Date:________________

UNIT 5: EXPONENTIAL FUNCTIONS

5.1 Laws of Exponents and Exponential Equations

Your study of calculus will require an ability to manipulate rational and negative exponents. The exponent laws enable us to simplify and evaluate expressions involving exponents. Here is a summary of the exponent laws.

Exponent Laws

$$a^m \times a^n = a^{m+n} \qquad a^{-m} = \left(\frac{1}{a}\right)^m \text{ or } \frac{1}{a^m} \qquad a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \frac{1}{a^{-m}} = a^m \qquad a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m \text{ or } \left(a^m\right)^{\frac{1}{n}}$$

$$\left(a^m\right)^n = a^{mn} \qquad \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m \qquad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m \text{ or } \sqrt[n]{a^m}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^0 = 1$$

Ex. 1. Evaluate each of the following:

a) $\dfrac{2^{-4}-2^{-6}}{2^{-5}-2^{-3}} \cdot \dfrac{2^6}{2^6}$

$= \dfrac{2^2-2^0}{2^1-2^3}$

$= \dfrac{4-1}{2-6}$

$= \dfrac{3}{-6}$

$= -\dfrac{1}{2}$

b) $\dfrac{9^{n+2} \times 27^{n-4}}{81^{n-1} \times 3^n}$

$= \dfrac{(3^2)^{n+2} \times (3^3)^{n-4}}{(3^4)^{n-1} \times 3^n}$

$= \dfrac{3^{2n+4} \times 3^{3n-12}}{3^{4n-4} \times 3^n}$

$= \dfrac{3^{5n-8}}{3^{5n-4}}$

$= 3^{(5n-8)-(5n-4)}$

$= 3^{-4}$

$= \dfrac{1}{81}$

c) $\dfrac{4^{-\frac{3}{2}} - 8^{\frac{1}{3}}}{16^{\frac{1}{4}} \times 25^{\frac{1}{2}}}$

$= \dfrac{\frac{1}{(\sqrt{4})^3} - \sqrt[3]{8}}{\sqrt[4]{16} \times \sqrt{25}}$

$= \dfrac{\frac{1}{8} - 2}{2 \times 5}$

$= \dfrac{\frac{1}{8} - \frac{16}{8}}{10}$

$= -\dfrac{15}{8} \div 10$

$= -\dfrac{\cancel{15}^{\,3}}{8} \times \dfrac{1}{\cancel{10}_{\,2}} = -\dfrac{3}{16}$

Ex. 2. Simplify each of the following, using the laws of exponents.

a) $\dfrac{\sqrt[3]{a^4} \times \sqrt{a}}{(\sqrt[4]{a})^3}$

$= \dfrac{a^{\frac{4}{3}} \times a^{\frac{1}{2}}}{a^{\frac{3}{4}}}$

$= a^{\frac{16}{12}+\frac{6}{12}-\frac{9}{12}}$

$= a^{\frac{13}{12}}$

b) $\dfrac{\left(\sqrt[3]{64x^6y^3}\right)\left(\sqrt{64x^{-6}y^4}\right)}{(-2x^2y^{-1})^3}$

$= \dfrac{(4x^2y)(8x^{-3}y^2)}{-8x^6y^{-3}}$

$= \dfrac{32x^{-1}y^3}{-8x^6y^{-3}}$

$= -4x^{-7}y^6$

$= -\dfrac{4y^6}{x^7}$

factor

c) $\dfrac{\left(x^2y - xy^2\right)^3}{(xy)^4}$

$= \dfrac{[xy(x-y)]^3}{(xy)^4}$

$= \dfrac{x^3y^3(x-y)^3}{x^4y^4}$

$= \dfrac{(x-y)^3}{xy}$

Exponential Equations

Ex. 3. Solve for *x*. ← to solve, need same (single) base on each side

a) $8^x = 16^{x-1}$

$(2^3)^x = (2^4)^{x-1}$

$2^{3x} = 2^{4x-4}$

$\because$ bases are equal

$\therefore 3x = 4x - 4$

$-x = -4$

$x = 4$

b) $3^{x^2+3} = 81^x$

$3^{x^2+3} = (3^4)^x$

$3^{x^2+3} = 3^{4x}$

$\because$ bases are equal

$\therefore x^2 + 3 = 4x$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\therefore x = 3$ or $x = 1$

c) $2^{x+5} - 2^{x+3} = 192$

$2^{x+3}(2^2 - 2^0) = 192$

$\frac{2^{x+3}(3)}{3} = \frac{192}{3}$

$2^{x+3} = 64$

$2^{x+3} = 2^6$

$\because$ bases are equal

$\therefore x + 3 = 6$

$x = 3$

d) $2^{2x} - 12(2^x) + 32 = 0$

$(2^x)^2 - 12(2^x) + 32 = 0$

Let $y = 2^x$

$y^2 - 12y + 32 = 0$

$(y-8)(y-4) = 0$

$y = 8$ or $y = 4$

$2^x = 2^3$ $\quad$ $2^x = 2^2$

$\because$ bases are equal

$\therefore x = 3$ or $x = 2$.

HW. Exercise 5.1

Date:______________

5.2 Investigating the Graphs of Exponential Functions

$$f(x) = b^x \ \ \& \ \ f(x) = a(b)^{k(x-d)} + c$$

I Graphing Exponential Functions of the Form $y = b^x$, where i) $b > 1$ & ii) $0 < b < 1$

Ex. 1. Complete the following table of values and graph the following exponential functions on the same axes.

i) $b > 1$ **ii)** $0 < b < 1$

$y = e^x$

x	y
-3	0.05
-2	0.14
-1	0.37
0	1
1	$\doteq 2.718$
2	7.4
3	20.1

a) $y = 2^x$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

b) $y = 3^x$

x	y
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

graph

a) $y = \left(\frac{1}{2}\right)^x$

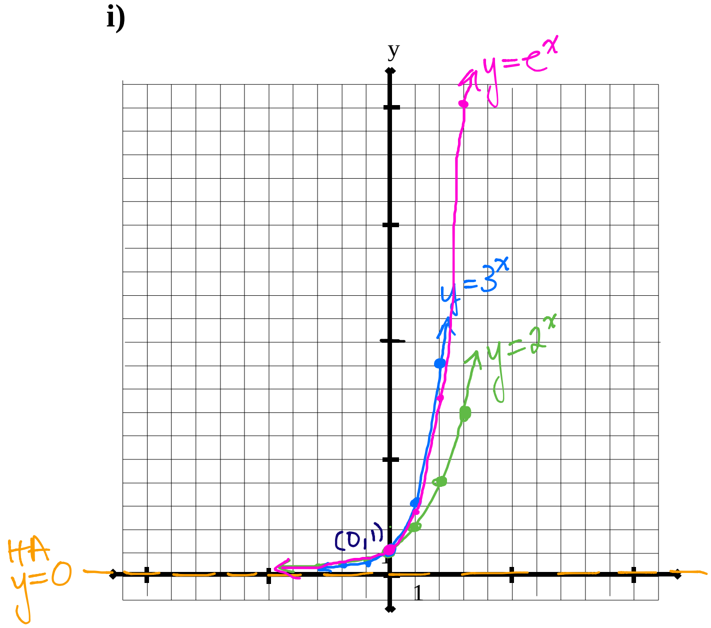
x	y
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$

b) $y = \left(\frac{1}{3}\right)^x$

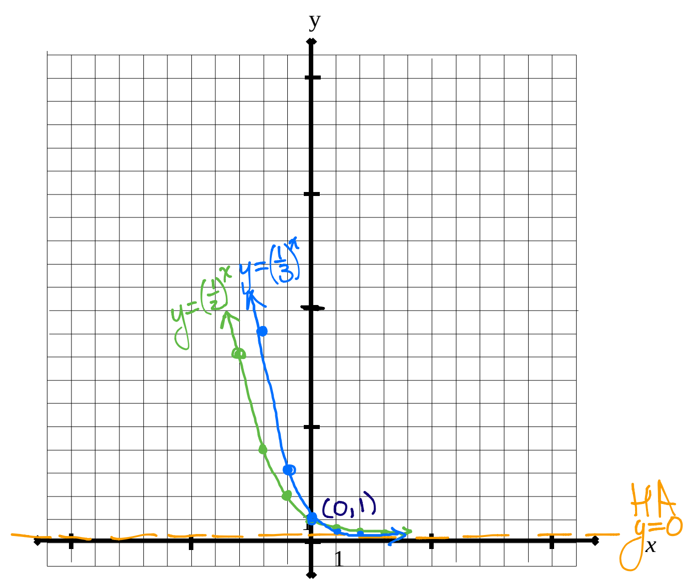
x	y
-3	27
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$

graph

i)



ii)



x – intercept none

y – intercept 1

Domain $\{x \in \mathbb{R}\}$

Range $\{y \in \mathbb{R} \mid y > 0\}$

Horizontal Asymptote is $y = 0$ for $x \to -\infty$

Function is always increasing & concave up

x – intercept none

y – intercept 1

Domain $\{x \in \mathbb{R}\}$

Range $\{y \in \mathbb{R} \mid y > 0\}$

Horizontal Asymptote is $y = 0$ for $x \to +\infty$

Function is always decreasing & concave up

II Transformations on the Exponential Function $f(x)=b^x$ **,where** $f(x)=a(b)^{k(x-d)}+c$

Recall:

$$(x,y)\rightarrow\left(\frac{1}{k}x+d, ay+c\right)$$

Ex. 2. Graph each of the following, by naming and applying transformations on an appropriate exponential function.

a) $y=\left(\frac{1}{3}\right)^{\frac{1}{2}x-1}-1 \longrightarrow y=\left(\frac{1}{3}\right)^{\frac{1}{2}(x-2)}-1$

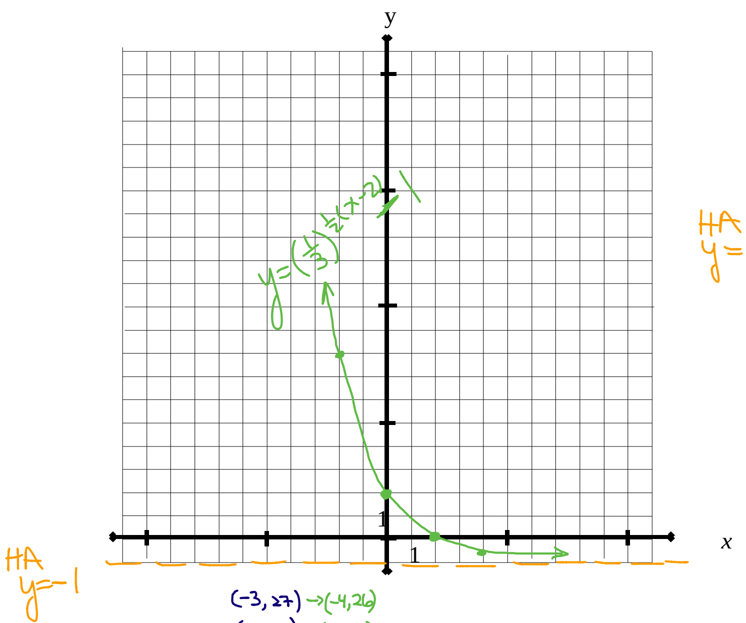
Transformations on $y=\left(\frac{1}{3}\right)^x$ *are:*

i) H.E. by a factor of 2

ii) H.T. 2 units right

iii) V.T 1 unit down

$(x,y)\rightarrow(2x+2, y-1)$



$(-3,27)\rightarrow(-4,26)$

$(-2,9)\rightarrow(-2,8)$

$(-1,3)\rightarrow(0,2)$

$(0,1)\rightarrow(2,0)$

$(1,\frac{1}{3})\rightarrow(4,-\frac{2}{3})$

$(2,\frac{1}{9})\rightarrow(6,-\frac{8}{9})$

$(3,\frac{1}{27})\rightarrow(8,-\frac{26}{27})$

Horizontal Asymptote: $y=-1$

Domain : $\{x\in\mathbb{R}\}$

Range: $\{y\in\mathbb{R} \mid y>-1\}$

y **– intercept :** 2

x **– intercept :** 2

b) $f(x)=-\frac{1}{2}(3)^{-x-3}+13 \rightarrow f(x)=-\frac{1}{2}(3)^{-(x+3)}+13$

Transformations on $y=3^x$ *are:*

i) V.R. in x-axis

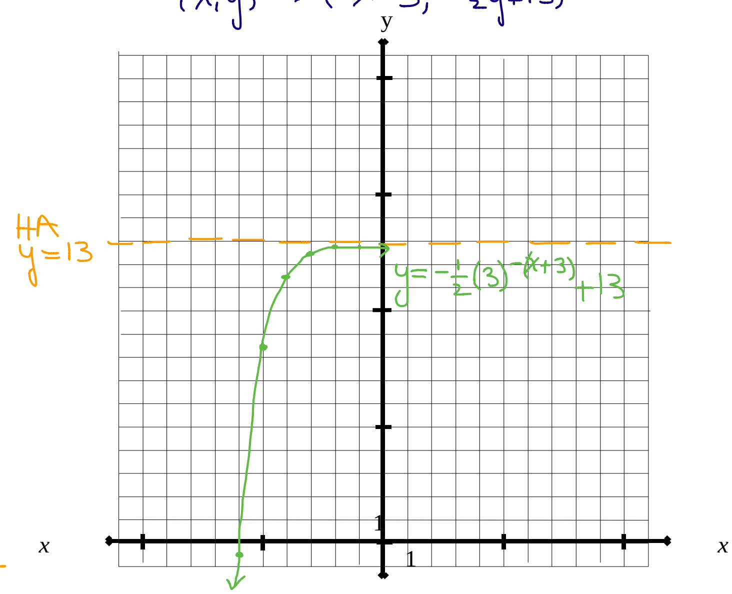
ii) V.C. by a factor of $\frac{1}{2}$

iii) H.R. in y-axis

iv) H.T. 3 units left

v) V.T. 13 units up.

$(x,y)\rightarrow(-x-3, -\frac{1}{2}y+13)$



$(-3,\frac{1}{27})\rightarrow(0,12\frac{53}{54})$

$(-2,\frac{1}{9})\rightarrow(-1,12\frac{17}{18})$

$(-1,\frac{1}{3})\rightarrow(-2,12\frac{5}{6})$

$(0,1)\rightarrow(-3,12\frac{1}{2})$

$(1,3)\rightarrow(-4,11\frac{1}{2})$

$(2,9)\rightarrow(-5,8\frac{1}{2})$

$(3,27)\rightarrow(-6,-\frac{1}{2})$

$3^{-x-3}=26$

Horizontal Asymptote: $y=13$

Domain : $\{x\in\mathbb{R}\}$

Range: $\{y\in\mathbb{R} \mid y<13\}$

y **– intercept :** $12\frac{53}{54}$

x **– intercept :** $\doteq -5\frac{29}{36} \doteq -5.967$

HW. Exercise 5.2

Date:_______________

5.3 Exponential Growth and Decay

Exponential growth *or* ***decay*** occurs when quantities increase or decrease at a rate proportional to the **initial** quantity present. This ***growth*** or ***decay*** occurs in savings accounts, the size of populations, appreciation, depreciation, and with radioactive chemicals. All problems of this type can be modeled by the **exponential function**

+ for growth $(1+r>1)$
− for decay $(0<1-r<1)$

i) $y = a \cdot b^x$ where

- y is the *final amount* or *number*
- a is the *initial amount* or *number*
- b is the *growth* or *decay* *factor*
- x is the *number of growth* or *decay periods*

or

ii) $A = A_0(1 \pm r)^n$ where

- A is the *final amount* or *number*
- A_0 is the *initial amount* or *number*
- r is the *growth* or *decay* *rate*
- n is the *number of growth* or *decay periods*

Ex. 1. An antique vase was purchased in 1980 for \$8000. If the vase appreciates in value by 6% per year, what is its estimated value in the year 2015, to the nearest hundred dollar?

Let V represent the value of the vase, in \$, t years after 1980.

$$V = V_0(1+r)^t$$
$$V = 8000(1.06)^t$$

Find V if $t = 35$:

$$V = 8000(1.06)^{35}$$
$$V \doteq 61\,500$$

∴ in 2015, the vase is worth approx \$61 500.

Ex. 2. A car depreciates by 15% per year. If you buy a car for \$20 000, find the value of the car in three years to the nearest hundred dollar and estimate when the car will be worth half of its original value.

Let V represent the value of the car, in \$, after t years

$$V = V_0(1-r)^t$$
$$V = 20000(0.85)^t$$

Find V if $t=3$:

$$V = 20000(0.85)^3$$
$$V \doteq 12\,300$$

∴ In 3 years, the car is worth approx. \$12 300.

t	V
4	$\doteq 10\,400$
4.25	$\doteq 10\,000$
5	$\doteq 8900$

∴ the car will be worth half its value between 4 & 5 years (4 years 3 months)

Ex. 3. The population of a town was 24 000 in 1980 and 29 000 in 1990.

a) Determine the annual growth rate for the town during this period.

b) Determine an expression for the population, *P*, at time *t* years after 1980.

c) Use this expression to estimate the population of the town in 2012.

a) Let P represent the population of the town, t years after 1980

$$P = P_0(1+r)^t$$

$$P = 24000(1+r)^t$$

Find r if t=10, P=29000.

$$29000 = 24000(1+r)^{10}$$

$$\frac{29}{24} = (1+r)^{10}$$

$$\sqrt[10]{\frac{29}{24}} = 1+r$$

$$r = \sqrt[10]{\frac{29}{24}} - 1$$

$$r = 0.019$$

∴ The annual growth rate is approx. 1.9%

b) $P = 24000(1.019)^t$

c) Find P if t=32

$$P = 24000(1.019)^{32}$$

$$P \doteq 44000$$

∴ in 2012 the population is approx. 44000.

Ex. 4. The population of the world was 6 billion in 1999. This population is growing exponentially and doubles every 51 years. Estimate the world population in ~~2015~~ 2017.

Let P represent the world population in billions, t years after 1999.

$$P = P_0(2)^{\frac{t}{d}}$$

t ← amount of time passed

d ← doubling period (51 years)

$$P = 6(2)^{\frac{t}{51}}$$

Find P if t=18

$$P = 6(2)^{\frac{18}{51}}$$

$$P \doteq 7.7$$

∴ in 2017 the world population is approx 7.7 billion

Ex. 5. A hospital uses cobalt-60 in its radiotherapy treatment for cancer patients. Cobalt-60 has a half-life of 5.2 years. This means that every 5.2 years, 50% of the original sample of cobalt-60 has decayed. The hospital has 80 g of cobalt-60. How much of the original sample will there be after 1 year to the nearest gram?

Let M represent the mass of Cobalt-60 remaining, ing, after t years.

$$M = M_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

t ← amount of time passed

h ← half-life (5.2 years)

$$M = 80\left(\frac{1}{2}\right)^{\frac{t}{5.2}}$$

Find M if t = 1,

$$M = 80\left(\frac{1}{2}\right)^{\frac{1}{5.2}}$$

$$M \doteq 70$$

∴ after one year, approx 70g remains.

Ex. 6. The isotope, radioactive strontium-90, decays to 25% of its original mass after approximately 58 years. Determine its half-life.

Let h represent the half-life, in years.

$$M = M_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$25 = 100\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

Find h if t = 58:

$$25 = 100\left(\frac{1}{2}\right)^{\frac{58}{h}}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{58}{h}}$$

$$\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{\frac{58}{h}}$$

∵ bases are equal

$$\therefore 2 = \frac{58}{h}$$

$$2h = 58$$

$$h = 29$$

∴ its half life is 29 years

Ex. 7. In a recent dig, a human skeleton was unearthed. It was later found that the amount of carbon-14 in it had decayed to $\frac{1}{\sqrt{8}}$ of its original amount.

If carbon -14 has a half-life of 5730 years, how old is the skeleton?

Let A_0 represent the initial amount of carbon-14.

Let t represent the age of the skeleton, in years

$$\frac{\frac{1}{\sqrt{8}}\cancel{A_0}}{A_0} = \frac{\cancel{A_0}\left(\frac{1}{2}\right)^{\frac{t}{h}}}{A_0}$$

Find t if h = 5730

$$\frac{1}{\sqrt{8}} = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$\left(\frac{1}{2}\right)^{\frac{3}{2}} = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

∵ bases are equal

$$\frac{3}{2} = \frac{t}{5730}$$

$$2t = 17190$$

$$t = 8595$$

∴ the skeleton is 8595 years old

HW. Exercise 5.3

Date:_______________ **5.4 Simplifying Using Exponent Laws**

Ex. 1. Rewrite each expression as the sum and/or difference of terms, where each term is of the form ax^n.

a) $\dfrac{4x^5-5x^4+6x-2}{2x^4}$

$= \dfrac{4x^5}{2x^4} - \dfrac{5x^4}{2x^4} + \dfrac{6x}{2x^4} - \dfrac{2x^0}{2x^4}$

$= 2x^1 - \frac{5}{2}x^0 + 3x^{-3} - x^{-4}$

or $= 2x - \frac{5}{2} + 3x^{-3} - x^{-4}$

b) $\dfrac{8x^3-27}{2x-3}$

$= \dfrac{\cancel{(2x-3)}(4x^2+6x+9)}{\cancel{2x-3}}$

$= 4x^2 + 6x^1 + 9x^0$

c) $\dfrac{2}{\sqrt{x^3}} + \dfrac{x}{\sqrt{3}} + 6\sqrt[3]{x} - \left(\dfrac{1}{3x}\right)^2$

$= 2x^{-\frac{3}{2}} + \frac{1}{\sqrt{3}}x^1 + 6x^{\frac{1}{3}} - \frac{1}{9}\cdot\frac{1}{x^2}$

$= 2x^{-\frac{3}{2}} + \frac{1}{\sqrt{3}}x^1 + 6x^{\frac{1}{3}} - \frac{1}{9}x^{-2}$

d) $\dfrac{1}{\sqrt{x}}\left(\sqrt{x} + \dfrac{1}{x}\right)^2$

$= x^{-\frac{1}{2}}(x^{\frac{1}{2}} + x^{-1})(x^{\frac{1}{2}} + x^{-1})$

$= x^{-\frac{1}{2}}(x^1 + 2x^{-\frac{1}{2}} + x^{-2})$

$= 1x^{\frac{1}{2}} + 2x^{-1} + 1x^{-\frac{5}{2}}$

e) $\left(\dfrac{x-3}{2\sqrt[3]{x^2}}\right)^3$

$= \dfrac{(x-3)(x-3)(x-3)}{8(x^{\frac{2}{3}})^3}$

$= \dfrac{(x^2-6x+9)(x-3)}{8x^2}$

$= \dfrac{x^3-6x^2+9x-3x^2+18x-27}{8x^2}$

$= \dfrac{1x^3}{8x^2} - \dfrac{9x^2}{8x^2} + \dfrac{27x^1}{8x^2} - \dfrac{27x^0}{8x^2}$

$= \frac{1}{8}x^1 - \frac{9}{8}x^0 + \frac{27}{8}x^{-1} - \frac{27}{8}x^{-2}$

f) $\dfrac{x-16}{4\sqrt{x}+x}$

$= \dfrac{(x^{\frac{1}{2}})^2 - 16}{4\sqrt{x^1}+x}$

$= \dfrac{(x^{\frac{1}{2}}-4)(x^{\frac{1}{2}}+4)}{4x^{\frac{1}{2}}+x^1}$

$= \dfrac{(x^{\frac{1}{2}}-4)\cancel{(x^{\frac{1}{2}}+4)}}{x^{\frac{1}{2}}\cancel{(4+x^{\frac{1}{2}})}}$

$= \dfrac{1x^{\frac{1}{2}}}{x^{\frac{1}{2}}} - \dfrac{4x^0}{x^{\frac{1}{2}}}$

$= 1x^0 - 4x^{-\frac{1}{2}}$

Ex. 2. Completely factor each of the following expressions.

a) $\underbrace{-2t(1-t)^4}+\underbrace{4t^2(1-t)^3}$

$=-2t(1-t)^3[(1-t)-2t]$

$=-2t(1-t)^3(1-3t)$

may need to be factored further.

b) $\underbrace{10(2x+1)^4(3x+2)^4}+\underbrace{12(2x+1)^3(3x+2)^5}$

$=2(2x+1)^3(3x+2)^4[5(2x+1)+6(3x+2)]$

$=2(2x+1)^3(3x+2)^4(10x+5+18x+12)$

$=2(2x+1)^3(3x+2)^4(28x+17)$

c) $\underbrace{12x(2x^2-1)^2(x^2-2)^{-2}}-\underbrace{4x(x^2-2)^{-3}(2x^2-1)^3}$

$=4x(2x^2-1)^2(x^2-2)^{-3}[3(x^2-2)-(2x^2-1)]$

$=4x(2x^2-1)^2(x^2-2)^{-3}(3x^2-6-2x^2+1)$

$=4x(2x^2-1)^2(x^2-2)^{-3}(x^2-5)$

$=\dfrac{4x(2x^2-1)^2(x^2-5)}{(x^2-2)^3}$

d) $\underbrace{-\dfrac{1}{2\sqrt{1-t}\cdot\sqrt{1+t}}}-\underbrace{\dfrac{\sqrt{1-t}}{2\sqrt{(1+t)^3}}}$

$=\underbrace{-\frac{1}{2}(1-t)^{-\frac{1}{2}}(1+t)^{-\frac{1}{2}}}-\underbrace{\frac{1}{2}(1-t)^{\frac{1}{2}}(1+t)^{-\frac{3}{2}}}$

$=-\frac{1}{2}(1-t)^{-\frac{1}{2}}(1+t)^{-\frac{3}{2}}[(1+t)+(1-t)]$

$=-\frac{1}{\cancel{2}_{1}}(1-t)^{-\frac{1}{2}}(1+t)^{-\frac{3}{2}}(\cancel{2}^{1})$

$=-1(1-t)^{-\frac{1}{2}}(1+t)^{-\frac{3}{2}}$

$=\dfrac{-1}{\sqrt{1-t}\sqrt{(1+t)^3}}$

HW. Exercise 5.4

Date:_______________ **Unit 5 Test Review**

$f(x) = -e^{-(x-3)} + 1$

1. ***Sketch*** the graph of $f(x) = -e^{-x+3} + 1$ by naming and applying transformations on an appropriate function. Determine the x and y-intercepts algebraically and mark these points on the graph.

The transformations on $y = e^x$ *are:*

i) V.R. across x-axis

ii) H.R. across y-axis

iii) H.T. 3 units right

iv) V.T. 1 unit up

For y-int:

$f(0) = -e^{-(0-3)} + 1$

$= -e^3 + 1$

$\doteq -19$

For x-int:

$0 = -e^{-x+3} + 1$

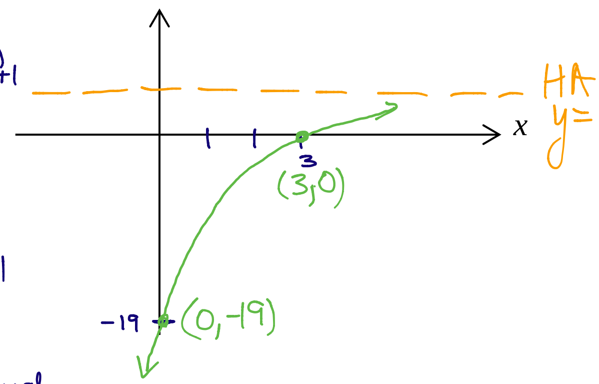
$e^{-x+3} = 1$

$e^{-x+3} = e^0$

$\therefore$ bases are equal

$-x + 3 = 0$

$\boxed{x = 3}$



2. Solve each of the following.

a) $3^x - 3^{x-1} = \frac{2}{\sqrt{27}}$

$3^x - 3^{x-1} = 2(27)^{-\frac{1}{2}}$

$3^x - 3^{x-1} = 2(3)^{-\frac{3}{2}}$

$3^{x-1}(3-1) = 2(3)^{-\frac{3}{2}}$

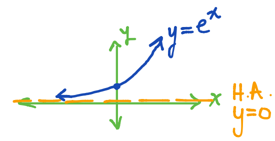
$3^{x-1} = 3^{-\frac{3}{2}}$

$\therefore$ bases are equal

$x - 1 = -\frac{3}{2}$

$x = -\frac{1}{2}$

b) $e^{2x} + 2e^x = 3$



$(e^x)^2 + 2(e^x) - 3 = 0$

Let $y = e^x$

$y^2 + 2y - 3 = 0$

$(y+3)(y-1) = 0$

$y + 3 = 0$ | $y - 1 = 0$

$y = -3$ | $y = 1$

$e^x = -3$ | $e^x = 1$

no solution | $e^x = e^0$

$e^x > 0$ for all $x \in \mathbb{R}$. | $\therefore$ bases are equal

$\boxed{\therefore x = 0}$

3. Evaluate.

a) $\frac{2^{-22} - 2^{-23}}{2^{-25} + 2^{-24}} \cdot \frac{2^{25}}{2^{25}}$

$= \frac{2^3 - 2^2}{2^0 + 2^1}$

$= \frac{8-4}{1+2}$

$= \frac{4}{3}$

or $= \frac{2^{-23}(2^1 - 1)}{2^{-25}(1 + 2^1)}$

$= 2^2 \cdot \frac{1}{3}$

$= \frac{4}{3}$

b) $\frac{3^{n+1} \times 9^{n-4}}{27^{n-2}}$

$= \frac{3^{n+1} \times (3^2)^{n-4}}{(3^3)^{n-2}}$

$= \frac{3^{n+1} \times 3^{2n-8}}{3^{3n-6}}$

$= \frac{3^{3n-7}}{3^{3n-6}}$

$= 3^{-1}$

$= \frac{1}{3}$

4. Completely factor the following expression.

$-16(1-4x)^3(x^2-1)^{-3} - 6x(x^2-1)^{-4}(1-4x)^4$

$= -2(1-4x)^3(x^2-1)^{-4}\left[8(x^2-1) + 3x(1-4x)\right]$ expand & collect like terms

$= -2(1-4x)^3(x^2-1)^{-4}(-4x^2 + 3x - 8)$ factor out -1.

$= \frac{2(1-4x)^3(4x^2 - 3x + 8)}{(x^2-1)^4}$

5. Rewrite each expression as the sum and/or difference of terms, where each term is of the form ax^n.

a) $\sqrt{\dfrac{\sqrt[3]{x}\cdot\sqrt[3]{x^2}}{\sqrt{x}}}$

$= \left(\dfrac{x^{\frac{1}{3}}\cdot x^{\frac{2}{3}}}{x^{\frac{1}{2}}}\right)^{\frac{1}{2}}$

$= \left(\dfrac{x^{1}}{x^{\frac{1}{2}}}\right)^{\frac{1}{2}}$

$= (x^{\frac{1}{2}})^{\frac{1}{2}}$

$= x^{\frac{1}{4}}$

b) $\dfrac{x-4}{x^{\frac{1}{2}}+2}$

$= \dfrac{(x^{\frac{1}{2}})^2-(2)^2}{x^{\frac{1}{2}}+2}$

$= \dfrac{(x^{\frac{1}{2}}-2)\cancel{(x^{\frac{1}{2}}+2)}^{1}}{\cancel{x^{\frac{1}{2}}+2}^{1}}$

$= 1x^{\frac{1}{2}} - 2x^{0}$

c) $\dfrac{x^3+8}{\sqrt{x^3}+2\sqrt{x}}$

$= \dfrac{(x+2)(x^2-2x+4)}{x^{\frac{3}{2}}+2x^{\frac{1}{2}}}$

$= \dfrac{\cancel{(x+2)}^{1}(x^2-2x+4)}{x^{\frac{1}{2}}\cancel{(x+2)}^{1}}$

$= \dfrac{x^2}{x^{\frac{1}{2}}} - \dfrac{2x^1}{x^{\frac{1}{2}}} + \dfrac{4x^0}{x^{\frac{1}{2}}}$

$= x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$

6. Joe's parents invested \$4000 in an account when he was born. The account pays interest at 6%/a, compounded quarterly. How much money will be in the account on Joe's 18th birthday?

Let A represent the amount of money, in \$, on Joe's 18th B-Day!

$P = 4000$

$i = \frac{0.06}{4} = 0.015$

$n = 18 \times 4 = 72$

$A = P(1+i)^n$

$A = 4000(1.015)^{72}$

$A \doteq 11684.63$

∴ on his 18th birthday Joe will have \$11684.63.

7. One bacterium divides into two bacteria every 5 days. Initially, there are 15 bacteria.

a) How many bacteria will there be in 10 days?

There will be 60 bacteria in 10 days.

5 days: $15 \times 2 = 30$

10 days: $30 \times 2 = 60$

b) What is the approximate growth rate per day?

Let r represent the growth rate per day.

$P = P_0(1+r)^t$

$60 = 15(1+r)^{10}$

$4 = (1+r)^{10}$

$\sqrt[10]{4} - 1 = r$

$r \doteq 0.15$

∴ the approx. growth rate is 15%.

8. A ***small*** self-contained forest was studied for squirrel population by a biologist. It was found that the forest population, P, was a function of time, t, where t was measured in weeks.

The function was $P = \dfrac{20}{1+3e^{-0.02t}}$.

a) Find the population at the start of the study and after one year.

If $t=0$:

$P = \dfrac{20}{1+3e^{-0.02(0)}}$

$= \dfrac{20}{1+3(1)} = \dfrac{20}{4} = 5$

If $t=52$:

$P = \dfrac{20}{1+3e^{-0.02(52)}}$

$P = \dfrac{20}{1+3e^{-1.04}} \doteq 9.7$

∴ there were 5 squirrels at the beginning and approximately 10 after one year.

b) The largest population the forest can sustain is represented mathematically by looking at the end behaviour of the function, P, specifically as $t \to \infty$.

Determine the largest squirrel population the forest can sustain.

$P = \dfrac{20}{1+3\cdot\frac{1}{e^{0.02t}}}$

As $t \to \infty$, $\dfrac{1}{e^{0.02t}} \to 0$

∴ $P \to \dfrac{20}{1+3(0)}$, so $P \to 20$

∴ the forest can sustain 20 squirrels.

HW. Unit 5 Review Exercise