Date:________________ **UNIT 5: EXPONENTIAL FUNCTIONS**

5.1 Laws of Exponents and Exponential Equations

1. Evaluate each of the following, using the laws of exponents.

a) $(-64)^{\frac{1}{3}}$ **b)** $8^{-\frac{1}{3}}$ **c)** $0.49^{\frac{1}{2}}$ **d)** $-25^{\frac{1}{2}}$

e) $81^{\frac{3}{4}}$ **f)** $\left(\frac{1}{8}\right)^{\frac{2}{3}}$ **g)** $(-32)^{\frac{-3}{5}}$ **h)** $\left(\frac{1}{16}\right)^{-0.75}$

i) $\left(\frac{9}{25}\right)^{-\frac{3}{2}}$ **j)** $\frac{1}{32^{-0.8}}$ **k)** $(-0.027)^{\frac{-2}{3}}$ **l)** $\sqrt[4]{16^3}$

m) $\frac{2^{-1}}{2^{-2}-2^{-3}}$ **n)** $\frac{3^{-13}-3^{-12}}{3^{-11}+3^{-12}}$ **o)** $\frac{2^{99}+2^{102}}{2^{101}-2^{103}}$

p) $(4^{-2}-4^{0})^{-1}$ **q)** $\frac{9^{-\frac{1}{2}}-4^{-1}}{2^{-1}-9^{-\frac{1}{2}}}$ **r)** $\left(8^{\frac{2}{3}}+4^{\frac{3}{2}}\right)\cdot\sqrt[3]{8^{-4}}$

2. Simplify to a single power and evaluate if possible.

a) $\left(3^{-4}\div 3\right)\times\left(\sqrt{3}\right)^{10}$ **b)** $(0.4)^3\div(0.4)^5$ **c)** $\frac{(5^3)^{-3}(5^{-4})}{(5^4)^{-4}}$

d) $\frac{27^3\times 3^{-4}}{9^2\times 3^3}$ **e)** $8\left[2^{\frac{3}{5}}\div\left(\frac{1}{16}\right)^{-\frac{2}{5}}\right]^2$ **f)** $\frac{(64^{x-2})(4^5)}{(4^x)(16^{x+1})}$

g) $\sqrt[3]{27x^6}+(-32x^{10})^{\frac{1}{5}}$ **h)** $\frac{\sqrt{t^3}}{\left(\sqrt[3]{t}\right)^2\times\sqrt[4]{t^3}}$ **i)** $\left[\frac{\left(\sqrt[4]{a^{2n-1}}\right)\left(\sqrt[4]{a}\right)}{\sqrt{a}}\right]^2$

3. Simplify each of the following using the laws of exponents, and express your final answer with positive exponents only.

a) $\frac{25x^3y^{-4}}{15x^{-2}y^2}$ **b)** $\frac{(4a^{-2})^2(a^3b^2)}{32a^4b^3}$ **c)** $(8c^{-3}d^6)^{\frac{2}{3}}\cdot\sqrt[3]{-27c^{-3}d^9}$

d) $(ab)^4\left(\frac{a^{-2}}{b^{-2}}\right)^2$ **e)** $\sqrt{\frac{(32x^{-5}y^2)(18x^2y)}{4xy^{-3}}}$ **f)** $a^{\frac{1}{2}}(2a^{\frac{1}{2}}+a^{-\frac{3}{2}})$

g) $\left(\sqrt[4]{m}+\sqrt[3]{m}\right)\left(\sqrt[4]{m}-\sqrt[3]{m}\right)$ **h)** $\frac{ab^2c+a^2bc}{abc}$ **i)** $\frac{(p^2q+pq^3)^3}{p^3q^4}$

4. Solve by first expressing each side of the equation as *simplified single powers* of the same base.

a) $6^{\frac{1}{2}x-\frac{1}{3}}=216^{x+1}$ **b)** $\left(3^{x-3}\right)^x=\frac{1}{9}$

c) $4(7^{2x-1})=28$ **d)** $3(5^{x^2+3x})=0.12$

e) $2^{x^2+5x}-5=59$ **f)** $4^{x+3}\cdot 8^{4-x}=\left(\frac{1}{16}\right)^{x-2}$

g) $(27\cdot 3^x)^x=27^x\cdot 3^{\frac{1}{x}}$ **h)** $\frac{\left(5^{x^2}\right)^x}{5^{13x}}=125^{5-x^2}$

5. Solve by factoring.

a) $5^{x+1}+5^{x}=750$ **b)** $7^{x+3}-7^{x}=342$

c) $2^{x-3}-33=-2^{x+2}$ **d)** $4^{x-2}+4^{x-5}-65=0$

e) $4^{2x}-20(4^{x})+64=0$ **f)** $2^{2x}-6(2^{x})-16=0$

g) $5^{2x}-5=4(5^{x})$ **h)** $3^{2x+1}-10(3^{x})+3=0$ ***Note:*** $3^{2x+1}=3^{1}\cdot 3^{2x}$

6. Solve the following linear exponential systems of equations.

a) $\begin{cases} 9^{3x+2y}=\dfrac{1}{9} \\ 3^{-2x-3y}=81 \end{cases}$ **b)** $\begin{cases} \dfrac{16^{x+2y}}{8^{x-y}}=32 \\ \dfrac{32^{x+3y}}{16^{x+2y}}=\dfrac{1}{8} \end{cases}$

Answers:

1. a) -4 **b)** $\frac{1}{2}$ **c)** 0.7 *or* $\frac{7}{10}$ **d)** -5 **e)** 27 **f)** $\frac{1}{4}$ **g)** $-\frac{1}{8}$ **h)** 8 **i)** $\frac{125}{27}$ **j)** 16 **k)** $\frac{100}{9}$ **l)** 8
m) 4 **n)** $-\frac{1}{6}$ **o)** $-\frac{3}{4}$ **p)** $-\frac{16}{15}$ **q)** $\frac{1}{2}$ **r)** $\frac{3}{4}$

2. a) $3^{0}=1$ **b)** $0.4^{-2}=\frac{25}{4}$ **c)** $5^{3}=125$ **d)** $3^{-2}=\frac{1}{9}$ **e)** $2^{1}=2$ **f)** $4^{-3}=\frac{1}{64}$ **g)** x^{2} **h)** $t^{\frac{1}{12}}$ **i)** a^{n-1}

3. a) $\frac{5x^{5}}{3y^{6}}$ **b)** $\frac{1}{2a^{5}b}$ **c)** $-\frac{12d^{7}}{c^{3}}$ **d)** b^{8} **e)** $\frac{12y^{3}}{x^{2}}$ **f)** $2a+\frac{1}{a}$ *or* $\frac{2a^{2}+1}{a}$ **g)** $m^{\frac{1}{2}}-m^{\frac{2}{3}}$ **h)** $a+b$ **i)** $\frac{(p+q^{2})^{3}}{q}$

4. a) $-\frac{4}{3}$ **b)** $1, 2$ **c)** 1 **d)** $-2, -1$ **e)** $-6, 1$ **f)** $-\frac{10}{3}$ **g)** $1, x\neq 0$ **h)** $-5, -1, 3$

5. a) 3 **b)** 0 **c)** 3 **d)** 5 **e)** $1, 2$ **f)** 3 **g)** 1 **h)** $-1, 1$

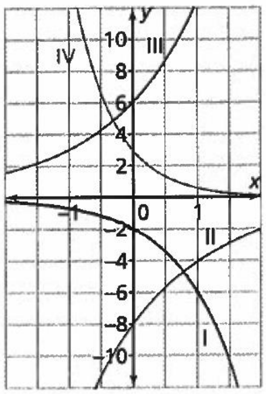
6. a) $x=1,\ y=-2$ **b)** $x=-17,\ y=2$

Date:______________

5.2 Investigating the Graphs of Exponential Functions

$f(x) = b^x$ **&** $f(x) = a(b)^{k(x-d)} + c$

1. Match each function to the corresponding graph, I, II, III or IV.



a) $y = 3(2)^{x+1}$

b) $y = -2(3)^x$

c) $y = 3(0.2)^x$

d) $y = -4(0.5)^{x-1}$

2. Graph each of the following, by naming and applying transformations on an appropriate exponential function.

a) $y = 5^{x+2} - 10$

b) $y = -0.25(10)^{-x+2}$

c) $y = -3(2)^{x+4} + 6$

d) $y = \left(\frac{1}{3}\right)^{2x-2} - 3$

3. The graph of the function $y = 4^x$ is transformed by vertically compressing it by a factor of 1/2, reflecting it in the y-axis, horizontally expanding it by a factor of 3, and vertically translating it 2 units up. Write the equation of the resulting function, graph and state the domain and range.

4. Graph $f(x) = 3^{-x} - 2$ and its reciprocal function $y = \dfrac{1}{f(x)}$ on the same grid. State the equation of the reciprocal function.

5. Graph $f(x) = \dfrac{1}{8}(4)^x - 4$ and its absolute value function $y = |f(x)|$ on the same grid. State a ***simplified*** equation of the absolute value function.

6. Graph the function $f(x) = \dfrac{2^{2x} - 2^x}{2^x - 1}$ by finding and labelling any holes, asymptotes and intercepts.

7. Given the piecewise function $f(x) = \begin{cases} -3^{\frac{1}{2}(x+4)} + 3 \ \ if \ \ x \in (-\infty, 0) \\ -0.5x^2 + 2x \ \ if \ \ x \in [0, +\infty) \end{cases}$,

a) graph

b) determine the value of x at which the function is discontinuous and state the type of discontinuity

c) examine how the function behaves near this discontinuity and at the ends of the graph

8. Determine the value of d such that the function $g(x) = \begin{cases} -2x - 1 & if \ \ x < 1 \\ 1 - (0.5)^{x-d} & if \ \ x \geq 1 \end{cases}$ is continuous for all $x \in R$ and then graph.

9. Solve the following inequalities using a ***number line strategy*** and answer using ***interval notation***.

a) $2^{2x} - 9(2^x) + 8 < 0$

b) $\dfrac{3^x - 9}{2x^3 + x^2 - 8x - 4} \geq 0$

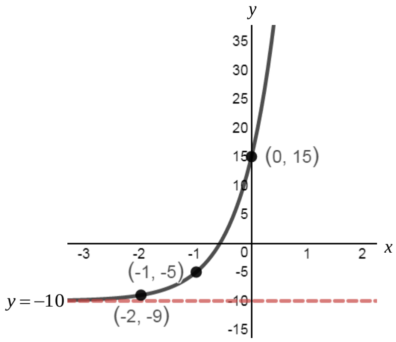
Answers:

1. a) III **b)** I **c)** IV **d)** II

2. a) *Transformations on* $y = 5^x$ *are:*

i) *horizontal translation left* 2 *units*

ii) *vertical translation down* 10 *units*



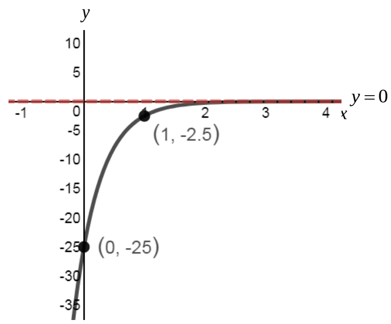
b) *Transformations on* $y = 10^x$ *are:*

i) *vertical reflection in the x-axis*

ii) *vertical compression by a factor of* $\frac{1}{4}$

iii) *horizontal reflection in the y-axis*

iv) *horizontal translation right* 2 *units*



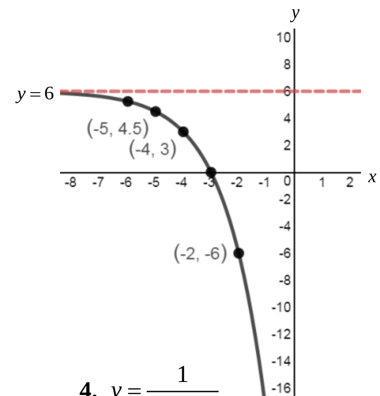
c) *Transformations on* $y = 2^x$ *are:*

i) *vertical reflection in the x-axis*

ii) *vertical expansion by a factor of* 3

iii) *horizontal translation left* 4 *units*

iv) *vertical translation up* 6 *units*

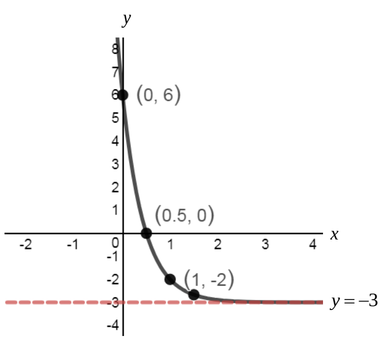


2. d) *Transformations on* $y = \left(\frac{1}{3}\right)^x$ *are:*

i) *horizontal compression by a factor of* $\frac{1}{2}$

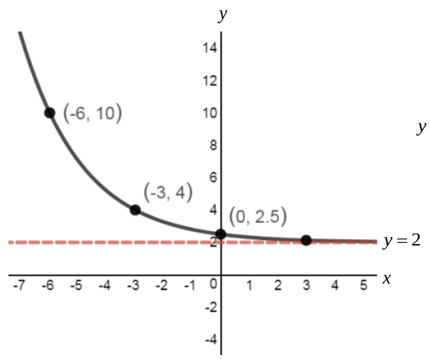
ii) *horizontal translation right* 1 *unit*

iii) *vertical translation down* 3 *units*

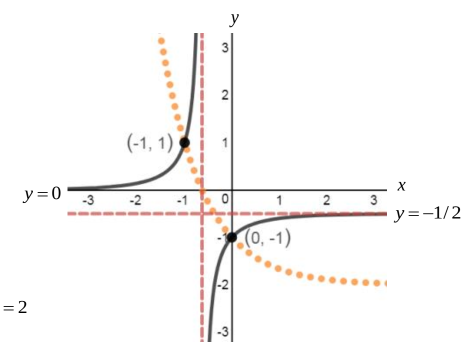


3. $y = \frac{1}{2}(4)^{-\frac{1}{3}x} + 2$

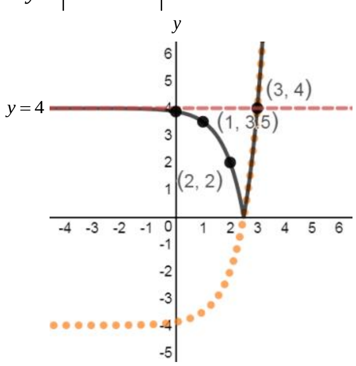
$D = \{x \in \Re\}$, $R = \{y \in \Re \mid y > 2\}$



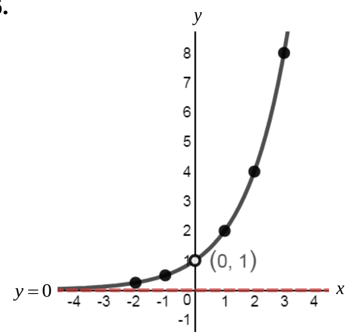
4. $y = \frac{1}{3^{-x} - 2}$



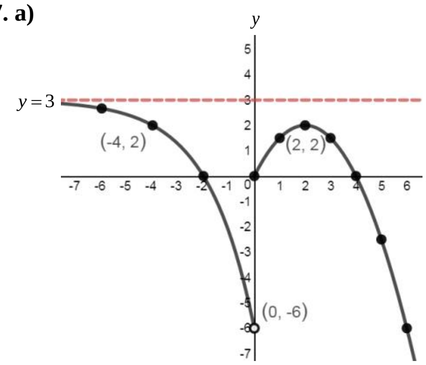
5. $y = \left| 2^{2x-3} - 4 \right|$



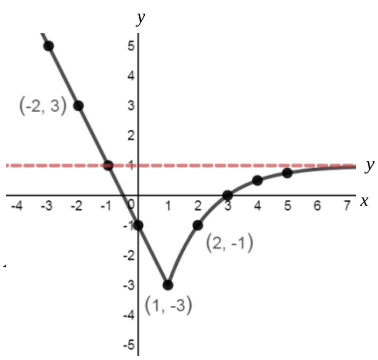
6.



7. a)



8. $d = 3$



9. a) $x \in (0, 3)$

b) $x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 2\right) \cup (2, +\infty)$

7. b) *jump discontinuity at* $x = 0$

7. c)

as $x \to 0^-$, $f(x) \to -6$ & *as* $x \to 0^+$, $f(x) \to 0$

as $x \to -\infty$, $f(x) \to 3$ & *as* $x \to +\infty$, $f(x) \to -\infty$

Date:_______________ **5.3 Exponential Growth and Decay**

1. The population of a city is 810 000. If it is increasing by 4% per year, estimate the population in four years to the nearest thousandth.

2. A painting, purchased for $10 000 in 1990, increased in value by 8% per year. Find the value of the painting in the year 2000 to the nearest hundred dollar.

3. A river is stocked with 5000 salmon. The population of salmon increases by 7% per year.
a) Write an expression for the population t years after the salmon were put into the river.
b) What will the population be in **i)** 3 years? **ii)** 15 years?
c) Approximately how many years does it take for the salmon population to double?

4. A house was bought six years ago for $175 000. If real-estate values have been increasing at the rate of 4% per year, what is the value of the house now to the nearest thousand?

5. A used-car dealer sells a five-year-old car for $4200. If the depreciation is 15% a year what was the original value of the car to the nearest hundred dollar?

6. In the early 1990s, the Canadian dollar was declining in value due to inflation at the rate of 8.3% per year. If the situation continued, what would the dollar be worth five years later?

7. To determine whether a pancreas is functioning normally, a tracer dye is injected. A normally functioning pancreas secretes 4% of the dye each minute. A doctor injects 0.50 g of the dye. Twenty minutes later, 0.35 g remain. If the pancreas were functioning normally, how much dye should remain?

8. If a bacteria population doubles in 5 d,
a) when will it be 16 times as large?
b) when was it $\frac{1}{2}$ of its present population?
c) when was it $\frac{1}{4}$ of its present population?
d) when was it $\frac{1}{32}$ of its present population?

9. Inflation is causing things to cost roughly 2% more per year.
a) A bag of milk costs $3.75 now. Estimate its cost in five years.
b) i) A movie ticket costs $8.50 now. If inflation continues at 2% per year, estimate when the ticket will cost $10.00.
ii) Estimate how long ago the movie ticket cost $4.25.

10. An element is decaying at the rate of 12%/h. Initially we have 100 g.
a) How much remains after 10 h to the nearest gram?
b) How much remains after 30 h to the nearest gram?
c) Approximately when will there be 40 g left?

11. A bacteria colony grows at the rate of 15%/h.
a) In approximately how many hours will the colony double in size?
b) In 10 h the bacteria population grows to 1.3×10^3. How many bacteria were there initially?

12. A research assistant made 160 mg of radioactive sodium (Na24) and found that there was only 20 mg left 45 h later.

a) What is the half-life of Na24?

b) Find a function that models the amount M left after t hours.

c) If the laboratory requires 100 mg of Na24 12 h from now, how much Na24 should the research assistant make now? (Ignore the 20 mg she currently has.)

d) How much of the original 20 mg would be left in 12 h?

13. The population of a city was estimated to be 125 000 in 1930 and 500 000 in 1998.

a) Determine the annual growth rate for the town during this period to the nearest hundredth of a percent.

b) Estimate the population of the city in 2020 using the same annual growth rate to the nearest thousand.

c) If the population continues to grow at the same rate, when will the population reach 1 million?

14. On the day his son is born, an excited father wants to give his new son a season's ticket to watch the father's favourite sports team. A season's ticket costs $900. The father realizes there is no point in buying tickets for a baby only a few hours old, so he decides to put the money aside until the boy is six years old. If inflation is assumed to be 3% per year, how much money should the father put aside so that he can purchase the season's ticket in six years?

15. Two different strains of cold virus were isolated and put in cultures to grow. Virus A triples every 8 h while virus B doubles every 4.8 h. If each culture has 1000 viruses to start, which has more after 24 h?

16. Use the compound interest formula, $A = P(1+i)^n$, to determine the amount of each loan.

a) $1000 at 4.5% per annum, compounded annually for 7 years.

b) $45 500 at 10.5% per annum, compounded semi-annually for 5 years.

17. Gabby hopes the $26 000 she is investing will be worth $40 000 in 6 years to upgrade the computers for her business. What rate of interest, to the nearest hundredth of a percent, compounded quarterly, would she need to achieve this goal?

18. Digital cable is being introduced into a certain city. The number of subscribers t months from now is expected to be $N(t) = \dfrac{80000}{1+10e^{-0.2t}}$.

a) How many subscribers will there be after six months?

b) How many subscribers will there eventually be?

Answers:

1. 948 000 **2.** $21 600

3. a) $P = 5000(1.07)^t$ **b) i)** 6125 **ii)** 13 795 **c)** between 10 and 11 years

4. $221 000 **5.** $9500

6. $0.65 **7.** 0.22 g

8. a) 20 days **b)** 5 days ago **c)** 10 days ago **d)** 25 days ago

9. a) $4.14 **b) i)** 8 years **ii)** 35 years ago

10. a) 28 g **b)** 2 g **c)** 7 h

11. a) 5 h **b)** 321

12. a) 15 h **b)** $M = 160\left(\frac{1}{2}\right)^{\frac{t}{15}}$ **c)** 174 mg **d)** 11.5 mg

13.a) 2.06% **b)** 783 000 **c)** 2032

14. $1075 **15.** B

16. a) $1360.86 **b)** $75 898.37

17. 7.24%

18. a) 19 940 **b)** 80 000

Date:_______________

5.4 Simplifying Using Exponent Laws

1. Rewrite each expression as the sum and/or difference of terms, where each term is of the form ax^n.

a) $\dfrac{x^{-2}-x^{-3}}{2x}$ **b)** $\dfrac{3t-2t^{-1}}{t^3}$ **c)** $\dfrac{x^{\frac{3}{2}}-x^{\frac{1}{2}}-x^{-1}}{x^{-\frac{1}{2}}}$ **d)** $\dfrac{4-\sqrt{x}}{x^{\frac{3}{2}}}$

e) $\dfrac{\left(\sqrt{x}-2\right)^2}{\sqrt[3]{x}}$ **f)** $\left(\sqrt{x}-2\right)\left(3\sqrt{x}+8\right)$ **g)** $\dfrac{t^2-9}{t-3}$ **h)** $\dfrac{x-9}{\sqrt{x}+3}$

i) $\dfrac{4x-1}{2\sqrt{x}-1}$ **j)** $\dfrac{8x^3+6x}{2x^3}$ **k)** $\dfrac{3\sqrt[3]{x^5}}{x}+\left(\dfrac{1}{5x^2}\right)^2-\sqrt{16x^3}$ **l)** $\left(\dfrac{x+2}{\sqrt[3]{x}}\right)^3$

2. Completely factor each of the following expressions. Express your final answer with positive exponents.

a) $3(x+4)^2(x-3)^6+6(x-3)^5(x+4)^3$ **b)** $3(1-x^2)^{-1}+2x(1-x^2)^{-2}(3x+5)$

c) $8(2x-1)^3(2-3x)^4-12(2-3x)^3(2x-1)^4$ **d)** $(6x+2)(x^2+1)^{-1}-2x(x^2+1)^{-2}(3x^2+2x)$

e) $3x^2(3x-5)^2+6x^3(3x-5)$ **f)** $4(2x-1)(x-2)^{-3}-3(x-2)^{-4}(2x-1)^2$

g) $4x^3(1-4x^2)^3-24x^5(1-4x^2)^2$ **h)** $8x(x^2-3)^3(x^2+3)^{-4}-8x(x^2+3)^{-5}(x^2-3)^4$

i) $6x(x^2+3)^2(x^3+3)^2+6x^2(x^3+3)(x^2+3)^3$ **j)** $(1+x^2)^{-\frac{1}{2}}-x^2(1+x^2)^{-\frac{3}{2}}$

k) $-36t^2(4-3t^3)^3(1-2t)^6-12(1-2t)^5(4-3t^3)^4$ **l)** $-x(1-x^2)^{-\frac{1}{2}}(1-x)^{-1}+(1-x)^{-2}(1-x^2)^{\frac{1}{2}}$

3. Completely factor the following expression.

$$\frac{2x(1-x)^3}{(1+x)^3}-\frac{3x^2(1-x)^2}{(1+x)^3}-\frac{3x^2(1-x)^3}{(1+x)^4}$$

Answers:

1. a) $\frac{1}{2}x^{-3}-\frac{1}{2}x^{-4}$ **b)** $3t^{-2}-2t^{-4}$ **c)** $x^2-x-x^{-\frac{1}{2}}$ **d)** $4x^{-\frac{3}{2}}-x^{-1}$ **e)** $x^{\frac{2}{3}}-4x^{\frac{1}{6}}+4x^{-\frac{1}{3}}$ **f)** $3x+2x^{\frac{1}{2}}-16$

g) $t+3$ **h)** $x^{\frac{1}{2}}-3$ **i)** $2x^{\frac{1}{2}}+1$ **j)** $4+3x^{-2}$ **k)** $3x^{\frac{2}{3}}+\frac{1}{25}x^{-4}-4x^{\frac{3}{2}}$ **l)** $x^2+6x+12+8x^{-1}$

2. a) $3(3x+5)(x+4)^2(x-3)^5$ **b)** $\dfrac{(3x+1)(x+3)}{\left(1-x^2\right)^2}$ **c)** $4(2x-1)^3(2-3x)^3(7-12x)$ **d)** $\dfrac{-2(x^2-3x-1)}{\left(x^2+1\right)^2}$ **e)** $15x^2(3x-5)(x-1)$

f) $\dfrac{-(2x-1)(2x+5)}{(x-2)^4}$ **g)** $4x^3(1-4x^2)^2(1-10x^2)$ **h)** $\dfrac{48x(x^2-3)^3}{\left(x^2+3\right)^5}$ **i)** $6x(2x^3+3x+3)(x^2+3)^2(x^3+3)$ **j)** $\dfrac{1}{\left(1+x^2\right)^{\frac{3}{2}}}$

k) $12(4-3t^3)^3(1-2t)^5(9t^3-3t^2-4)$ **l)** $\dfrac{1}{(1-x)\sqrt{1-x^2}}$ **3.** $\dfrac{-2x(x^2+3x-1)(1-x)^2}{(1+x)^4}$

Date:______________ **Unit 5 Test Review**

Part I: *Complete each of the following without a calculator.*

1. Evaluate.

a) $\left(11\frac{1}{9}\right)^{-1.5}$ **b)** $\left(8^{-\frac{1}{3}}+9^{-\frac{1}{2}}\right)^{2}$ **c)** $\frac{4^{-18}-4^{-16}}{4^{-16}+4^{-17}}$ **d)** $\frac{4^{3x-1}\cdot 16^{x}}{32^{2x}\cdot 8}$

2. Simplify using the exponent laws and express the final answer with positive exponents.

a) $\left(-\frac{4^3a^5b}{4^2a^6b^{-4}}\right)^3$ **b)** $(64a^3b^6)^{\frac{1}{3}}\sqrt{25a^{-8}b^{-4}}$ **c)** $\sqrt[5]{\frac{\sqrt{x}\cdot\sqrt[4]{x^5}}{\sqrt[3]{x^2}\cdot x^{-1}}}$

3. Solve each of the following exponential equations where $x \in R$.

a) $4^{2x+1}=8^{2x^2}$ **b)** $\frac{125^{x+2}}{25^{x-3}}=625^{x-3}$ **c)** $3^{x-1}\cdot 9^{\frac{3}{2x^2}}=27$

d) $10^{x+4}=11-10^{x+3}$ **e)** $3^{x-1}-3^{x-3}=24$ **f)** $5^{2x}-30(5^x)+125=0$

g) $2^{2x}-2^{x+1}=8$ ***Hint:*** $2^{x+1}=2^1\cdot 2^x$ **h)** $6^x-7(6^{-x})+6=0$ ***Hint:*** $6^{-x}=\frac{1}{6^x}$

4. Graph each of the following, by naming and applying transformations on an appropriate exponential function. Find and label any holes, asymptotes and intercepts and state the domain and range.

a) $f(x)=4-(2)^{2x+4}$ **b)** $y=0.5\left(\frac{1}{3}\right)^{-x+1}-5$ **c)** $g(x)=\frac{e^{2x}-e^x}{1-e^x}$

5. Rewrite each expression as the sum and/or difference of terms, where each term is of the form ax^n.

a) $\left(\frac{\sqrt{x}}{-2}\right)^3(1-2\sqrt{x})^2$ **b)** $\frac{2-2x^3}{\sqrt[4]{x}-\sqrt[4]{x^5}}$

6. Completely factor each of the following expressions and then rewrite with positive exponents.

a) $x^{\frac{3}{2}}-25x^{-\frac{1}{2}}$ **b)** $1+8x^{-1}+15x^{-2}$ ***Hint:*** $1=1x^0$

c) $8x(x^2+3)^3(4x-5)^3+12(4x-5)^2(x^2+3)^4$ **d)** $-2(5+3t)^{-\frac{1}{3}}(1-t^2)^{-2}+4t(1-t^2)^{-3}(5+3t)^{\frac{2}{3}}$

7. Jamie works in a lab that uses radioactive substances. The lab received a shipment of 200 g of radioactive radon and 16 days later 12.5 g of the radon remained. Determine the half-life of radon.

8. Salmonella bacteria, found on almost all chicken and eggs, grow rapidly in a nice warm place. If just a few hundred bacteria are left on the cutting board when chicken is cut up and they get into the potato salad, the population begins compounding. Suppose initially there were 500 bacteria in the potato salad and the population doubles every 20 minutes after being left out on the table.

a) Write an equation to represent the number of bacteria, $N(t)$, in the potato salad after t hours .

b) How quickly will the number of bacteria increase to 32 000?

Part II: *Complete each of the following with a calculator.*

9. An old stamp is currently worth \$60. The stamp's value will grow exponentially by 15% each year.
a) What will the stamp's value be in 5 years to the nearest dollar?
b) When will the stamp be worth three times its initial value?

10. The town of Greenleaf is growing exponentially at a rate of 4.5% each year. What is the population of Greenleaf now, to the nearest hundred, if 5000 people will be living there 6 years from now?

11. Emily bought a new car for \$35 000 and sold it five years later for \$18 475. If the value of the vehicle depreciates exponentially, calculate the annual rate of depreciation to the nearest percent.

12. Sam wants to have \$40 000 available for a down payment on a condo in 10 years. How much should she invest now at 6.25% per annum compounded semi-annually to the nearest cent?

13. A rumour spreads through a school. After the rumour has begun, $N(t)=\dfrac{50}{1+49e^{-t}}$ people have heard the rumour where t is in hours.
a) How many people will have heard it after 4 h?
b) How many people will eventually hear it?

Answers:

1. a) $\frac{27}{1000}$ **b)** $\frac{25}{36}$ **c)** $-\frac{3}{4}$ **d)** $\frac{1}{32}$ **2. a)** $-\frac{64b^{15}}{a^3}$ **b)** $\frac{20}{a^3}$ **c)** $x^{\frac{5}{12}}$

3. a) $-\frac{1}{3}, 1$ **b)** 8 **c)** $\frac{3-\sqrt{21}}{2}, 1, \frac{3+\sqrt{21}}{2}, x \neq 0$ **d)** -3 **e)** 4 **f)** $1, 2$ **g)** 2 **h)** 0

4. a) *Transformations on* $y=2^x$ *are:*

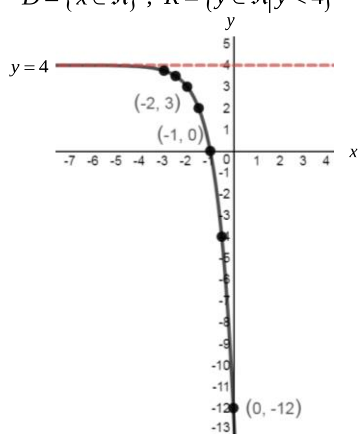
i) *vertical reflection in the x-axis*

ii) *horizontal compression by a factor of* $\frac{1}{2}$

iii) *horizontal translation left* 2 *units*

iv) *vertical translation up* 4 *units*

$D=\{x \in \Re\}$, $R=\{y \in \Re \mid y<4\}$



b) *Transformations on* $y=\left(\frac{1}{3}\right)^x$ *are:*

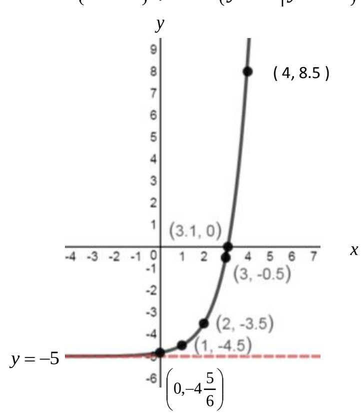
i) *vertical compression by a factor of* $\frac{1}{2}$

ii) *horizontal reflection in the y-axis*

iii) *horizontal translation right* 1 *unit*

iv) *vertical translation down* 5 *units*

$D=\{x \in \Re\}$, $R=\{y \in \Re \mid y>-5\}$

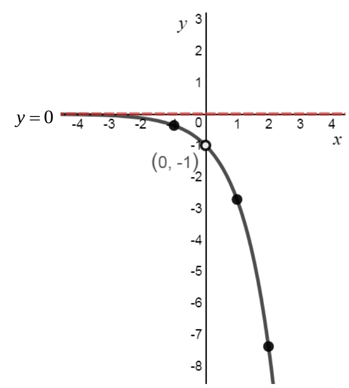


c) *Transformations on* $y=e^x$ *are:*

i) *vertical reflection in the x-axis*

$D=\{x \in \Re \mid x \neq 0\}$,

$R=\{y \in \Re \mid y<-1 \; or \; -1<y<0\}$



5. a) $-\frac{1}{8}x^{\frac{3}{2}}+\frac{1}{2}x^2-\frac{1}{2}x^{\frac{5}{2}}$ **b)** $2x^{-\frac{1}{4}}+2x^{\frac{3}{4}}+2x^{\frac{7}{4}}$

6. a) $\frac{(x+5)(x-5)}{x^{\frac{1}{2}}}$ **b)** $\frac{(x+5)(x+3)}{x^2}$ **c)** $4(x^2+3)^3(4x-5)^2(11x^2-10x+9)$ **d)** $\frac{2(7t^2+10t-1)}{(5+3t)^{\frac{1}{3}}(1-t^2)^3}$

7. 4 days **8. a)** $N(t)=500(2)^{3t}$ **b)** 2 hours **9. a)** \$121 **b)** 8 years **10.** 3800 **11.** 12% **12.** \$21 616.27 **13. a)** 26 **b)** 50