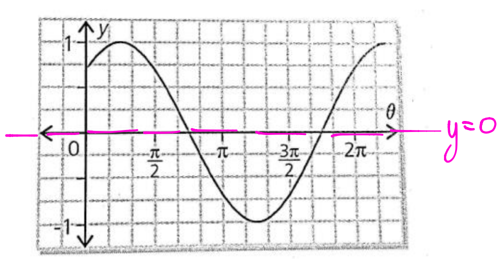
- the ***reflection*** of $y=\sin\theta$ or $y=\cos\theta$ is in the θ- axis if $a<0$
- the ***reflection*** of $y=\sin\theta$ or $y=\cos\theta$ is in the y- axis if $k<0$
- the ***amplitude*** is $|a|$
- the ***period*** is $\frac{1}{|k|}\times 2\pi$ or $\frac{2\pi}{|k|}$ ← $P=\frac{2\pi}{k} \longleftrightarrow K=\frac{2\pi}{P}$
- the ***phase shift*** is **right** $|d|$ units if $d>0$ or **left** $|d|$ units if $d<0$, and
- the ***vertical translation*** is **up** $|c|$ units if $c>0$ or **down** $|c|$ units if $c<0$

Ex. 2. For each of the following graphs determine:

i) the amplitude, period, phase shift and vertical translation

ii) the sine function $y=a\sin k(\theta-d)+c$ and the cosine function $y=a\cos k(\theta-d)+c$

a)



A: 1 → $|a|=1$

P: 2π → $K=1$

P.S: vary

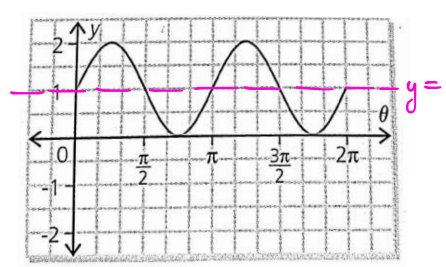
V.T.: none

$\therefore\ y=\cos(\theta-\frac{\pi}{4})$

or $y=\sin(\theta+\frac{\pi}{4})$

or $y=-\sin(x-\frac{3\pi}{4})$

b)



A: 1 → $|a|=1$

P: π → $K=\frac{2\pi}{P}=\frac{2\pi}{\pi}=2$

P.S.: varies

V.T.: up 1 unit → $c=+1$

$y=\cos 2(\theta-\frac{\pi}{4})+1$

or $y=\sin 2\theta+1$

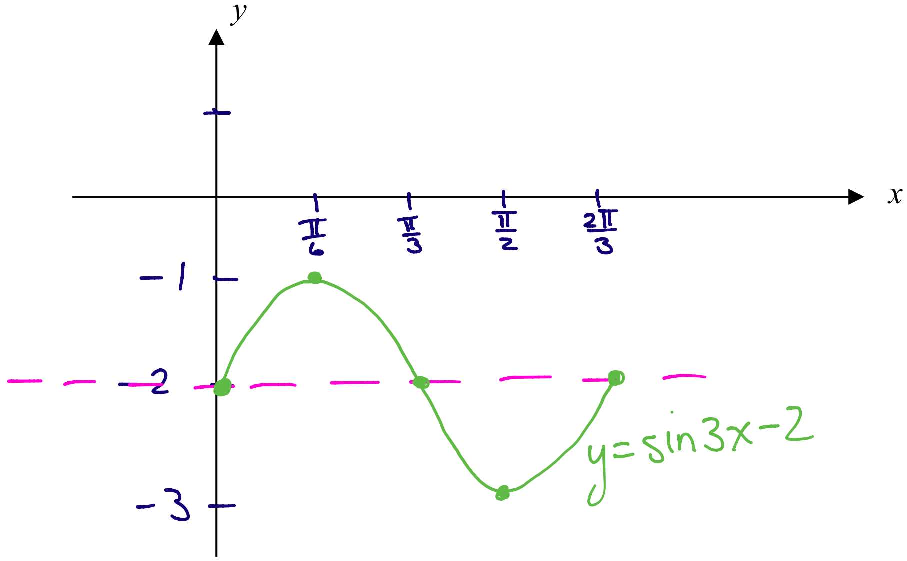
Ex. 3. State the amplitude, period, phase shift, and vertical translation for each of the following functions and graph for one period.

a) $f(x) = \sin 3x - 2$

A: 1
P: $\frac{2\pi}{3}$
P.S: none
V.T.: down 2 units

Period scale:
$\frac{1}{4} \times \text{Period}$
$= \frac{1}{\cancel{4}_2} \cdot \frac{\cancel{2}^1\pi}{3}$
$= \frac{\pi}{6}$

$y = -2$

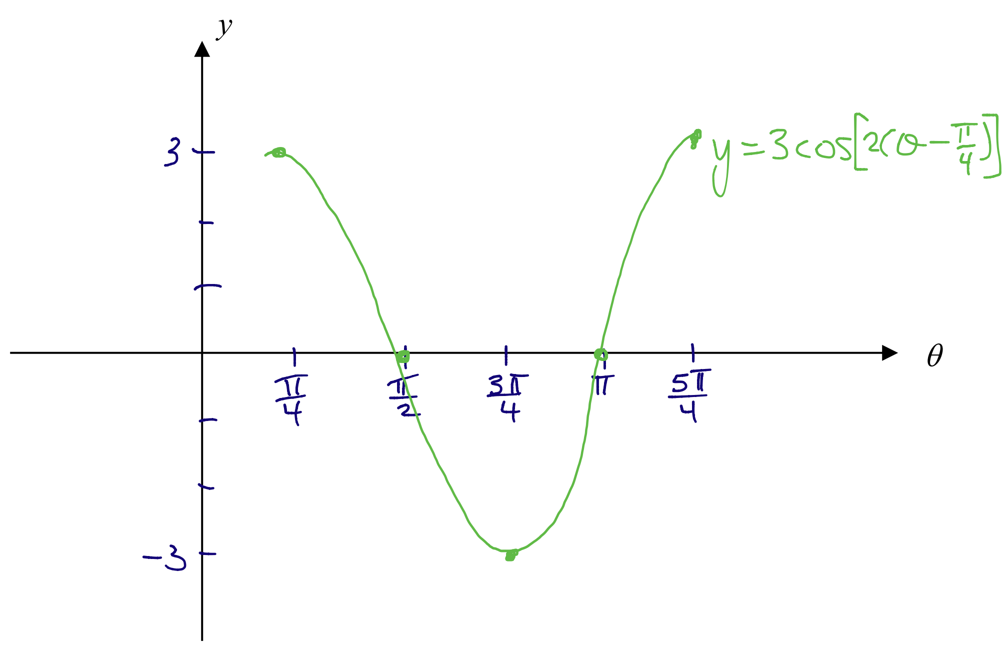


b) $y = 3\cos\left(2\theta - \frac{\pi}{2}\right)$ ** $\longrightarrow y = 3\cos\left[2\left(\theta - \frac{\pi}{4}\right)\right]$

A: 3
P: π
P.S.: $\frac{\pi}{4}$ right
V.T.: none

Period Scale:
$\frac{1}{4} \times \text{Period}$
$= \frac{1}{4} \cdot \pi$
$= \frac{\pi}{4}$

* must have L.C.D with P.S.



HW. Exercise 4.5

Date:______________ **4.6 Combinations of Transformations of the Sine and Cosine Functions**

Ex. 1. For each of the following state any reflections, the amplitude (V.S.), period (H.S.), phase shift (H.T.) and vertical translation. Graph the curve for one cycle and state the domain and range.

a) $y = -2\cos\left(x + \frac{\pi}{4}\right) + 2$

V.R. in x-axis

A: 2 P: 2π

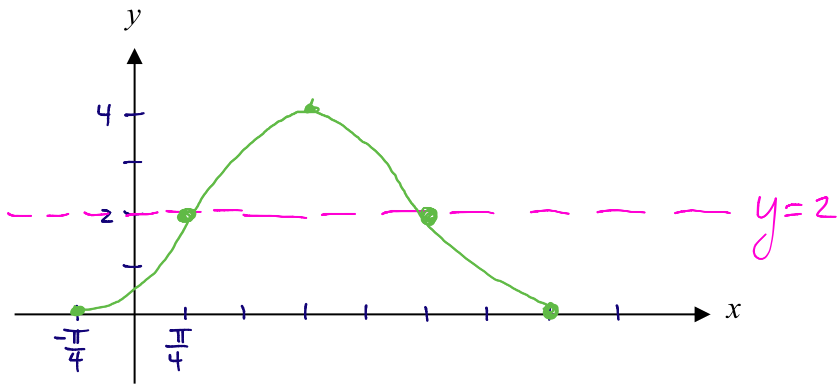
P.S.: $\frac{\pi}{4}$ left V.T.: 2 units up

start

Period scale:

$\frac{1}{4} \cdot 2\pi = \frac{\pi}{2} = \frac{2\pi}{4}$ # of spots between key points

(x-scale: $\frac{\pi}{4}$)



D: $\{x \in \mathbb{R} \mid -\frac{\pi}{4} \le x \le \frac{7\pi}{4}\}$

R: $\{y \in \mathbb{R} \mid 0 \le y \le 4\}$

b) $y = \frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right) \longrightarrow y = \frac{1}{2}\sin\left[2\left(x - \frac{\pi}{6}\right)\right]$

V.R.: none

A: $\frac{1}{2}$ P: $\frac{2\pi}{2} = \pi$

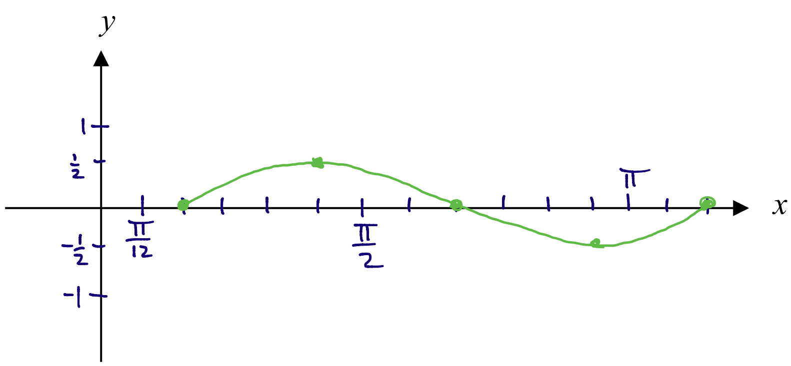
P.S.: $\frac{\pi}{6}$ right V.T.: none

$= \frac{2\pi}{12}$ start

Period Scale:

$\frac{1}{4} \cdot \pi = \frac{\pi}{4} = \frac{3\pi}{12}$ count to key points

x-scale $= \frac{\pi}{12}$



D: $\{x \in \mathbb{R} \mid \frac{\pi}{6} \le x \le \frac{7\pi}{6}\}$

R: $\{y \in \mathbb{R} \mid -\frac{1}{2} \le y \le \frac{1}{2}\}$

Ex. 2. For each of the following state any reflections, the amplitude, period, phase shift and vertical translation. Graph the curve for the specified domain and then state the range.

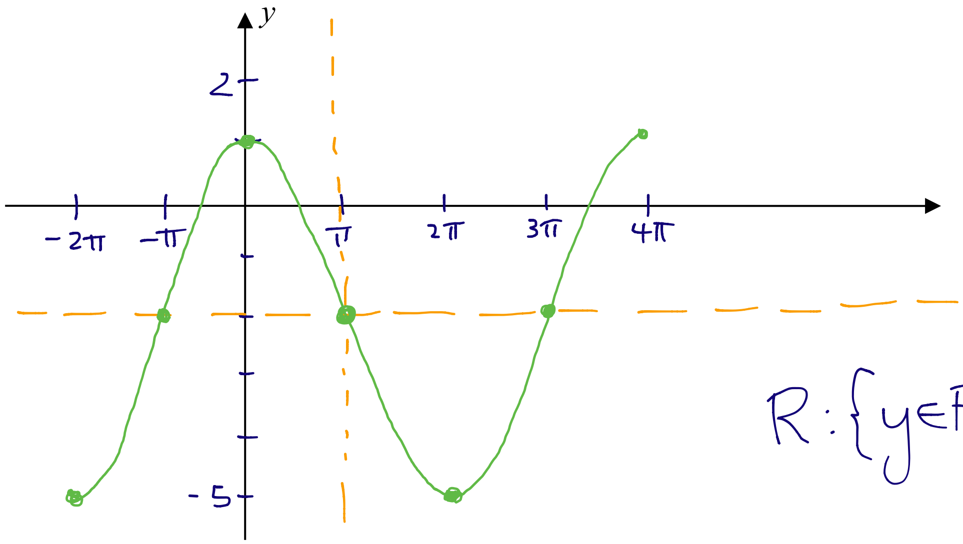
a) $y=-3\sin\left(\frac{1}{2}x-\frac{\pi}{2}\right)-2,\ -2\pi\le x\le 4\pi$

$y=-3\sin\left[\frac{1}{2}(x-\pi)\right]-2$

V.R. in x-axis
A: 3
P.S.: π units right
VT: 2units down

$P=\frac{2\pi}{k}$
$=\frac{2\pi}{\left(\frac{1}{2}\right)}$
$=2\pi\times\frac{2}{1}$
$=4\pi$

Period Scale:
$\frac{1}{4}\times 4\pi$
$=\pi$



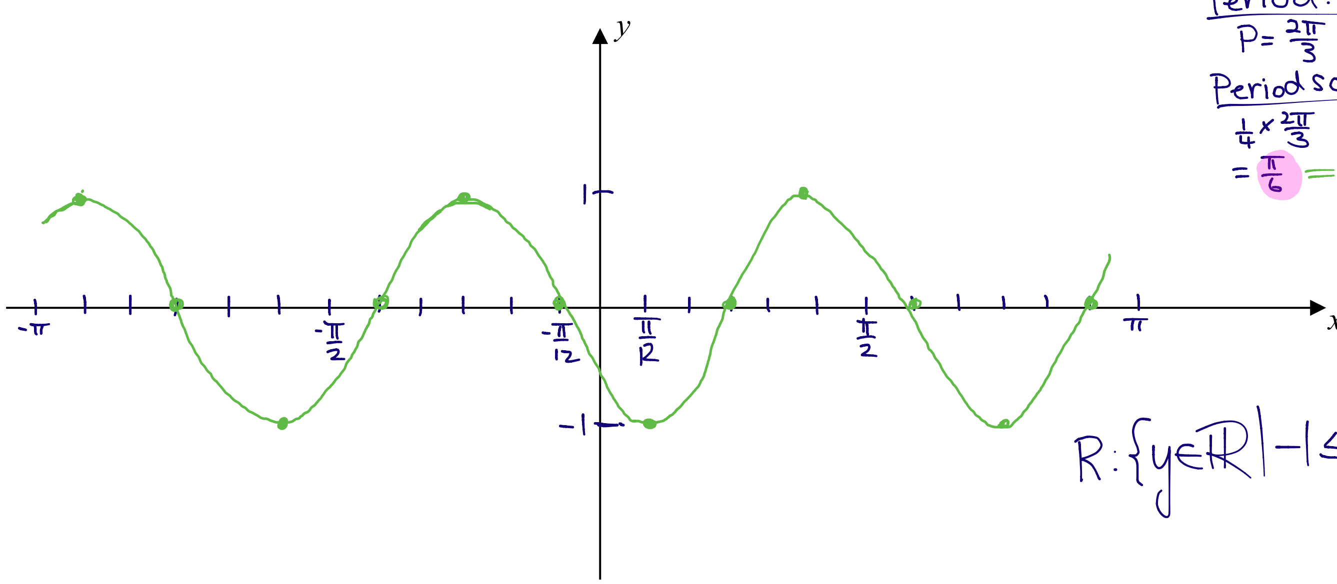
R: $\{y\in\mathbb{R}\mid -5\le y\le 1\}$

b) $y=\cos 3\left(x+\frac{\pi}{4}\right),\ -\pi\le x\le\pi$

Reflection: none
A: 1
P.S.: $\frac{\pi}{4}$ units left ($\frac{3\pi}{12}$)
V.T.: none

Period:
$P=\frac{2\pi}{3}$

Period scale:
$\frac{1}{4}\times\frac{2\pi}{3}$
$=\frac{\pi}{6}=\frac{2\pi}{12}$



R: $\{y\in\mathbb{R}\mid -1\le y\le 1\}$

HW. Exercise 4.6

Date:_________________ **4.7 Applications of Trigonometric Functions**

Ex. 1. A carnival Ferris wheel with a radius of 20 m makes three complete revolutions in 2 minutes. Passengers get on at the lowest point which is 1 m above the ground.

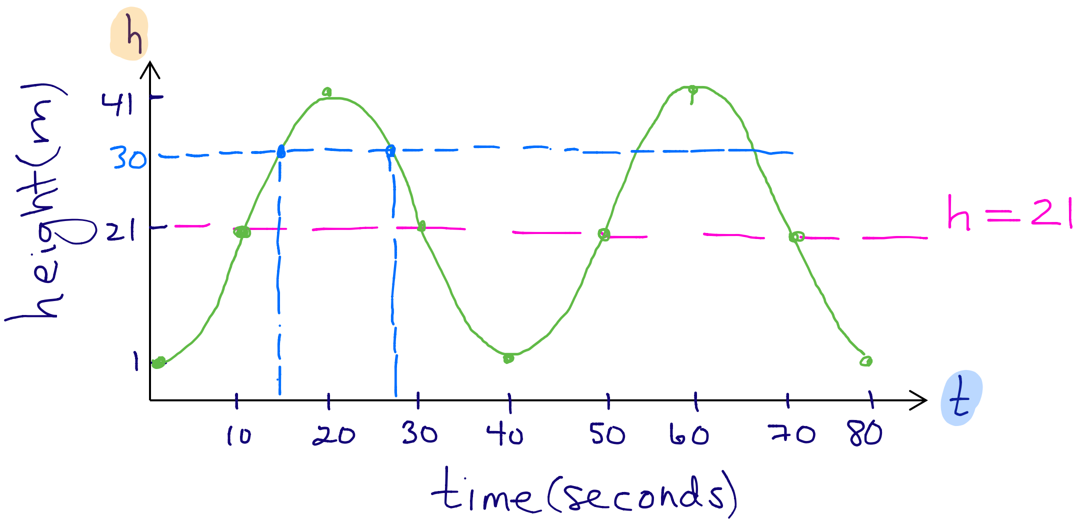
a) Draw a graph to show how a person's height, *h*, above the ground in metres, varies with time, *t*, in seconds, for two revolutions.

3 rev in 120 sec

1 rev in 40 sec ← P

Period scale:

$\frac{1}{4} \cdot 40 = 10$ s.



b) Write an equation which expresses your height as a function of time on the ride.

$h(t) = -20\cos\frac{\pi}{20}t + 21$

or $h(t) = 20\sin\frac{\pi}{20}(t-10) + 21$

$A = 20$, $k = \frac{2\pi}{P} = \frac{2\pi}{40} = \frac{\pi}{20}$

c) Calculate your height above the ground after 15 s.

$h(15) = -20\cos\left[\frac{\pi}{20} \cdot 15\right] + 21$

$= -20\cos\left(\frac{3\pi}{4}\right) + 21$

$= -20\left(-\frac{1}{\sqrt{2}}\right) + 21 = \frac{20}{\sqrt{2}} + 21 \doteq 35.1$

d) At what times will the rider be 30 m above the ground?

Find t if $h = 30$ m:

$-20\cos\frac{\pi}{20}t + 21 = 30$

$\cos\frac{\pi}{20}t = -\frac{9}{20}$

Let $\theta = \frac{\pi}{20}t$

$\cos\theta = -\frac{9}{20}$

$raa \doteq \cos^{-1}\left(+\frac{9}{20}\right)$

$\doteq 1.104$

In QII:

$\theta = \pi - raa$

$\theta \doteq 2.038$

$\frac{20}{\pi}\left(\frac{\pi}{20}t\right) = (2.038)\frac{20}{\pi}$

$t \doteq 13.0$

In QIII:

$\theta = \pi + raa$

$\theta \doteq 4.246$

$\frac{\pi}{20}t = 4.246$

$t \doteq 27.0$

∴ the rider is 30 m above ground at the times $(13 + 40n)$ and $(27 + 40n)$, $n \in W$ seconds

Ex. 2. The daily high temperature of the city of Waterloo, in degrees Celsius, as a function of the number of days into the year, can be described by the function $T(d) = -20\cos\frac{2\pi}{365}(d+10)+25$

117

a) Use the function to determine today's temperature to the nearest degree Celsius.

$T(117) = -20\cos\left[\frac{2\pi}{365}(117+10)\right]+25$

$\doteq 37°$

$\therefore$ the temperature is approx $37°$ on April 27.

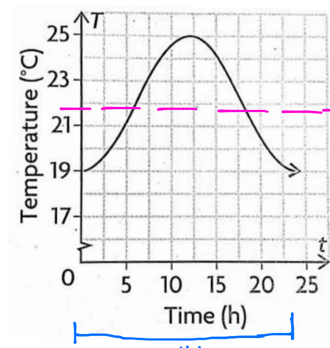
b) Determine the range of this function . Explain the meaning of the range in this application.

$25-20 \le T \le 25+20$

$5 \le T \le 45$

$\therefore$ the temperature ranges between $5°$ and $45°$.

Ex. 3. The temperature, T, in degrees Celsius, of the surface water in a swimming pool varies according to the following graph, where t is the number of hours since sunrise at 6 a.m.



T=22

24h.

a) Find possible cosine and sine equations for the temperature of the surface water as a function of time.

$T(t) = -3\cos\frac{\pi}{12}t + 22$

or

$T(t) = 3\sin\frac{\pi}{12}(t-6)+22$

$k = \frac{2\pi}{P} = \frac{2\pi}{24} = \frac{\pi}{12}$, Period Scale: $\frac{1}{4}$Period $= \frac{1}{4}\cdot 24 = 6$, $A=3$

b) At what times is the temperature of the surface water at least $23\,^\circ C$?

Find t if T=23

$-3\cos\frac{\pi}{12}t + 22 = 23$

$\cos\frac{\pi}{12}t = -\frac{1}{3}$

raa $\doteq \cos(+\frac{1}{3})$

$\doteq 1.231$

In QII:

$\frac{\pi}{12}t \doteq 1.911$

$t \doteq 7.3$

In QIII:

$\frac{\pi}{12}t \doteq 4.373$

$t \doteq 16.7$

$\therefore$ the temperature is at least $23°C$ between 1:20pm and 10:42pm

HW. Exercise 4.7

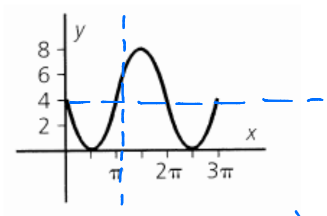
Date:__________________

Unit 4 Test Review

Warmup

1. Each of the diagrams below is the graph of a sinusoidal function.

a) Express as a sine function.



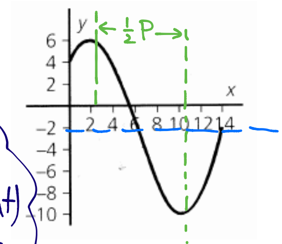
$y = 4\sin(x-\pi)+4$

or

$y = -4\sin x + 4$

A: 4
P = 2π (k=1)
P.S varies (π units right)
V.T up 4 units

b) Express as a cosine function.



P = 16

$K = \frac{2\pi}{P}$

$K = \frac{2\pi}{16}$

$K = \frac{\pi}{8}$

$y = 8\cos\frac{\pi}{8}(x-2)-2$

2. The function $y = \sin(x-c)+d$ has been vertically translated 3 units down and passes through the point $\left(\frac{\pi}{6}, -2\right)$. Determine the values of c and d.

d

x y

Find c if $d=-3$, $x=\frac{\pi}{6}$, $y=-2$

$-2 = \sin\left(\frac{\pi}{6}-c\right) - 3$

let $\frac{\pi}{6} - c = z$

$1 = \sin\left(\frac{\pi}{6}-c\right) \longrightarrow \sin z = 1$

$\therefore \frac{\pi}{6} - c = \frac{\pi}{2}$

$-c = \frac{\pi}{2} - \frac{\pi}{6}$

$-c = \frac{3\pi}{6} - \frac{\pi}{6}$

$-c = \frac{2\pi}{6}$

$-c = \frac{\pi}{3}$

$\therefore c = -\frac{\pi}{3}$

$\therefore c = -\frac{\pi}{3}$, $d = -3$

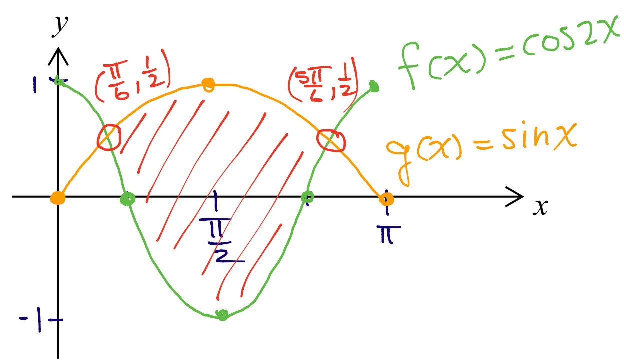
$y = \sin\left(x+\frac{\pi}{3}\right) - 3$

3. Solve the following trigonometric inequality for x in the domain $[0, \pi]$ and state your final answer in a solution set.

$\cos 2x < \sin x$

transformations: $k=2 \therefore P=\pi$

Let $f(x) = \cos 2x$ & $g(x) = \sin x$
and graph for $0 \le x \le \pi$



Find x, if $\cos 2x = \sin x$

$1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

raa $= \frac{\pi}{6}$

In QI: $x = \frac{\pi}{6}$

In QII: $x = \frac{5\pi}{6}$

$\therefore x = \frac{3\pi}{2}$

not in domain

$\therefore SS = \left\{x \in \mathbb{R} \mid \frac{\pi}{6} < x < \frac{5\pi}{6}\right\}$

HW. Unit 4 Review Exercise