Date:________________

UNIT 5: EXPONENTIAL FUNCTIONS

5.1 Laws of Exponents and Exponential Equations

Your study of calculus will require an ability to manipulate rational and negative exponents. The exponent laws enable us to simplify and evaluate expressions involving exponents. Here is a summary of the exponent laws.

Exponent Laws

$a^m \times a^n = a^{m+n}$	$a^{-m} = \left(\frac{1}{a}\right)^m$ or $\frac{1}{a^m}$	$a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{1}{a^{-m}} = a^m$	$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$ or $\left(a^m\right)^{\frac{1}{n}}$
$\left(a^m\right)^n = a^{mn}$	$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$
$(ab)^m = a^m b^m$		
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$	$a^0 = 1$

Ex. 1. Evaluate each of the following:

a) $\dfrac{2^{-4} - 2^{-6}}{2^{-5} - 2^{-3}}$ **b)** $\dfrac{9^{n+2} \times 27^{n-4}}{81^{n-1} \times 3^n}$ **c)** $\dfrac{4^{\frac{3}{2}} - 8^{\frac{1}{3}}}{16^{\frac{1}{4}} \times 25^{\frac{1}{2}}}$

Ex. 2. Simplify each of the following, using the laws of exponents.

a) $\dfrac{\sqrt[3]{a^4} \times \sqrt{a}}{\left(\sqrt[3]{a}\right)^4}$ **b)** $\dfrac{\left(\sqrt[3]{64x^6y^3}\right)\left(\sqrt{64x^{-6}y^4}\right)}{\left(-2x^2y^{-1}\right)^3}$ **c)** $\dfrac{\left(x^2y - xy^2\right)^3}{(xy)^4}$

Exponential Equations

Ex. 3. Solve for *x*.

a) $8^{x} = 16^{x-1}$

b) $3^{x^2+3} = 81^{x}$

c) $2^{x+5} - 2^{x+3} = 192$

d) $2^{2x} - 12\left(2^{x}\right) + 32 = 0$

HW. Exercise 5.1

Date:______________ **5.2 Investigating the Graphs of Exponential Functions**

$f(x) = b^x$ **&** $f(x) = a(b)^{k(x-d)} + c$

I Graphing Exponential Functions of the Form $y = b^x$**, where i)** $b > 1$ **& ii)** $0 < b < 1$

Ex. 1. Complete the following table of values and graph the following exponential functions on the same axes.

i) $b > 1$

a) $y = 2^x$

x	y

b) $y = 3^x$

x	y

ii) $0 < b < 1$

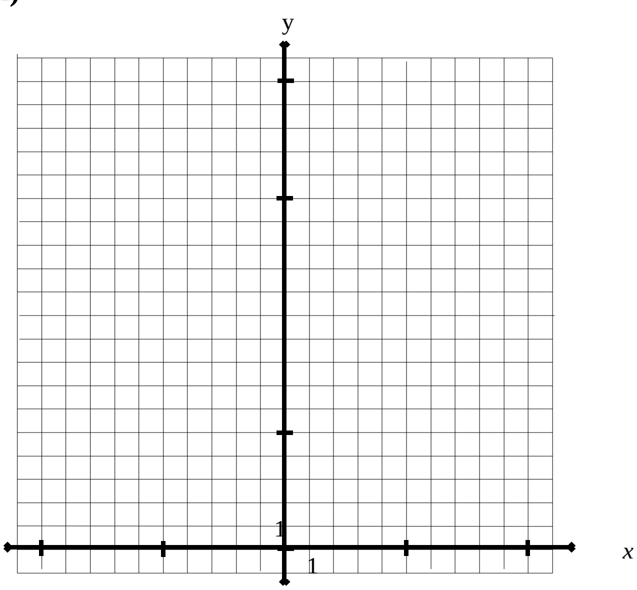
a) $y = \left(\frac{1}{2}\right)^x$

x	y

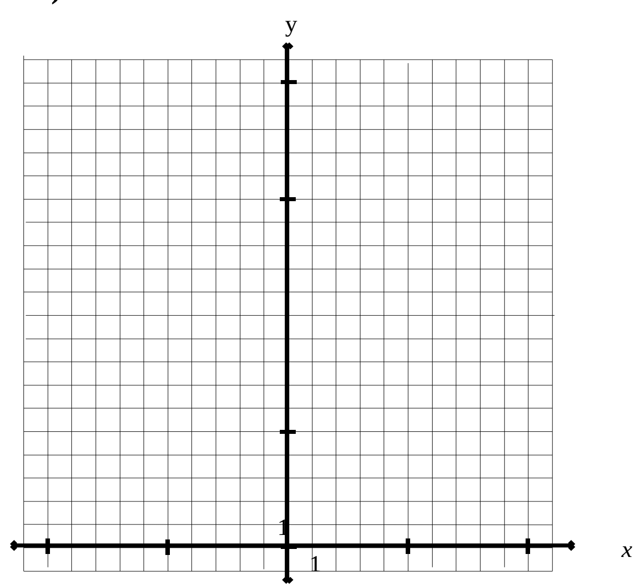
b) $y = \left(\frac{1}{3}\right)^x$

x	y

i)



ii)



x **– intercept** ____________________	***x*** **– intercept** ____________________
y **– intercept** ____________________	***y*** **– intercept** ____________________
Domain ____________________	**Domain** ____________________
Range ____________________	**Range** ____________________
Horizontal Asymptote is ____________	**Horizontal Asymptote is** ____________
Function is ____________________	**Function is** ____________________

II Transformations on the Exponential Function $f(x)=b^x$,where $f(x)=a(b)^{k(x-d)}+c$

Recall: $(x,y)\rightarrow\left(\frac{1}{k}x+d, ay+c\right)$

Ex. 2. Graph each of the following, by naming and applying transformations on an appropriate exponential function.

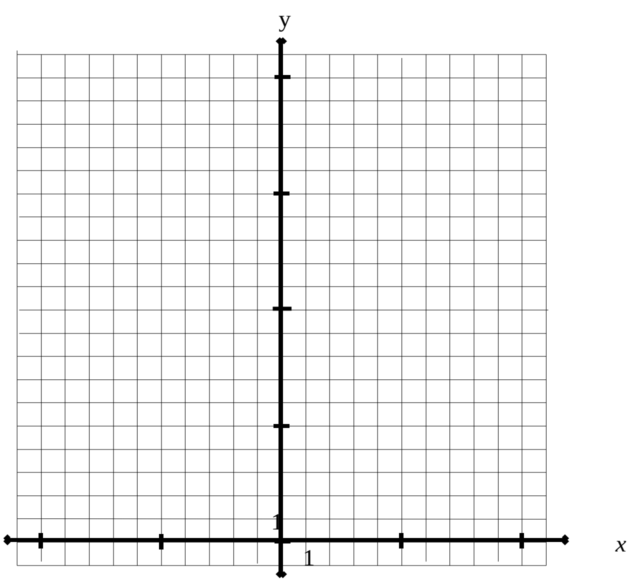
a) $y=\left(\frac{1}{3}\right)^{\frac{1}{2}x-1}-1$

Transformations on __________ *are:*

i) __________________________________

ii) __________________________________

iii) __________________________________



Horizontal Asymptote: _________________

Domain : ______________________________

Range: ______________________________

***y* – intercept :** __________________________

***x* – intercept :** __________________________

b) $f(x)=-\frac{1}{2}(3)^{-x-3}+13$

Transformations on __________ *are:*

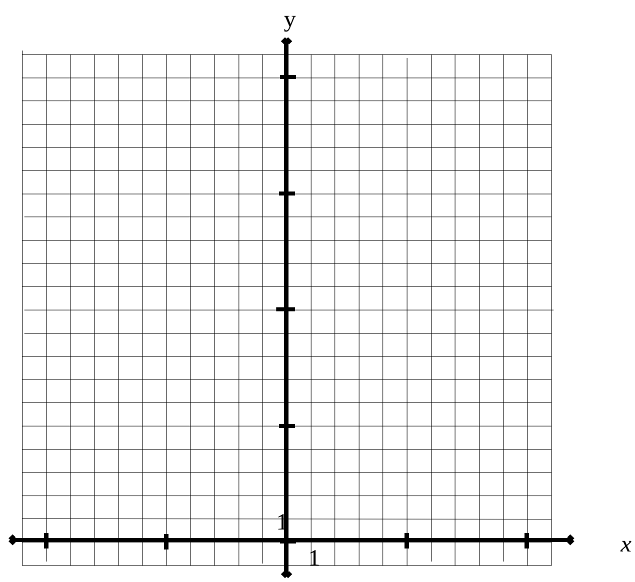
i) ______________________________________

ii) ______________________________________

iii) ______________________________________

iv) ______________________________________

v) ______________________________________



Horizontal Asymptote: _________________

Domain : ______________________________

Range: ______________________________

***y* – intercept :** __________________________

***x* – intercept :** __________________________

HW. Exercise 5.2

Date:_______________ **5.3 Exponential Growth and Decay**

Exponential growth *or* ***decay*** occurs when quantities increase or decrease at a rate proportional to the **initial** quantity present. This ***growth*** or ***decay*** occurs in savings accounts, the size of populations, appreciation, depreciation, and with radioactive chemicals. All problems of this type can be modeled by the **exponential function**

i) $y = a \cdot b^x$ where

- y is the *final amount* or *number*
- a is the *initial amount* or *number*
- b is the *growth* or *decay* <u>*factor*</u>
- x is the *number of growth* or *decay periods*

or

ii) $A = A_0(1 \pm r)^n$ where

- A is the *final amount* or *number*
- A_0 is the *initial amount* or *number*
- r is the *growth* or *decay* <u>*rate*</u>
- n is the *number of growth* or *decay periods*

Ex. 1. An antique vase was purchased in 1980 for $8000. If the vase appreciates in value by 6% per year, what is its estimated value in the year 2015, to the nearest hundred dollar?

Ex. 2. A car depreciates by 15% per year. If you buy a car for $20 000, find the value of the car in three years to the nearest hundred dollar and estimate when the car will be worth half of its original value.

Ex. 3. The population of a town was 24 000 in 1980 and 29 000 in 1990.

a) Determine the annual growth rate for the town during this period.

b) Determine an expression for the population, *P*, at time *t* years after 1980.

c) Use this expression to estimate the population of the town in 2012.

Ex. 4. The population of the world was 6 billion in 1999. This population is growing exponentially and doubles every 51 years. Estimate the world population in 2018.

Ex. 5. A hospital uses cobalt-60 in its radiotherapy treatment for cancer patients. Cobalt-60 has a half-life of 5.2 years. This means that every 5.2 years, 50% of the original sample of cobalt-60 has decayed. The hospital has 80 g of cobalt-60.
How much of the original sample will there be after 1 year to the nearest gram?

Ex. 6. The isotope, radioactive strontium-90, decays to 25% of its original mass after approximately 58 years. Determine its half-life.

Ex. 7. In a recent dig, a human skeleton was unearthed. It was later found that the amount of carbon-14 in it had decayed to $\frac{1}{\sqrt{8}}$ of its original amount.
If carbon -14 has a half-life of 5730 years, how old is the skeleton?

HW. Exercise 5.3

Date:________________ **5.4 Simplifying Using Exponent Laws**

Ex. 1. Rewrite each expression as the sum and/or difference of terms, where each term is of the form ax^n.

a) $\dfrac{4x^5-5x^4+6x-2}{2x^4}$

b) $\dfrac{8x^3-27}{2x-3}$

c) $\dfrac{2}{\sqrt{x^3}}+\dfrac{x}{\sqrt{3}}+6\sqrt[3]{x}-\left(\dfrac{1}{3x}\right)^2$

d) $\dfrac{1}{\sqrt{x}}\left(\sqrt{x}+\dfrac{1}{x}\right)^2$

e) $\left(\dfrac{x-3}{2\sqrt[3]{x^2}}\right)^3$

f) $\dfrac{x-16}{4\sqrt{x}+x}$

Ex. 2. Completely factor each of the following expressions.

a) $-2t(1-t)^4+4t^2(1-t)^3$

b) $10(2x+1)^4(3x+2)^4+12(2x+1)^3(3x+2)^5$

c) $12x(2x^2-1)^2(x^2-2)^{-2}-4x(x^2-2)^{-3}(2x^2-1)^3$

d) $-\dfrac{1}{2\sqrt{1-t}\cdot\sqrt{1+t}}-\dfrac{\sqrt{1-t}}{2\sqrt{(1+t)^3}}$

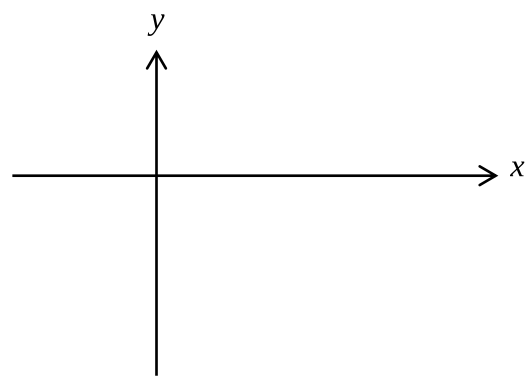
HW. Exercise 5.4

Date:______________

Unit 5 Test Review

1. ***Sketch*** the graph of $f(x) = -e^{-x+3} + 1$ by naming and applying transformations on an appropriate function. Determine the x and y-intercepts algebraically and mark these points on the graph.

The transformations on are:



i) ______________________________

ii) ______________________________

iii) ______________________________

iv) ______________________________

2. Solve each of the following.

a) $3^x - 3^{x-1} = \dfrac{2}{\sqrt{27}}$

b) $e^{2x} + 2e^x = 3$

3. Evaluate.

a) $\dfrac{2^{-22} - 2^{-23}}{2^{-25} + 2^{-24}}$

b) $\dfrac{3^{n+1} \times 9^{n-4}}{27^{n-2}}$

4. Completely factor the following expression.

$-16(1-4x)^3(x^2-1)^{-3} - 6x(x^2-1)^{-4}(1-4x)^4$

5. Rewrite each expression as the sum and/or difference of terms, where each term is of the form ax^n.

a) $\sqrt{\dfrac{\sqrt[3]{x}\cdot\sqrt[3]{x^2}}{\sqrt{x}}}$ **b)** $\dfrac{x-4}{x^{\frac{1}{2}}+2}$ **c)** $\dfrac{x^3+8}{\sqrt{x^3}+2\sqrt{x}}$

6. Joe's parents invested \$4000 in an account when he was born. The account pays interest at 6%/a, compounded quarterly. How much money will be in the account on Joe's 18th birthday?

7. One bacterium divides into two bacteria every 5 days. Initially, there are 15 bacteria.
a) How many bacteria will there be in 10 days? **b)** What is the approximate growth rate per day?

8. A ***small*** self-contained forest was studied for squirrel population by a biologist. It was found that the forest population, *P*, was a function of time, *t*, where *t* was measured in weeks.

The function was $P=\dfrac{20}{1+3e^{-0.02t}}$.

a) Find the population at the start of the study and after one year.

b) The largest population the forest can sustain is represented mathematically by looking at the end behaviour of the function, *P*, specifically as $t \to$.
Determine the largest squirrel population the forest can sustain.

HW. Unit 5 Review Exercise