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UNIT 7: COMBINING FUNCTIONS

7.1 Composite Functions

1. Let $f = \{(1,2), (2,3), (3,5), (4,7)\}$ and $g = \{(1,4), (2,3), (3,1)\}$. Determine

a) f^{-1} **b)** $(f \circ g)(1)$ **c)** $(g \circ f)(3)$

d) $(g \circ g)(3)$ **e)** $(f \circ f^{-1})(5)$ **f)** $(f^{-1} \circ g \circ f)(1)$

g) $g \circ f$ **h)** $f \circ g$ **i)** $g \circ f^{-1}$

2. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find the following:

a) $f(g(1))$ **b)** $g(f(1))$ **c)** $(g \circ f)(0)$

d) $(f \circ g)(-4)$ **e)** $f(g(x))$ **f)** $(g \circ f)(x)$

3. Use the functions $f(x) = 3x + 1$, $g(x) = x^3$, $h(x) = \dfrac{1}{x+1}$, and $u(x) = \sqrt{x}$ to find expressions for the indicated composite function.

a) $f \circ u$ **b)** $g \circ f$ **c)** $h \circ u$

d) $h \circ (f \circ u)$ **hint:** refer to part **a)** **e)** $g \circ (h \circ u)$ **hint:** refer to part **c)**

4. Express h as the composition of two functions f and g, such that $h(x) = f(g(x))$.

a) $h(x) = (2x^2 - 1)^4$ **b)** $h(x) = \sqrt{5x-1}$ **c)** $h(x) = \dfrac{1}{x-4}$

d) $h(x) = (2-3x)^{\frac{5}{2}}$ **e)** $h(x) = x^4 + 5x^2 + 6$ **f)** $h(x) = (x+1)^2 - 9(x+1)$

5. If $f(x) = \dfrac{1}{1-x}$ and $g(x) = 1 - x$, determine

a) $g(f(x))$ **b)** $f(g(x))$

6. If $f(x) = 3x + 5$ and $g(x) = x^2 + 2x - 3$, determine x such that $f(g(x)) = g(f(x))$.

Answers

1. a) $\{(2,1), (3,2), (5,3), (7,4)\}$ **b)** 7 **c)** *undefined* **d)** 4 **e)** 5 **f)** 2 **g)** $\{(1,3), (2,1)\}$ **h)** $\{(1,7), (2,5), (3,2)\}$ **i)** $\{(2,4), (3,3), (5,1)\}$

2. a) 0 **b)** 0 **c)** -1 **d)** $\sqrt{15}$ **e)** $\sqrt{x^2-1}$ **f)** $x-1$

3. a) $3\sqrt{x}+1$ **b)** $(3x+1)^3$ **c)** $\dfrac{1}{\sqrt{x}+1}$ **d)** $\dfrac{1}{3\sqrt{x}+2}$ **e)** $\dfrac{1}{\left(\sqrt{x}+1\right)^3}$

4. a) $f(x) = x^4,\ g(x) = 2x^2 - 1$ **b)** $f(x) = \sqrt{x},\ g(x) = 5x - 1$ **c)** $f(x) = \dfrac{1}{x},\ g(x) = x - 4$ **d)** $f(x) = x^{\frac{5}{2}},\ g(x) = 2 - 3x$

e) $f(x) = x^2 + 5x + 6,\ g(x) = x^2$ *or* $f(x) = x(x+1),\ g(x) = x^2 + 2$ **f)** $f(x) = x^2 - 9x,\ g(x) = x + 1$

5. a) $\dfrac{x}{x-1}$ **b)** $\dfrac{1}{x}$ **6.** $-2, -3$

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7.2 Composition of Functions continued

1. For each of the following pairs of functions, find the composite functions $f \circ g$ and $g \circ f$. What is the domain of each composite function? Are the composite functions equal?

a) $f(x) = \frac{1}{x}$ and $g(x) = x^2 + 1$ **b)** $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x+2}$

2. Use the functions $f(x) = 3x + 1$, $g(x) = x^3$, $h(x) = \frac{1}{x+1}$, and $u(x) = \sqrt{x}$ to find expressions for the indicated composite function.

a) $u \circ h$ **b)** $u \circ g$ **c)** $f \circ g$ **d)** $(f \circ g) \circ u$ **hint:** refer to part **c)**

3. If $f(x) = 2x - 7$ and $g(x) = 5 - 2x$,

a) determine **i)** f^{-1} **ii)** g^{-1} **iii)** $f \circ f^{-1}$ **iv)** $g^{-1} \circ g$ **v)** $f \circ g$

b) show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$ **hint:** *Let* $L.S. = (f \circ g)^{-1}$ and $R.S. = g^{-1} \circ f^{-1}$

4. Given $f(x) = \frac{x+3}{2}$ and $g(x) = \sqrt{-4x+1}$, find:

a) $(f \circ g)(-2)$ **b)** $(g \circ f)(-1)$ **c)** x, if $(g \circ f)(x) = 5$

5. Given $f(x) = \frac{x}{x-3}$,

a) find $f^{-1}(x)$ **b)** show that $(f \circ f^{-1})(x) = x$

6. Given $f(x) = x^2 - 4x + 3$, $g(x) = \sin x$ and $h(x) = e^x$, find:

a) $(g \circ f)(x)$ **b)** x**,** if $(f \circ h)(x) = 3$ **c)** x for $-2\pi \le x \le 2\pi$, if $(f \circ g)(x) = 0$

7. Given $f(x) = 4^{x+3}$ and $g(x) = \log_2(2x)$, find a simplified expression for $(g \circ f)(x)$.

8. Given $f(x) = 3^x$ and $g(x) = 2\log_3(x+1)$, find x if $(f \circ g)(x) = 16$.

Answers

1. a) $f(g(x)) = \frac{1}{x^2+1}, D_{f\circ g} = \{x \in R\}$; $g(f(x)) = \frac{1}{x^2} + 1$, $D_{g\circ f} = \{x \in R | x \neq 0\}$; $f \circ g \neq g \circ f$

b) $f(g(x)) = \frac{1}{\sqrt{x+2}}$, $D_{f\circ g} = \{x \in R | x > -2\}$; $g(f(x)) = \sqrt{\frac{1+2x}{x}}$, $D_{g\circ f} = \left\{x \in R | x \le -\frac{1}{2} \text{ or } x > 0\right\}$; $f \circ g \neq g \circ f$

2. a) $\frac{1}{\sqrt{x+1}}$ **b)** $\sqrt{x^3}$ **c)** $3x^3 + 1$ **d)** $3x\sqrt{x} + 1$

3. a) i) $f^{-1}(x) = \frac{x+7}{2}$ **ii)** $g^{-1}(x) = \frac{5-x}{2}$ **iii)** x **iv)** x **v)** $f(g(x)) = -4x + 3$

4. a) 3 **b)** d.n.e. **c)** -15 **5. a)** $f^{-1}(x) = \frac{3x}{x-1}$

6. a) $g(f(x)) = \sin(x^2 - 4x + 3)$ **b)** $\ln 4$ **c)** $-\frac{3\pi}{2}, \frac{\pi}{2}$ **7.** $2x + 7$ **8.** 3

Date:________________ **7.3 Applications of Composite Functions**

1. The function $C(F)=\frac{5}{9}(F-32)$ relates Celsius temperatures, *C*, and Fahrenheit temperatures, *F*.
The function $K(C)=C+273.15$ relates Celsius temperatures, *C*, and Kelvin temperatures, *K*.
a) Use composition of functions to write a function to relate Fahrenheit temperatures to Kelvin temperatures.
b) What temperature is 25°F in Kelvin?

2. An oil tanker in trouble at sea is discharging oil at a constant rate of 500 m^2/min.
a) Express the area, *A*, in square metres, of the oil slick as a function of time, *t*, in minutes.
b) Express the radius, *r*, in metres, of the oil slick as a function of its area, *A*.
c) Determine a formula for $r \circ A$ and explain what it represents.
d) What is the radius of the oil slick 1 h after the spill begins?

3. A certain honeydew melon is approximately spherical and grows so that its volume increases at a rate of 50 cm^3/day (on average).
a) Express the radius, *r*, in centimetres, of the melon as a function of time, *t*, in days.
b) Determine the radius of the melon eight weeks after it begins growing.

4. In a newspaper delivery department, the number of subscribers is *x*, the number of delivery personnel is $p(x)=\frac{x}{45}+1$, and the number of supervisors is approximately $s(p)=\frac{p}{12}$.
a) Express the number of supervisors needed as a function of *x*, the number of subscribers.
b) Approximately how many supervisors are needed if there are 5000 subscribers?

5. A supernova explosion creates a spherical shock wave that travels outward at a speed of approximately 3000 km/s.
a) Express the radius, *r*, in kilometres, of the supernova as a function of time, *t*, in seconds, if the radius of the supernova is 1 000 000 km before it explodes.
b) If *V* is the volume of the supernova as a function of the radius, find $V \circ r$ and explain what it represents.
c) What is the volume of the supernova 15 s after it explodes?

6. The number of bicycles, *n*, sold at one store in a week is a function of the price, *p*, in dollars.
So $n(p)=\frac{5(360-p)}{p-80}$ for $p>80$.
The store's cost, *c*, in dollars for each bike is a function of the number of bikes the store sells each week. So $c(n)=0.002(n+2)^2+80$.
a) Evaluate $n(100)$ and $n(180)$. Why are these values reasonable in this situation?
b) Evaluate $c(8)$ and $c(48)$. Why are these values reasonable in this situation?
c) Evaluate the cost of each bicycle if the selling price is $120.
d) Determine the store's profit per bicycle if the selling price is $120.
e) Evaluate the total profit if the selling price is $120.
f) Express the cost of each bike to the store as a function of the selling price.
g) Express the total profit in terms of the functions *c* and *n* and the variable *p*.

Answers

1. a) $K(F)=\frac{5}{9}(F-32)+273.15$ **b)** $269.26^{\circ}K$

2. a) $A(t)=500t$ **b)** $r(A)=\sqrt{\frac{A}{\pi}}$ **c)** $r(A(t))=\sqrt{\frac{500t}{\pi}}$ *or* $r(t)=\sqrt{\frac{500t}{\pi}}$ represents radius (m) as a function of time (min) **d)** 97.72 m

3. a) $r(t)=\sqrt[3]{\frac{75t}{2\pi}}$ **b)** 8.74 cm

4. a) $s(x)=\frac{x+45}{540}$ **b)** 10 (you need more than 9)

5. a) $r(t)=3000t+1\,000\,000$ **b)** $V(r(t))=V(t),\ V(t)=\frac{4}{3}\pi(3000t+1\,000\,000)^3$ represents volume (km^3) as a function of time (sec)

c) $4.780\times10^{18}\ km^3$

6. a) $n(100)=65;\ n(180)=9;$ A lower price will result in increased sales.

b) $c(8)=80.2;\ c(48)=85;$ Costs will increase if sales increase.

c) \$82.05
d) \$37.95
e) \$1138.50

f) $c(n(p))=0.002\left[\frac{5(360-p)}{p-80}+2\right]^2+80$ *or* $c(p)=0.002\left[\frac{5(360-p)}{p-80}+2\right]^2+80$

g) $total\ profit=n(p)\left[p-c(n(p))\right]$

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7.4 Applications of Combining Functions

1. In a certain manufacturing process, when the level of production is x units, the cost of production (in dollars) is $C(x) = 3000 + 9x + 0.05x^2$, $1 \le x \le 300$ and the unit cost is $U(x) = \frac{C(x)}{x}$.

a) Write a function for the unit cost in terms of the production level, x units.
b) Determine the unit cost for each of the following production levels:
i) 1 unit **ii)** 244 units **iii)** 245 units **iv)** 246 **v)** 300 units
c) What level of production in **b)** will minimize the unit cost?

2. The number, N, of bacteria in a culture at time t is given by $N(t) = 2000\left[30 + te^{-\frac{t}{20}}\right]$.

If the bacterial culture is placed into a colony of mice, the number of mice, M, that become infected is related to the number of bacteria present by the equation $M(N) = \sqrt[3]{N + 1000}$.
After ten days, how many mice are infected to 1 decimal place?

3. The pH value of a chemical solution measures the acidity or alkalinity of the solution. The formula is $pH = -\log(H)$, where H is the concentration of hydrogen ions in the solution (in moles per litre).
a) Tomatoes have $H = 6.3 \times 10^{-5}$. Find the pH value.
b) Recipe ingredients are being added to a bowl of tomatoes, so that the concentration of hydrogen ions in the whole mixture is given by $H(t) = 29 - 5t - 25\left(e^{-\frac{t}{5}} - 1\right)$ moles per litre, where t is measured in seconds. Determine the pH value after 10 s.

4. A firm can sell x units of a product daily at p dollars per unit, where $p = 1000 - x$.
The cost, C of producing x units per day is $C(x) = 3000 + 20x$.
a) Find the revenue function, $R(x)$.
b) Find the profit function, $P(x)$.
c) What price per unit will maximize profit? What is the maximum daily profit?

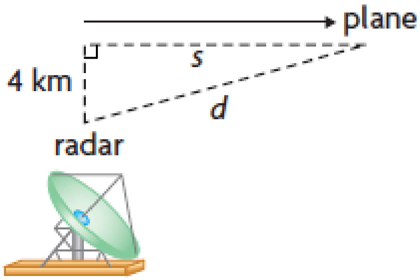
5. A deli owner has found that his revenue from producing x pounds of cream cheese is given by $R(x) = -x^2 + 30x$, while the cost in dollars is given by $C(x) = 5x + 100$.
a) Find the minimum break-even quantity. ***Note:*** $R(x) = C(x)$
b) Find the maximum revenue.
c) Find the maximum profit.

6. A diner is open from 6 a.m. to 6 p.m., and the average number of customers in the diner at any time can be modelled by the function $C(h) = -30\cos\left(\frac{\pi}{6}h\right) + 34$, where h is the number of hours after 6 a.m. opening time. The average amount of money, in dollars, that each customer in the diner will spend can be modelled by the function $D(h) = -3\sin\left(\frac{\pi}{6}h\right) + 7$.
a) Write the function that represents the diner's average revenue from the customers.
b) Graph the function you wrote in part a).
c) What is the average revenue from the customers in the diner at 2 p.m.?

7. An airplane passes directly over a radar station at time $t = 0$. The plane maintains an altitude of 4 km and is flying at a speed of 560 km/h. Let *d* represent the distance from the radar station to the plane, and let *s* represent the horizontal distance travelled by the plane since it passed over the radar station.

a) Express *d* as a function of *s*, and *s* as a function of *t*.

b) Use composition to express the distance between the plane and the radar station as function of time.



8. The cost **,** in thousands of dollars, for a company to produce *x* thousand of its product is given by $C(x) = 10x + 30$. The revenue from the sales of the product is given by the function $R(x) = -5x^2 + 150x$.

a) Write the function that represents the company's profit on sales of *x* thousand of its product.

b) Graph the cost, revenue, and profit functions on the same grid, where $0 \le x \le 40$.

c) What is the company's profit on the sale of 7500 of its product?

9. For a car travelling a constant speed of 80 km/h, the distance driven, *d* kilometres, is represented by $d(t) = 80t$, where *t* is the time in hours. The cost of gasoline, in dollars, for the drive is represented by $C(d) = 0.09d$.

a) Determine $C(d(5))$ and interpret your result.

b) Describe the relationship represented by $C(d(t))$.

Answers

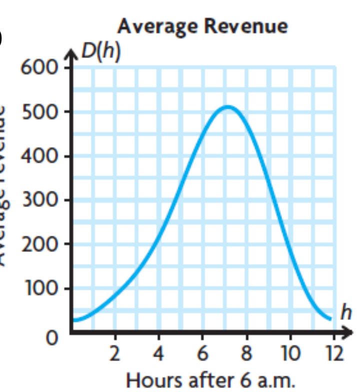
1. a) $U(x) = 3000x^{-1} + 9 + 0.05x$ **b) i)** \$3009.05 **ii)** \$33.50 **iii)** \$33.49 **iv)** \$33.50 **v)** \$34 **c)** 245 units

2. 41.8 **3. a)** 4.2 **b)** 0.2 **4. a)** $R(x) = -x^2 + 1000x$ **b)** $P(x) = -x^2 + 980x - 3000$ **c)** \$510 if $x = 490$, \$237 100

5. a) 5 lb of cream cheese **b)** \$225 **c)** \$56.25 if 12.5 lb of cream cheese is sold

6. a) $R(h) = 45\sin\left(\frac{\pi}{3}h\right) - 102\sin\left(\frac{\pi}{6}h\right) - 210\cos\left(\frac{\pi}{6}h\right) + 238$

b)



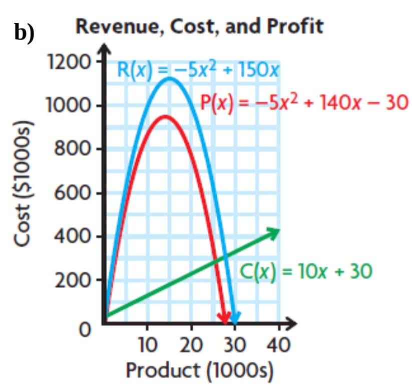
c) about \$470.30

7. a) $d(s) = \sqrt{16 + s^2}$; $s(t) = 560t$

b) $d(s(t)) = \sqrt{16 + 313600t^2}$, where *t* is the time in hours and $d(s(t))$ is the distance in kilometres

8. a) $P(x) = -5x^2 + 140x - 30$

b)



c) \$738.75

9. a) $C(d(5)) = 36$ It costs \$36 to travel for 5 h.

b) $C(d(t))$ represents the relationship between the time driven and the cost of gasoline.

Date:________________ **7.5 Composition Involving Trigonometric Functions**

1. Given $f(x)=2x$, $g(x)=\tan x$ and $h(x)=x^2-x$ find

a) $(h\circ g)\left(\frac{5\pi}{6}\right)$ **b)** $(g\circ f)\left(\frac{\pi}{4}\right)$ **c)** $(h\circ g\circ f)\left(\frac{\pi}{2}\right)$

d) x for $x\in R$, if $\frac{8}{f(x)}=h(x)$

e) x for $0\le x\le 2\pi$, if $(g\circ f)(x)=\sqrt{3}$

f) x for $-2\pi\le x\le 0$, if $(h\circ g)(x)=0$

2. Find the point(s) of intersection of each pair of curves in the interval, algebraically. Illustrate your results graphically.

a) $f(x)=\sin 2x,\ \ g(x)=\sin x,\ \ 0\le x\le 2\pi$

b) $f(x)=\sin 2x,\ \ g(x)=\tan x,\ \ 0\le x\le 2\pi$

c) $f(x)=\cos 2x,\ \ g(x)=\sin x,\ \ -\pi\le x\le \pi$

d) $f(x)=\tan 2x,\ \ g(x)=2\sin x,\ \ -\frac{\pi}{4}\le x\le \frac{3\pi}{4}$

e) $f(x)=\sin 3x,\ \ g(x)=\sin x,\ \ 0\le x\le 2\pi$

Answers

1. a) $\frac{1+\sqrt{3}}{3}$ **b)** d.n.e. **c)** 0 **d)** 2 **e)** $\frac{\pi}{6},\frac{2\pi}{3},\frac{7\pi}{6},\frac{5\pi}{3}$ **f)** $-2\pi,-\frac{7\pi}{4},-\pi,-\frac{3\pi}{4},0$

2. a) $(0,0),\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right),(\pi,0),\left(\frac{5\pi}{3},-\frac{\sqrt{3}}{2}\right),(2\pi,0)$

b) $(0,0),\left(\frac{\pi}{4},1\right),\left(\frac{3\pi}{4},-1\right),(\pi,0),\left(\frac{5\pi}{4},1\right),\left(\frac{7\pi}{4},-1\right),(2\pi,0)$

c) $\left(-\frac{\pi}{2},-1\right),\left(\frac{\pi}{6},\frac{1}{2}\right),\left(\frac{5\pi}{6},\frac{1}{2}\right)$

d) $(0,0),\left(\frac{2\pi}{3},\sqrt{3}\right)$

e) $(0,0),\left(\frac{\pi}{4},\frac{1}{\sqrt{2}}\right),\left(\frac{3\pi}{4},\frac{1}{\sqrt{2}}\right),(\pi,0),\left(\frac{5\pi}{4},-\frac{1}{\sqrt{2}}\right),\left(\frac{7\pi}{4},-\frac{1}{\sqrt{2}}\right),(2\pi,0)$

Date:____________ **7.6 Combining Functions Using Algebraic Operations**

1. Given the functions $f(x)$ and $g(x)$,

i) State the domain of $f(x)$.
ii) State the domain of $g(x)$.
iii) Find $(f+g)(x)$ and state the domain.
iv) Find $(f-g)(x)$ and state the domain.
v) Find $(g-f)(x)$ and state the domain.

a) $f(x)=-3x+2$ and $g(x)=x^3$

b) $f(x)=\frac{1}{x}$ and $g(x)=x^2-3x+2$

c) $f(x)=\sqrt{x}$ and $g(x)=x^2+1$

d) $f(x)=\frac{x+3}{x-3}$ and $g(x)=\frac{x-1}{x^2-3x-10}$

2. Given the functions $f(x)$ and $g(x)$,

i) State the domain of $f(x)$.
ii) State the domain of $g(x)$.
iii) Find $(fg)(x)$ and state the domain.
iv) Find $\left(\frac{f}{g}\right)(x)$ and state the domain.
v) Find $\left(\frac{g}{f}\right)(x)$ and state the domain.

a) $f(x)=4x+8$ and $g(x)=(x+2)(x-4)(x+3)$

b) $f(x)=\sqrt{x^2-1}$ and $g(x)=\sqrt{x+1}$

c) $f(x)=\frac{x+6}{x+2}$ and $g(x)=\frac{x^2-9}{x^2+8x+12}$

Answers

1. a) $D_f=\{x\in R\}$; $D_g=\{x\in R\}$; $(f+g)(x)=x^3-3x+2$, $D_{f+g}=\{x\in R\}$;
$(f-g)(x)=-x^3-3x+2$, $D_{f-g}=\{x\in R\}$; $(g-f)(x)=x^3+3x-2$, $D_{g-f}=\{x\in R\}$

b) $D_f=\{x\in R \mid x\neq 0\}$; $D_g=\{x\in R\}$; $(f+g)(x)=x^2-3x+2+\frac{1}{x}$, $D_{f+g}=\{x\in R \mid x\neq 0\}$;
$(f-g)(x)=-x^2+3x-2+\frac{1}{x}$, $D_{f-g}=\{x\in R \mid x\neq 0\}$; $(g-f)(x)=x^2-3x+2-\frac{1}{x}$, $D_{g-f}=\{x\in R \mid x\neq 0\}$

c) $D_f=\{x\in R \mid x\geq 0\}$; $D_g=\{x\in R\}$; $(f+g)(x)=\sqrt{x}+x^2+1$, $D_{f+g}=\{x\in R \mid x\geq 0\}$;
$(f-g)(x)=\sqrt{x}-x^2-1$, $D_{f-g}=\{x\in R \mid x\geq 0\}$; $(g-f)(x)=x^2+1-\sqrt{x}$, $D_{g-f}=\{x\in R \mid x\geq 0\}$

d) $D_f=\{x\in R \mid x\neq 3\}$; $D_g=\{x\in R \mid x\neq -2,5\}$; $(f+g)(x)=\frac{x+3}{x-3}+\frac{x-1}{(x-5)(x+2)}$, $D_{f+g}=\{x\in R \mid x\neq -2,3,5\}$;
$(f-g)(x)=\frac{x+3}{x-3}-\frac{x-1}{(x-5)(x+2)}$, $D_{f-g}=\{x\in R \mid x\neq -2,3,5\}$; $(g-f)(x)=\frac{x-1}{(x-5)(x+2)}-\frac{x+3}{x-3}$, $D_{g-f}=\{x\in R \mid x\neq -2,3,5\}$

2. a) $D_f=\{x\in R\}$; $D_g=\{x\in R\}$; $(fg)(x)=4(x+2)^2(x-4)(x+3)$, $D_{fg}=\{x\in R\}$;
$\left(\frac{f}{g}\right)(x)=\frac{4}{(x-4)(x+3)}$, $D_{\frac{f}{g}}=\{x\in R \mid x\neq -3,-2,4\}$; $\left(\frac{g}{f}\right)(x)=\frac{(x-4)(x+3)}{4}$, $D_{\frac{g}{f}}=\{x\in R \mid x\neq -2\}$

b) $D_f=\{x\in R \mid x\leq -1 \text{ or } x\geq 1\}$; $D_g=\{x\in R \mid x\geq -1\}$; $(fg)(x)=\sqrt{(x-1)(x+1)^2}$, $D_{fg}=\{x\in R \mid x=-1 \text{ or } x\geq 1\}$;
$\left(\frac{f}{g}\right)(x)=\sqrt{x-1}$, $D_{\frac{f}{g}}=\{x\in R \mid x\geq 1\}$; $\left(\frac{g}{f}\right)(x)=\frac{1}{\sqrt{x-1}}$, $D_{\frac{g}{f}}=\{x\in R \mid x>1\}$

c) $D_f=\{x\in R \mid x\neq -2\}$; $D_g=\{x\in R \mid x\neq -6,-2\}$; $(fg)(x)=\frac{(x-3)(x+3)}{(x+2)^2}$, $D_{fg}=\{x\in R \mid x\neq -6,-2\}$;
$\left(\frac{f}{g}\right)(x)=\frac{(x+6)^2}{(x-3)(x+3)}$, $D_{\frac{f}{g}}=\{x\in R \mid x\neq -6,-3,-2,3\}$; $\left(\frac{g}{f}\right)(x)=\frac{(x-3)(x+3)}{(x+6)^2}$, $D_{\frac{g}{f}}=\{x\in R \mid x\neq -6,-2\}$

Date:____________ 7.7 Combining Functions Using Algebraic Operations continued

1. Given the functions $f(x)=3(2)^x-1$ and $g(x)=-(2)^{-x-1}+4$,
a) State the domain of $f(x)$. **b)** State the domain of $g(x)$.
c) Determine $(f+g)(x)$ and state the domain. **d)** Find $(f+g)(0)$.
e) Determine $(f-g)(x)$ and state the domain. **f)** Find $(f-g)(-1)$.

2. Given the functions $f(x)=8(3)^{2x+1}$ and $g(x)=2(3)^{-x-1}$,
a) State the domain of $f(x)$. **b)** State the domain of $g(x)$.
c) Determine $(fg)(x)$ and state the domain. **d)** Find $(fg)(2)$.
e) Determine $\left(\frac{f}{g}\right)(x)$ and state the domain. **f)** Find $\left(\frac{f}{g}\right)(-1)$.
g) Determine $\left(\frac{g}{f}\right)(x)$ and state the domain. **h)** Find $\left(\frac{g}{f}\right)(0)$.

3. Given the functions $f(x)=\log_2(x+2)$ and $g(x)=\log_2(x^2-4)$,
a) State the domain of $f(x)$. **b)** State the domain of $g(x)$.
c) Determine $(f+g)(x)$ and state the domain. **d)** Find $(f+g)(6)$.
e) Determine $(f-g)(x)$ and state the domain. **f)** Find $(f-g)(10)$.
g) Determine $(g-f)(x)$ and state the domain.
h) Find the exact value of $(g-f)(5)$. Then, find an approximate value to 2 decimal places.

4. Given the functions $f(x)=\log_3(x+4)$ and $g(x)=\log_3(6-x)$,
a) State the domain of $f(x)$. **b)** State the domain of $g(x)$.
c) Determine $(fg)(x)$ and state the domain. **d)** Find $(fg)(-1)$ exactly and approximately.
e) Determine $\left(\frac{f}{g}\right)(x)$ and state the domain. **f)** Find $\left(\frac{f}{g}\right)(2)$ exactly and approximately.
g) Determine $\left(\frac{g}{f}\right)(x)$ and state the domain. **h)** Find $\left(\frac{g}{f}\right)(-1)$ exactly and approximately.

1. a) $D_f=\{x\in R\}$ **b)** $D_g=\{x\in R\}$ **c)** $(f+g)(x)=3(2)^x-(2)^{-x-1}+3$, $D_{f+g}=\{x\in R\}$
d) $(f+g)(0)=\frac{11}{2}$ **e)** $(f-g)(x)=3(2)^x+(2)^{-x-1}-5$, $D_{f-g}=\{x\in R\}$ **f)** $(f-g)(-1)=-\frac{5}{2}$

2. a) $D_f=\{x\in R\}$ **b)** $D_g=\{x\in R\}$ **c)** $(fg)(x)=16(3)^x$, $D_{fg}=\{x\in R\}$ **d)** $(fg)(2)=144$
e) $\left(\frac{f}{g}\right)(x)=4(3)^{3x+2}$, $D_{\frac{f}{g}}=\{x\in R\}$ **f)** $\left(\frac{f}{g}\right)(-1)=\frac{4}{3}$ **g)** $\left(\frac{g}{f}\right)(x)=\frac{1}{4}(3)^{-3x-2}$, $D_{\frac{g}{f}}=\{x\in R\}$ **h)** $\left(\frac{g}{f}\right)(0)=\frac{1}{36}$

3. a) $D_f=\{x\in R\,|\,x>-2\}$ **b)** $D_g=\{x\in R\,|\,x<-2 \text{ or } x>2\}$ **c)** $(f+g)(x)=\log_2\left[(x+2)^2(x-2)\right]$, $D_{f+g}=\{x\in R\,|\,x>2\}$
d) $(f+g)(6)=8$ **e)** $(f-g)(x)=-\log_2(x-2)$, $D_{f-g}=\{x\in R\,|\,x>2\}$ **f)** $(f-g)(10)=-3$
g) $(g-f)(x)=\log_2(x-2)$, $D_{f-g}=\{x\in R\,|\,x>2\}$ **h)** $(g-f)(5)=\log_2 3=\frac{\log 3}{\log 2}\doteq 1.58$

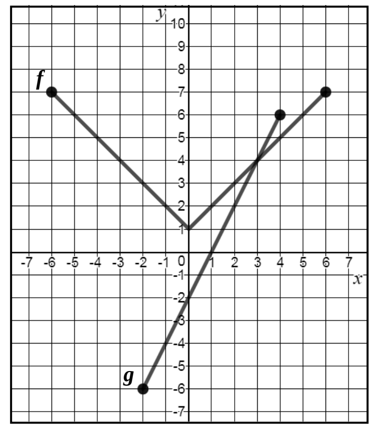
4. a) $D_f=\{x\in R\,|\,x>-4\}$ **b)** $D_g=\{x\in R\,|\,x<6\}$ **c)** $(fg)(x)=\log_3(x+4)\log_3(6-x)$, $D_{fg}=\{x\in R\,|\,-4<x<6\}$
d) $(fg)(-1)=\log_3 7=\frac{\log 7}{\log 3}\doteq 1.77$ **e)** $\left(\frac{f}{g}\right)(x)=\frac{\log_3(x+4)}{\log_3(6-x)}$, $D_{\frac{f}{g}}=\{x\in R\,|\,-4<x<6, x\neq 5\}$ **f)** $\left(\frac{f}{g}\right)(2)=\log_4 6=\frac{\log 6}{\log 4}\doteq 1.29$
g) $\left(\frac{g}{f}\right)(x)=\frac{\log_3(6-x)}{\log_3(x+4)}$, $D_{\frac{g}{f}}=\{x\in R\,|\,-4<x<6, x\neq -3\}$ **h)** $\left(\frac{g}{f}\right)(-1)=\log_3 7=\frac{\log 7}{\log 3}\doteq 1.77$

Date:_______________ **Unit 7 Test Review**

1. Given $f=\{(0,4),(1,2),(2,1),(4,5)\}$ and $g=\{(0,0),(1,1),(4,2),(9,3)\}$, determine each of the following, if possible.

a) $(f\circ f)(0)$ **b)** $(g-f)(1)$ **c)** $(f\circ g)(4)$ **d)** g^{-1} **e)** $(g^{-1}\circ g)(3)$ **f)** $f\circ g$

2. Use the graphs of f and g to determine each of the following, if possible.



a) $(g\circ f)(-2)$ **b)** $(f\circ g)(-2)$ **c)** $(g\circ g)(0)$

d) $(fg)(-1)$ **e)** $\left(\frac{f}{g}\right)(1)$ **f)** $(f+g)(3)$

g) D_{f-g} **h)** the ***graph*** of $h=f-g$

i) the equation of h expressed as a ***piecewise*** function

3. Given $f(x)=\frac{1}{x-1}$, $g(x)=6x^2-x-1$ and $h(x)=\sin x$, find

a) $f^{-1}(x)$ and show that $(f^{-1}\circ f)(x)=x$ **b)** $(g\circ h)\left(\frac{11\pi}{6}\right)$

c) $(f\circ h)\left(-\frac{3\pi}{2}\right)$ **d)** x, if $(g\circ f)(x)=0$

e) x for $-2\pi\le x\le 0$**,** if $(f\circ h)(x)=-2$ **f)** x, if $f(x)=g(x)$

g) x for $0\le x\le 2\pi$, if $(g\circ h)(x)=0$ **h)** $(h\circ f)\left(\frac{12}{\pi}+1\right)$

4. Given $f(x)=x^2-x$, $g(x)=x^2-8x+12$, $h(x)=\sqrt{x-2}$, $p(x)=2^x$ and $q(x)=\log_2 x$, find

a) $h^{-1}(x)$ and its domain **b)** $(q\circ g)(2)$ **c)** $(q\circ h)(34)$ **d)** $(f\circ p\circ q)(3)$

e) $(q\circ g)(x)$ and its domain **f)** x, if $(f\circ h)(x)=2$ **g)** x, if $(q\circ g)(x)=4$ **h)** x, if $(g\circ q)(x)=0$

i) x, if $(f\circ p)(x)=0$ **j)** $r(x)=(q\circ h)(x)$ **k)** $r^{-1}(x)=(q\circ h)^{-1}(x)$

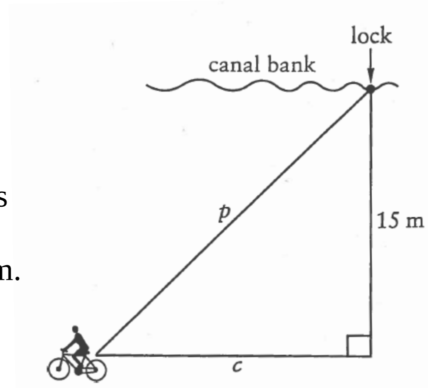
l) graph $r(x)=(q\circ h)(x)$ using transformations on $q(x)=\log_2 x$

5. Given $f(x)=\ln x$, $g(x)=x^2$, $h(x)=\sqrt{x-4}$ and $p(x)=e^{2x}$, find

a) $(f\circ p)(x)$ **b)** $(f\circ h)(5)$ **c)** $(g\circ h)(0)$

d) $q(x)=(f\circ g)(x)$ and its domain**,** then graph **e)** $r(x)=(h\circ g)(x)$ and its domain**,** then graph

6. A scenic bicycle path is parallel to a canal. The canal and path are 15 m apart. A cyclist travelling at 15 km/h passes a lock on the canal at 1:00 p.m.



a) Express the distance, p, in metres, between the lock and the cyclist as a function of c, the distance, in metres, the cyclist has travelled since 1:00 p.m.

b) Express c as a function of the time, t, in minutes, since 1:00 p.m. **Hint:** 15 km/h is ? m/min

c) Find $p\circ c$. What does this function represent?

d) How far from the lock will the cyclist be 10 s after 1:00 p.m.?

7. A sphere has radius r.

a) Write a function for the sphere's surface area in terms of r.

b) Write a function for the radius in terms of the volume, V.

c) Determine $(A \circ r)(V)$.

d) A mother wrapped a ball in wrapping paper and gave it to her son on his birthday. The volume of the ball was 0.75 m^3. Assuming that she used the minimum amount of wrapping paper possible to cover the ball, how much wrapping paper did she use?

8. An environmental scientist measures the pollutant in a lake. The concentration, $C(P)$, in parts per million (ppm), of pollutant can be modeled as a function of the population, P, of the lakeside city, by $C(P) = 1.28P + 53.12$. The city's population, in ten thousands, can be modeled by the function $P(t) = 12.5 \times 2^{\frac{t}{20}}$, where t is the time, in years.

a) Determine an equation for the concentration of pollutant as a function of time.

b) How long will it take for the concentration to reach 100 ppm?

9. A company produces a product for \$9.45 per unit, plus a fixed operating cost of \$52 000. The company sells the product for \$15.80 per unit.

a) Determine a function, $C(n)$, to represent the cost of producing n units.

b) Determine a function, $R(n)$, to represent revenue from sales of n units.

c) Determine a function, $P(n)$ that represents profit.

10. The dances at a community centre produce revenue $R(x) = -60x^2 + 600x$, where x is the ticket price in dollars. It is found that the cost of organizing an event varies as $C(x) = 1000 - 90x$. Determine the profit function, $P(x)$, and use it to determine the maximum possible profit.

11. Find the points of intersection of the curves $f(x) = \cos x$ and $g(x) = \cos 3x$ for $0 \le x \le 2\pi$ algebraically. Illustrate your solution graphically.

12. Given $h(x) = \dfrac{1}{\sin x} - 2\sin x$, find two ***simplified*** functions $f(x)$ and $g(x)$ such that,

a) $h(x) = (g \circ f)(x)$ **b)** $h(x) = f(x) - g(x)$ **c)** $h(x) = \dfrac{g(x)}{f(x)}$

13. Given $f(x) = x^2 - 3x + 2$, $g(x) = x - 1$ and $h(x) = \sqrt{x+4}$ find:

a) functions $p(x)$ and $q(x)$ such that **i)** $h(x) = (p \circ q)(x)$ **ii)** $f(x) = (p \circ q)(x)$

c) $(f - g)\left(\dfrac{1}{2}\right)$ **d)** $(gh)(4)$ **e)** $\left(\dfrac{g}{h}\right)(-4)$

f) the domains, equations and graphs of : **i)** $(fg)(x)$ **ii)** $\left(\dfrac{f}{g}\right)(x)$ **iii)** $\left(\dfrac{g}{f}\right)(x)$

14. The graph of the function $f(x)$ is a line passing through the point $(2, -3)$ with a slope of -2. The graph of the function $g(x)$ is the graph of the function $y = \dfrac{1}{x}$ vertically reflected across the x-axis, vertically stretched by a factor of 5, horizontally compressed by a factor of $\frac{1}{2}$, horizontally translated 3 units to the left, and vertically translated 1 unit up.

a) Determine ***simplified*** equations for: **i)** $f(x)$ **ii)** $g(x)$ **iii)** $(f \times g)(x)$

b) Solve $(f \times g)(x) \ge 0$ for x using the ***number line strategy***. Answer using ***interval notation***.

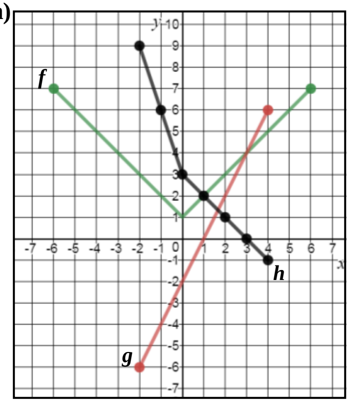
15. If $u(x) = ax - 6$ and $v(x) = b^x$, where $b > 1$, find values for a and b if $\left(\frac{u}{v}\right)(-1) = -10$ and $\left(\frac{u}{v}\right)(2) = -2$.

16. If $f(x) = px + q$ and $(f \circ f \circ f)(x) = 8x + 21$, where p and q are real numbers, find values for p and q.

Answers

1. a) 5 **b)** -1 **c)** 1 **d)** $g^{-1} = \{(0,0), (1,1), (2,4), (3,9)\}$ **e)** undefined **f)** $f \circ g = \{(0,4), (1,2), (4,1)\}$

2. a) 4 **b)** 7 **c)** -6
d) -8 **e)** undefined **f)** 8
g) $D_{f-g} = \{x \in R \mid -2 \le x \le 4\}$
h)



i) $h(x) = \begin{cases} -3x + 3 \ \ if \ -2 \le x \le 0 \\ -x + 3 \ \ if \ 0 < x \le 4 \end{cases}$

3. a) $f^{-1}(x) = \frac{1+x}{x}$ **b)** 1 **c)** d.n.e. **d)** -2, 3 **e)** $-\frac{7\pi}{6}, -\frac{11\pi}{6}$
f) $0, \frac{7}{6}$ **g)** $3.48, 5.94, \frac{\pi}{6}, \frac{5\pi}{6}$ **h)** $\frac{\sqrt{6} - \sqrt{2}}{4}$

4. a) $h^{-1}(x) = x^2 + 2,\ D_{h^{-1}} = \{x \in R \mid x \ge 0\}$ **b)** d.n.e. **c)** $\frac{5}{2}$ **d)** 6
e) $(q \circ g)(x) = \log_2(x^2 - 8x + 12),\ D_{q \circ g} = \{x \mid x \in R, x < 2, x > 6\}$ **f)** 6
g) $4 \pm 2\sqrt{5}$ **h)** 4, 64 **i)** 0 **j)** $r(x) = \frac{1}{2}\log_2(x-2)$
k) $r^{-1}(x) = 2^{2x} + 2 \ or \ r^{-1}(x) = 4^x + 2$

5. a) $2x$ **b)** 0 **c)** d.n.e. **d)** $q(x) = \ln x^2,\ D_q = \{x \in R \mid x \ne 0\}$ **e)** $r(x) = \sqrt{x^2 - 4},\ D_r = \{x \in R \mid x \le -2 \ or \ x \ge 2\}$

6. a) $p(c) = \sqrt{c^2 + 225}$ **b)** $c(t) = 250t$ **c)** $p \circ t$ is $p(t) = \sqrt{62500t^2 + 225}$ **d)** 44.3 m

7. a) $A(r) = 4\pi r^2$ **b)** $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$ **c)** $A(r(V)) = 4\pi\left(\frac{3V}{4\pi}\right)^{\frac{2}{3}}$ **d)** $4\pi\left(\frac{3(0.75)}{4\pi}\right)^{\frac{2}{3}} \doteq 4\ m^2$

8. a) $C(t) = 16 \times 2^{\frac{t}{20}} + 53.12$ **b)** approximately 31 years

9. a) $C(n) = 9.45n + 52\,000$ **b)** $R(n) = 15.8n$ **c)** $P(n) = 6.35n - 52\,000$

10. $P(x) = -60x^2 + 690x - 1000$; \$983.75

11. $(0,1), \left(\frac{\pi}{2}, 0\right), (\pi, -1), \left(\frac{3\pi}{2}, 0\right), (2\pi, 1)$

12. a) $f(x) = \sin x,\ g(x) = \frac{1}{x} - 2x$ **b)** $f(x) = \csc x,\ g(x) = 2\sin x$ **c)** $f(x) = \sin x,\ g(x) = \cos 2x$

13. a) i) $p(x) = \sqrt{x}$, $q(x) = x + 4$ **ii)** $p(x) = x(x+1)$, $q(x) = x - 2$ **c)** $\frac{5}{4}$ **d)** $6\sqrt{2}$ **e)** d.n.e

f) i) $D_{fg} = \{x \in R\},\ (fg)(x) = (x-1)^2(x-2)$

ii) $D_{\frac{f}{g}} = \{x \in R \mid x \ne 1\},\ \left(\frac{f}{g}\right)(x) = x - 2 \ with\ a\ hole\ (1, -1)$

iii) $D_{\frac{g}{f}} = \{x \in R \mid x \ne 1, 2\},\ \left(\frac{g}{f}\right)(x) = \frac{1}{x-2} \ with\ a\ hole\ (1, -1)$

14. a) i) $f(x) = -2x + 1$ **ii)** $g(x) = \frac{-5}{2(x+3)} + 1$ **iii)** $(f \times g)(x) = \frac{1 - 4x^2}{2x + 6}$ **b)** $x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \frac{1}{2}\right]$

15. $a = -1, b = 2 \ or \ a = -4 + 2\sqrt{6}, b = -1 + \sqrt{6}$

16. $p = 2, q = 3$