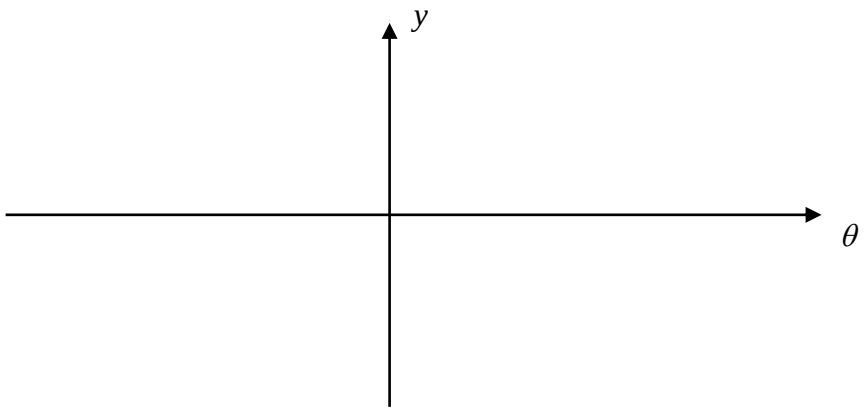
Date:_____ **UNIT 4: GRAPHING TRIGONOMETRIC FUNCTIONS & SOLVING TRIGONOMETRIC EQUATIONS**

4.1 Graphing the Primary and Reciprocal Trigonometric Functions

1. Graph $y = \sin\theta$ and its ***reciprocal*** function on the same grid for $-2\pi \le \theta \le 2\pi$.



Complete:

Properties of the function $y = \sin\theta$

i) Length of the period:_____

ii) Domain: _____

iii) Range: _____

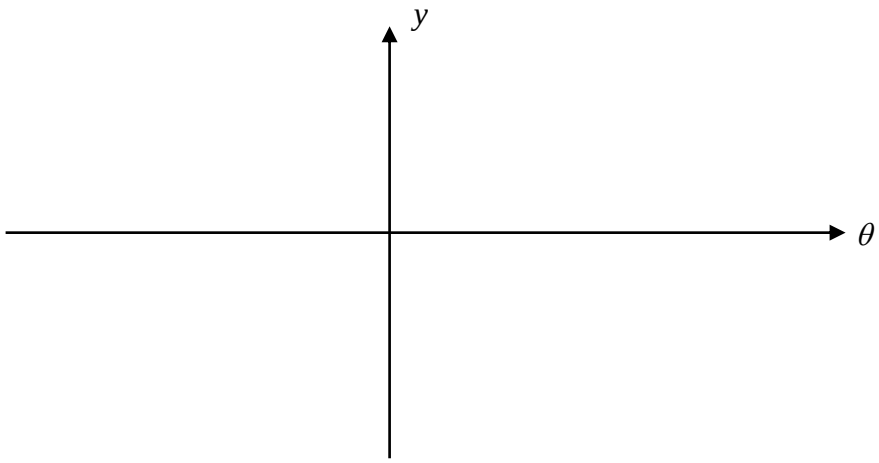
Properties of the function $y = \csc\theta$

i) Length of the period:_____

ii) Domain: _____

iii) Range: _____

2. Graph $y = \cos\theta$ and its ***reciprocal*** function on the same grid for $-2\pi \le \theta \le 2\pi$.



Complete:

Properties of the function $y = \cos\theta$

i) Length of the period:_____

ii) Domain: _____

iii) Range: _____

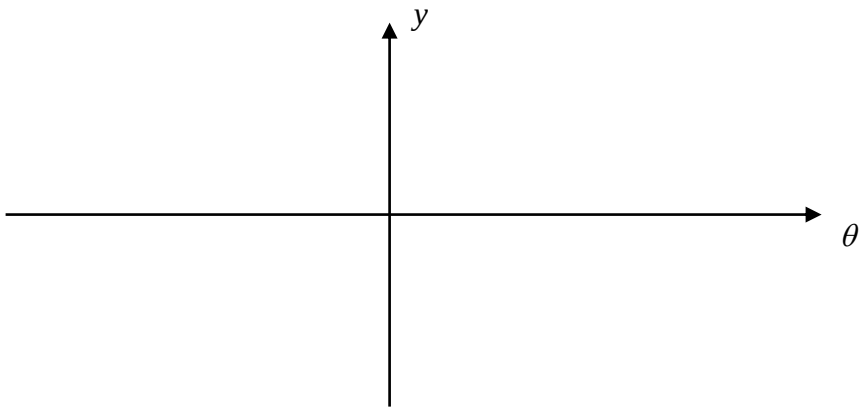
Properties of the function $y = \sec\theta$

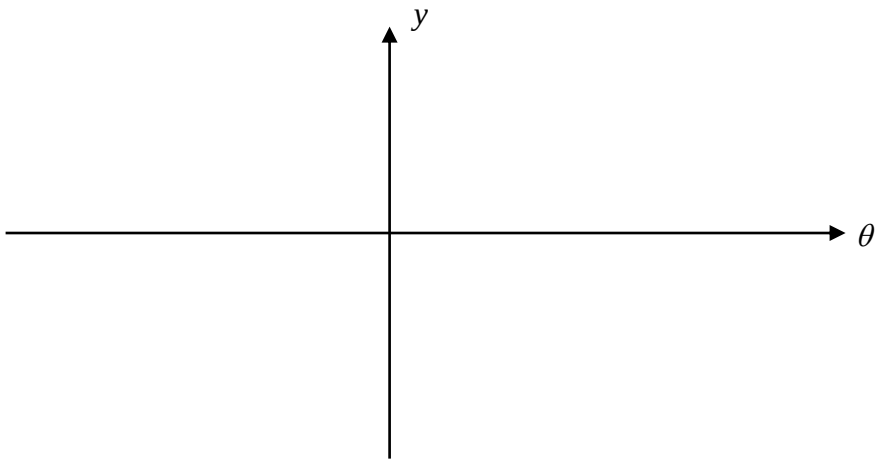
i) Length of the period:_____

ii) Domain: _____

iii) Range: _____

3. Graph $y = \tan\theta$ and its ***reciprocal*** function on separate grids for $-2\pi \le \theta \le 2\pi$.





Complete:

Properties of the function $y = \tan\theta$

i) Length of the period:_____

ii) Domain: _____

iii) Range: _____

iv) Asymptotes:_____

Properties of the function $y = \cot\theta$

i) Length of the period:_____

ii) Domain: _____

iii) Range: _____

iv) Asymptotes:_____

HW. ***Memoriz***e the graphs of all trigonometric functions for $-2\pi \le \theta \le 2\pi$ and complete **Exercise 4.1**

Date:_____ **4.2 Solving Linear & Quadratic Trigonometric Equations**

Ex. 1. Determine ***exact*** solutions for each equation in the interval $x \in [0, 2\pi]$.

a) $2\sin x - \sqrt{3} = 0$ **b)** $4\sec^2 x - 8 = 0$

Ex. 2. Determine ***approximate*** solutions for each equation in the interval $x \in [0, 2\pi]$, to the nearest hundredth of a radian.

a) $2\tan x + 1 = 0$ **b)** $\cot^2 x - 0.64 = 0$

Ex. 3. Determine exact solutions where possible for each equation in the interval $x \in [0, 2\pi]$. Round approximate solutions to the nearest hundredth of a radian.

a) $\sin x \cos x + \sin x = 0$ **b)** $2\csc^2 x + \csc x - 1 = 0$

c) $6\cos^2 x = 7\cos x - 2$

HW. Exercise 4.2

Date:_____ **4.3 Solving Trigonometric Equations With Compound Angles**

Ex. 1. Solve for $0 \le \theta \le 2\pi$ and $0 \le A \le 4\pi$.

a) $\cos^2 2\theta + \cos 2\theta = 0$

b) $\tan\frac{A}{2} + 1 = 0$

c) $2\sin 2\theta + \sqrt{3} = 0$

Ex. 2. Solve for θ. Give exact solutions where possible and round approximate solutions to the nearest hundredth of a radian.

a) $-5\sin\theta = \cos 2\theta + 2,\ -2\pi \le \theta \le 0$

b) $-5\cos\theta - \cos 2\theta = 2,\ 0 \le \theta \le 2\pi$

HW. Exercise 4.3

Date:_____ **4.4 Solving Trigonometric Equations Involving Absolute Value**
&
Solving Trigonometric Inequalities by Graphing

Warmup

Solve each of the following equations. State exact answers where possible.
Otherwise round your answers to two decimal places.

a) $2\cos\left(x+\frac{5\pi}{6}\right)+4=3,\ 0\le x\le 2\pi$

b) $5\tan\left(x-\frac{\pi}{3}\right)+3=1,\ -\pi\le x\le\pi$

Ex. 1. Find all values of x in the interval $[0,2\pi]$ that satisfy the equation $|\sin x|=1$.

Ex. 2. Find all values of x in the interval $[0, 2\pi]$ that satisfy the inequality $\sqrt{2} - 2\cos x > 0$.

Ex. 3. Find all values of x in the interval $[-2\pi, 0]$ that satisfy the inequality $\sin x - \cos x \leq 0$.

HW. Exercise 4.4

Date:_____ **4.5 Transformations of Sine and Cosine Graphs**

Given $y = a\,f\,[k(x-d)]+c$, the **transformations** on the graphs of $y = f(x)$ where $f(x) = \sin x$ or $f(x) = \cos x$ are as follows:

i) ***vertical reflection*** in the x-axis if $a < 0$

ii) ***vertical stretch*** by a factor of $|a|$

*Note: A stretch is an **expansion** if the stretch factor is more than* 1 *or a **compression** if the stretch factor is between* 0 *and* 1.

iii) ***horizontal reflection*** in the y-axis if $k < 0$

iv) ***horizontal stretch*** by a factor of $\frac{1}{|k|}$

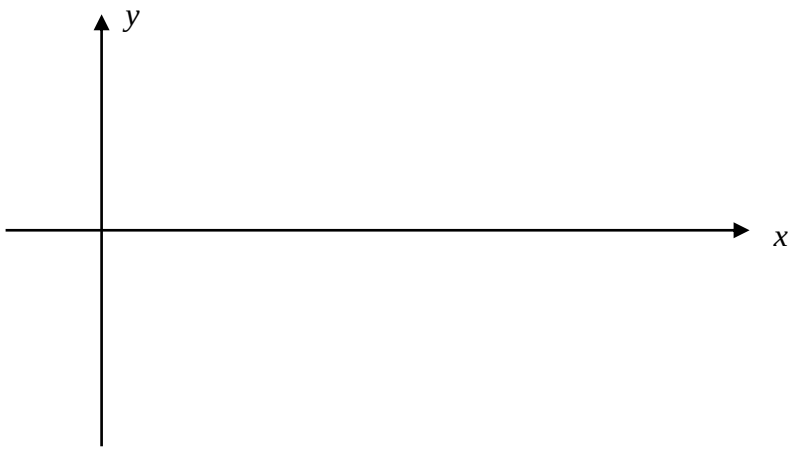
v) ***horizontal translation*** **right** $|d|$ units if $d > 0$ or **left** $|d|$ units if $d < 0$

vi) ***vertical translation*** **up** $|c|$ units if $c > 0$ or **down** $|c|$ units if $c < 0$

$$(x, y) \rightarrow \left(\frac{1}{k}x + d, ay + c\right)$$

Ex. 1. Graph each of the following functions by naming and using transformations on $y = \sin x$.

a) $y = -2\sin x$, $0 \le x \le 2\pi$



b) $y = \sin 2x$, $0 \le x \le 2\pi$

c) $y = \sin\left(x + \frac{\pi}{4}\right) + 1, \quad 0 \le x \le 2\pi$

Summary of Transformations on the Periodic Functions $y = \sin\theta$ ***and*** $y = \cos\theta$

For $y = a\sin k(\theta - d) + c$ and $y = a\cos k(\theta - d) + c$,

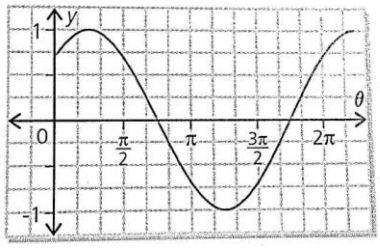
- the ***reflection*** of $y = \sin\theta$ or $y = \cos\theta$ is in the θ - axis if $a < 0$
- the ***reflection*** of $y = \sin\theta$ or $y = \cos\theta$ is in the y - axis if $k < 0$
- the ***amplitude*** is $|a|$
- the ***period*** is $\frac{1}{|k|} \times 2\pi$ or $\frac{2\pi}{|k|}$
- the ***phase shift*** is **right** $|d|$ units if $d > 0$ or **left** $|d|$ units if $d < 0$, and
- the ***vertical translation*** is **up** $|c|$ units if $c > 0$ or **down** $|c|$ units if $c < 0$

Ex. 2. For each of the following graphs determine:

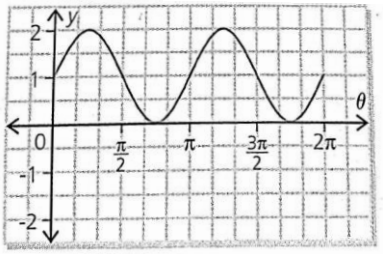
i) the amplitude, period, phase shift and vertical translation

ii) the sine function $y = a\sin k(\theta - d) + c$ and the cosine function $y = a\cos k(\theta - d) + c$

a)

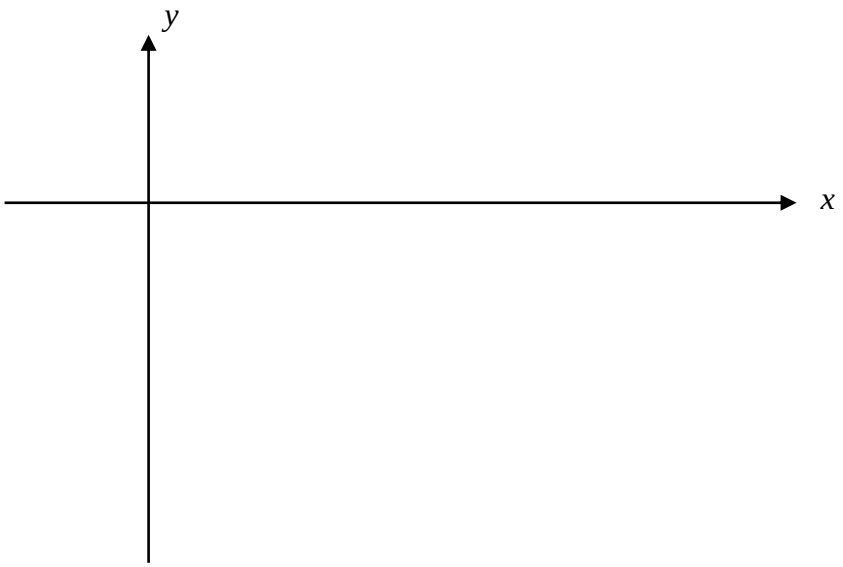


b)

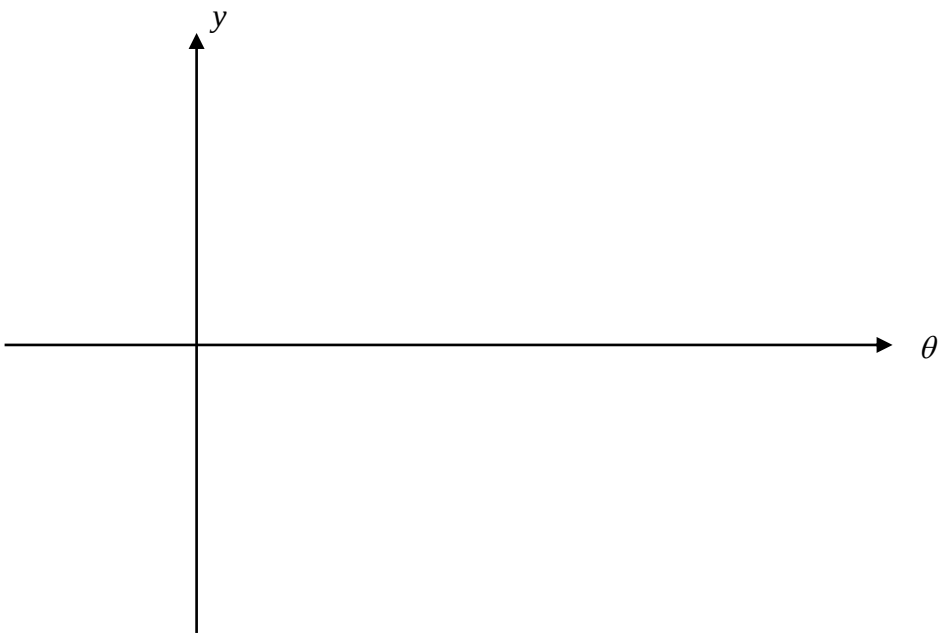


Ex. 3. State the amplitude, period, phase shift, and vertical translation for each of the following functions and graph for one period.

a) $f(x) = \sin 3x - 2$



b) $y = 3\cos\left(2\theta - \frac{\pi}{2}\right)$



HW. Exercise 4.5

Date:_____ **4.6 Combinations of Transformations of the Sine and Cosine Functions**

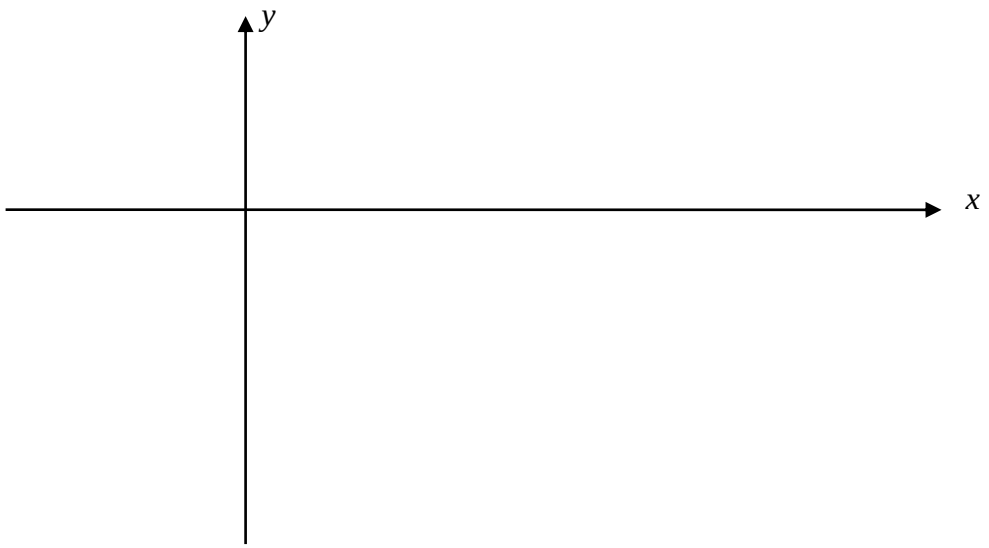
Ex. 1. For each of the following state any reflections, the amplitude, period, phase shift and vertical translation. Graph the curve for one cycle and state the domain and range.

a) $y = -2\cos\left(x + \frac{\pi}{4}\right) + 2$

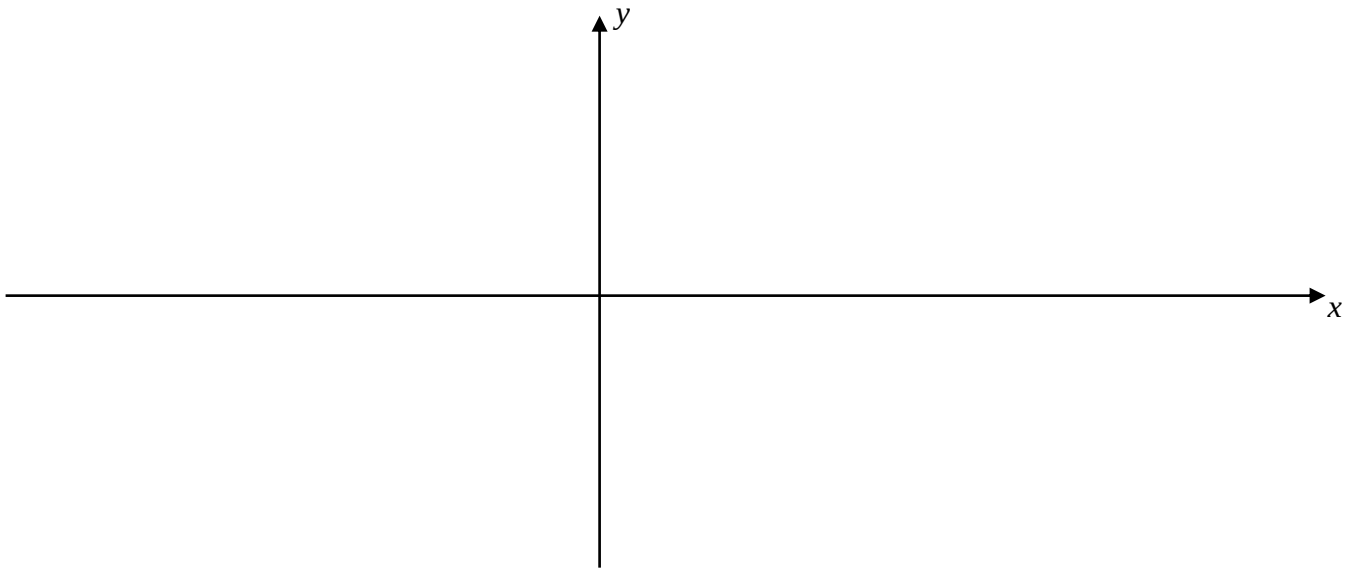
b) $y = \frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right)$

Ex. 2. For each of the following state any reflections, the amplitude, period, phase shift and vertical translation. Graph the curve for the specified domain and then state the range.

a) $y = -3\sin\left(\frac{1}{2}x - \frac{\pi}{2}\right) - 2,\ -2\pi \le x \le 4\pi$



b) $y = \cos 3\left(x + \frac{\pi}{4}\right),\ -\pi \le x \le \pi$



HW. Exercise 4.6

Date:_____ **4.7 Applications of Trigonometric Functions**

Ex. 1. A carnival Ferris wheel with a radius of 20 m makes three complete revolutions in 2 minutes. Passengers get on at the lowest point which is 1 m above the ground.

a) Draw a graph to show how a person's height, *h*, above the ground in metres, varies with time, *t*, in seconds, for two revolutions.

b) Write an equation which expresses your height as a function of time on the ride.

c) Calculate your height above the ground after 15 s.

d) At what times will the rider be 30 m above the ground?

Ex. 2. The daily high temperature of the city of Waterloo, in degrees Celsius, as a function of the number of days into the year, can be described by the function $T(d) = -20\cos\frac{2\pi}{365}(d+10)+25$

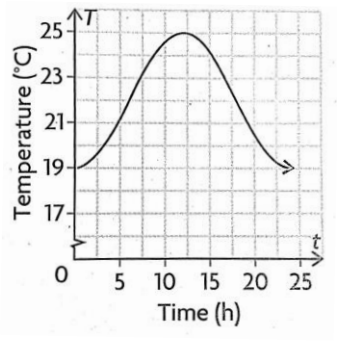
.

a) Use the function to determine today's temperature to the nearest degree Celsius.

b) Determine the range of this function . Explain the meaning of the range in this application.

Ex. 3. The temperature, *T*, in degrees Celsius, of the surface water in a swimming pool varies according to the following graph, where *t* is the number of hours since sunrise at 6 a.m.

a) Find possible cosine and sine equations for the temperature of the surface water as a function of time.



b) At what times is the temperature of the surface water at least $23^\circ C$?

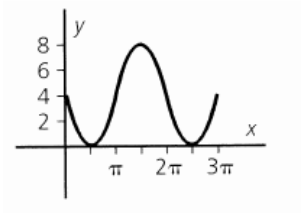
HW. Exercise 4.7

Date:_____ **Unit 4 Test Review**

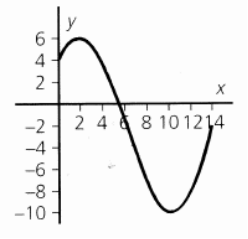
Warmup

1. Each of the diagrams below is the graph of a sinusoidal function.

a) Express as a sine function.



b) Express as a cosine function.



2. The function $y = \sin(x - c) + d$ has been vertically translated 3 units down and passes through the point $\left(\frac{\pi}{6}, -2\right)$. Determine the values of *c* and *d*.

3. Solve the following trigonometric inequality for *x* in the domain $[0, \pi]$ and state your final answer in a solution set.

$\cos 2x < \sin x$

HW. Unit 4 Review Exercise