Date: Dec 12/14

COMPOSITION INVOLVING TRIGONOMETRIC FUNCTIONS

Ex. 1. Given $f(x)=\frac{1}{x}$, $g(x)=\sin x$ and $h(x)=2x^2-5x+2$ find:

a) x, if $h(x)+f(x)=0$

$x(2x^2-5x+2)+x\left(\frac{1}{x}\right)=(0)x \quad , x\neq 0$

$2x^3-5x^2+2x+1=0$

$(x-1)(2x^2-3x-1)=0$

$x=1$ or $x=\frac{3\pm\sqrt{(-3)^2-4(2)(-1)}}{2(2)}$

$x=\frac{3\pm\sqrt{17}}{4}$

$$\begin{array}{r} 2x^2-3x-1 \\ x-1\overline{)\,2x^3-5x^2+2x+1} \\ \underline{2x^3-2x^2} \\ -3x^2+2x \\ \underline{-3x^2+3x} \\ -x+1 \\ \underline{-x+1} \\ 0 \end{array}$$

$\therefore x=1,\ x=\frac{3+\sqrt{17}}{4},\ x=\frac{3-\sqrt{17}}{4}$

b) $(g\circ f)\left(-\frac{3}{2\pi}\right)$

$=g\left(f\left(-\frac{3}{2\pi}\right)\right)$

$=g\left(-\frac{2\pi}{3}\right)$ QIII

$=\sin\left(-\frac{2\pi}{3}\right)$

$=-\sin\left(\frac{\pi}{3}\right)$

$=-\frac{\sqrt{3}}{2}$

c) $(f\circ g)(-\pi)$

$=f(g(-\pi))$

$=f(\sin(-\pi))$

$=f(0)$

d.n.e or undefined.

d) $(f\circ g)(x)$

$=f(g(x))$

$=f(\sin x)$

$=\frac{1}{\sin x}$

$=\csc x$.

e) x, for $0\le x\le 2\pi$ if $(f\circ g)(x)+5=0$

From d) $\quad \csc x+5=0$

$\sin x=-\frac{1}{5}$

$\text{raa}=\sin^{-1}\left(+\frac{1}{5}\right)$

$\doteq 0.2014$

In QIII

$x=\pi+0.2014$

$x\doteq 3.34$

In QIV

$x=2\pi-0.2014$

$x\doteq 6.08$

f) x for $0\le x\le 2\pi$, if $(h\circ f\circ g)(x)=0$

from d)

$h(f(g(x)))=0$

$h(\csc x)=0$

$2\csc^2 x-5\csc x+2=0$

$(2\csc x-1)(\csc x-2)=0$

$\csc x=\frac{1}{2}$ or $\csc x=2$

$\sin x=2$

no solution

$-1\le \sin x\le 1$

$\sin x=\frac{1}{2}$

$\text{raa}=\frac{\pi}{6}$

In QI: $x=\frac{\pi}{6}$

In QII: $x=\frac{5\pi}{6}$

Recall: $\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$= \cos^2 A - \sin^2 A$

Ex. 2. Find the points of intersection of the curves $f(x) = \cos x$ and $g(x) = \cos 2x$ for $0 \le x \le 2\pi$. Illustrate your solution graphically.

$P = \frac{2\pi}{2} = \pi$

For points of intersection:

$f(x) = g(x)$

$\cos x = \cos 2x$

$\cos x = 2\cos^2 x - 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

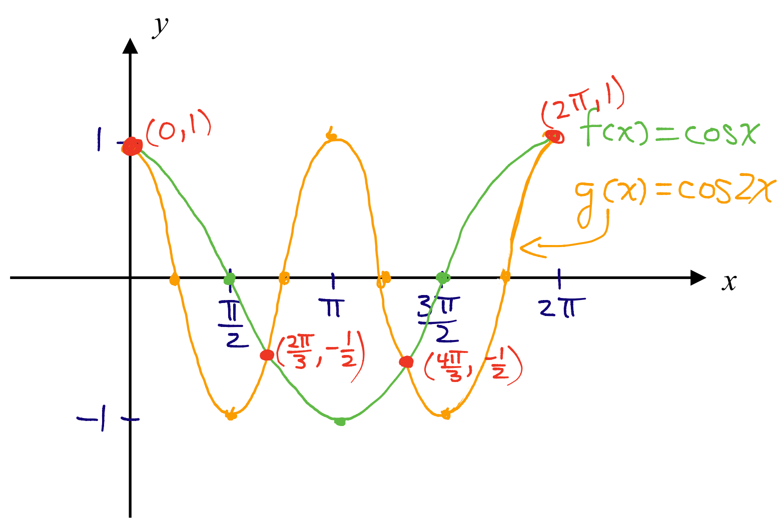
raa $= \frac{\pi}{3}$ $\quad\therefore x = 0,\ x = 2\pi$

In QII: $x = \frac{2\pi}{3}$

In QIII: $x = \frac{4\pi}{3}$

$\therefore$ the points of intersection are

$(0,1)\ \left(\frac{2\pi}{3}, -\frac{1}{2}\right)\ \left(\frac{4\pi}{3}, -\frac{1}{2}\right)(2\pi, 1)$



HW. Worksheet on Composition Involving Trigonometric Functions

Date: Dec 15/14

COMBINING FUNCTIONS USING ALGEBRA

Two real-valued functions f and g can be combined to form new functions $f+g$, $f-g$, fg and $\dfrac{f}{g}$ in a manner similar to the way we add, subtract, multiply and divided real numbers.

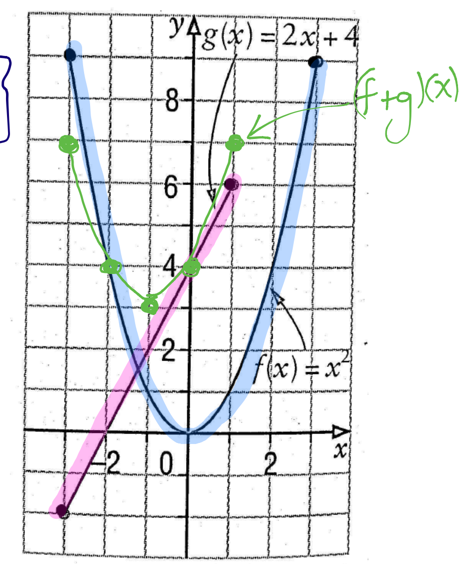
Operations on Functions

Let the domain of f be A and the domain of g be B.

intersection (overlap)

Addition	$(f+g)(x)=f(x)+g(x)$	*Domain* $=A\cap B$
Subtraction	$(f-g)(x)=f(x)-g(x)$	Domain $=A\cap B$
Multiplication	$(fg)(x)=f(x)g(x)$	Domain $=A\cap B$
Division	$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}$	$\mathrm{D}=\{x\in A\cap B \mid g(x)\neq 0\}$

Ex. 1. Use the graphs of f and g to determine the following.



a) $D_f=\{x\in\mathbb{R}\mid -3\le x\le 3\}$, $D_g=\{x\in\mathbb{R}\mid -3\le x\le 1\}$

b) $D_{f+g}=\{x\in\mathbb{R}\mid -3\le x\le 1\}$, $D_{f-g}=\{x\in\mathbb{R}\mid -3\le x\le 1\}$

c) $D_{fg}=\{x\in\mathbb{R}\mid -3\le x\le 1\}$, $D_{\frac{f}{g}}=\{x\in\mathbb{R}\mid -3\le x\le 1, x\neq -2\}$ or $\{x\in\mathbb{R}\mid -3\le x<-2, -2<x\le 1\}$

d) $(f+g)(-1)$

$=f(-1)+g(-1)$

$=1+2$

$(f+g)(-1)=3$

e) $(f-g)(-3)$

$=f(-3)-g(-3)$

$=9-(-2)$

$(f-g)(-3)=11$

f) $(fg)(1)$

$=f(1)g(1)$

$=1\cdot 6$

$(fg)(1)=6$

g) $\left(\dfrac{f}{g}\right)(-2)$ → -2 is not in the domain ∴ undefined

$=\dfrac{f(-2)}{g(-2)}$

$=\dfrac{4}{0}$

undefined

h) the graph and equation of $(f+g)(x)$

$(f+g)(x)=(x^2)+(2x+4)$

$=x^2+2x+4$

For the vertex: $x=\dfrac{-b}{2a}$

$=\dfrac{-2}{2(1)}$

$=-1$

$(f+g)(-1)=3$ ∴ vertex $(-1,3)$

x	$f(x)$	$g(x)$	$(f+g)(x)$
-3	9	-2	7
-2	4	0	4
-1	1	2	3
0	0	4	4
1	1	6	7

$(x, f+g(x))$

$(-3, 7)$

$(-2, 4)$

⋮

etc.

$D_f: \{x \in \mathbb{R}\}$
$D_g: \{x \in \mathbb{R}\}$

Ex. 2. If $f(x) = x + 3$ and $g(x) = x^2 + 8x + 15$ determine each of the following.

a) the domain of $f + g$

$D_{f+g}: \{x \in \mathbb{R}\}$

b) the domain of $\frac{f}{g}$

$D_{\frac{f}{g}}: \{x \in \mathbb{R} \mid x \neq -5, x \neq -3\}$

Find x if $g(x) = 0$
$x^2 + 8x + 15 = 0$
$(x+5)(x+3) = 0$
$x = -5, x = -3$

c) $(f - g)(-5)$

$= f(-5) - g(-5)$
$= -2 - 0$
$= -2$

d) $(fg)(0)$

$= f(0) \cdot g(0)$
$= 3 \cdot 15$
$= 45$

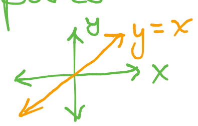
e) the equation and graph of $(fg)(x)$

$(fg)(x) = (x+3)(x^2 + 8x + 15)$

$\boxed{\therefore (fg)(x) = x^3 + 11x^2 + 39x + 45}$

$(fg)(x) = (x+3)(x+5)(x+3)$
$(fg)(x) = (x+3)^2(x+5)$
$\therefore$ x-ints are -3 & -5
double single

Compares to: $y = x$



f) the equation and graph of $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

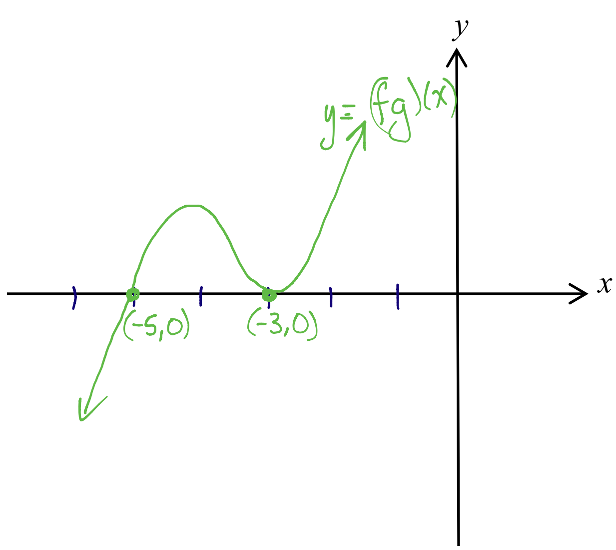
$\left(\frac{f}{g}\right)(x) = \frac{x+3}{x^2 + 8x + 15}$

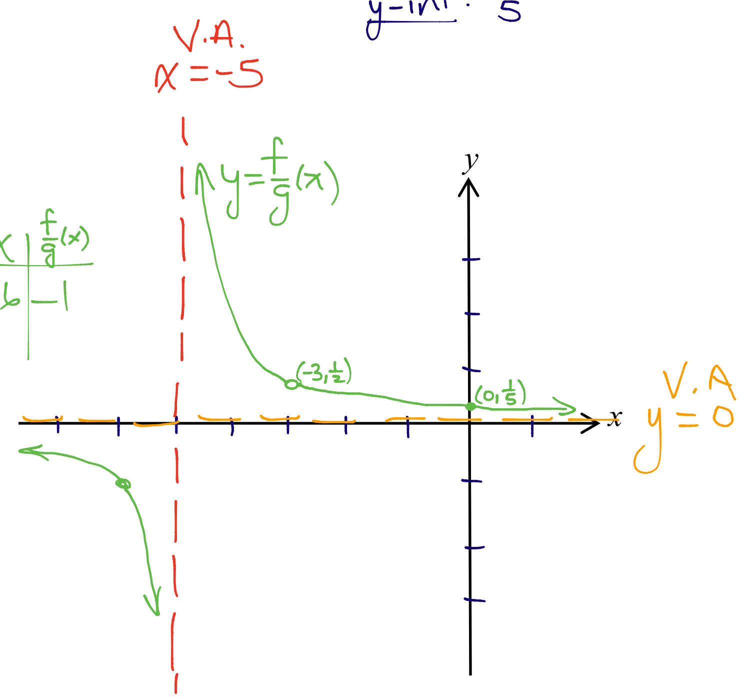
$\left(\frac{f}{g}\right)(x) = \frac{\cancel{x+3}\,1}{\cancel{(x+3)}(x+5)} \quad x \neq -3, -5$

$\boxed{\therefore \left(\frac{f}{g}\right)(x) = \frac{1}{x+5}}$

hole @ $\left(-3, \frac{f}{g}(-3)\right)$
$\left(-3, \frac{1}{2}\right)$

V.A. $x = -5$
For HA, as $x \to \pm\infty$
$\left(\frac{f}{g}\right)(x) \to 0$
$\therefore$ H.A. is $y = 0$
x-int: none
y-int: $\frac{1}{5}$
hole $\left(-3, \frac{1}{2}\right)$





x	$\frac{f}{g}(x)$
-6	-1

Ex. 3. If $f(x)=\sqrt{9-x^2}$ and $g(x)=\sqrt{x+3}$, determine the following.

a) the domain of f

$9-x^2 \geq 0$

$(3-x)(3+x) \geq 0$

$D_f: \{x \in \mathbb{R} \mid -3 \leq x \leq 3\}$

b) the domain of g

$x+3 \geq 0$

$x \geq -3$

$D_g: \{x \in \mathbb{R} \mid x \geq -3\}$

c) the domain of $f-g$

$D_{f-g} \{x \in \mathbb{R} \mid -3 \leq x \leq 3\}$

d) the domain of $\frac{g}{f}$

$D_{\frac{g}{f}}: \{x \in \mathbb{R} \mid -3 < x < 3\}$

e) the domain, equation and graph of $\left(\frac{f}{g}\right)(x)$

$D_{\frac{f}{g}}: \{x \in \mathbb{R} \mid -3 < x \leq 3\}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{\sqrt{9-x^2}}{\sqrt{x+3}}$

$= \sqrt{\frac{9-x^2}{x+3}}$

$= \sqrt{\frac{(3-x)\cancel{(3+x)}^{1}}{\cancel{x+3}_{1}}}$ $x \neq -3$

$\boxed{\therefore \left(\frac{f}{g}\right)(x) = \sqrt{3-x}}$

hole @ $(-3, \sqrt{6})$

$\frac{f}{g}(x) = \sqrt{-(x-3)}$

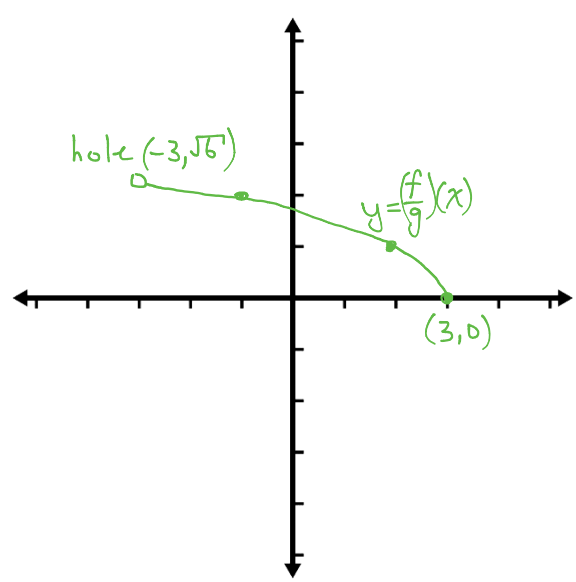
f) $(f+g)(-1)$

$= f(-1) + g(-1)$

$= \sqrt{8} + \sqrt{2}$

$= 2\sqrt{2} + 1\sqrt{2}$

$= 3\sqrt{2}$



HW. Day 6 Part I #1, Part II #1

Date: Dec 16/14 **COMBINING FUNCTIONS USING ALGEBRA continued**

Two real-valued functions f and g can be combined to form new functions $f+g$, $f-g$, fg and $\dfrac{f}{g}$ in a manner similar to the way we add, subtract, multiply and divided real numbers.

Operations on Functions

Let the domain of f be A and the domain of g be B.

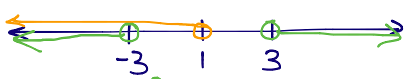
Addition	$(f+g)(x)=f(x)+g(x)$	*Domain* $=A\cap B$
Subtraction	$(f-g)(x)=f(x)-g(x)$	Domain $=A\cap B$
Multiplication	$(fg)(x)=f(x)g(x)$	Domain $=A\cap B$
Division	$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}$	$D=\{x\in A\cap B \mid g(x)\neq 0\}$

Ex. 1. If $f(x)=\log_4(x^2-9)$ and $g(x)=\log_4(1-x)$, determine the following.

a) the domain of f

$x^2-9>0$

$(x-3)(x+3)>0$



$D_f:\{x\in\mathbb{R} \mid x<-3 \text{ or } x>3\}$

b) the domain of g

$1-x>0$

$-x>-1$

$x<1$

$D_g:\{x\in\mathbb{R} \mid x<1\}$

c) $(g-f)(x)$ and its domain

$(g-f)(x)=g(x)-f(x)$

$=\log_4(1-x)-\log_4(x^2-9)$

$\therefore (g-f)(x)=\log_4\left[\dfrac{1-x}{x^2-9}\right]$

$D_{g-f}:\{x\in\mathbb{R} \mid x<-3\}$

d) $(f+g)(x)$ and its domain

$(f+g)(x)=f(x)+g(x)$

$=\log_4(x^2-9)+\log_4(1-x)$

$=\log_4[(x^2-9)(1-x)]$

$\therefore (f+g)(x)=\log_4(-x^3+x^2+9x-9)$

$D_{f+g}:\{x\in\mathbb{R} \mid x<-3\}$

e) $(f+g)(-7)$

$=f(-7)+g(-7)$

$=\log_4 40+\log_4 8$

$=\log_4 320$

$\left(=\dfrac{\log 320}{\log 4}\doteq \ldots\right.$ using change of base formula$\left.\right)$

f) $\left(\dfrac{g}{f}\right)(-4)$

$=\dfrac{g(-4)}{f(-4)}$

$=\dfrac{\log_4 5}{\log_4 7}$

$=\log_7 5$ (change of base)

Ex. 2. If $f(x)=4^{-x}$ and $g(x)=3(2)^{5x-1}$, determine the following.

a) the domain of f

$D_f: \{x \in \mathbb{R}\}$

b) the domain of g

$D_g: \{x \in \mathbb{R}\}$

c) $(fg)(x)$ and its domain

$(fg)(x) = f(x) \cdot g(x)$

$= 4^{-x} \cdot 3(2)^{5x-1}$

$= 2^{-2x} \cdot 3(2)^{5x-1}$

$\therefore (fg)(x) = 3(2)^{3x-1}$

$D_{fg}: \{x \in \mathbb{R}\}$

d) $\left(\frac{g}{f}\right)(x)$ and its domain

$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$= \frac{3(2)^{5x-1}}{4^{-x}}$

$= \frac{3(2)^{5x-1}}{2^{-2x}}$

$\therefore \left(\frac{g}{f}\right)(x) = 3(2)^{7x-1}$

$D_{\frac{g}{f}}: \{x \in \mathbb{R}\}$

e) $(f-g)(0)$

$(f-g)(0) = f(0) - g(0)$

$= 4^{-(0)} - 3(2)^{5(0)-1}$

$= 1 - 3\left(\frac{1}{2}\right)$

$= \frac{2}{2} - \frac{3}{2}$

$\therefore (f-g)(0) = -\frac{1}{2}$

f) $(fg)(-1)$

$(fg)(-1) = f(-1)g(-1)$

$= \left[4^{-(-1)}\right]\left[3(2)^{5(-1)-1}\right]$

$= 4^{1} \cdot 3 \cdot 2^{-6}$

$= \frac{12}{64}$

$\therefore (fg)(-1) = \frac{3}{16}$

HW. Day 6 Part I #2, 3 , Part II #2, 3

For Unit 7 Test: do Day 7 Unit 7: Review #1 to 7