Date:______________

UNIT 7: COMBINING FUNCTIONS

7.1 Composite Functions

Definition of a Composite Function
For two functions *f* and *g*, the composite function $f(g(x))$ is formed by evaluating *f* at *g*. This new ***composite function*** $f \circ g$, is called the **composition of *f* and *g***, and is defined by $(f \circ g)(x) = f(g(x))$. In this situation, the domain of f is the range of *g*, provided that *g* is defined.

Ex. 1. Let $f = \{(3,2), (5,1), (7,4), (9,3), (11,5)\}$ and $g = \{(1,3), (2,5), (3,7), (4,9), (5,6)\}$. Determine

a) $(f \circ g)(3)$ **b)** $(g \circ f)(3)$ **c)** $(f \circ f)(9)$ **d)** $(g \circ g)(3)$

e) $f \circ g$ **e)** $g \circ f$ **f)** $D_{f \circ g}$

Ex. 2. If $f(x) = \sqrt{x}$, $g(x) = x + 5$ and $h(x) = x^2 - \frac{1}{x}$, find each of the following:

a) $(f \circ g)(4)$ **b)** $(g \circ f)(4)$ **c)** $(h \circ f \circ g)(-5)$

d) $(f \circ g)(x)$ **e)** $g \circ f$ **f)** $h \circ g$

Recall: $f(x)=\sqrt{x}$, $g(x)=x+5$, $h(x)=x^2-\frac{1}{x}$ and $x\in R$

g) x, if $(f\circ g)(x)=(g\circ f)(x)$

h) x, if $(h\circ g)(x)=0$

Ex. 3. From the functions listed in the box, find two whose composite function is $h(x)$.

$p(x)=x-3$	$f(x)=x^2$
$g(x)=\sqrt{x}$	$q(x)=x+1$

a) $h(x)=(x+1)^2$

b) $h(x)=\sqrt{x-3}$

c) $h(x)=x^2-6x+9$

Ex. 2. Express *h* as the composition of two functions *f* and *g*, such that $h(x)=f(g(x))$.

a) $h(x)=\sqrt{x^2-4}$

b) $h(x)=2\sin^2 x-3\sin x+1$

c) $h(x)=\frac{1}{1-x^2}$

d) $h(x)=2^{(6x+7)}$

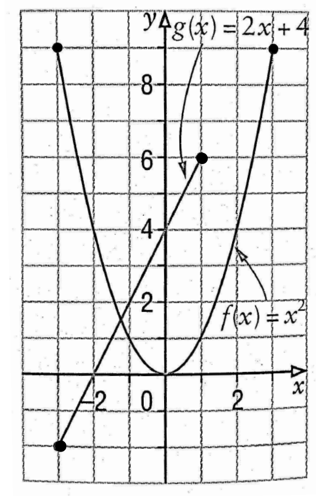
HW. Exercise 7.1

Date:________________ **7.2 Composition of Functions continued**

Definition of a Composite Function

For two functions *f* and *g*, the composite function $f \circ g$ is formed by evaluating *f* at *g*. This new ***composite function*** $f \circ g$, is called the **composition of *f* and *g***, and is defined by $(f \circ g)(x) = f(g(x))$. In this situation, the domain of *f* is the range of *g*, provided that *g* is defined.

Ex. 1. State the domain and range of *f* and *g*. Use the graphs of *f* and *g* to evaluate each expression, if it exists. If it does not exist, explain why.



$D_f =$ ________________________ $D_g =$ ________________________

$R_f =$ ________________________ $R_g =$ ________________________

a) $(g \circ f)(-1)$ **b)** $(g \circ g)(-2)$ **c)** $(f \circ g)(0)$

Ex. 2. If $f(x) = 2x - 1$, $h(x) = x^2 - 2x$, $g(x) = \ln(x-1)$ and $p(x) = \cos x$ determine:

a) $f^{-1}(x)$ **b)** g^{-1}

c) $(f \circ f^{-1})(x)$ **d)** $g^{-1} \circ g$

Recall: $f(x)=2x-1$, $h(x)=x^2-2x$, $g(x)=\ln(x-1)$ and $p(x)=\cos x$

e) $h\circ f$

f) $(h\circ p)(\frac{\pi}{4})$

g) x, if $(h\circ g)(x)=0$

h) x, for $0\le x\le 2\pi$ if $(f\circ p)(x)=-2$

HW. Exercise 7.2

Date:________________ **7.3 Applications of Composite Functions**

Warmup

1. Given $f(x) = \log_3 x$ and $g(x) = x^3 - 5x^2 + 2x + 8$, determine

a) $h(x) = (f \circ g)(x)$ **b)** D_h

2. Given $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x+2}$, determine

a) $h(x) = (g \circ f)(x)$ **b)** D_h

Applications of Composite Functions

Ex. 1. Refrigeration slows down the growth of bacteria in food. The number of bacteria in a certain food is approximated by $B(T) = 15T^2 - 70T + 600$, where T represents the temperature in degrees Celsius and $3 \leq T \leq 12$. Once the food is removed from refrigeration, the temperature, $T(t)$, is given by $T(t) = 3.5t + 3$, where t is the time in hours and $0 \leq t \leq 3$.

a) Write the expression for the number of bacteria in the food, t hours after it is removed from refrigeration.

b) At 1.5 h, about how many bacteria are in the food?

c) When will the bacteria count reach about 1200?

Ex. 2. A circle has radius *r*.

a) Write a function for the circle's area in terms of *r*.

b) Write a function for the radius in terms of the circumference, *C*.

c) Determine $(A \circ r)(C)$.

d) A tree's circumference is 3.6 m. What is the area of the cross section?

Ex. 3. A spherical hailstone grows in a cloud. The hailstone maintains a spherical shape while its radius increases at a rate of 0.2 mm/min.

a) Express the radius, *r*, in millimetres, of the hailstone, as a function of the time, *t*, in minutes.

b) Express the volume, *V*, in cubic millimetres, of the hailstone, in terms of *r*.

c) Determine $(V \circ r)(t)$ and explain what it means.

d) What is the volume of the hailstone 1 h after it begins to form?

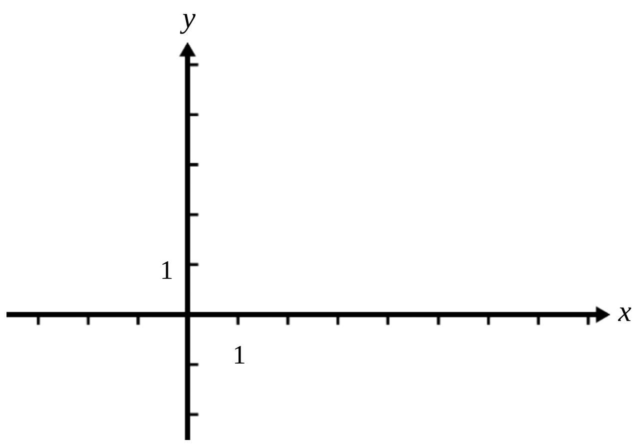
HW. Exercise 7.3

Date:________________ **7.4 Applications of Composite Functions**

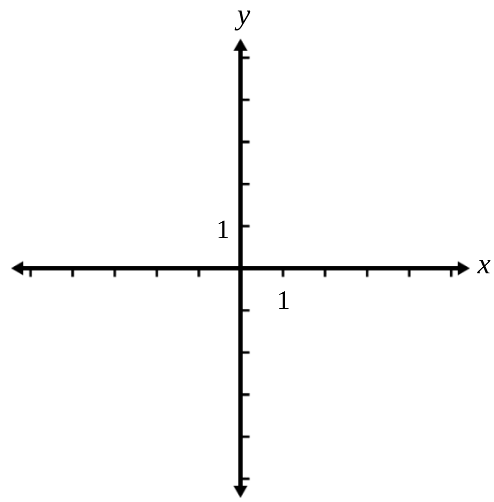
Warmup

Ex. 1. Given $f(x)=\frac{1}{x}$ and $g(x)=\sqrt{x+2}$, find

a) $h(x)=(f\circ g)(x)$, graph and state the domain and range.



b) x, if $f(x)=g(x)$ and illustrate your solution graphically.



Ex. 2. Given $f(x)=\dfrac{x}{x+2}$ and $g(x)=2x+1$, determine:

a) $g^{-1}(x)$

b) $f^{-1}(x)$

c) $(g\circ f)(x)$

d) $(g\circ f)^{-1}(x)$

Ex. 3. The demand function for a new magazine is $p(x)=-6x+40$, where $p(x)$ represents the selling price of the magazine, in thousands of dollars, and x is the number sold, in thousands. The cost function is $C(x)=4x+48$.

a) Determine the revenue function, $R(x)$, and profit function, $P(x)$, in simplified form.

b) Calculate the maximum profit and number of magazines sold that will produce it.

HW. **i)** Using your answers to **Ex. 2.**, show that $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$.

ii) Exercise 7.4

Date:________________ **7.5 Composition Involving Trigonometric Functions**

Ex. 1. Given $f(x)=\frac{1}{x}$, $g(x)=\sin x$ and $h(x)=2x^2-5x+2$ find:

a) x, if $h(x)+f(x)=0$

b) $(g\circ f)\left(-\frac{3}{2\pi}\right)$

c) $(f\circ g)(-\pi)$

d) $(f\circ g)(x)$

e) x, for $0\le x\le 2\pi$ if $(f\circ g)(x)+5=0$

f) x for $0\le x\le 2\pi$, if $(h\circ f\circ g)(x)=0$

Recall: $\sin(A+B) =$

$\cos(A+B) =$

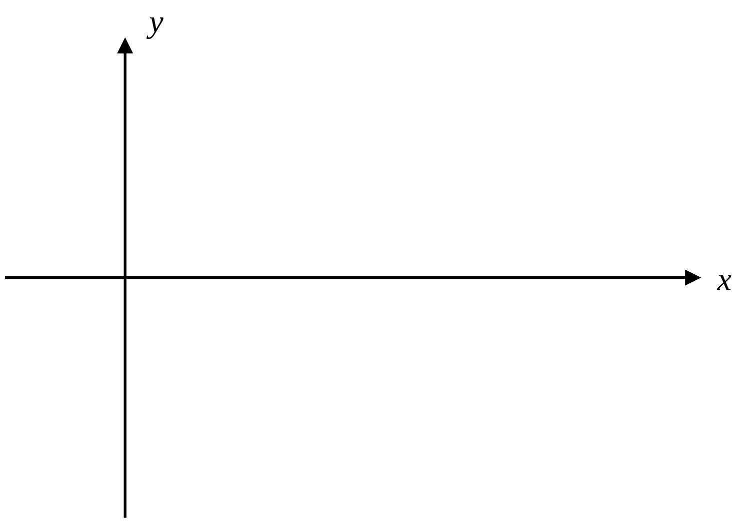
$\sin 2A =$

$\cos 2A =$

$=$

$=$

Ex. 2. Find the points of intersection of the curves $f(x) = \cos x$ and $g(x) = \cos 2x$ for $0 \le x \le 2\pi$.
Illustrate your solution graphically.



HW. Exercise 7.5

Date:____________ **7.6 Combining Functions Using Algebraic Operations**

Two real-valued functions f and g can be combined to form new functions $f+g$, $f-g$, fg and $\dfrac{f}{g}$ in a manner similar to the way we add, subtract, multiply and divide real numbers.

Operations on Functions

Let the domain of f be A and the domain of g be B.

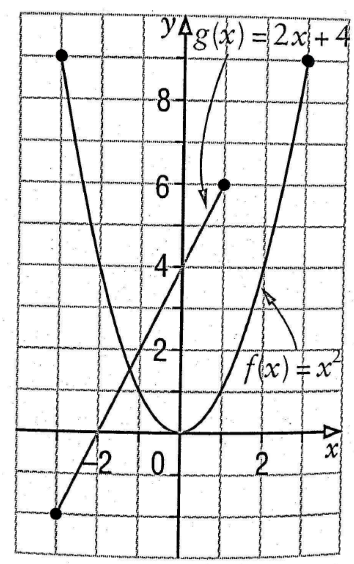
Addition	$(f+g)(x)=f(x)+g(x)$	Domain $=A\cap B$
Subtraction	$(f-g)(x)=f(x)-g(x)$	Domain $=A\cap B$
Multiplication	$(fg)(x)=f(x)g(x)$	Domain $=A\cap B$
Division	$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}$	$\text{D}=\{x\in A\cap B \mid g(x)\neq 0\}$

Ex. 1. Use the graphs of f and g to determine the following.

a) $D_f=\{x\in R \mid -3\le x\le 3\}$, $D_g=\{x\in R \mid -3\le x\le 1\}$

b) $D_{f+g}=$ __________________, $D_{f-g}=\{x\in R \mid -3\le x\le 1\}$

c) $D_{fg}=\{x\in R \mid -3\le x\le 1\}$, $D_{\frac{f}{g}}=$ ____________________



d) $(f+g)(-1)$

e) $(f-g)(-3)$

f) $(fg)(1)$

g) $\left(\dfrac{f}{g}\right)(-2)$

h) the graph and equation of $(f+g)(x)$

x	$f(x)$	$g(x)$	$(f+g)(x)$

Ex. 2. If $f(x) = x + 3$ and $g(x) = x^2 + 8x + 15$ determine each of the following.

a) $(f - g)(-5)$

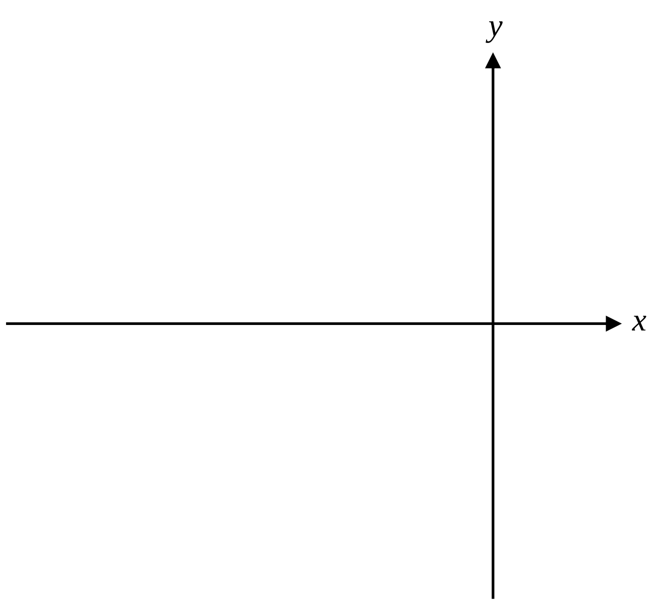
b) $(fg)(0)$

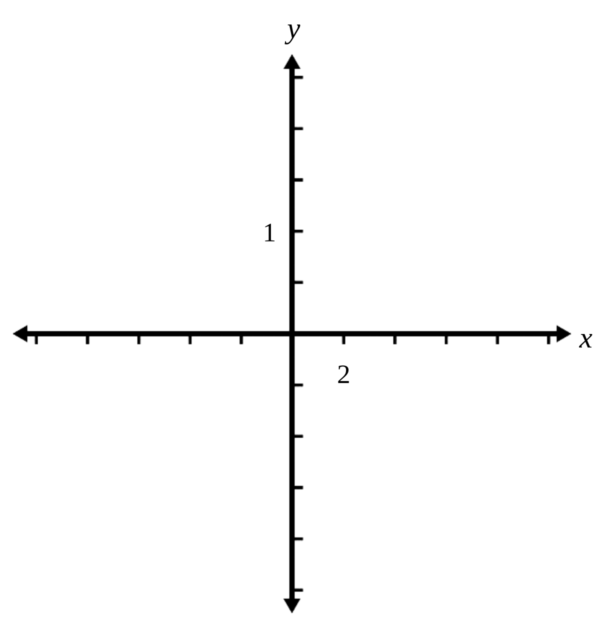
c) the domain of $f + g$

d) the domain of $\frac{f}{g}$

e) the equation and graph of $(fg)(x)$

f) the equation and graph of $\left(\frac{f}{g}\right)(x)$





Ex. 3. If $f(x)=\sqrt{9-x^2}$ and $g(x)=\sqrt{x+3}$, determine the following.

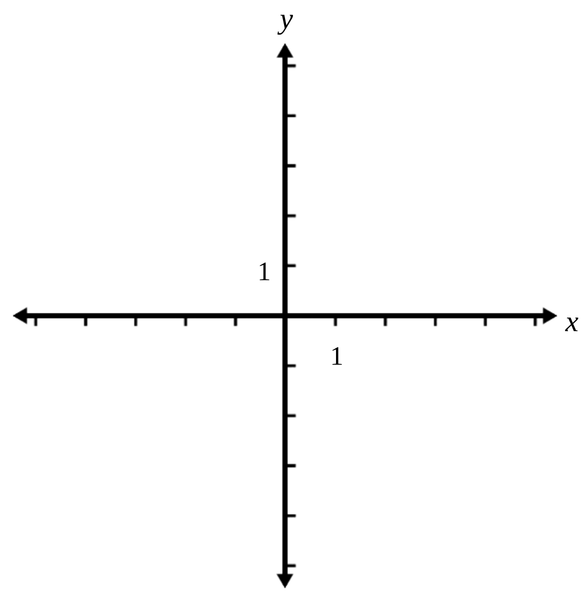
a) $(f+g)(-1)$

b) the domain of g

c) the domain of f

d) the domain of $f-g$

e) the domain, equation and graph of $\left(\frac{f}{g}\right)(x)$



HW. Exercise 7.6

Date:____________ **7.7 Combining Functions Using Algebraic Operations continued**

Two real-valued functions *f* and *g* can be combined to form new functions $f+g$, $f-g$, fg and $\frac{f}{g}$ in a manner similar to the way we add, subtract, multiply and divide real numbers.

Operations on Functions

Let the domain of f be *A* and the domain of *g* be *B*.

Addition	$(f+g)(x)=f(x)+g(x)$	Domain $=A\cap B$
Subtraction	$(f+g)(x)=f(x)+g(x)$	Domain $=A\cap B$
Multiplication	$(fg)(x)=f(x)g(x)$	Domain $=A\cap B$
Division	$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$	D $=\{x\in A\cap B \mid g(x)\neq 0\}$

Ex. 1. If $f(x)=\log_4(x^2-9)$ and $g(x)=\log_4(1-x)$, determine the following.

a) $(f+g)(-7)$

b) $\left(\frac{g}{f}\right)(-4)$

c) the domain of f

d) the domain of g

e) $(g-f)(x)$ and its domain

f) $(f+g)(x)$ and its domain

Ex. 2. If $f(x) = 4^{-x}$ and $g(x) = 3(2)^{5x-1}$, determine the following.

a) $(f - g)(0)$

b) $(f \circ g)(0)$

c) $(fg)(-1)$

d) the domain of fg

e) the domain of $\frac{g}{f}$

c) the equation of $(fg)(x)$

d) the equation of $\left(\frac{g}{f}\right)(x)$

HW. Exercise 7.7
For Unit 7 Test: do Unit 7 Review Exercise