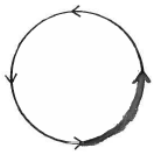
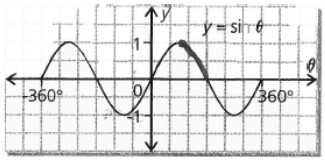
Date:_____

UNIT 4: GRAPHING TRIGONOMETRIC FUNCTIONS & SOLVING TRIGONOMETRIC EQUATIONS

4.1 Graphing the Primary and Reciprocal Trigonometric Functions

1. Point $P(x, y)$ is on the terminal arm of an angle θ in standard position.
It could be anywhere on the highlighted part of the unit circle.
Match each unit circle to its graph of $y = \sin\theta$ and restrict θ in both degrees and radians.

(a) 

i. 

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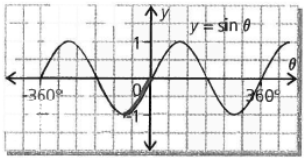
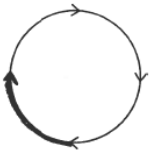
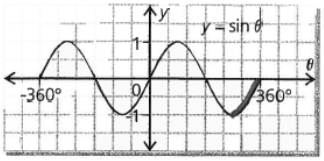
ii. 

(c) 

iii. 

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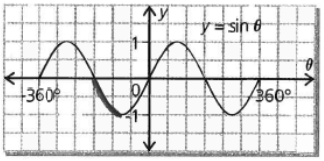
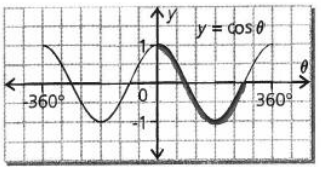
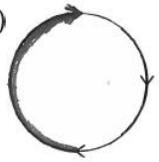
iv. 

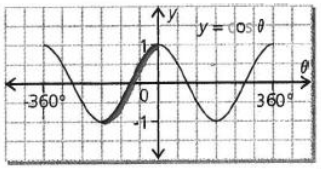
2. Repeat question **1** for the the graph of $y = \cos\theta$.

3. Repeat question **1** for the the graph of $y = \tan\theta$.

(a) 

i. 

(b) 

ii. 

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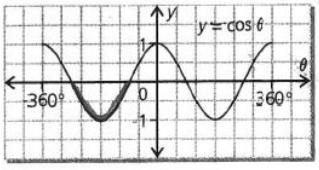
iii. 

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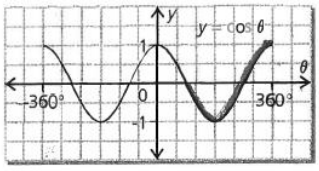
iv. 

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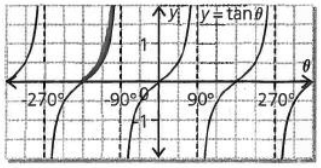
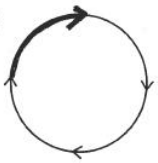
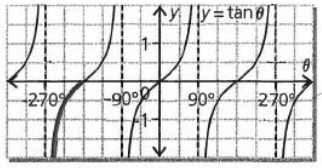
i. 

(b) 

ii. 

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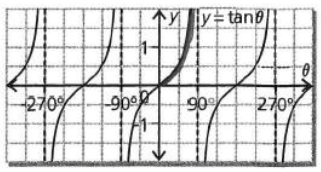
iii. 

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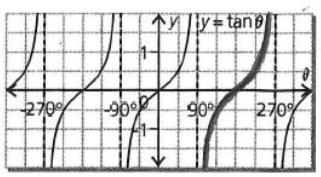
iv. 

Date:_____ **4.2 Solving Linear & Quadratic Trigonometric Equations**

1. Determine approximate solutions for each equation in the interval $x \in [0, 2\pi]$, to the nearest hundredth of a radian.

a) $\sin x = 0.3124$ **b)** $\cos x = -0.1476$ **c)** $5\tan x - 3 = 3\tan x + 7$

d) $2\sec x - 8 = 0$ **e)** $3\cot x + 2 = 0$ **f)** $\csc x + 2.5 = 0$

2. Determine exact solutions for each equation in the interval $x \in [0, 2\pi]$.

a) $2\sin x + \sqrt{3} = 0$ **b)** $\sec x - 2 = 0$ **c)** $\sqrt{3}\tan x - 1 = 0$

d) $\cot x + 1 = 0$ **e)** $2 - 3\cos x = 2$ **f)** $3\sin x - 1 = 4\sin x$

3. Determine approximate solutions for each equation in the interval $x \in [0, 2\pi]$, to the nearest hundredth of a radian.

a) $\sin^2 x - 0.64 = 0$ **b)** $9\cos^2 x - 4 = 0$ **c)** $\sec^2 x - 2.5 = 0$ **d)** $\cot^2 x - 1.21 = 0$

4. Determine exact solutions for each equation in the interval $x \in [0, 2\pi]$.

a) $\sin^2 x - \frac{1}{4} = 0$ **b)** $\cos^2 x - \frac{3}{4} = 0$ **c)** $\tan^2 x - 3 = 0$ **d)** $3\csc^2 x - 4 = 0$

5. Determine exact solutions where possible for each equation in the interval $x \in [0, 2\pi]$. Round approximate solutions to the nearest hundredth of a radian.

a) $\sin^2 x = \sin x$ **b)** $\sec x \csc x - 2\csc x = 0$

c) $2\cot x \sec x - 2\sqrt{3}\cot x = 0$ **d)** $\sin^2 x - 2\sin x - 3 = 0$

e) $\csc^2 x = 2 + \csc x$ **f)** $2\sec^2 x + \sec x - 1 = 0$

g) $\tan x = 6 - \tan^2 x$ **h)** $6\cos^2 x + 5\cos x = 6$

6. Determine exact solutions where possible for each equation in the interval $x \in [-2\pi, 0]$. Round approximate solutions to the nearest hundredth of a radian.

a) $2\cos^2 x + \sqrt{3}\cos x = 0$ **b)** $\tan x \csc x = -\tan x$

c) $\csc x \sec x + 2\sec x = 0$ **d)** $3\csc^2 x - 5\csc x = 2$

e) $\sec^2 x + 5\sec x + 6 = 0$ **f)** $5\tan x + 3 = 2\tan^2 x$

g) $2\cos^2 x - 2\sqrt{2}\cos x + 1 = 0$ **h)** $3\sin^2 x + \sin x - 1 = 0$

(Use the quadratic formula to find $\sin x$ ***to four decimal places and then solve for*** x***.)***

7. Determine exact solutions where possible for each equation in the indicated interval. Round approximate solutions to the nearest hundredth of a radian.

a) $\sin^3 x + \sin^2 x + \sin x + 1 = 0,\ x \in [-2\pi, 2\pi]$ **b)** $4\cot x \cos x - \cot x - 4\cos x + 1 = 0,\ x \in [0, 2\pi]$

Date:_____ **4.3 Solving Trigonometric Equations With Compound Angles**

1. Solve for $0 \le \theta \le 2\pi$ and $0 \le A \le 4\pi$.

a) $\sin 2\theta = 1$ **b)** $\sin 2\theta = -1$ **c)** $\sin 2\theta = \frac{1}{2}$

d) $2\cos 2\theta = 1$ **e)** $\cos \frac{1}{2} A = 1$ **f)** $\sin \frac{A}{2} = -1$

2. Solve the following equations by using an appropriate compound angle formula, $0 \le \theta \le 2\pi$.

a) $\sin\theta \cos\theta = 0.25$ **b)** $\cos^2\theta - \sin^2\theta = -\frac{1}{2}$

c) $2\cos^2\theta + 3\cos 2\theta = 3$ **d)** $3\cos 2\theta + \cos\theta + 1 = 0$

e) $3 + \sin\theta = 5\cos 2\theta$ **f)** $3\sin\theta + 3\cos 2\theta = 2$

3. The Pythagorean identities are $\sin^2 x + \cos^2 x = 1$, $\sec^2 x = 1 + \tan^2 x$ and $\csc^2 x = 1 + \cot^2 x$. Use an appropriate Pythagorean identity to create a quadratic trigonometric equation in terms of one trigonometric ratio and then solve for x in the indicated interval.

a) $2\sec^2 x - 3 + \tan x = 0,\ x \in [-2\pi, 0]$ **b)** $2\cot x + \csc^2 x = 0,\ x \in [0, 2\pi]$

c) $\frac{1}{1 + \tan^2 x} = -\cos x,\ x \in [-2\pi, 0]$ **d)** $6\sin^2 x = 17\cos x + 11,\ x \in [-\pi, \pi]$

e) $\cos^2 x + \sqrt{2}\sin x = \sin^2 x - \sqrt{2}\sin x + 2,\ x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ **f)** $\sqrt{3}\sin x = \sqrt{3} - \cos x,\ x \in [0, 2\pi]$

Hint: Square both sides first and check for extraneous roots.

Date:_____ **4.4 Solving Trigonometric Equations Involving Absolute Value & Solving Trigonometric Inequalities by Graphing**

1. Find all values of x in the indicated interval for each of the following. Include applicable graphs.

a) $|\cot x| = 1,\ -2\pi < x < 0$ **b)** $|\cos x| - 1 = 0,\ x \in [0, 2\pi]$ **c)** $\frac{1}{2} - \frac{|\csc x|}{2} = 0,\ -2\pi < x < 2\pi$

d) $\sin x \le \frac{1}{2},\ x \in [-\pi, \pi]$ **e)** $2\cos x + 1 > 0,\ x \in [0, 2\pi]$

f) $-1 \le \tan x \le 1,\ -\frac{\pi}{2} < x < \frac{3\pi}{2}$ **g)** $\sin x > \cos x,\ x \in [0, 2\pi]$

2. Solve each of the following equations.

a) $2\sin\left(x + \frac{\pi}{4}\right) + 1 = 0,\ 0 \le x \le 2\pi$, exact answers

b) $3\cos\left(x - \frac{\pi}{3}\right) + 3 = 5,\ 0 \le x \le 2\pi$, correct to two decimal places

3. Determine exact solutions where possible for each equation in the interval $x \in [0, 2\pi]$. Round approximate solutions to the nearest hundredth of a radian.

a) $3\cot^2 x = 1$ **b)** $\sin 2x = \cos x$

c) $\sin x = \tan x$ **d)** $2 + \cos 2x = 3\cos x$

e) $\csc^4 x - 4\csc^2 x + 4 = 0$ **f)** $-5\sin x = \cos x$

g) $3\sec\frac{x}{2} + 5 = 0$ **h)** $2\sin 3x\cos x - 2\cos 3x\sin x + 1 = 0$

4. Determine exact solutions where possible for each equation in the interval $x \in [-2\pi, 0]$. Round approximate solutions to the nearest hundredth of a radian. Include any restrictions on the variable.

a) $3\tan^3 x = \tan x$ **b)** $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 2$

c) $2\sin x - \cos 2x = 0$ **d)** $\sin\left(x + \frac{\pi}{4}\right) = \cos x$

5. Find the exact points of intersection of $\begin{cases} f(x) = 4\sin^2 x + 7\sin x + 6 \\ g(x) = 2\cos^2 x - 4\sin x + 11 \end{cases}$ on the domain $x \in [0, 3\pi]$.

Date:_____ **4.5 Transformations of Sine and Cosine Graphs**

1. For each of the following state any reflections, the amplitude, period, phase shift and vertical translation.

a) $y = 20\sin 3\theta + 10$

b) $f(x) = \frac{1}{5}\cos\left[4\left(x + \frac{\pi}{2}\right)\right] - 3$

c) $y = \sin\left(-2\theta + \frac{\pi}{8}\right)$

d) $g(x) = -\frac{5}{2}\sin\left(\frac{2}{3}x - \frac{\pi}{6}\right) - \frac{4}{5}$

e) $f(x) = 5\sin[\pi(x+2)] + 1$

f) $P(t) = -20\cos\frac{\pi}{12}t + 100$

2. State any reflections, the amplitude, period, phase shift, and vertical translation for the following functions and graph for one cycle.

a) $y = 2\sin\left(x + \frac{\pi}{2}\right)$

b) $f(x) = -3\sin 3x + 2$

c) $y = \cos\left(2\theta - \frac{\pi}{2}\right) - 1$

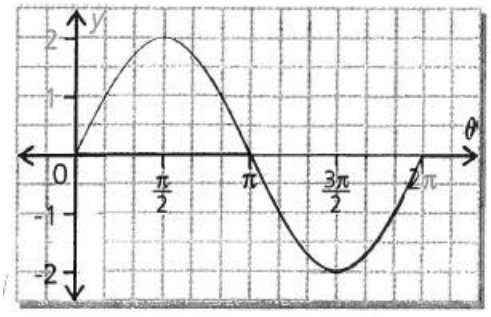
d) $h(t) = -8\cos\frac{\pi}{18}t + 10$

3. Graph $f(x) = 2\sin(-x)$ and its reciprocal function $y = \frac{1}{f(x)}$ on the same grid for $x \in [-2\pi, 0]$.

4. Graph $f(x) = -3\cos x - 2$ and its absolute value function $y = |f(x)|$ on the same grid for $x \in [0, 2\pi]$.

5. The following graphs are sine curves. Determine the equation of each graph.

a)



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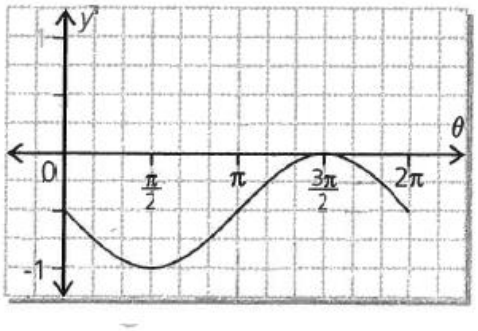
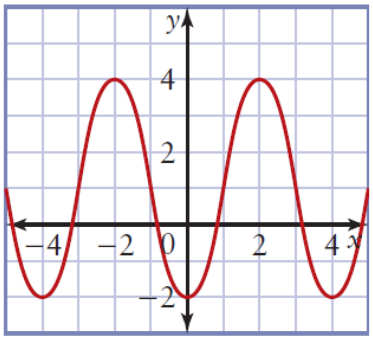


6. The following graphs are cosine curves. Determine the equation of each graph.

a)



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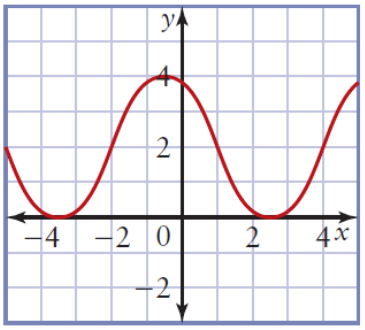


7. A sine function has a maximum value of 7, a minimum value of -1, a phase shift of $\frac{2\pi}{4}$ radians to the left, and a period of $\frac{\pi}{2}$.

a) Write an equation for the function.

b) Graph the function for two cycles to verify that it has the properties given.

8. A cosine function has a maximum value of 1, a minimum value of -5, a phase shift of 2 radians to the right, and a period of 8.

a) Write an equation for the function.

b) Graph the function for two cycles to verify that it has the properties given.

9. The function $y = \cos x$ is reflected in the *y*-axis, vertically stretched by a factor of 4, horizontally expanded by a factor of 2, horizontally translated 2π units to the left.

a) Write an equation for the transformed function.

b) Graph the function for two cycles to verify that it has the properties given.

10. The function $y = \sin x$ is reflected in the *x*-axis, vertically compressed by a factor of 0.5, horizontally compressed by a factor of $\frac{2}{3}$ and vertically translated 2.5 units down.

a) Write an equation for the transformed function.

b) Graph the function for two cycles to verify that it has the properties given.

Date:_____ **4.6 Combinations of Transformations of the Sine and Cosine Functions**

1. State any reflections, the amplitude, period, phase shift, and vertical translation for the following functions and graph for one cycle. State the domain and range.

a) $y = 2\sin 2\left(x - \frac{\pi}{4}\right)$ **b)** $y = 3\cos 2\left(x + \frac{\pi}{4}\right) - 1$ **c)** $y = -\sin\frac{1}{3}(x - 2\pi)$

d) $y = 2\cos\frac{1}{2}(x - \pi) + 1$ **e)** $y = \frac{1}{2}\sin\left(\frac{1}{2}x - \pi\right) - 2$ **f)** $y = -2\cos(3x - \pi) + 2$

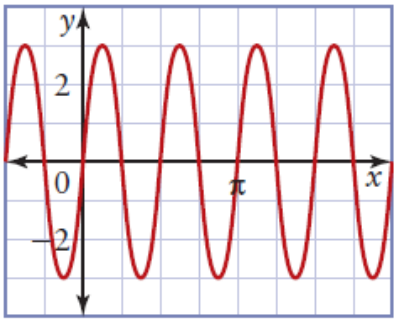
2. For each of the following state any reflections, the amplitude, period, phase shift and vertical translation. Graph the curve for the specified domain and then state the range.

a) $y = 3\cos x - 2,\ \ 0 \le x \le 3\pi$ **b)** $y = -2\sin 2x + 2,\ -\pi \le x \le \pi$

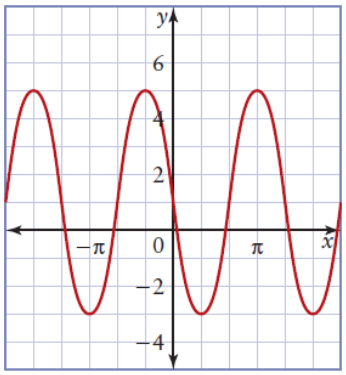
c) $y = \frac{1}{2}\sin\left(x + \frac{\pi}{4}\right),\ -2\pi \le x \le \pi$ **d)** $y = 5\cos 2\left(x - \frac{\pi}{2}\right) + 1,\ -2\pi \le x \le 2\pi$

e) $y = -2\cos\left(2x + \frac{\pi}{3}\right) - 1,\ -\pi \le x \le \pi$ **f)** $y = -4\sin\left(\frac{1}{3}x - \pi\right),\ -\pi \le x \le 3\pi$

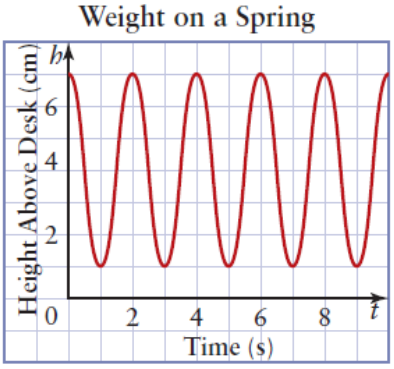
3. Determine an equation for the cosine function graphed below.



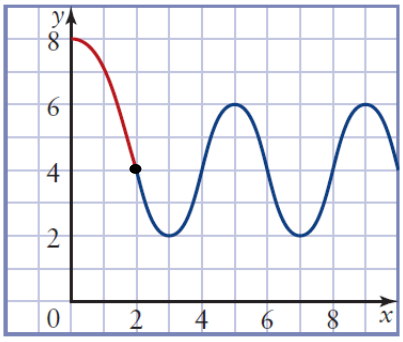
4. Determine three equations for the sine function graphed below.



5. A weight is suspended on a spring and set in motion such that it bobs up and down vertically. The graph shows the height, *h*, in centimetres, of the weight above the desk after time, *t*, in seconds. Use the graph to determine a model of the height versus time using a cosine function.



6. A rollercoaster at a theme park starts with a vertical drop that leads into two pairs of identical valleys and hills, as shown. Use the graph to determine the equation of this *quadratic/sinusoidal* piecewise function.



Date:_____ **4.7 Applications of Trigonometric Functions**

Sound

1. A pure tone made by a tuning fork can be observed as a sine curve on a piece of scientific equipment called an oscilloscope. For the note A, above middle C, the equation of the curve is $y = 10\sin 880\pi x$.

a) Determine the amplitude and the period.

b) Determine how $y = \sin x$ could be transformed onto $y = 10\sin 880\pi x$.

Electricity

2. The voltage, *V*, in volts, given by $V(t) = 170\sin 120\pi t$, where *t* is the time in seconds.

A component in the circuit can safely withstand a voltage of more than 120 volts for 0.01 s or less.

a) Determine the amplitude and the period for this function.

b) The number of cycles completed in 1 s is the frequency of the current. Determine the frequency.

c) Graph two cycles of the function.

d) Determine the length of time that the voltage is greater than 120 V on each half cycle. Is it safe to use this component in this circuit? Just your answer.

Spring

3. An object attached to the end of a spring is oscillating up and down. The displacement of the object, *d*, in centimetres, is a function of the time, *t*, is seconds, and is given by

$d(t) = 2.4\cos\left(12t + \frac{\pi}{6}\right)$.

a) Graph two cycles of the function.

b) What is the maximum distance through which the object oscillates?

c) What is the period of the function? Give your answer as an exact number of seconds in terms of π, and as an approximate number of seconds, to the nearest hundredth.

Ocean Cycles

4. The water depth in a harbor is 21 m at high tide and 11 m at low tide. One cycle is completed approximately every 12 h.

a) Find an equation for the water depth, *h*, as a function of time, *t* hours, after low tide.

b) Graph two cycles of the function.

5. A buoy bobs up and down in the water. The distance between the highest and lowest points is 1.5 m. It takes 6 s for the buoy to move from its highest point to its lowest point and back to its highest point.

a) Model the vertical displacement, *v*, in metres, of the buoy as a function of time, *t*, in seconds. Assume that the buoy is at its equilibrium point at $t = 0$ s and that the buoy is on its way down at that time.

b) Graph *v* versus *t* over two cycles.

Ferris Wheel

6. A carnival Ferris wheel with a radius of 7 m makes one complete revolution every 16 s. The bottom of the wheel is 1.5 m above the ground.

a) Draw a graph to show how a person's height above the ground varies with time for two revolutions, starting when the person gets onto the Ferris wheel at its lowest point.

b) Find both a sine and cosine equation for the graph.

7. The height, *h*, in metres, above the ground of a rider on a Ferris wheel can be modeled by the equation $h(t) = -10\cos\left(\frac{\pi}{15}t\right) + 12$, where *t* is the time, in seconds.

a) At $t = 0$ s, the rider is at the lowest point. Determine the first two times that the rider is 20 m above the ground, to the nearest hundredth of a second.

b) Use a graph to illustrate your solutions to part **a)**.

Archery

8. The range of an arrow shot from a particular bow can be modeled by the equation $r = 100\sin 2\theta$, where *r*, is the range, in metres, and θ is the angle, in radians, above the horizontal that the arrow is released. A target is placed 80 m away.

a) What are the restrictions on the angle θ? Justify your answer.

b) Determine the angle or angles that the archer should use to hit the target, to the nearest hundredth of a radian and to the nearest degree.

Blood Pressure

9. Each person's blood pressure is different. But there is a range of blood pressure values that is considered healthy. The function $p(t) = -20\cos\frac{5\pi}{3}t + 100$ models blood pressure, *p*, in millimetres of mercury, at time, *t*, in seconds of a person at rest.

a) What is the period of the function? What does the period represent for an individual?

b) How many times does this person's heart beat each minute?

c) What is the range of the function? Explain the meaning of the range in terms of a person's blood pressure.

Date:_____ **Unit 4 Review**

1. Solve each of the following equations to two decimal places, $0 \le \theta \le 2\pi$.

a) $\sin\theta = -0.1357$ **b)** $3\cos\frac{\theta}{2} + 1 = 0$ **c)** $\frac{1}{2}\cot 2\theta - \frac{3}{4} = 0$

2. Solve each of the following. Give exact answers only, $0 \le x \le 2\pi$.

a) $\sec x - \sqrt{2} = 0$ **b)** $\frac{2}{\sin x} - 10 = -6$ **c)** $\frac{5\cot x}{2} + \frac{7}{3} = -\frac{1}{6}$ **d)** $3\csc^2 x - 4 = 0$

3. Solve each of the following equations.

a) $3\cos\left(x - \frac{\pi}{3}\right) + 3 = 1$, $-2\pi \le x \le 0$, correct to two decimal places.

b) $\tan\left(x + \frac{5\pi}{6}\right) = \sqrt{3}$, $-2\pi \le x \le 0$, exact answers.

4. Solve each of the following equations for θ where $0 \le \theta \le 2\pi$. Answer to the nearest hundredth of a radian if necessary.

a) $2\cos^2\theta + \sin\theta - 1 = 0$ **b)** $2\cos^2\left(\frac{\theta}{2}\right) + \cos\left(\frac{\theta}{2}\right) = 0$ **c)** $2\cos\theta\sin\theta + 1 = 0$

d) $2\tan\theta + 3 = -\cot\theta$ **e)** $3\cos 2\theta + \cos\theta + 1 = 0$ **f)** $2\cos\theta + \sin 2\theta = 0$

g) $\sqrt{3}\sin 2\theta = 2\cos^2\theta - 1$ **h)** $\cos\theta - 2\cot\theta = 0$ **i)** $\sin\theta + \cos\theta = 1$ ***(Square both sides.)***

j) $2\sec 2\theta + \csc\theta = 0$ **k)** $\sin 2\theta + \cos 2\theta + \sin\theta + \cos\theta + 1 = 0$ **l)** $2\cos 3\theta + \cos 2\theta + 1 = 0$

5. The quadratic trigonometric equation $a\cos^2 x + b\cos x - 1 = 0$ has solutions $\frac{\pi}{3}$, π, and $\frac{5\pi}{3}$ in the interval $0 \le x \le 2\pi$. What are the values of *a* and *b*?

6. Solve the following trigonometric inequalities graphically. State your final answer in a solution set.

a) $|\cot x| \le 1$, $x \in (0, 2\pi)$ **b)** $\csc x > 2$, $x \in (0, 2\pi)$

c) $\cos 2x < \sin x$, $x \in [0, \pi]$ **d)** $\sin 2x \ge \tan x$, $x \in [-\pi, 0]$

7. Graph $f(x) = -2\cos x$ and its reciprocal function $y = \frac{1}{f(x)}$ on the same grid for $-2\pi \le x \le 0$.

8. Graph $f(x) = 2\sin x - 1$ and its absolute value function $y = |f(x)|$ on the same grid for $0 \le x \le 2\pi$.

9. A cosine function has a maximum value of 3, a minimum value of -11, a phase shift of 3 radians to the right, and a period of 8. Write the equation of the resulting function, graph for one cycle and state the domain and range.

10. The graph of the function $y = \sin x$ is transformed by vertically expanding it by a factor of 2, reflecting it in the *y*-axis, horizontally compressing it by a factor of 1/3, horizontally translating it $\pi/4$ units to the left, and vertically translating it 3 units down. Write the equation of the resulting function, graph for one cycle and state the domain and range.

11. For each of the following state any reflection, the amplitude, period, phase shift and vertical translation.

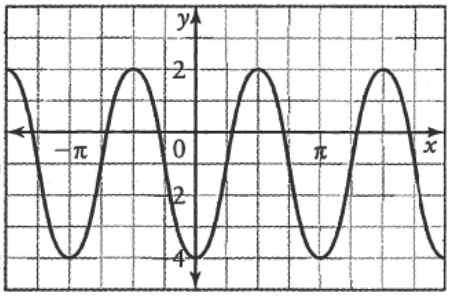
a) Graph $y = \frac{1}{2}\sin 3\left(\theta - \frac{\pi}{4}\right)$ for $-\pi \le \theta \le \frac{\pi}{2}$.

b) Graph $y = -2\cos\left(2\theta + \frac{\pi}{3}\right) - 1$ for $-\frac{\pi}{2} \le \theta \le \pi$.

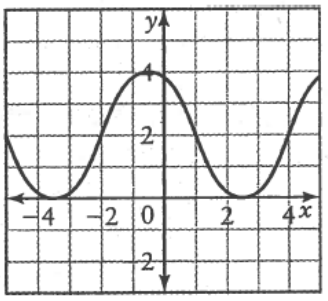
c) Graph $y = -3\sin\left(\frac{1}{2}\theta - \frac{\pi}{6}\right) + 3$ for $-2\pi \le \theta \le 3\pi$.

12. Determine an equation for each sine function.

a)

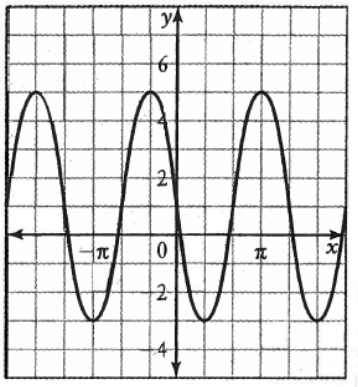


b)

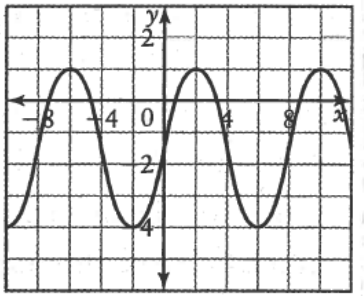


13. Determine an equation for each cosine function.

a)



b)



14. The range of a "human-cannonball" at a circus can be modeled by the equation $r = \frac{v^2}{g}\sin 2\theta$, where r is the range, in metres; v is the launch speed, in metres per second; g is the acceleration due to gravity, 9.8 m /s^2; and θ is the angle above the horizontal that the cannon is aimed. The target is placed 20 m away. If the human cannonball is launched at 15 m /s, determine the angle or angles at which the cannon could be aimed to hit the target.

15. Today, the high tide in Matthews Cove, New Brunswick, is at midnight. The water level at high tide is 7.5 m. The depth, d in metres, of water in the cove at time t hours is modelled by the equation $d(t) = 4 + 3.5\cos\frac{\pi}{6}t$. Jenny is planning a day trip to the cove tomorrow, but the water needs to be at least 2 m deep for her to manoeuvre her sailboat safely. Graph the function for two cycles and determine the time interval for her to safely sail in Matthews Cove.

16. A Ferris wheel has a diameter of 20 m and rotates at the rate of one revolution every 48 s. At the bottom of the ride, the passenger is 2 m above the ground. You start your ride from the bottom of the wheel. Determine a function that represents your height h above the ground at any time t, and use it to determine your height above the ground 88 seconds into the ride.
Graph for two cycles.

Answers 4.1

1. a) iii, $270^\circ \le \theta \le 360^\circ$, $\frac{3\pi}{2} \le \theta \le 2\pi$ **b)** i, $90^\circ \le \theta \le 180^\circ$, $\frac{\pi}{2} \le \theta \le \pi$ **c)** iv, $-180^\circ \le \theta \le -90^\circ$, $-\pi \le \theta \le -\frac{\pi}{2}$

d) ii, $-90^\circ \le \theta \le 0^\circ$, $-\frac{\pi}{2} \le \theta \le 0$ **2. a)** ii, $-180^\circ \le \theta \le 0^\circ$, $-\pi \le \theta \le 0$ **b)** iii, $-270^\circ \le \theta \le -90^\circ$, $-\frac{3\pi}{2} \le \theta \le -\frac{\pi}{2}$

c) iv, $90^\circ \le \theta \le 360^\circ$, $\frac{\pi}{2} \le \theta \le 2\pi$ **d)** i, $0^\circ \le \theta \le 270^\circ$, $0 \le \theta \le \frac{3\pi}{2}$ **3. a)** i, $-180^\circ \le \theta < -90^\circ$, $-\pi \le \theta < -\frac{\pi}{2}$

b) ii, $-270^\circ < \theta \le -180^\circ$, $-\frac{3\pi}{2} < \theta \le -\pi$ **c)** iii, $0^\circ \le \theta < 90^\circ$, $0 \le \theta < \frac{\pi}{2}$ **d)** iv, $90^\circ < \theta < 270^\circ$, $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$

Answers 4.2

1. a) 0.32, 2.82 **b)** 1.72, 4.56 **c)** 1.37, 4.51 **d)** 1.32, 4.97 **e)** 2.16, 5.30 **f)** 3.55, 5.87

2. a) $\frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $\frac{\pi}{3}, \frac{5\pi}{3}$ **c)** $\frac{\pi}{6}, \frac{7\pi}{6}$ **d)** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **e)** $\frac{\pi}{2}, \frac{3\pi}{2}$ **f)** $\frac{3\pi}{2}$

3. a) 0.93, 2.21, 4.07, 5.36 **b)** 0.84, 2.30, 3.98, 5.44 **c)** 0.89, 2.26, 4.03, 5.40 **d)** 0.74, 2.40, 3.88, 5.55

4. a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **c)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **d)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

5. a) $0, \frac{\pi}{2}, \pi, 2\pi$ **b)** $\frac{\pi}{3}, \frac{5\pi}{3}$ **c)** $0.96, \frac{\pi}{2}, \frac{3\pi}{2}, 5.33$ **d)** $\frac{3\pi}{2}$ **e)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **f)** π **g)** 1.11, 1.89, 4.25, 5.03 **h)** 0.84, 5.44

6. a) $-\frac{3\pi}{2}, -\frac{7\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2}$ **b)** $-2\pi, -\pi, -\frac{\pi}{2}, 0$ **c)** $-\frac{5\pi}{6}, -\frac{\pi}{6}$ **d)** $-\frac{11\pi}{6}, -\frac{7\pi}{6}$ **e)** $-4.37, -\frac{4\pi}{3}, -\frac{2\pi}{3}, -1.91$

f) $-5.03, -3.60, -1.89, -0.46$ **g)** $-\frac{7\pi}{4}, -\frac{\pi}{4}$ **h)** $-5.83, -3.59, -2.27, -0.88$ **7. a)** $-\frac{\pi}{2}, \frac{3\pi}{2}$ **b)** $\frac{\pi}{4}, 1.32, \frac{5\pi}{4}, 4.96$

Answers 4.3

1. a) $\frac{\pi}{4}, \frac{5\pi}{4}$ **b)** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **c)** $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ **d)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **e)** $0, 4\pi$ **f)** 3π

2. a) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ **b)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **c)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **d)** $\frac{\pi}{3}, \frac{5\pi}{3}, 2.30, 3.98$ **e)** $\frac{7\pi}{6}, \frac{11\pi}{6}, 0.42, 2.73$

f) 0.82, 2.32, 3.37, 6.05

3. a) $-\frac{\pi}{4}, -2.68, -\frac{5\pi}{4}, -5.82$ **b)** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **c)** $-\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}$ **d)** $-1.91, 1.91$ **e)** $\frac{3\pi}{4}$ **f)** $\frac{\pi}{6}, \frac{\pi}{2}$ ***Note:*** $\frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ *are extraneous roots*

Answers 4.4

1. a) $-\frac{7\pi}{4}, -\frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4}$ **b)** $0, \pi, 2\pi$ **c)** $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ **d)** $-\pi \le x \le \frac{\pi}{6}, \frac{5\pi}{6} \le x \le \pi$ **e)** $0 \le x < \frac{2\pi}{3}, \frac{4\pi}{3} < x \le 2\pi$

f) $-\frac{\pi}{4} \le x \le \frac{\pi}{4}, \frac{3\pi}{4} \le x \le \frac{5\pi}{4}$ **g)** $\frac{\pi}{4} < x < \frac{5\pi}{4}$ **2. a)** $\frac{11\pi}{12}, \frac{19\pi}{12}$ **b)** $0.21, 1.89$

3. a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **c)** $0, \pi, 2\pi$ **d)** $0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$ **e)** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ **f)** 2.94, 6.09

g) 4.42 **h)** $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

4. a) $-2\pi, -\frac{11\pi}{6}, -\frac{7\pi}{6}, -\pi, -\frac{5\pi}{6}, -\frac{\pi}{6}, 0$ *Note:* $x \ne -\frac{3\pi}{2}, -\frac{\pi}{2}$ **b)** $-2\pi, 0$ *Note:* $x \ne -\frac{3\pi}{2}, -\frac{\pi}{2}$

c) $-6.07, -3.35$ **d)** $-\frac{15\pi}{8}, -\frac{11\pi}{8}, -\frac{7\pi}{8}, -\frac{3\pi}{8}$ **5.** $\left(\frac{\pi}{6}, \frac{21}{2}\right), \left(\frac{5\pi}{6}, \frac{21}{2}\right), \left(\frac{13\pi}{6}, \frac{21}{2}\right), \left(\frac{17\pi}{6}, \frac{21}{2}\right)$

Answers 4.5

1. a) amplitude: 20, period: $\frac{2\pi}{3}$, phase shift: none, vertical translation: up 10

b) amplitude: $\frac{1}{5}$, period: $\frac{\pi}{2}$, phase shift: left $\frac{\pi}{2}$, vertical translation: down 3

c) horizontal reflection in the θ -axis, amplitude: 1 , period: π , phase shift: right $\frac{\pi}{16}$, vertical translation: none

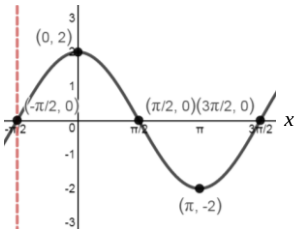
d) vertical reflection in the x -axis, amplitude: 2.5 , period: 3π , phase shift: right $\frac{\pi}{4}$, vertical translation: down 0.8

e) amplitude: 5 , period: 2 , phase shift: left 2 , vertical translation: up 1

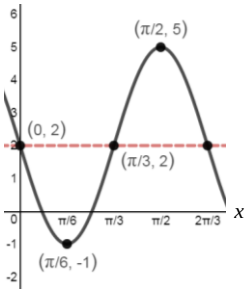
f) vertical reflection in the t -axis, amplitude: 20 , period: 24 , phase shift: none, vertical translation: up 100

y *y*

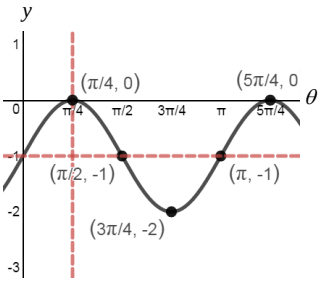
2. a) amplitude: 2
period: 2π
phase shift: left $\pi/2$
vertical translation: none



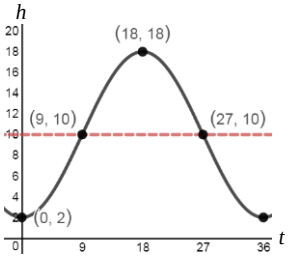
b) reflection in x-axis
amplitude: 3
period: $2\pi/3$
vertical translation: up 2



c) amplitude: 1
period: π
phase shift: right $\pi/4$
vertical translation: down 1



d) reflection in t-axis
amplitude: 8
period: 36
vertical translation: up 10



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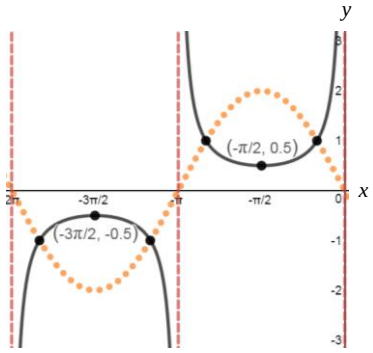

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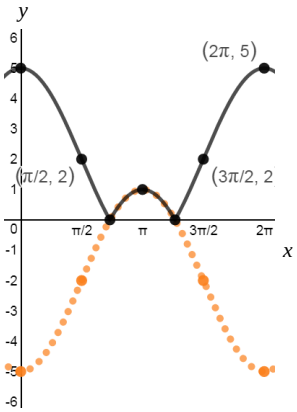
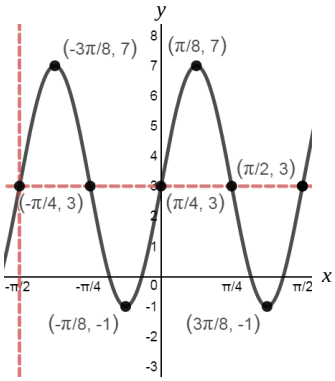


5. a) $y = 2\sin\theta$ **b)** $y = -\frac{1}{2}\sin\theta - \frac{1}{2}$

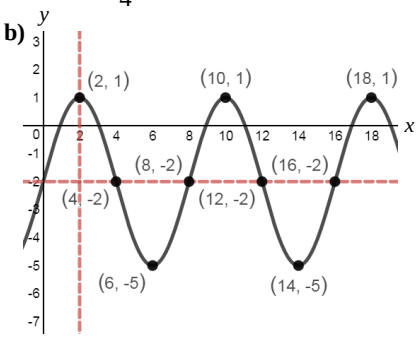
6. a) $y = -3\cos\frac{\pi}{2}x + 1$ **b)** $y = 2\cos\frac{\pi}{3}\left(x + \frac{1}{2}\right) + 2$

7. a) $y = 4\sin 4\left(x + \frac{\pi}{2}\right) + 3$



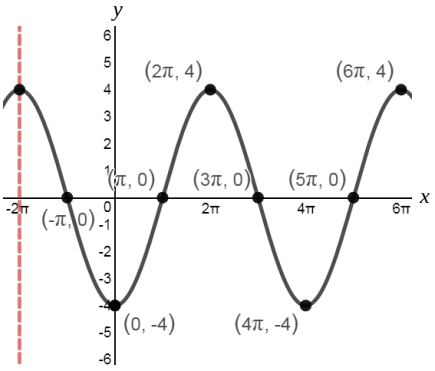
8. a) $y = 3\cos\frac{\pi}{4}(x-2) - 2$

b)



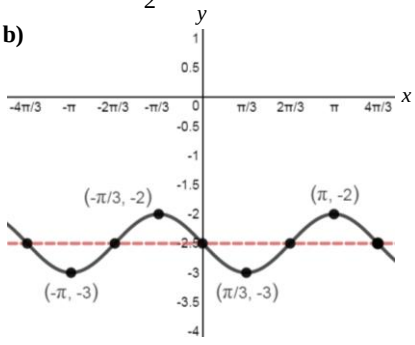
9. a) $y = 4\cos\left[-\frac{1}{2}(x + 2\pi)\right]$

b)



10. a) $y = -0.5\sin\frac{3}{2}x - 2.5$

b)



Answers 4.6

1. a) domain: $\frac{\pi}{4} \le x \le \frac{5\pi}{4}$, range: $-2 \le y \le 2$, amplitude: 2, period: π, phase shift: $\frac{\pi}{4}$ to the right

b) domain: $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, range: $-4 \le y \le 2$, amplitude: 3, period: π, phase shift: $\frac{\pi}{4}$ to the left, vertical translation: down 1

c) domain: $2\pi \le x \le 8\pi$, range: $-1 \le y \le 1$, reflection: in x-axis, amplitude: 1, period: 6π, phase shift: 2π to the right

d) domain: $\pi \le x \le 5\pi$, range: $-1 \le y \le 3$, amplitude: 2, period: 4π, phase shift: π to the right, vertical translation: up 1

e) domain: $2\pi \le x \le 6\pi$, range: $-\frac{5}{2} \le y \le -\frac{3}{2}$, amplitude: $\frac{1}{2}$, period: 4π, phase shift: 2π to the right, vertical translation: down 2

f) domain: $\frac{\pi}{3} \le x \le \pi$, range: $0 \le y \le 4$, reflection: in the x-axis, amplitude: 2, period: $\frac{2\pi}{3}$, phase shift: $\frac{\pi}{3}$ to the right, vertical translation: up 2

2. a) range: $-5 \le y \le 1$, amplitude: 3, period: 2π, phase shift: none, vertical translation: down 2

b) range: $0 \le y \le 4$, reflection: in the x-axis, amplitude: 2, period: π, phase shift: none, vertical translation: up 2

c) range: $-\frac{1}{2} \le y \le \frac{1}{2}$, amplitude: $\frac{1}{2}$, period: 2π, phase shift: $\frac{\pi}{4}$ to the left

d) range: $-4 \le y \le 6$, amplitude: 5, period: π, phase shift: $\frac{\pi}{2}$ to the right, vertical translation: up 1

e) range: $-3 \le y \le 1$, reflection: in the x-axis, amplitude: 2, period: π, phase shift: $\frac{\pi}{6}$ to the left, vertical translation: down 1

f) range: $-2\sqrt{3} \le y \le 4$, reflection: in the x-axis, amplitude: 4, period: 6π, phase shift: 3π to the right

3. $y = 3\cos 4\left(x - \frac{\pi}{8}\right)$

4. $y = 4\sin\frac{3}{2}\left(x + \frac{2\pi}{3}\right) + 1,\ y = -4\sin\frac{3}{2}x + 1,\ y = 4\sin\frac{3}{2}\left(x - \frac{2\pi}{3}\right) + 1$

5. $h(t) = 3\cos(\pi t) + 4$

6. $f(x) = \begin{cases} -x^2 + 8 & if\ 0 \le x < 2 \\ 2\sin\left(\frac{\pi}{2}x\right) + 4 & if\ 2 \le x \le 10 \end{cases}$ *or* $f(x) = \begin{cases} -x^2 + 8 & if\ 0 \le x \le 2 \\ 2\sin\left(\frac{\pi}{2}x\right) + 4 & if\ 2 < x \le 10 \end{cases}$

Answers 4.7

1. a) amplitude: 10, period: $\frac{1}{440}$ **b)** vertical expansion by a factor of 10, horizontal compression by a factor of $\frac{1}{880\pi}$

2. a) amplitude: 170, period: $\frac{1}{60}$ **b)** 60 Hz **d)** 0.004 seconds; Yes, since 0.004 s < 0.01 s

3. b) 4.8 cm **c)** $\frac{\pi}{6}$ or 0.52 s **4.** $h = 5\sin\frac{\pi}{6}(t-3) + 16$, $h = -5\cos\frac{\pi}{6}t + 16$

5. b) $v = -\frac{3}{4}\sin\frac{\pi}{3}t$ **6. b)** $h = 7\sin\frac{\pi}{8}(t-4) + 8.5$, $h = -7\cos\frac{\pi}{8}t + 8.5$

7. a) 11.93 s, 18.07 s **8. a)** $0 \le \theta \le \frac{\pi}{2}$ **b)** 0.46, 1.11 or 26°, 64°

9. a) 1.2 s; time between consecutive heartbeats **b)** 50 **d)** $80 \le p(t) \le 120$; The person's blood pressure measures 120 over 80

Answers Review

1. a) 3.28, 6.15 **b)** 3.82 **c)** 0.29, 3.44, 1.87, 5.01 **2. a)** $\frac{\pi}{4}, \frac{7\pi}{4}$ **b)** $\frac{\pi}{6}, \frac{5\pi}{6}$ **c)** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **d)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

3. a) −2.94, −1.25 **b)** $-\frac{3\pi}{2}, -\frac{\pi}{2}$ **4. a)** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $\pi, \frac{4\pi}{3}$ **c)** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **d)** 2.68, 5.82, $\frac{3\pi}{4}, \frac{7\pi}{4}$ **e)** 2.30, 3.98, $\frac{\pi}{3}, \frac{5\pi}{3}$

f) $\frac{\pi}{2}, \frac{3\pi}{2}$ **g)** $\frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$ **h)** $\frac{\pi}{2}, \frac{3\pi}{2}$ **i)** $0, \frac{\pi}{2}, 2\pi$ *Note:* $\pi, \frac{3\pi}{2}$ *are extraneous* **j)** 3.52, 5.91

k) $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{4}, \frac{7\pi}{4}$ **l)** $0.72, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 5.56$ **5.** $a = 2, b = 1$

6. a) $\left\{x \in R \,\middle|\, \frac{\pi}{4} \le x \le \frac{3\pi}{4},\ \frac{5\pi}{4} \le x \le \frac{7\pi}{4}\right\}$ **b)** $\left\{x \in R \,\middle|\, 0 < x < \frac{\pi}{6},\ \frac{5\pi}{6} < x < \pi\right\}$ **c)** $\left\{x \in R \,\middle|\, \frac{\pi}{6} < x < \frac{5\pi}{6}\right\}$

d) $\left\{x \in R \,\middle|\, -\pi \le x \le -\frac{3\pi}{4},\ -\frac{\pi}{2} < x \le -\frac{\pi}{4},\ x = 0\right\}$ **9.** $y = 7\cos\frac{\pi}{4}(x-3) - 4$, $\{x \in R \,|\, 3 \le x \le 11\}$, $\{y \in R \,|\, -11 \le y \le 3\}$

10. $y = 2\sin\left[-3\left(x + \frac{\pi}{4}\right)\right] - 3$, $\left\{x \in R \,\middle|\, -\frac{11\pi}{12} \le x \le -\frac{\pi}{4}\right\}$, $\{y \in R \,|\, -5 \le y \le -1\}$

12. a) $y = 3\sin 2\left(x - \frac{\pi}{4}\right) - 1$ **b)** $y = 2\sin\frac{\pi}{3}(x+2) + 2$ **13. a)** $y = 4\cos\frac{3}{2}\left(x + \frac{\pi}{3}\right) + 1$ **b)** $y = 2.5\cos\frac{\pi}{4}(x-2) - 1.5$

14. 0.53, 1.04 **15.** During the day between 7:50 a.m. and 4:10 p.m. **16.** $h(t) = -10\cos\frac{\pi}{24}t + 12$ or $h(t) = 10\sin\frac{\pi}{24}(t-12) + 12$, 7 m