Date:____________ **UNIT 8: SECANTS, TANGENTS & RATES OF CHANGE**

8.1 Review of Prerequisite Skills For Unit 8

1. Find the slope of the line through each pair of points.

a) $(2,7)$ & $(-3,-8)$ **b)** $\left(\frac{1}{2},\frac{3}{2}\right)$ & $\left(\frac{7}{2},-\frac{7}{2}\right)$ **c)** $(6.3,-2.6)$ & $(1.5,-1)$

2. What is the slope of a line perpendicular to the following?

a) $y=3x-5$ **b)** $13x-7y-11=0$

3. State the equation and sketch the graph of the following straight lines:

a) passing through $(-4,-4)$ and $\left(-\frac{5}{3},-\frac{5}{3}\right)$

b) having slope 8 and *y*-intercept 6

c) having *x*-intercept 5 and *y*-intercept -3

d) passing through $(5,6)$ and $(5,-9)$

4. Simplify each of the following:

a) $\frac{(2+h)^2-4}{h}$ **b)** $\frac{(5+h)^3-125}{h}$ **c)** $\frac{(3+h)^4-81}{h}$ **d)** $\frac{\frac{1}{1+h}-1}{h}$

e) $\frac{3(1+h)^2-3}{h}$ **f)** $\frac{(2+h)^3-8}{h}$ **g)** $\frac{\frac{3}{4+h}-\frac{3}{4}}{h}$ **h)** $\frac{\frac{-1}{2+h}+\frac{1}{2}}{h}$

5. Rationalize each of the following numerators to obtain an equivalent expression.

a) $\frac{\sqrt{16+h}-4}{h}$ **b)** $\frac{\sqrt{h^2+5h+4}-2}{h}$ **c)** $\frac{\sqrt{5+h}-\sqrt{5}}{h}$

6. Rationalize each of the following denominators to obtain an equivalent expression.

a) $\frac{x}{2-\sqrt{4+x}}$ **b)** $\frac{x}{\sqrt{x+5}-\sqrt{5-x}}$ **c)** $\frac{2x-4}{\sqrt{7x+2}-\sqrt{6x+4}}$

Answers

1. **a)** 3 **b)** $-\frac{5}{3}$ **c)** $-\frac{1}{3}$

2. **a)** $-\frac{1}{3}$ **b)** $-\frac{7}{13}$

3. **a)** $y=x$ or $x-y=0$ **b)** $y=8x+6$ or $8x-y+6=0$ **c)** $y=\frac{3}{5}x-3$ or $3x-5y-15=0$ **d)** $x=5$ or $x-5=0$

4. **a)** $4+h$ **b)** $75+15h+h^2$ **c)** $108+54h+12h^2+h^3$
d) $\frac{-1}{1+h}$ **e)** $6+3h$ **f)** $12+6h+h^2$ **g)** $\frac{-3}{4(4+h)}$ **h)** $\frac{1}{4+2h}$

5. **a)** $\frac{1}{\sqrt{16+h}+4}$ **b)** $\frac{h+5}{\sqrt{h^2+5h+4}+2}$ **c)** $\frac{1}{\sqrt{5+h}+\sqrt{5}}$

6. **a)** $-2-\sqrt{4+x}$ **b)** $\frac{\sqrt{x+5}+\sqrt{5-x}}{2}$ **c)** $2\left(\sqrt{7x+2}+\sqrt{6x+4}\right)$

Date:________________ **8.2 Slopes of Lines Given Two Points**

1. Determine the slope of the line passing through each of the following pairs of points:

a) $(2,5)$ and $(6,-7)$ **b)** $(-6,-1)$ and $(-5,11)$ **c)** $(-3,6)$ and $(3,2)$

d) through the origin and $(1,4)$ **e)** $(0,1)$ and $(-6,6)$ **f)** $(-2.1,4.41)$ and $(-2,4)$

2. Find the equation of a line determined by the given information.

a) slope 4, y-intercept -2 **b)** slope 0, y-intercept -5

c) through $(-1,6)$ and $(4,12)$ **d)** through $(0,2)$ and $(-1,-4)$

e) slope 7, through $(4,-1)$ **e)** vertical, through $(-3,5)$

3. Find the slope m, in simplified form, of each pair of points.

a) $P(-2,0),\ Q\left(-2+h, 4-(-2+h)^2\right)$ **b)** $P(1,4),\ Q\left(1+h, (1+h)^2-6(1+h)+9\right)$

c) $P(-1,2),\ Q\left(-1+h, \sqrt{3-(-1+h)}\right)$ **d)** $P\left(-3, -\frac{1}{3}\right),\ Q\left(-3+h, \frac{1}{-3+h}\right)$

e) $P(2,5),\ Q\left(2+h, \frac{2(2+h)+1}{(2+h)-1}\right)$ **f)** $P(1,1),\ Q\left(1+h, \frac{1}{\sqrt{1+h}}\right)$

Answers

1. **a)** -3 **b)** 12 **c)** $-\frac{2}{3}$ **d)** 4 **e)** $-\frac{5}{6}$ **f)** $-\frac{41}{10}$

2. **a)** $4x-y-2=0$ or $y=4x-2$ **b)** $y=-5$ or $y+5=0$ **c)** $6x-5y+36=0$ or $y=\frac{6}{5}x+\frac{36}{5}$

d) $6x-y+2=0$ or $y=6x+2$ **e)** $7x-y-29=0$ or $y=7x-29$ **f)** $x+3=0$ or $x=-3$

3. **a)** $4-h$ **b)** $h-4$ **c)** $\frac{-1}{\sqrt{4-h}+2}$ **d)** $\frac{1}{3(h-3)}$ **e)** $\frac{-3}{1+h}$ **f)** $\frac{-1}{\left(\sqrt{1+h}\right)\left(1+\sqrt{1+h}\right)}$

Date:______________

8.3 Slopes of Secants and Tangents
Average and Instantaneous Rates of Change

1. Determine the slope of the line passing through each pairs of points.

a) $(2,4)$ and $(6,8)$ **b)** $(-3,-2)$ and $(1,10)$

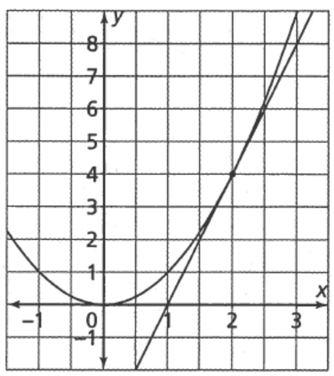
c) $(0,5)$ and $(-5,15)$ **d)** $(-4,-2)$ and $(-9,-17)$

2. Determine the equation of a line that has a slope of 4 and passes through $(3,-8)$.

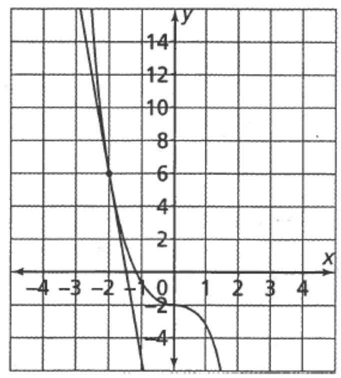
3. Determine the equation of a line that passes through $(6,-1)$ and $(4,-2)$.

4. Estimate the slope of the tangent line in the graph of each function.

a)



b)



5. Find the slope of the secant to the graph of $y = x^3 - 4x$ between the points where $x = 1.5$ and $x = 2.5$.

6. **a)** Sketch the graph of $f(x) = -x^2$ for $-3 \le x \le 3$.

b) Draw the secant line that passes through $(1,-1)$ and $(3,-9)$ and calculate its slope.

c) Draw the tangent line to the curve at $(-2,-4)$ and use the graph to estimate the slope of the tangent.

7. For each curve,

i) find the slope of the tangent at the given point

ii) find the equation of the tangent at the given point

iii) graph the curve and the tangent

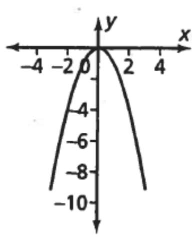
a) $f(x) = x^2 - 6x + 9$ at $P(-1,16)$ **b)** $f(x) = x^3$ at $P(2,8)$

c) $f(x) = \sqrt{x-2}$ at $P(11,3)$ **d)** $f(x) = \frac{1}{x+1}$ at $P\left(-\frac{1}{2}, 2\right)$

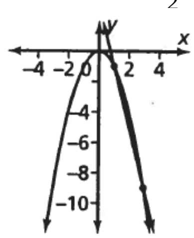
Answers

1. a) 1 **b)** 3 **c)** −2 **d)** 3 **2.** $y = 4x - 20$ **3.** $y = \frac{1}{2}x - 4$ **4. a)** 4 **b)** −12 **5.** 8.25

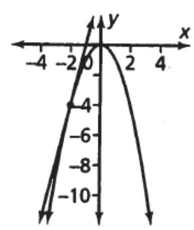
6. a)



b) −4



c) 4



7. a) i) −8 **ii)** $y = -8x + 8$ **b) i)** 12 **ii)** $y = 12x - 16$ **c) i)** $\frac{1}{6}$ **ii)** $y = \frac{1}{6}x + \frac{7}{6}$ **d) i)** −4 **ii)** $y = -4x$

Date:______________ **8.4 Applications of Rates of Change**

1. Find the equation of the tangent of the curve at the given point. Illustrate graphically.

a) $f(x) = x^2 + 1$ at (2, 5)

b) $g(x) = \sqrt{x+3}$ at $x = 6$

c) $f(x) = \dfrac{1}{x-2}$ at $x = 3$

2. If a ball is dropped from the top of a 150-m cliff, then its height after *t* seconds, and before it hits the ground, is $s(t) = 150 - 4.9t^2$.

a) Find the average velocity of the ball for the following time periods.

i) $t \in [2,3]$ **ii)** $t \in [2,2.1]$ **iii)** $t \in [2,2.01]$

b) Find the instantaneous velocity when $t = 2$.

3. A ball of paper falls from the top of the CN Tower. Its height, *h*, above the ground in metres after *t* seconds is given by $s(t) = 603 - \sqrt{t^2 + 9},\ t \geq 0$.

a) Determine the average velocity of the ball during the first four seconds after its release.

b) Determine the initial velocity (at $t = 0$).

4. The radius of a circular juice blot on a piece of paper towel *t* seconds after it was first seen is modelled by $r(t) = \dfrac{1+2t}{1+t}$, where *r* is measured in centimetres. Calculate

a) the radius of the blot when it was first observed

b) the time at which the radius of the blot was 1.5 cm

c) the rate of increase of the radius of the blot when the radius was 1.5 cm

d) if the radius of the blot ever reach 2 cm? Explain your answer.

5. The position function of a cheetah moving across level ground in a straight line chasing after prey is given by $s(t) = t^3 - 6t^2 + 9t$, $t \in [0, 6]$, where *t* is measured in seconds and *s*, in metres.

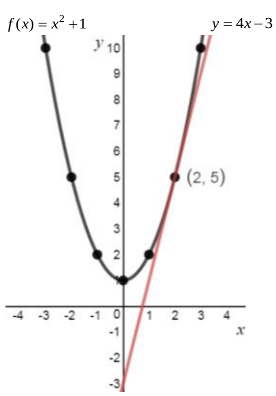
a) Determine the average velocity of the cheetah during each of the first three seconds after it starts chasing after its prey.

b) Determine the total distance travelled by the cheetah during the first 3 seconds.

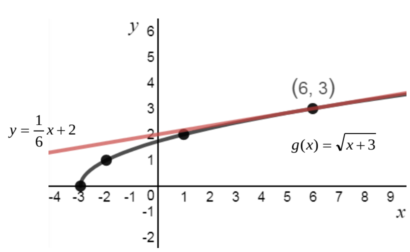
c) Determine the velocity of the cheetah at $t = 3$ seconds.

Answers

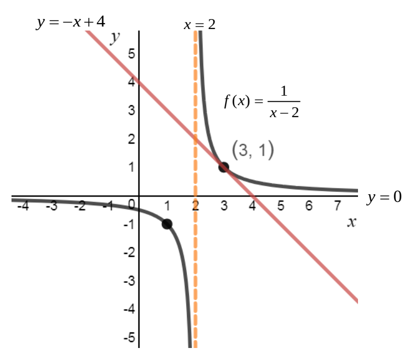
1. a) $y = 4x - 3$



b) $y = \dfrac{1}{6}x + 2$



c) $y = -x + 4$



2. a) i) -24.5 m/s **ii)** -20.09 m/s **iii)** -19.649 m/s **b)** -19.6 m/s **3. a)** -0.5 m/s **b)** 0 m/s

4. a) 1 cm **b)** $t = 1$ s **c)** 0.25 cm/s **d)** no; $r = 2$ is the horizontal asymptote of the graph

5. a) i) 4 m/s **ii)** -2 m/s **iii)** -2 m/s **b)** 8 m **c)** 0 m/s

Date:________________ **8.5 Applications of Rates of Change continued**

1. Find the equation of the tangent of the curve at the given point. Illustrate graphically.

a) $f(x) = x^2 + 4x + 4$ at (1, 9)

b) $f(x) = 1 - x^3$ at $x = 0$

c) $g(x) = \dfrac{1}{\sqrt{x}}$ at $x = 1$

2. A population of raccoons moves into a wooded, urban area. At *t* months, the number of raccoons , *P*, can be modelled by $P(t) = 100 + 30t + 4t^2$.

a) How long does it take for the initial population to double?

b) Determine the rate at which the raccoon population is changing when the initial population has doubled in size.

3. During a chemical reaction, the mass, in grams, of a compound being formed is modelled by the function $M(t) = \dfrac{5.8t}{t + 1.9}$, where *t* is the time after the start of the reaction, in seconds.

a) What is the average rate of change of the mass over the time interval $t \in [0,5]$?

b) What is the rate of change of the mass after 5 s?

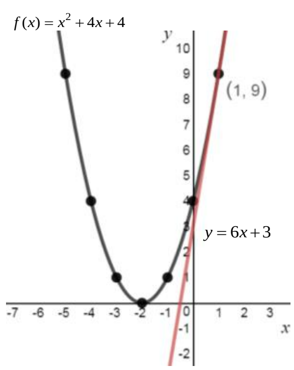
4. A grocery store determines that after *t* hours on the job, a new cashier can ring up $N(t) = 20 - \dfrac{30}{\sqrt{9 + t^2}}$ items per minute.

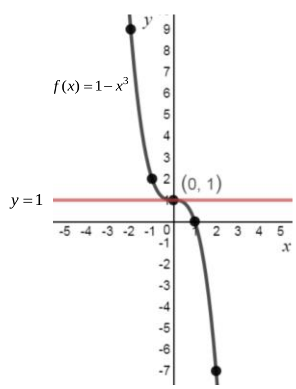
a) Find the average rate at which the cashier's productivity is changing over the first four hours.

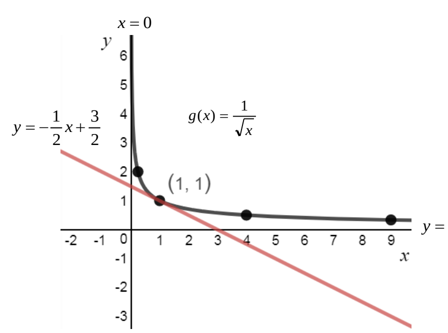
b) Find the rate at which the cashier's productivity is changing after four hours on the job.

Answers

1. a) $y = 6x + 3$ **b)** $y = 1$ **c)** $y = -\dfrac{1}{2}x + \dfrac{3}{2}$







2. a) 2.5 months **b)** 50 raccoons/month

3. a) 0.84 06 g/s **b)** 0.2315 g/s

4. a) 1 item per minute/hour **b)** 0.96 items per minute/hour

Date:____________ **8.6 Expressions For Slopes of Tangents & Rates of Change**

1. Given the function $y=\sqrt{x+2}$, find:

a) an expression for the slope of the tangent to the curve at any point (*x*, *y*).
b) the point on the curve where the tangent has a slope of 1.
c) the equation of the normal to the curve at $x=-1$.
Illustrate your solutions graphically.

2. Given the function $g(x)=\frac{1}{x^2}$, find:

a) an expression for the slope of the tangent to the curve at any point (*x*, *y*).
b) the point on the curve where the tangent has a slope of $\frac{1}{4}$.
c) the equation of the normal to the curve at $x=1$.
Illustrate your solutions graphically.

3. During a hockey game, a forward is skating hard toward the net along the path $y=2x^2-8x+12$. When the forward reaches the point where $x=2.5$, he falls and slides along a tangent to his path. The goal line of the net extends along the *x*-axis between $x=-1$ and $x=1$, and the goaltender is located somewhere along the goal line.

a) Determine an expression for the slope of the tangent to the curve at any point (*x*, *y*).
b) Determine an equation for the tangent the forward slides along after he falls.
c) Should the goaltender take evasive action to avoid being hit by the sliding forward? Explain.
Illustrate your solutions graphically.

4. The position function of a cheetah moving across level ground in a straight line chasing after prey is given by $s(t)=t^3-6t^2+9t$, $t\in[0,6]$, where *t* is measured in seconds and *s*, in metres.

a) Sketch the graph of the function. (Scale the horizontal axis only.)
b) Find an expression for the **velocity** (slope of the tangent) at any time, *t*.
c) When is the cheetah momentarily stopped?
d) What are the positions of the cheetah in part **c)**?
e) Graph the velocity function in part **b)** and use it to determine when the cheetah is moving in:
i) the positive direction **ii)** the negative direction
f) Find the total distance travelled by the cheetah during the 6 seconds it takes to capture his prey.

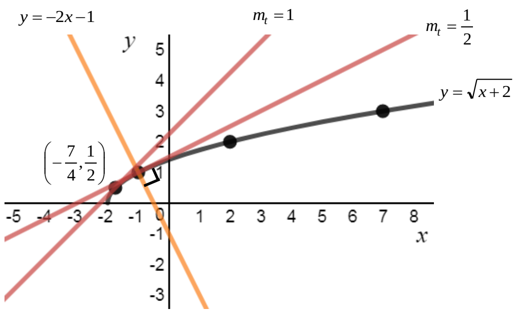
5. A small town committee estimates that, with the introduction of a large new business in the area, the population, *P*, of the town will change with respect to time, *t*, in years, according to the function $P(t)=8000\sqrt{1+t}$.

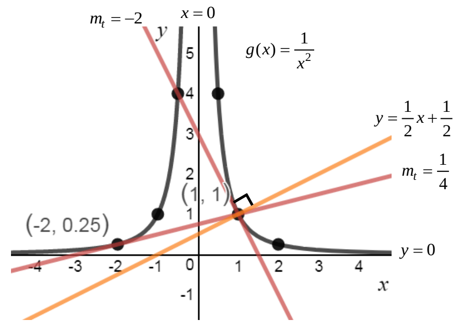
a) What is the population of the town when the new business is introduced? (initial population)
b) Find an expression for the rate at which the population of the town will be changing with respect to time.
c) When will the population of the town be increasing at a rate of 1000 people/year?

Answers

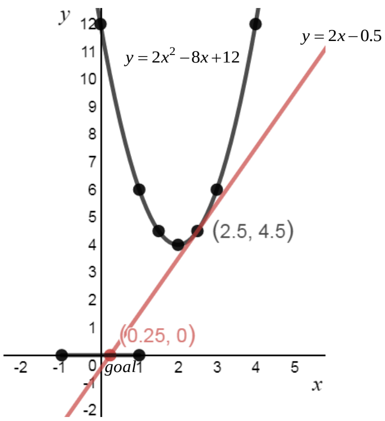
1. a) $m_t = \dfrac{1}{2\sqrt{x+2}}$ **b)** $\left(-\dfrac{7}{4}, \dfrac{1}{2}\right)$ **c)** $y = -2x - 1$

2. a) $m_t = -\dfrac{2}{x^3}$ **b)** $\left(-2, \dfrac{1}{4}\right)$ **c)** $y = \dfrac{1}{2}x + \dfrac{1}{2}$

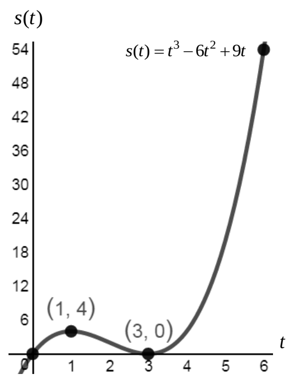




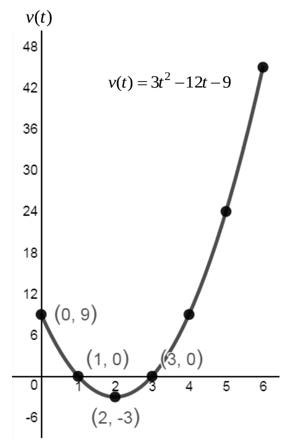
3. a) $m_t = 4x - 8$ **b)** $y = 2x - 0.5$ **c)** *yes*



4. a)



e) i) $t \in [0,1) \cup (3,6]$ **ii)** $t \in (1,3)$



b) $v(t) = 3t^2 - 12t + 9$ **c)** $t = 1, 3$

d) 4 m, 0 m from starting position **f)** 62 m

5. a) 8000 **b)** $\dfrac{4000}{\sqrt{1+t}}$ **c)** 15 years

Date:_______________

Unit 8 Test Review

1. Find the equations of the tangent and normal to the curve at the given point. Illustrate graphically.

a) $f(x)=(x-2)^3$ at $x=1$ **b)** $g(x)=\sqrt{3-x}$ at $x=-1$ **c)** $f(x)=\dfrac{1}{1-x}$ at $x=2$

2. An object moves in a straight line, and its position *s*, in metres after *t* seconds is $s(t)=8-7t+t^2$.

a) Find the ***average*** velocity between $t=2$ and $t=3$.

b) Find the velocity when $t=3$.

3. Suppose the function $V(t)=\dfrac{50\,000+6t}{1+0.4t}$ represents the value, in dollars, of a new car *t* years after it is purchased.

a) What was the initial value of the car?

b) What will the value of the car eventually be according to this model?

c) If the car is sold after ten years, determine the ***average*** rate of change of the value of the car.

d) What is the rate of change of the value of the car at 1 year?

4. The estimated number of houses in a new subdivision after *t* months is given by the function $N(t)=\dfrac{1000t}{14+t}$.

a) Find an expression for the slope of the tangent to the curve at any point in terms of *t*.

b) Use the expression obtained in part **a** to determine when the number of houses in the subdivision will be increasing at a rate of 35 houses/month.

5. Given the function $f(x)=\dfrac{2(x-1)}{\sqrt{x+3}-2}$,

a) simplify the function by rationalizing the denominator and identify any holes.

b) determine the slope of the of the tangent to the curve at $x=-2$ and illustrate graphically.

6. Given the function $f(x)=\dfrac{1}{x+2}\left[\dfrac{1}{(3+x)^2}-\dfrac{1}{3+x}\right]$,

a) determine a simplified rational function of the form $f(x)=\dfrac{p(x)}{q(x)}$ and identify any holes.

b) determine the equation of the tangent to the curve at $x=-5$ and illustrate graphically.

7. Given the function $g(x)=\dfrac{x^3-x^2-8x+12}{x^2-4}$,

a) simplify the function and identify any holes.

b) determine the equations of the tangent and normal to the curve at $x=0$ and illustrate graphically.

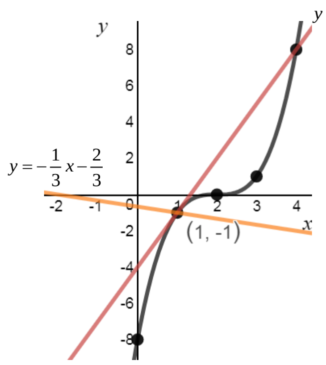
8. Given the function $g(x)=\dfrac{x^3-x^2-2x}{|x+1|}$,

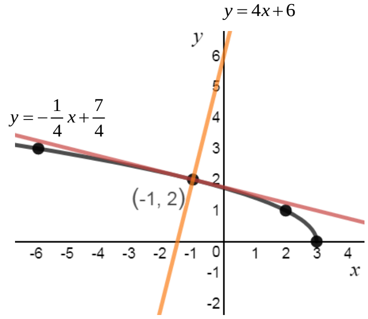
a) rewrite the function as a piecewise function and graph.

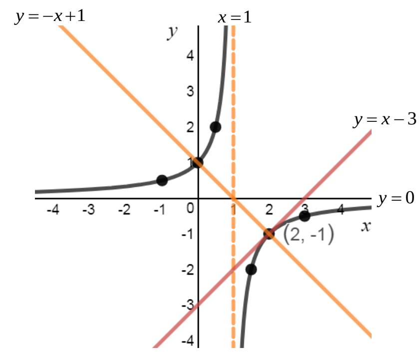
b) determine the point(s) on the graph where the tangent is parallel to the line $9x+3y-2=0$ and illustrate graphically.

Answers

1. a) $y = 3x - 4, \; y = -\frac{1}{3}x - \frac{2}{3}$ **b)** $y = -\frac{1}{4}x + \frac{7}{4}, \; y = 4x + 6$ **c)** $y = x - 3, \; y = -x + 1$







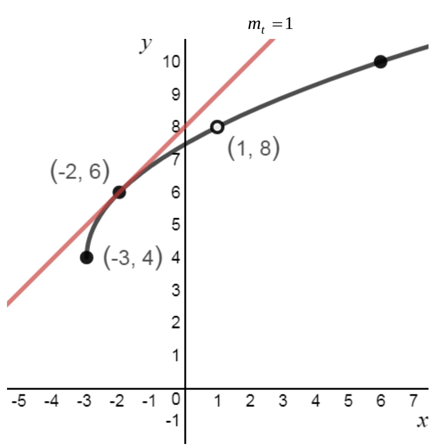
2. a) -2 m/s **b)** -1 m/s

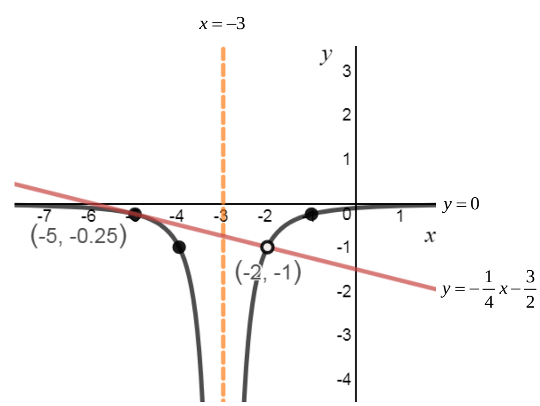
3. a) \$50 000 **b)** \$15 **c)** $-$\$399880/ year **d)** $-$\$10201.02/ year

4. a) $m_t = \frac{14000}{(14+t)^2}$ **b)** after 6 months

5. a) $f(x) = 2\sqrt{x+3} + 4, \; hole\,at\,(1,8)$ **b)** $m_t = 1$

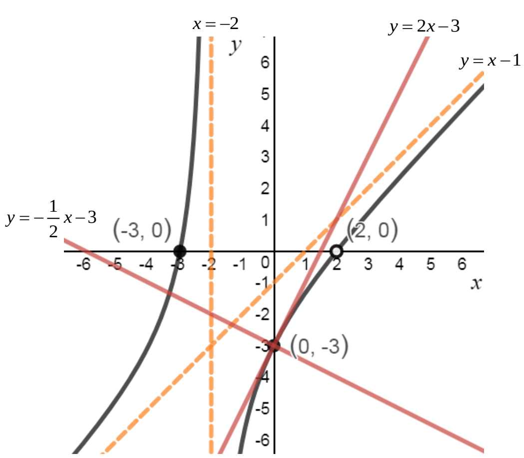
6. a) $f(x) = -\frac{1}{(x+3)^2}, \; hole\,at\,(-2,-1)$ **b)** $y = -\frac{1}{4}x - \frac{3}{2}$

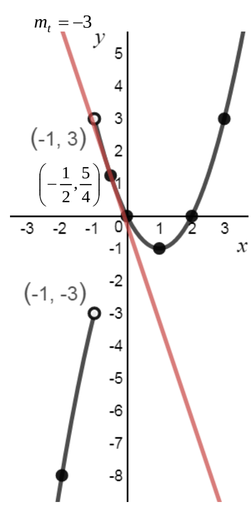




7. a) $g(x) = \frac{x^2 + x - 6}{x+2}, \; hole\,at\,(2,0)$ **b)** $y = 2x - 3, \; y = -\frac{1}{2}x - 3$

8. a) $g(x) = \begin{cases} -x^2 + 2x, \; if\; x < -1 \\ x^2 - 2x, \; if\; x > -1 \end{cases}$ **b)** $\left(-\frac{1}{2}, \frac{5}{4}\right)$





Date:______________ **EXAM REVIEW UNITS 1 & 2**

UNIT 1

1. When a polynomial $f(x)$ is divided by $2x-1$ the quotient is x^2-2x-1 and the remainder is -4. Find $f(x)$ in expanded form.

2. Find an exact value for the remainder when $-6x^3-2x^2+x+1$ is divided by $3x+1$ without using long division.

3. The polynomial px^3-x^2+qx-2 has no remainder when divided by $x-1$ and a remainder of -18 when divided by $x+2$. Determine the values of p and q.

4. If $-\frac{2}{3}$ is one root of $9x^3-3x^2+kx-6=0$, find *k*.

5. Determine the quartic equation in expanded form with roots 0, 0, $1-3i\sqrt{2}$ and $1+3i\sqrt{2}$.

6. Factor completely.

a) x^6-64 **b)** $2x^3-3x^2-5x+6$ **c)** $16x^4-24x^3+2x-3$

7. Solve, $x \in C$.

a) $\frac{3x+1}{x}-\frac{x}{6}=2$ **b)** $x^3-x^2+9x-9=0$ **c)** $x^3-5x+2=0$

d) $4x^4+31x^2-8=0$ **e)** $\left(x+\frac{2}{x}\right)^2+2\left(x+\frac{2}{x}\right)-3=0$

8. Solve each of the following. (Check for ***extraneous*** roots)

a) $\sqrt{3x+1}-\sqrt{x+1}=2$ **b)** $\sqrt{3x-2}-2\sqrt{x}-1=0$

9. State the ***solution set*** for each of the following. (Use cases to solve **b** and **c**)

a) $|7-x|=11$ **b)** $|x-1|<x$ **c)** $|2x+4|\geq 12x$

Answers

1. $f(x)=2x^3-5x^2-3$ **2.** $\frac{2}{3}$ **3.** $p=1,\ q=2$ **4.** $k=-15$ **5.** $x^4-2x^3+19x^2=0$

6. a) $(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$ **b)** $(x-2)(2x+3)(x-1)$ **c)** $(2x-3)(2x+1)(4x^2-2x+1)$

7. a) $3-\sqrt{15},\ 3+\sqrt{15}$ **b)** $1,\ -3i,\ 3i$ **c)** $-1-\sqrt{2},\ -1+\sqrt{2},\ 2$ **d)** $-\frac{1}{2},\ \frac{1}{2},-2i\sqrt{2},\ 2i\sqrt{2}$

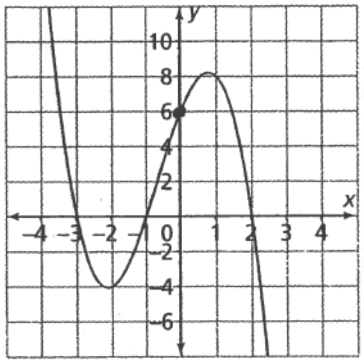
e) $-2,\ -1,\ \frac{1+i\sqrt{7}}{2},\ \frac{1-i\sqrt{7}}{2}$

8. a) $x=8$ **b)** *no solution* **9. a)** $\{-4, 18\}$ **b)** $\left\{x \in R \middle| x>\frac{1}{2}\right\}$ **c)** $\left\{x \in R \middle| x \leq \frac{2}{5}\right\}$

UNIT 2

1. Determine an ***exact*** equation of each function graphed below in ***factored*** form.

a)



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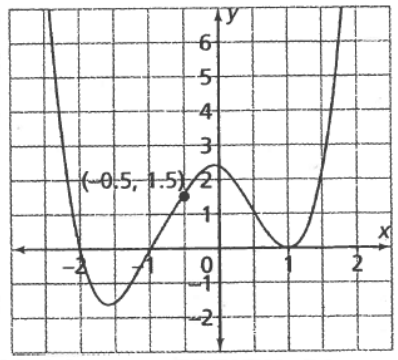


2. Sketch the following functions, clearly labeling all x-intercepts and identifying these intercepts as single, double or triple roots.

a) $f(x)=(x+1)(2-x)(x-4)$ **b)** $f(x)=x^2(x-3)^3$ **c)** $f(x)=(1-x)(x+2)(x+1)^3$

d) $f(x)=-x^4-x^3+6x^2$ **e)** $f(x)=-x^4+5x^2-4$

3. Solve the following inequalities ***graphically*** where $x\in R$. Answer using a ***solution set***.

a) $x^3-3x^2-9x+27\le 0$ **b)** $x^3-x^2-9x+9>0$

4. Solve the following inequalities using a ***number line strategy*** where $x\in R$. State your final answer using ***interval notation***.

a) $(2-x)^3(x+2)^2(1-3x)\ge 0$ **b)** $2x^3-x^2<13x+6$ **c)** $\dfrac{6x^2-5x+1}{2x+1}>0$ **d)** $\dfrac{x-4}{x+1}\le\dfrac{3x-8}{2x-1}$

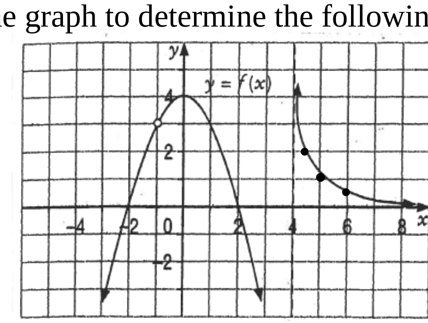
5. Graph the following rational functions by finding and labeling any holes, intercepts, asymptotes and points where the function crosses the horizontal or linear oblique asymptotes. Include a table of values for a more accurate graph.

a) $g(x)=\dfrac{6}{x^2+2x-3}$ **b)** $f(x)=\dfrac{-x^2}{x-2}$ **c)** $f(x)=\dfrac{2x^3-8x}{x^3+2x^2+x+2}$

6. Solve the following inequalities ***graphically***. State your final answer in a ***solution set***.

a) $\dfrac{2x-1}{x+2}\le 3$ **b)** $\dfrac{x^2-x-2}{x-1}>0$

7. The graph of a piecewise function ***f*** is shown. Use the graph to determine the following:



a) $f(1)=$ ______________

b) $f(-1)=$ ______________

c) the equation of this piecewise function

d) the value(s) of x at which the function is discontinuous are _________________

e) as $x\to -1^-$, $f(x)\to$ _____ and as $x\to 4^+$, $f(x)\to$ _____

f) the end behavior of ***f***: as $x\to-\infty$, $f(x)\to$ ______ and as $x\to+\infty$, $f(x)\to$ ______

8. Graph the piecewise function $f(x)=\begin{cases} x+5 & \text{if } x<-3 \\ (x+1)^2 & \text{if } x\geq -3 \end{cases}$ and determine the value(s) of x at which the function is discontinuous.

9. Rewrite the function $f(x)=\dfrac{x^2-1}{|x-1|}$ as a piecewise function.

10. Find constants a and b such that the function $f(x)=\begin{cases} \dfrac{ax-b}{x-2} & \text{if } x\in(-\infty,1] \\ 3x & \text{if } x\in(1,2) \\ bx^2-a & \text{if } x\in[2,\infty) \end{cases}$

is continuous for all $x\in R$.

11. Given $f(x)=-\left|\dfrac{1}{2}x+1\right|+2$,

a) graph the function $y=f(x)$ by naming and applying transformations on an appropriate function.

b) sketch the graph of its reciprocal function $y=\dfrac{1}{f(x)}$ on the same grid.

12. Given $f(x)=2\sqrt{x-1}\;-4$,

a) graph the function $y=f(x)$ by naming and applying transformations on an appropriate function.

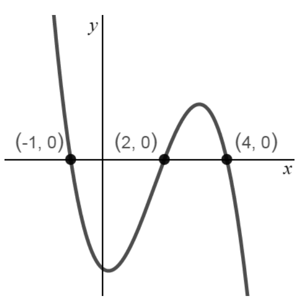
b) sketch the graph of its absolute value function $y=|f(x)|$ on the same grid.

Answers

1. a) $f(x)=-(x+3)(x+1)(x-2)$

b) $f(x)=\dfrac{8}{9}(x+2)(x+1)(x-1)^2$

2. a)



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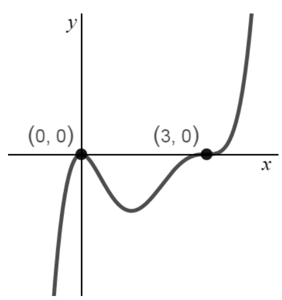

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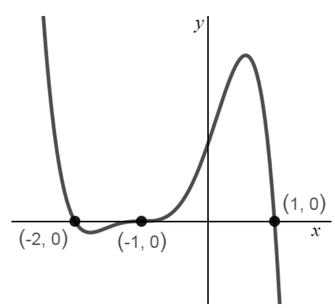

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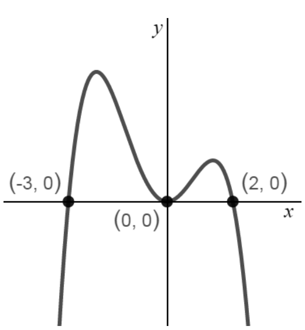

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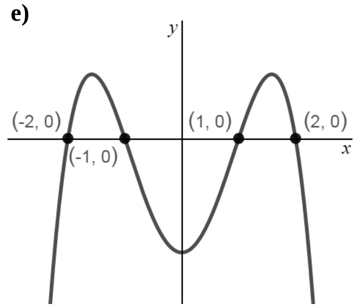
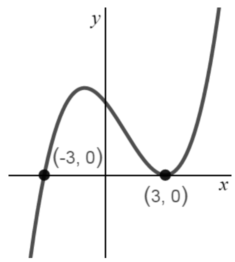
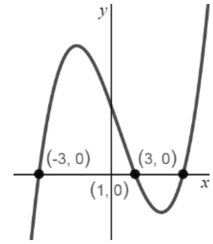


3. a) $\{x \in R \mid x \le -3 \text{ or } x = 3\}$



b) $\{x \in R \mid -3 < x < 1 \text{ or } x > 3\}$



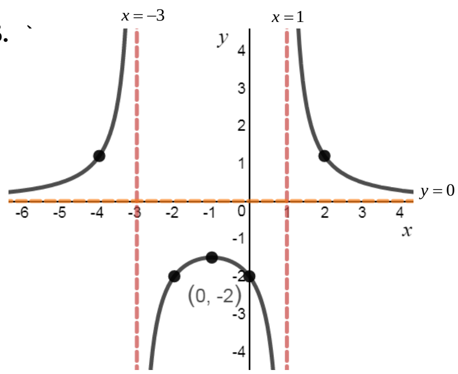
4. a) $x \in \left(-\infty, \frac{1}{3}\right] \cup [2, +\infty)$

b) $x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 3\right)$

c) $x \in \left(-\frac{1}{2}, \frac{1}{3}\right) \cup \left(\frac{1}{2}, +\infty\right)$

d) $x \in (-\infty, -6] \cup \left(-1, \frac{1}{2}\right) \cup [2, +\infty)$

5. `



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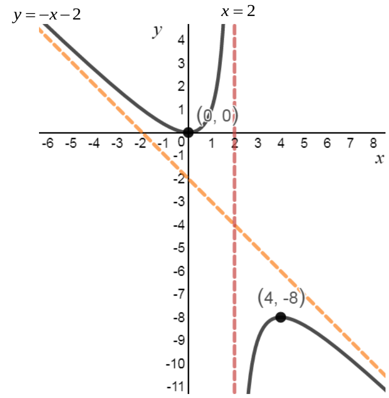

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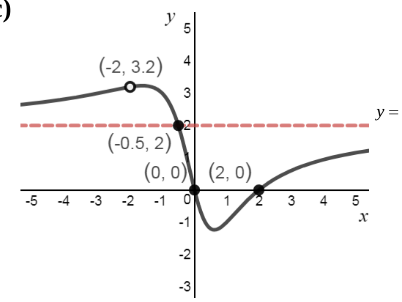
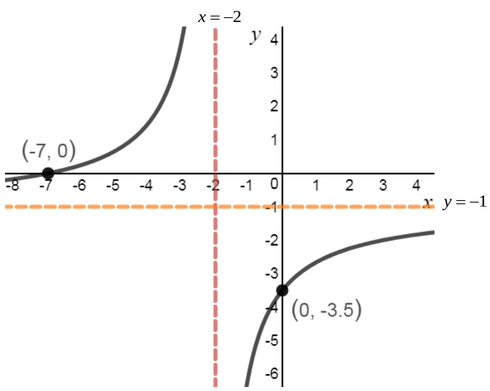
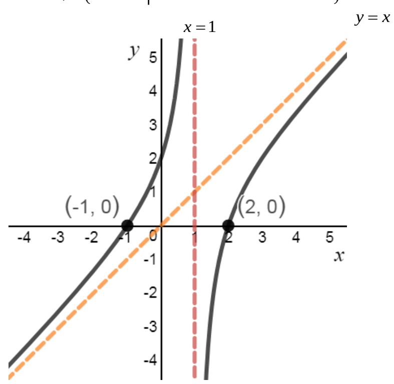


6. a) $\{x \in R \mid x \le -7 \text{ or } x > -2\}$

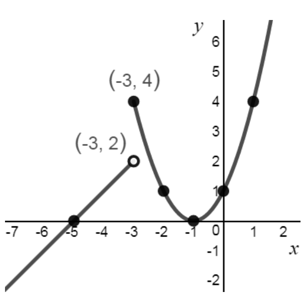


b) $\{x \in R \mid -1 < x < 1 \text{ or } x > 2\}$

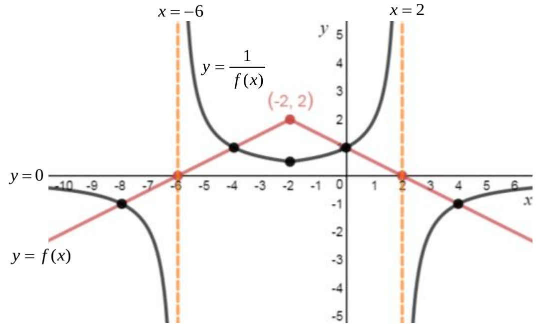


7. a) 3 **b)** dne **c)** $f(x) = \begin{cases} \dfrac{-x^3 - x^2 + 4x + 4}{x+1}, \text{ if } x < 4 \\ \dfrac{1}{x-4} \quad ,\text{if } x > 4 \end{cases}$ **d)** $x = -1,\ x = 4$ **e)** $3,\ +\infty$ **f)** $-\infty,\ 0$

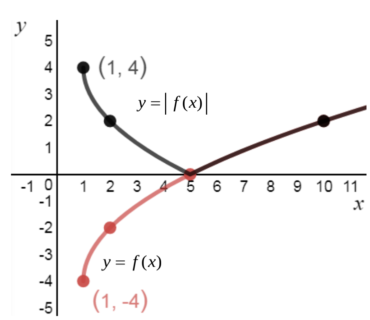
8. a) $x = -3$



11.



12.



9. $f(x) = \begin{cases} -x - 1, \text{ if } x < 1 \\ +x + 1, \text{ if } x > 1 \end{cases}$

10. $a = -2,\ b = 1$

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EXAM REVIEW UNITS 3 & 4

1. Write out the *addition*, *subtraction*, and *double-angle formulas* for *sine, cosine*, and *tangent*.

2. **a)** Graph $y = \sin\theta$ and its ***reciprocal*** function on the same grid for $-2\pi \le \theta \le 2\pi$.
b) Graph $y = \cos\theta$ and its ***reciprocal*** function on the same grid for $-2\pi \le \theta \le 2\pi$.
c) Graph **i)** $y = \tan\theta$ and **ii)** its ***reciprocal*** function on separate grids for $-2\pi \le \theta \le 2\pi$.

3. Convert the following.
a) 195° to exact radians in terms of π
b) 6.85 radians to the nearest degree

4. Determine the exact measure of each of the following.
a) a positive angle co-terminal with $\frac{\pi}{3}$
b) a negative angle co-terminal with 228°
c) the principal angle, if -520° is the angle in standard position
d) the principal angle, if the terminal arm lies in the third quadrant and the related acute angle is $\frac{\pi}{6}$

5. Calculate exact values for each of the following. (Include a well-labeled diagram for each.)
a) $\sin\frac{3\pi}{2}$ **b)** $\sec\frac{-5\pi}{4}$ **c)** $\cot(-2\pi)$ **d)** $\tan\frac{8\pi}{3}$ **e)** $\csc\left(\frac{7\pi}{2}\right)$ **f)** $\cot\left(-\frac{7\pi}{3}\right)$

6. Simplify to a single trigonometric ratio using the appropriate identity from **1.** and evaluate.
a) $\sin 34^\circ \cos 4^\circ - \cos 34^\circ \sin 4^\circ$ **b)** $\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}$ **c)** $\sin\left(\frac{-5\pi}{8}\right)\cos\left(\frac{-5\pi}{8}\right)$

7. Use an appropriate formula to find the exact value of : **a)** $\sin\left(\frac{11}{12}\pi\right)$ **b)** $\cos\left(\frac{7}{12}\pi\right)$

8. If $\csc\beta = \frac{-13}{5}$ where $270^\circ \le \beta \le 360^\circ$, determine the following. (Include a detailed diagram.)
a) the measure of the related acute angle to the nearest degree
b) the measure of β to the nearest degree.
c) exact primary trigonometric values of β
d) an exact value for $\cos 2\beta$
e) an exact value for $\sin 2\beta$

9. If $\cot\alpha = \frac{-4}{3}$, and α is a second quadrant angle, determine the following.
(Include a detailed diagram.)

a) the related acute angle to the nearest degree
b) the principal angle to the nearest degree
c) a negative co-terminal angle with the principal angle
d) exact values for the primary trigonometric ratios
e) an exact value for $\tan 2\alpha$
f) an exact value for $\sin\left(\frac{\pi}{3} - \alpha\right)$

10. Prove the following identities:

a) $\dfrac{\cos x}{1-\sin x}-\tan x=\sec x$ **b)** $\dfrac{1}{1+\cos\theta}=\csc^2\theta-\csc\theta\cot\theta$

c) $\dfrac{\cos 2x}{1+\sin 2x}=\dfrac{\cot x-1}{\cot x+1}$ **d)** $\dfrac{\cos\theta-\sin 2\theta}{\cos 2\theta+\sin\theta-1}=\cot\theta$

11. Solve for x where $0\le x\le 2\pi$. Answer to the nearest hundredth of a radian if necessary.

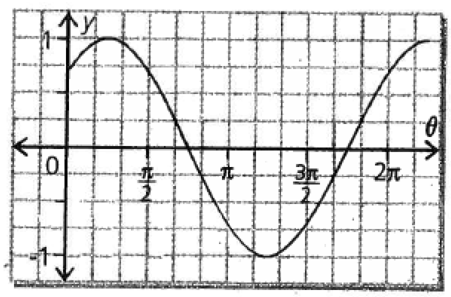
a) $\sec x=-6.658$ **b)** $\tan^2\left(\dfrac{x}{2}\right)-1=0$ **c)** $\sin 2x-1=0$

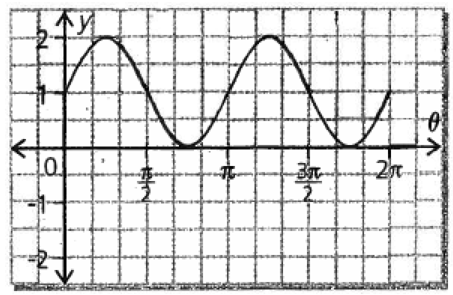
d) $\sin 2x=\cos x$ **e)** $7\sin x+\cos 2x=-3$ **f)** $3\cos 2x+\cos x+1=0$

12. Solve for x where $x\in[-2\pi,0]$. Answer to the nearest hundredth of a radian if necessary.

a) $4\sin x-3\cos^2 x=0$ **b)** $2\sin\left(x-\dfrac{2\pi}{3}\right)=1$

13. For each of the following graphs determine an equation of the sine function $y=a\sin k(\theta-d)+c$ and the cosine function $y=a\cos k(\theta-d)+c$ by finding appropriate values for a, k, d and c.

a) 

b) 

14. For each of the following state any reflections, the amplitude, period, phase shift and vertical translation. Graph the curve as indicated.

a) $y=3\sin\left(x+\dfrac{\pi}{3}\right)$ for one cycle and state the domain and range

b) $y=-\dfrac{1}{2}\sin 3\left(x-\dfrac{\pi}{4}\right)-\dfrac{1}{2},\ -\pi\le x\le\dfrac{\pi}{2}$

c) $y=-2\cos\left(2x-\dfrac{\pi}{3}\right)-1$ for $-\dfrac{\pi}{2}\le x\le\pi$

15. A Ferris wheel has a diameter of 18 m and rotates at the rate of 5 revolutions every 2 minutes. Passengers get on at the lowest point which is 2 m above the ground.

a) Draw a graph showing the height, h, in metres above the ground after t seconds for two cycles.
b) Determine an equation for the function graphed in **a**.
c) Use the equation to determine to 1 decimal place:
i) a passenger's height above the ground after 1.75 minutes
ii) the times in the first rotation the rider will be 18 m above the ground

16. Find the points of intersection of the curves $f(x)=\cos 2x$ and $g(x)=\sin x$ for $-\pi\le x\le\pi$. Illustrate your solution graphically and then state the solution set for $f(x)<g(x)$.

17. Graph each of the following on the same grid for the indicated domain.

a) $f(x)=-2\sin 2x$ & $y=\dfrac{1}{f(x)},\ x\in[-\pi,0]$ **b)** $f(x)=3\cos\dfrac{1}{2}x+1$ & $y=|f(x)|,\ x\in[-2\pi,2\pi]$

Answers

3. a) $\frac{13\pi}{12}$ **b)** 392° **4. a)** $\frac{7\pi}{3}$ **b)** -132° **c)** 200° **d)** $\frac{7\pi}{6}$ **5. a)** -1 **b)** $-\sqrt{2}$ **c)** dne **d)** $-\sqrt{3}$ **e)** -1 **f)** $-\frac{1}{\sqrt{3}}$

6. a) $\sin 30^\circ = \frac{1}{2}$ **b)** $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ **c)** $\frac{1}{2}\sin\left(-\frac{5\pi}{4}\right) = \frac{\sqrt{2}}{4}$ **7. a)** $\frac{\sqrt{6}-\sqrt{2}}{4}$ **b)** $\frac{\sqrt{2}-\sqrt{6}}{4}$

8. a) 23° **b)** 337° **c)** $\sin\beta = -\frac{5}{13}, \cos\beta = \frac{12}{13}, \tan\beta = -\frac{5}{12}$ **d)** $\frac{119}{169}$ **e)** $-\frac{120}{169}$

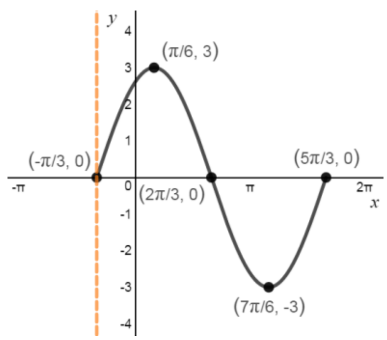
9. a) 37° **b)** 143° **c)** -217° **d)** $\sin\alpha = \frac{3}{5}, \cos\alpha = -\frac{4}{5}, \tan\alpha = -\frac{3}{4}$ **e)** $-\frac{24}{7}$ **f)** $\frac{-4\sqrt{3}-3}{10}$

11. a) 1.72, 4.56 **b)** $\frac{\pi}{2}, \frac{3\pi}{2}$ **c)** $\frac{\pi}{4}, \frac{5\pi}{4}$ **d)** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **e)** $\frac{7\pi}{6}, \frac{11\pi}{6}$ **f)** $\frac{\pi}{3}$, 2.30, 3.98, $\frac{5\pi}{3}$

12. a) $-5.72, -3.70$ **b)** $-\frac{7\pi}{6}, -\frac{\pi}{2}$

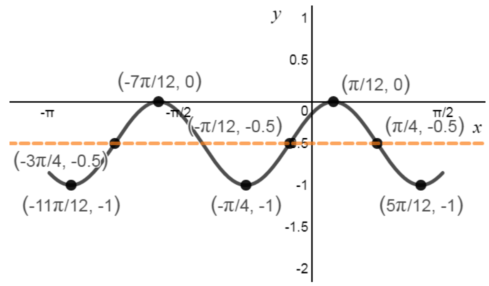
13. a) $y = \sin\left(\theta + \frac{\pi}{4}\right), y = \cos\left(\theta - \frac{\pi}{4}\right)$ **b)** $y = \sin 2\theta + 1, y = \cos 2\left(\theta - \frac{\pi}{4}\right) + 1$

14. a)

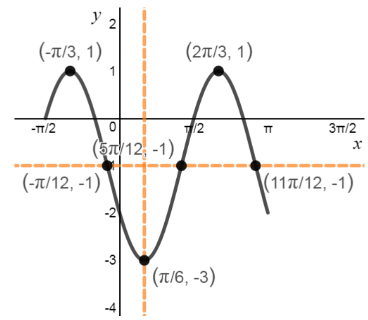


$D = \left\{x \in R \,\middle|\, -\frac{\pi}{3} \le x \le \frac{5\pi}{3}\right\}, R = \{y \in R \mid -3 \le y \le 3\}$

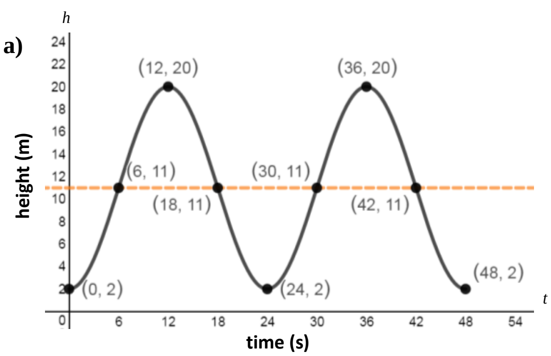
b)



c)



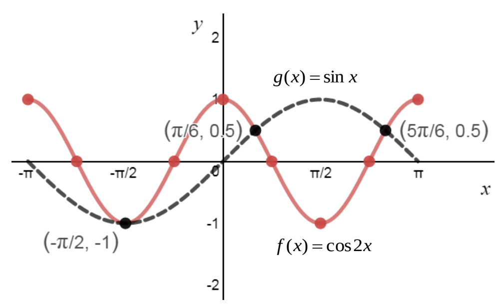
15. a)



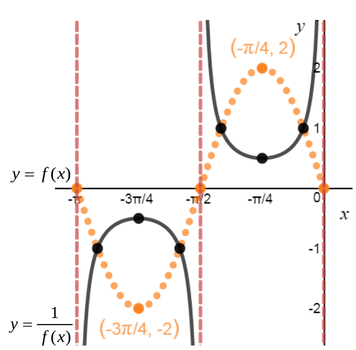
b) $h(t) = -9\cos\left(\frac{\pi}{12}t\right) + 11$ *or* $h(t) = 9\sin\left[\frac{\pi}{12}(t-6)\right] + 11$

c) i) 17.4 m **ii)** 9.4 seconds, 14.6 seconds

16. $\left(-\frac{\pi}{2}, -1\right), \left(\frac{\pi}{6}, \frac{1}{2}\right), \left(\frac{5\pi}{6}, \frac{1}{2}\right)$; $\left\{x \in R \,\middle|\, \frac{\pi}{6} < x < \frac{5\pi}{6}\right\}$



17. a)



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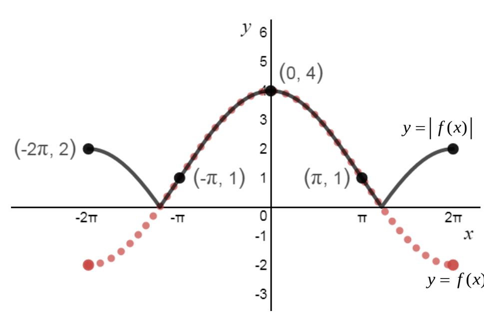


Date:______________ **EXAM REVIEW UNITS 5 & 6**

1. Review the ***Exponent Laws*** from Unit 5: Day 1.

2. Evaluate each of the following:

a) $\frac{5^{-2}}{3^{-3}}$ **b)** $\frac{2^{-1}+4^{-1}}{3^{-2}}$ **c)** $\frac{4^{\frac{3}{2}}-8^{\frac{1}{3}}}{16^{\frac{1}{4}}\times 25^{\frac{1}{2}}}$ **d)** $\left(\frac{1}{4}-\frac{9}{100}\right)^{-\frac{3}{2}}$ **e)** $\frac{3^{-12}+3^{-11}}{3^{-12}-3^{-13}}$

3. Simplify to a single power and evaluate.

a) $\frac{3^{n+1}\times 9^{n-4}}{27^{n-2}}$ **b** $\frac{4^{3x-1}\times 16^{x}}{32^{2x}\times 8}$

4. Simplify and write your final answer with positive exponents.

a) $\left(2x^4y^{-2}\right)^2\cdot\left(x^4y^3\right)^{-1}$ **b)** $\sqrt[3]{\frac{\sqrt{x}\cdot\sqrt[3]{x^2}}{x^{-1}}}$

5. Rewrite each expression as the sum and/or difference of terms, where each term is of the form ax^n.

a) $\frac{4x^5-5x^4+6x-2}{2x^4}$ **b)** $\frac{x^3+27}{x+3}$ **c)** $\frac{1}{\sqrt{x}}\left(x^2+\frac{1}{2x}\right)^2$ **d)** $\frac{x-4}{\sqrt{x}-2}$

6. Completely factor each of the following expressions.

a) $-2t(1-t)^4+4t^2(1-t)^3$ **b)** $10(2x+1)^4(3x+2)^4+12(2x+1)^3(3x+2)^5$

c) $12x(2x^2-1)^2(x^2-2)^{-2}-4x(x^2-2)^{-3}(2x^2-1)^3$

7. Review the ***Logarithm Laws and Properties*** from Unit 6: Day 3

8. Change to exponential form.

a) $\log_9 3=\frac{1}{2}$ **b)** $\log\left(\frac{1}{100}\right)=-2$

9. Change to logarithmic form.

a) $3^2=9$ **b)** $e^0=1$

10. Evaluate each of the following exactly.

a) $\log_{\sqrt{3}}1$ **b)** $\log_4\left(\frac{1}{64}\right)$ **c)** $\log_5 5^{\frac{1}{2}}$ **d)** $e^{\ln 3^{-2}}$ **e)** $\log_2(-1)$

11. Use the properties of logarithms to write each of the following as a sum and/or a difference of logarithms.

a) $\log_8\sqrt[4]{m}$ **b)** $\ln\left[\frac{x^3(x-2)^4}{\sqrt{x^2+1}}\right]$

12. Simplify to a single logarithm using the logarithm laws and evaluate.

a) $\log_6 3+\log_6 12$ **b)** $\log_2 15-\log_2 24-\log_2 5$ **c)** $\frac{\log_7\sqrt{8}}{\log_7 2}$

(Use change of base formula)

13. Solve each of the following. ($*$ *means answer to 2 decimal places, otherwise answer exactly*)

a) $3\left(5^{x^2+3x}\right)=\frac{3}{25}$ **b)** $5^{x-3}-5^{x-1}+120=0$ **c)** $3^{2x}-3^x-12=0$ $*$

d) $\log_x 27=\frac{3}{2}$ **e)** $\log_7 x=\frac{1}{3}\log_7 27+\frac{2}{3}\log_7 8$ **f)** $4\left(7^{x-2}\right)=8$ $*$

g) $\ln\left(\log_4 x^2\right)=0$ **h)** $\log_9(x-5)+\log_9(x+3)=1$ **i)** $\log_2(x+4)-\log_2(x-3)=3$

j) $(\ln x)^2+\ln x^2=0$ **k)** $e^{2x}-5e^x-24=0$ **l)** $x=\log_2 80$ $*$
(Use change of base formula)

14. A photocopier, which originally cost $500 000, depreciates exponentially in value by 10% each year. What will be the photocopier's value in five years to the nearest hundred dollars?

15. In 1947, an investor bought Vincent van Gogh's painting *Irises* for $84 000. In 1987, she sold it for $49 million. What was the annual exponential growth rate for this investment? (to the nearest percent)

16. In a recent dig, a human skeleton was unearthed. It was later found that the amount of Carbon 14 in it had decayed to $\frac{1}{\sqrt{8}}$ of its original amount. If Carbon-14 has a half-life of 5760 years, how old is the skeleton? (*Use "a" for its original amount.*)

17. A radioactive substance decays from 20 g to 15 g in 7 hours. Determine the half-life of the substance, to the nearest minute.

18. Graph the following by naming and applying transformations on an appropriate function. Then, find the domain, range, intercept(s) and asymptote(s). (A table of values may be included.)

a) $f(x)=-\frac{1}{2}\left(\frac{1}{3}\right)^x+2$ **b)** $y=2e^{x-1}-2$ **c)** $y=\log_2\left(\frac{x^3}{8}\right)$ **d)** $f(x)=\ln(2-x)$

Answers

2. a) $\frac{27}{25}$ **b)** $\frac{27}{4}$ **c)** $\frac{3}{5}$ **d)** $\frac{125}{8}$ **e)** 6 **3.a)** $3^{-1}=\frac{1}{3}$ **b)** $2^{-5}=\frac{1}{32}$ **4. a)** $\frac{4x^4}{y^7}$ **b)** $x^{\frac{13}{18}}$

5. a) $2x-\frac{5}{2}+3x^{-3}-x^{-4}$ **b)** x^2-3x+9 **c)** $x^{\frac{7}{2}}+x^{\frac{1}{2}}+\frac{1}{4}x^{-\frac{5}{2}}$ **d)** $x^{\frac{1}{2}}+2$

6. a) $-2t(1-t)^3(1-3t)$ **b)** $2(2x+1)^3(3x+2)^4(28x+17)$ **c)** $4x\left(2x^2-1\right)^2\left(x^2-2\right)^{-3}\left(x^2-5\right)$

8. a) $9^{\frac{1}{2}}=3$ **b)** $10^{-2}=\frac{1}{100}$ **9. a)** $2=\log_3 9$ **b)** $0=\log_e 1$ *or* $0=\ln 1$ **10. a)** 0 **b)** -3 **c)** $\frac{1}{2}$ **d)** $\frac{1}{9}$ **e)** dne

11. a) $\frac{1}{4}\log_8 m$ **b)** $3\ln x+4\ln(x-2)-\frac{1}{2}\ln\left(x^2+1\right)$ **12. a)** $\log_6 36=2$ **b)** $\log_2\left(\frac{1}{8}\right)=-3$ **c)** $\log_2\sqrt{8}=\frac{3}{2}$

13. a) $-2,\ -1$ **b)** 4 **c)** 1.26 **d)** 9 **e)** 12 **f)** 2.36 **g)** $-2,\ 2$ **h)** 6 **i)** 4 **j)** $1,\ \frac{1}{e^2}$ **k)** $\ln 8$ **l)** 6.32

14. $295 200 **15.** 17% **16.** 8 640 years **17.** 16 hours & 52 minutes

18. a) Transformations on $y=\left(\frac{1}{3}\right)^x$ are:

i) reflection in the x-axis

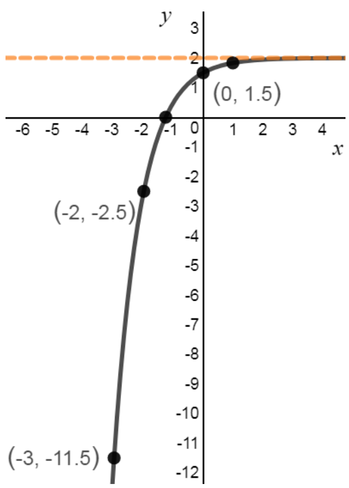
ii) vertical stretch by a factor of $\frac{1}{2}$

iii) vertical translation up 2 units

$D=\{x\in R\},\ R=\{y\in R|y<2\}$

x-int is $\log_3 4$ $or \doteq -1.3$, y-int is 1.5

horizontal asymptote is $y=2$



b) Transformations on $y=e^x$ are:

i) vertical stretch by a factor of 2

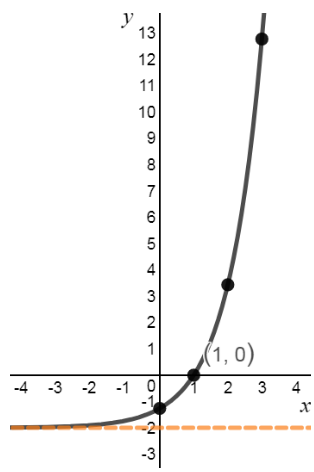
ii) horizontal translation right 1 unit

iii) vertical translation down 2 units

$D=\{x\in R\},\ R=\{y\in R|y>-2\}$

x-int is 0, y-int $\doteq -1.3$

horizontal asymptote is $y=-2$



c) Transformations on $y=\log_2 x$ are:

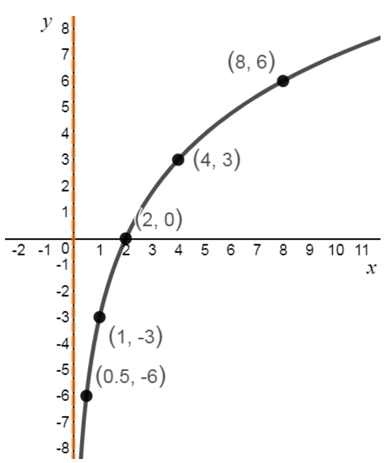
i) vertical stretch by a factor of 3

ii) vertical translation down 3 units

$D=\{x\in R|\ x>0\},\ R=\{y\in R\}$

x-int is 2, no y-int

vertical asymptote is $x=0$



d) Transformations on $y=\ln x$ are:

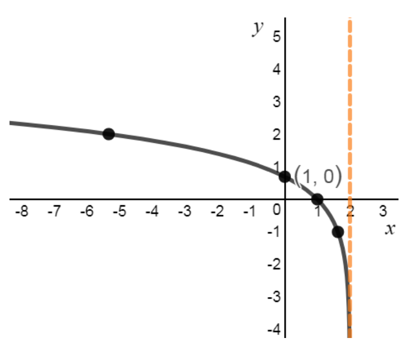
i) reflection in the y-axis

ii) horizontal translation right 2 units

$D=\{x\in R|\ x<2\},\ R=\{y\in R\}$

x-int is1, y-int is $\ln 2$ $or \doteq 0.7$

vertical asymptote is $x=2$



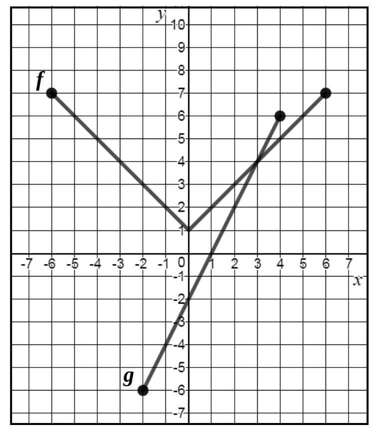
Date:______________ **EXAM REVIEW UNITS 7 & 8**

UNIT 7

1. Given $f = \{(0,4), (1,2), (2,1), (4,5)\}$ and $g = \{(0,0), (1,1), (4,2), (9,3)\}$, determine each of the following, if possible.

a) $(f \circ f)(0)$ **b)** $(g - f)(1)$ **c)** $(f \circ g)(4)$ **d)** g^{-1} **e)** $(g^{-1} \circ g)(3)$ **f)** $f \circ g$

2. Use the graphs of f and g to determine each of the following, if possible.



a) $(g \circ f)(-2)$ **b)** $(f \circ g)(-2)$ **c)** $(g \circ g)(0)$

d) $(fg)(-1)$ **e)** $\left(\frac{f}{g}\right)(1)$ **f)** $(f+g)(3)$

g) D_{f-g} **h)** the ***graph*** of $h = f - g$

i) the equation of h expressed as a ***piecewise*** function

3. Given $f(x) = \frac{2+x}{x}$, $g(x) = x^2 + 2x$ **,** $h(x) = \sqrt{4-x}$ **,** $p(x) = \csc x$ and $q(x) = \log_2 x$, find:

a) $g(f(-1))$ **b)** $(f \circ g)(-2)$ **c)** $(f+h)(-12)$ **d)** $(q \circ h)(-60)$

e) D_{q+h} **f)** $f^{-1}(x)$ and show that $(f \circ f^{-1})(x) = x$ **g)** x, if $f(x) = g(x)$

h) x, if $(g \circ h)(x) = 0$ **i)** x for $-2\pi \le x \le 0$, if $(f \circ p)(x) = 2$ **j)** $(f \times g)(x)$ and ***graph***

4. The number, N, of bacteria in a culture at time t is given by $N(t) = 2000\left[30 + te^{-\frac{t}{20}}\right]$.

If the bacterial culture is placed into a colony of mice, the number of mice, M, that become infected is related to the number of bacteria present by the equation $M(N) = \sqrt[3]{N+1000}$. After ten days, how many mice are infected?

5. A firm can sell x units of a product daily at p dollars per unit, where $x = 1000 - p$.
The cost, C of producing x units per day is $C(x) = 3000 + 20x$.

a) Find the revenue function, $R(x)$.
b) Find the profit function, $P(x)$.
c) What price per unit will maximize profit? What is the maximum daily profit?

UNIT 8

Redo **Unit 8 Test Review.**

Answers

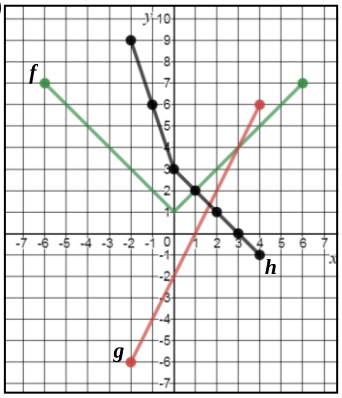
1. a) 5 **b)** -1 **c)** 1 **d)** $g^{-1} = \{(0,0), (1,1), (2,4), (3,9)\}$ **e)** undefined **f)** $f \circ g = \{(0,4), (1,2), (4,1)\}$

2. a) 4 **b)** 7 **c)** -6

d) -8 **e)** undefined **f)** 8

g) $D_{f-g} = \{x \in R \mid -2 \le x \le 4\}$

h)

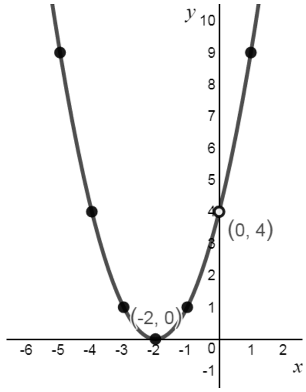


i) $h(x) = \begin{cases} -3x+3 & if\ -2 \le x \le 0 \\ -x+3 & if\ 0 < x \le 4 \end{cases}$

3. a) -1 **b)** dne **c)** $\frac{29}{6}$ **d)** 3 **e)** $D_{q+h} = \{x \in R \mid 0 < x \le 4\}$

f) $f^{-1}(x) = \frac{2}{x-1}$ **g)** $-2, -1, 1$ **h)** 4 **i)** $-\frac{11\pi}{6}, -\frac{7\pi}{6}$

j) $(f \times g)(x) = (x+2)^2,\ hole\ at\ (0,4)$



4. 41.8

5. a) $R(x) = -x^2 + 1000x$ **b)** $P(x) = -x^2 + 980x - 3000$ **c)** \$510 if $x = 490$, \$237 100