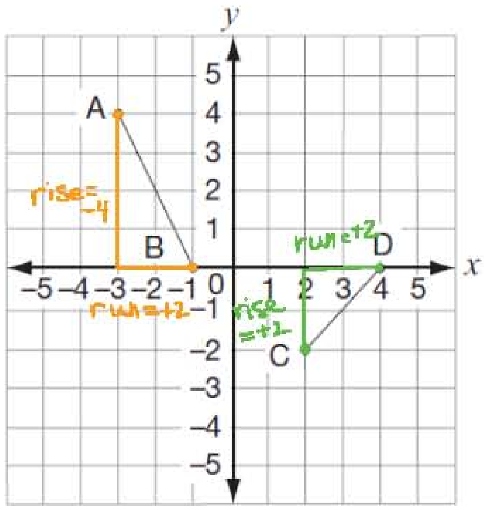
Slopes and Rates of Change

Rate of Change: The ***rate of change*** of a linear relation tells us how one variable changes as the other variable changes.

i) Graphically, the ***rate of change*** is the ***steepness*** of the line where $slope = \frac{rise}{run}$.

ii) Algebraically, the ***rate of change*** is the ***slope***, m, in the equation $y = mx + b$.

Ex. 1. Use the graph to determine the **slope**, ***(rate of change)*** of each line segment.



$m_{AB} = \frac{rise}{run}$

$= \frac{-4}{2}$

$\therefore m_{AB} = -2$

For AB, the rate of change (slope) is −2

$m_{CD} = \frac{rise}{run}$

$= \frac{2}{2}$

$\therefore m_{CD} = 1$

For CD, the rate of change (slope) is 1.

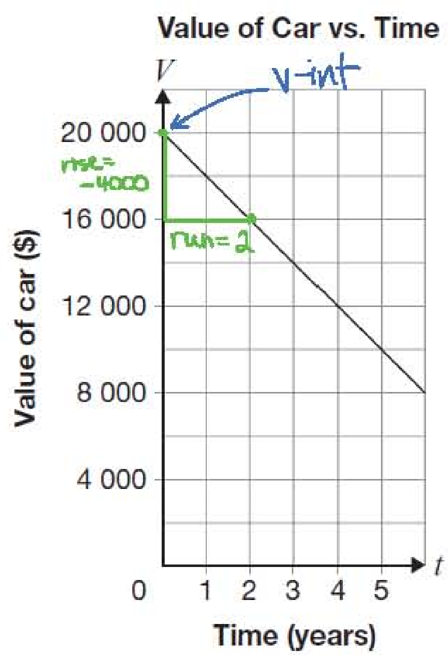
Ex. 2. Use the graph to determine the:

a) ***rate of change*** in the value of the car (m)

b) ***V*– *intercept*** and explain what it means

c) ***equation*** of the linear relation

d) ***t*– *intercept*** and explain what it means



a) rate of change = m

$= \frac{rise}{run}$

$= \frac{-4000 \text{ \$}}{2 \text{ years}}$ ← per

$= -2000$ \$/years

∴ the rate of change in the value of the car is −\$2000/year.

b) V-intercept is 20000. This value means the car's initial value is \$20 000.

c) $y = mx + b$

$V = -2000t + 20000$

d) For t-int:
Let $V = 0$

$0 = -2000t + 20000$

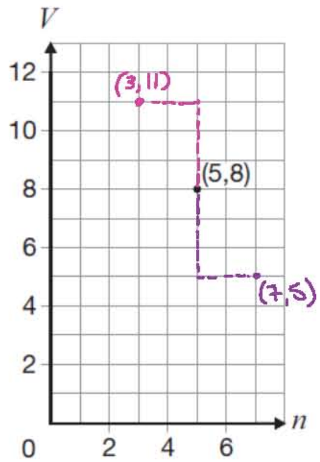
$\frac{2000t}{2000} = \frac{20\,000}{2000}$

$t = 10$

∴ the t-intercept is 10.
This means that after 10 years the car's value is 0

Ex. 3. The point on the grid below belongs to a linear relation that has $-\frac{3}{2}$ as its ***rate of change***. m

Write the coordinates of two other points that would be on the line passing through this point.



rate of change $= m$

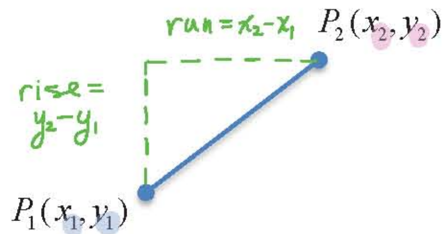
$= \frac{-3}{2}$ rise / run

$= \frac{3}{-2}$ rise / run

$\therefore$ two other points are $(3,11)$ & $(7,5)$

The Slope Formula:

Using the endpoints $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ for a line segment, (or any two points on a line) develop a formula for the slope.



$slope = \frac{rise}{run}$

$m = \frac{\Delta y}{\Delta x}$ ← delta y ← delta x

$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ memorize!!

Ex. 4. Use the ***slope formula*** to calculate the slope of the line through each pair of points.

a) $A(3,-5)$ and $B(-1,-3)$ (x_1 y_1, x_2 y_2)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-3-(-5)}{-1-3}$

$m = \frac{-3+5}{-1-3}$

$m = \frac{2}{-4}$

$m = -\frac{1}{2}$

b) $C(-0.5, 5.4)$ and $D(0.7, 2.1)$ (x_1 y_1, x_2 y_2)

$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{CD} = \frac{2.1 - 5.4}{0.7-(-0.5)}$

$m_{CD} = \frac{-3.3 \times 10}{1.2 \times 10}$

$m_{CD} = -\frac{33}{12}$ → $m_{CD} = -\frac{11}{4}$

c) $E(-3,-2)$ and $F(1,-2)$ (x_1 y_1, x_2 y_2)

$m_{EF} = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{EF} = \frac{-2-(-2)}{1-(-3)}$

$m_{EF} = \frac{0}{4}$

$\therefore m_{EF} = 0$

Ex. 5. The point $(-1,-1)$ lies on a line with slope $\frac{1}{2}$. Determine the $y-$coordinate of the point on the line with $x-$coordinate 9.

Let $A(-1,-1)$ and $B(9,K)$ be points on the line (x_1, y_1; x_2, y_2)

Find k if

$m_{AB} = \frac{1}{2}$

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{2}$

$\frac{K-(-1)}{9-(-1)} = \frac{1}{2}$

$\frac{\cancel{10}^{1}}{1}\left(\frac{K+1}{\cancel{10}_{1}}\right) = \left(\frac{1}{\cancel{2}_{1}}\right)\frac{\cancel{10}^{5}}{1}$

→ $K+1=5$

$k=4$

$\therefore$ the y-coordinate is 4.

Collinear: Three or more points are collinear if they all lie on the same line.
ie. Points A, B, and C are ***collinear*** if $m_{AB} = m_{BC}$.

Ex. 6. Determine whether the points $P(-6,12)$, $Q(3,6)$ and $R(12,0)$ are collinear.

$x_1\ y_1$ $x_2\ y_2$ $x_1\ y_1$ $x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{PQ} = \frac{6-12}{3-(-6)} = \frac{-6}{9} = -\frac{2}{3}$$

$$m_{QR} = \frac{0-6}{12-3} = -\frac{6}{9} = -\frac{2}{3}$$

$\because m_{PQ} = m_{QR}$

$\therefore P, Q, R$ are collinear.

Ex. 7. The table below represents the linear relationship between cost and repair time at an appliance store. Determine the ***rate of change*** and explain what it means.

Repair time, t (h)	Cost, C (\$)
3 t_1	205 C_1
6 t_2	385 C_2
8	505

(t, C)

$$\text{rate of change} = m = \frac{C_2 - C_1}{t_2 - t_1}$$

$$m = \frac{385-205}{6-3} \frac{\$}{h}$$

$$m = \frac{180}{3}$$

$$m = 60\ \$/h$$

$\therefore$ the rate of change is 60 and this means that the hourly rate is \$60/h.

Ex. 8. A bathtub is filling with water at a constant rate. After 3 minutes the water is 7.5 cm deep, and after 8 minutes the water is 15 cm deep. At what rate is the depth of water increasing?

linear relation

Let x represent the time in minutes.
Let y represent the depth of water, in cm.

(x, y): $(3, 7.5)$ and $(8, 15)$

$$\text{rate of change} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-7.5}{8-3} = \frac{7.5}{5} \frac{\text{cm}}{\text{min}} = 1.5$$

$\therefore$ the depth is increasing at a rate of 1.5 cm/min.

Using a Point and Slope to Determine the Equation of a Line

Warmup:

1. If the equation $y = mx + 5$ represents a line that passes through the point $(3,7)$, find m.

Find m if $x=3$, $y=7$

$7 = m(3) + 5$

$7 = 3m + 5$

$-3m = 5 - 7$

$\frac{-3m}{-3} = \frac{-2}{-3}$

$\therefore m = \frac{2}{3}$

2. If the equation $y = -\frac{5}{6}x + b$ represents a line that passes through the point $(-2,1)$, find b.

Find b if $x=-2$, $y=1$

$1 = -\frac{5(-2)}{6} + b$

$1 = \frac{5}{3} + b$

$-b = \frac{5}{3} - 1$

$-b = \frac{5}{3} - \frac{3}{3}$

$-b = \frac{2}{3}$

$\therefore b = -\frac{2}{3}$

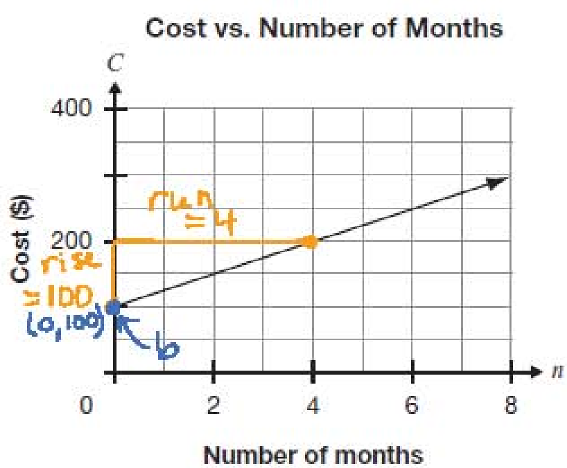
$3(1) = \frac{3}{1}\left(\frac{5}{3}\right) + 3(b)$

$3 = 5 + 3b$

$-3b = 5 - 3$

$-3b = 2$

Ex. 1. The graph below represents the cost to belong to a local gym. Determine the equation that represents the graph in the form $y = mx + b$.



Find m:

$m = \frac{rise}{run}$

$= \frac{100}{4}$ (\$) (months)

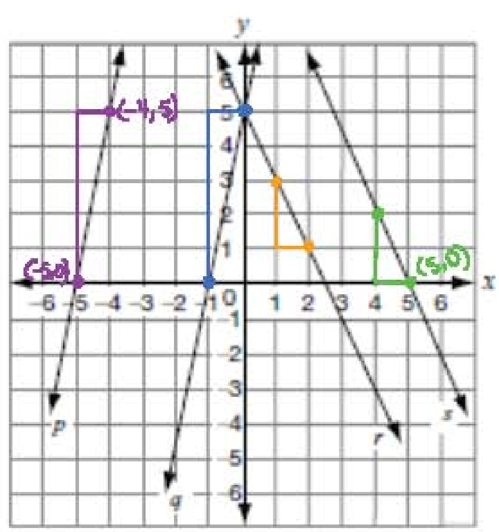
$= 25$ (\$/month)

Find b:

$b = 100$

$\therefore$ the equation is $C = 25n + 100$.

Ex. 2. Determine the equation of each line in the form $y = mx + b$.



For line q:

$m = \frac{5}{1}$, $b = 5$

$= 5$

$\therefore y = 5x + 5$ is the equation for q

For line r:

$m = \frac{-2}{1}$ $b = 5$

$= -2$

$\therefore y = -2x + 5$ is the equation for r.

For line p:

$m = \frac{5}{1}$, $x = -5$

$= 5$ $y = 0$

$y = 5x + b$

Find b:

$0 = 5(-5) + b$

$0 = -25 + b$

$-b = -25$

$b = 25$

$\therefore y = 5x + 25$ is the equation for p

For line s:

$m = \frac{-2}{1}$ $x = 5$

$= -2$ $y = 0$

$y = -2x + b$

Find b:

$0 = -2(5) + b$

$0 = -10 + b$

$-b = -10$

$b = 10$

$\therefore y = -2x + 10$ is the equation for s

Ex. 3. Determine the equation of each line in the form $y = mx + b$ described below.

a) has a $y-$intercept of 0 and a slope of $\frac{1}{2}$

$m=\frac{1}{2}$ $\quad b=0$

$\therefore y=\frac{1}{2}x+0$ or $y=\frac{1}{2}x$

b) passing through $A(0,-3)$, with a slope of -1

$\because x=0$
$y=-3$ is the y-int

$m=-1$ $\quad b=-3$

$\therefore y=-x-3$

c) with a slope of -5, passing through $A(7,-3)$

$m=-5$ $\quad b=\_\_\_$

$x=7$ $\quad y=-3$

Find b:

$y=mx+b$

$-3=-5(7)+b$

$-3=-35+b$

$-b=-35+3$

$-b=-32$

$b=32$

$\therefore y=-5x+32$

d) passing through $A(-2,1)$, with a slope of $\frac{3}{2}$

$m=\frac{3}{2}$ $\quad b=\_\_\_$

$x=-2$ $\quad y=1$

$1=\frac{3}{\cancel{2}_1}(\cancel{-2}^{-1})+b$

$1=-3+b$

$1+3=b$

$4=b$

$\therefore y=\frac{3}{2}x+4$ is the equation.

e) has an $x-$intercept of $1\frac{1}{4}$ and a slope of $-\frac{2}{5}$

$m=-\frac{2}{5}$ $\quad b=\_\_\_$

$x=1\frac{1}{4}=\frac{5}{4}$ $\quad y=0$

Find b:

$0=-\frac{\cancel{2}^1}{\cancel{5}_1}\left(\frac{\cancel{5}^1}{\cancel{4}_2}\right)+b$

$0=-\frac{1}{2}+b$

$\frac{1}{2}=b$

$\therefore y=-\frac{2}{5}x+\frac{1}{2}$

f) with a slope of $\frac{3}{5}$, passing through $(4, 2)$

$m=\frac{3}{5}$ $\quad b=\_\_\_$

$x=4$ $\quad y=2$

$2=\frac{3}{5}(4)+b$ $\quad$ or $\quad 5(2)=\cancel{5}^1\left[\frac{3}{\cancel{5}_1}(4)\right]+5b$

$10=12+5b$

$10-12=5b$

$-2=5b$

$-\frac{2}{5}=b$

$\frac{2}{1}=\frac{12}{5}+b$

$\frac{10}{5}-\frac{12}{5}=b$

$-\frac{2}{5}=b$

$\therefore y=\frac{3}{5}x-\frac{2}{5}$ is the equation.

Using Two Points to Determine the Equation of a Line

Ex. 1. Find the equation of the line that passes through the point $(-3, 2)$ and has the following slope:

a) $m = \frac{5}{6}$, $b =$ ___, $x=-3$, $y=2$

Find b: $y=\frac{5}{6}x+b$

$2=\frac{5(-3)}{6}+b$

$2(2)=\frac{2}{1}\left(-\frac{5}{2}\right)+(b)2$

$4=-5+2b$

$9=2b$

$b=\frac{9}{2}$

$\therefore y=\frac{5}{6}x+\frac{9}{2}$ is the equation.

b) $m = 0$, $b=$ ___, $x=-3$, $y=2$

Find b: $y=0x+b$

$2=0(-3)+b$

$2=b$

$\therefore y=2$ is the equation.

Ex. 2. Determine the value of k if the points $X(2, 3)$, $Y(8, k)$ and $Z(29, k+7)$ are ***collinear***.

$x_1\ y_1$ $x_2\ y_2$

$\because X, Y, Z$ are collinear; $\therefore m_{XY}=m_{YZ}$

$$\frac{k-3}{8-2}=\frac{k+7-k}{29-8}$$

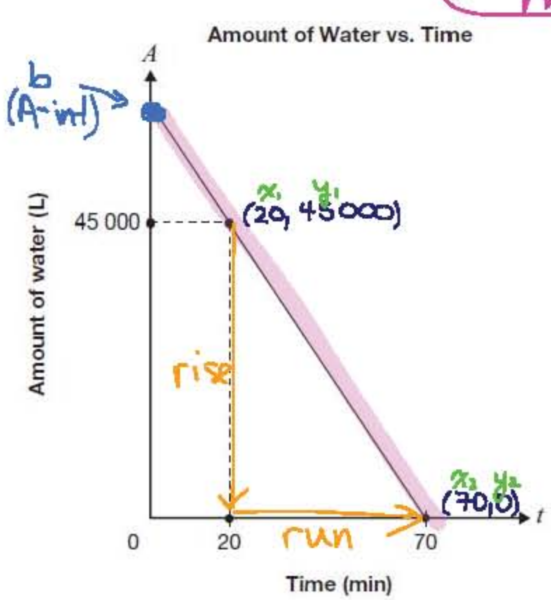
$$\frac{k-3}{6}=\frac{7}{21}$$

$$\frac{6}{1}\left(\frac{k-3}{6}\right)=\left(\frac{1}{3}\right)\frac{6}{1}$$

$$k-3=2$$

$$\therefore k=5$$

Ex. 3. The graph below represents the relationship between the amount of water, A, in a pool as it drains and time, t. Determine the ***equation*** of this relation by first determining the ***initial amount*** (b) of water in the pool and the ***rate of change*** (m) of this relation.



$y=mx+b$

Rate of change:

$=m$

$=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{0-45000}{70-20}$

$=\frac{-45000}{50}$

$=\frac{-4500}{5}\frac{L}{min}$

$=-900$

$\therefore$ the rate of change is -900. This means water is draining at a rate of 900L/min.

Find Initial Amount:

Find b:

$m=-900$, $b=$ ___, $x=70$, $y=0$

$y=mx+b$

$0=-900(70)+b$

$0=-63000+b$

$-b=-63000$

$\therefore b=63000$

$\therefore$ the pool has an initial amount 63000L of water

$\therefore A=-900t+63000$ is the equation

Ex. 4. Determine the equation of the line that passes through the given two points.

a) $(-6, 10)$ and $(2, 4)$

$m=$ ___ $b=$ ___

Find m: $m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{4-10}{2-(-6)}$

$m=\frac{-6}{8}$

$m=-\frac{3}{4}$

Find b: $x=2, y=4$

$4=-\frac{3}{\cancel{4}_2}\left(\frac{\cancel{2}^1}{1}\right)+b$

$4=-\frac{3}{2}+b$

$\frac{8}{2}+\frac{3}{2}=b$

$b=\frac{11}{2}$

$\therefore y=-\frac{3}{4}x+\frac{11}{2}$

b) y-intercept is -4 $(0,-4)$ and $(-5, -4)$

$m=$ ___ $b=-4$

Find m: $m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{-4-(-4)}{-5-0}$

$m=\frac{0}{-5}$

$m=0$

$\therefore y=-4$

Ex. 5. Determine the equation of the line with the same x-intercept as the line $5x-3y+15=0$ ① and the same y-intercept as the line $x+8y+2=0$. ② ③

① $5x-3y+15=0$

For x-int: let $y=0$

$5x-3(0)+15=0$

$5x+15=0$

$\frac{5x}{5}=\frac{-15}{5}$

$x=-3$

$\boxed{\text{Point: }(-3,0)}$

② $x+8y+2=0$

For y-int: let $x=0$

$(0)+8y+2=0$

$8y+2=0$

$\frac{8y}{8}=\frac{-2}{8}$

$y=-\frac{1}{4}$

$\boxed{\text{point: }(0,-\frac{1}{4})}$

③ $(-3,0)$, $(0,-\frac{1}{4})$ are points on our line

x_1 y_1, x_2 y_2

$m=$ ___ $b=-\frac{1}{4}$

$m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{-\frac{1}{4}-0}{0-(-3)}$

$m=\frac{-\frac{1}{4}}{3}\cdot\frac{4}{4}=-\frac{1}{12}$

$m=-\frac{1}{4}\div\left(\frac{3}{1}\right)$

$m=-\frac{1}{4}\times\frac{1}{3}$

$m=-\frac{1}{12}$

$\therefore y=-\frac{1}{12}x-\frac{1}{4}$

Horizontal and Vertical Lines & Parallel and Perpendicular Lines

PART A: Horizontal and Vertical Lines

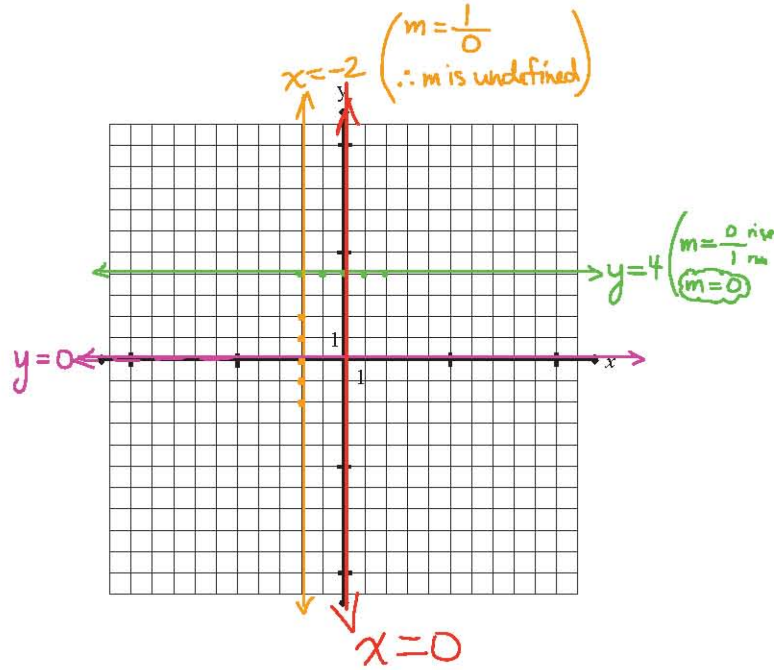
Ex. 1. Graph using a table of values. State the slope.

a) $y = 4$

x	y
-2	4
-1	4
0	4
1	4
2	4

b) $x = -2$

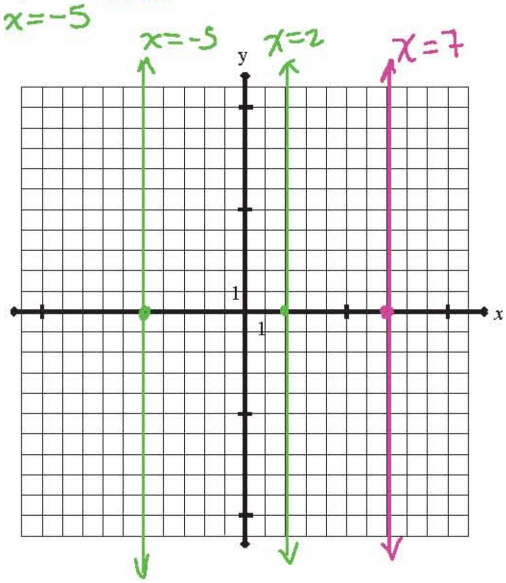
x	y
-2	-2
-2	-1
-2	0
-2	1
-2	2



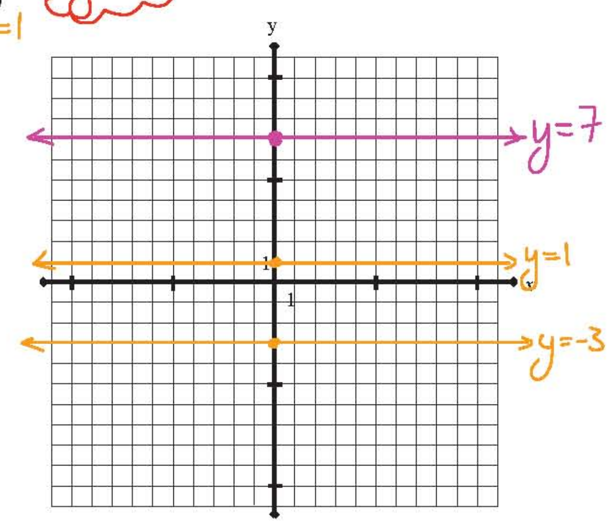
Summary	Horizontal Lines	Vertical Lines
Equation	$y = \#$ (no x in equation)	$x = \#$ (no y in equation)
Slope	0	undefined
x-intercept	none (except for $y=0$)	$x=\#$
y-intercept	$y=\#$	none (except $x=0$)

Ex. 2. Graph the following. ① plot the intercept ② Draw a straight line through it.

a) $x = 2$
$x + 5 = 0$
↳ $x=-5$
$x=7$



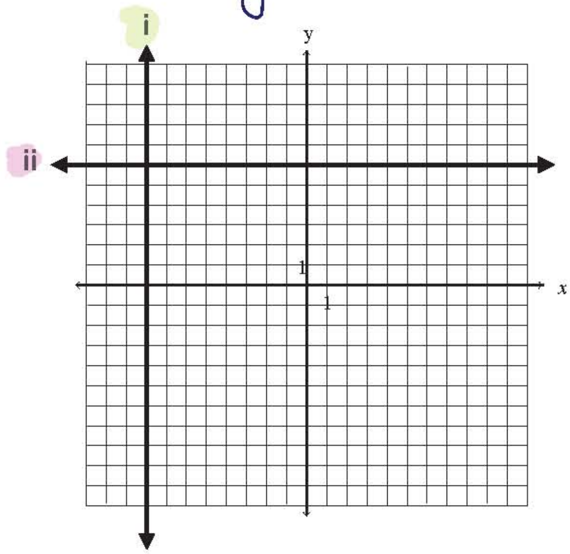
b) $y = -3$
$y - 1 = 0$
↳ $y=1$
$y=7$



Ex. 3. Find the equation of each of the following lines given:

a) i) $x=-8$

ii) $y=6$



b) $(-1,\ 9)$ and $(-1,\ -2)$ are points on the line

vertical

$\therefore x=-1$

c) the line is horizontal and passes through $(3,\ 5)$

$y=5$

d) the slope is undefined and $(-3,\ 1)$ is on the line

vertical $\therefore x=-3$

e) the line is parallel to $y=-3$ and passes through $(2,\ 6)$

horizontal $\therefore y=6$

f) the line is perpendicular to the x-axis and passes through $(-3,\ -8)$

vertical

$\therefore x=-3$

g) perpendicular to $x-5=0$ with a y-intercept of 1

horizontal $\therefore y=1$

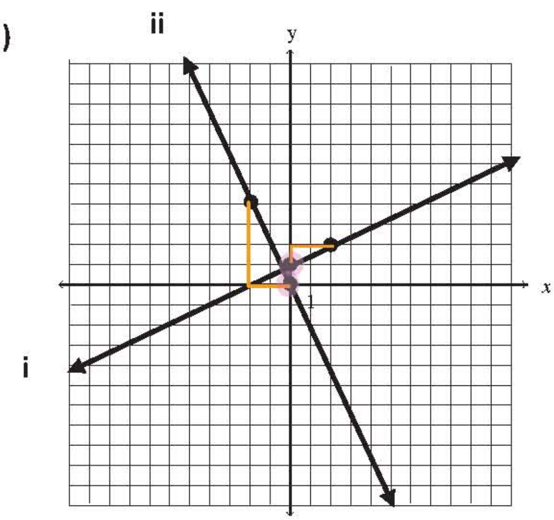
h) passes through $(5,\ 7)$ and $(-2,\ 7)$

$\therefore y=7$

PART B: Parallel and Perpendicular Lines

Ex. 4. Find the equation of each line below in slope, y-intercept form.

a)

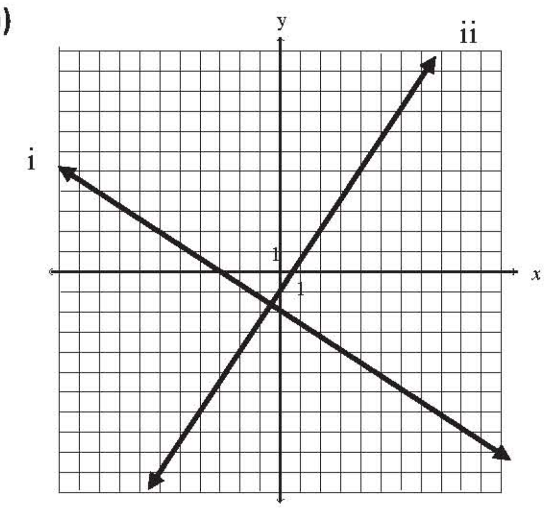


i) $\boxed{m=\frac{1}{2}}$ $b=1$ $\therefore y=\frac{1}{2}x+1$

ii) $\boxed{m=-\frac{2}{1}}$ $b=0$ $\therefore y=-2x$

$=-2$

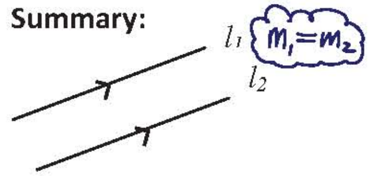
b)



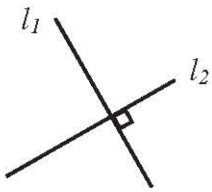
i) $\boxed{m=-\frac{2}{3}}$ $b=-2$ $\therefore y=-\frac{2}{3}x-2$

ii) $\boxed{m=\frac{3}{2}}$ $b=-1$ $\therefore y=\frac{3}{2}x-1$

Summary:



Parallel Lines have slopes that are equal.



Perpendicular Lines have slopes that are "negative reciprocals" of each other.

"flip & change sign"

Ex. 5. State whether the following lines are ***parallel***, ***perpendicular*** or ***neither*** by comparing slopes.

a) ① $y = -\frac{1}{4}x + 3$ and ② $y = \frac{-2}{8}x - 1$

$m_1 = -\frac{1}{4}$

$m_2 = -\frac{2}{8} = -\frac{1}{4}$

$\because m_1 = m_2$

$\therefore$ the lines are parallel

b) ① $y = -\frac{5}{2}x$ and ② $y = -0.4x + 2$

$m_1 = -\frac{5}{2}$

$m_2 = -0.4 = -\frac{4}{10} = -\frac{2}{5}$

$\because m_1 \neq m_2$ and $m_1 \neq \frac{-1}{m_2}$

$\therefore$ they are neither.

c) ① $y = -x - 3$ and ② $y = x + 5$

$m_1 = -\frac{1}{1}$ $\quad m = \frac{1}{1}$

$\because m_1 = -\frac{1}{m_2}$ $\therefore$ the slopes are negative reciprocals.

$\therefore$ the lines are perpendicular.

Ex. 6. Determine if lines k and l are ***parallel***, ***perpendicular*** or ***neither*** by comparing slopes.

k: through points $(-3, -7)$, $(9, 2)$ and l: through points $(-1, 5)$, $(2, 1)$

$m = \frac{y_2 - y_1}{x_2 - x}$

$m_k = \frac{2-(-7)}{9-(-3)}$

$m_k = \frac{9}{12}$

$m_k = \frac{3}{4}$

$m_l = \frac{1-5}{2-(-1)}$

$= \frac{-4}{3}$

$\therefore$ the slopes are negative reciprocals

$\therefore$ k and l are perpendicular.

③

Ex. 7. Determine the equation of the line in slope, y-intercept form that is:

a) ① perpendicular to $2x - y - 4 = 0$ and has ② the same y-intercept as $\frac{2}{3}x + \frac{3}{4}y + 6 = 0$.

① $2x - y - 4 = 0$

$\frac{-y}{-1} = \frac{-2x}{-1} + \frac{4}{-1}$

$y = 2x - 4$

$m = 2$

$\boxed{\therefore m_\perp = -\frac{1}{2} \text{ or } m_{perp.} = -\frac{1}{2}}$

② $\frac{2}{3}x + \frac{3}{4}y + 6 = 0$

For y-int: let $x = 0$

$\frac{2}{3}(0) + \frac{3}{4}y + 6 = 0$

$\cancel{4}\left(\frac{3}{\cancel{4}}y\right) + (6)4 = (0)4$

$3y + 24 = 0$

$\frac{3y}{3} = \frac{-24}{3}$

$y = -8$

③ $m = -\frac{1}{2}$, $b = -8$

$\therefore y = -\frac{1}{2}x - 8$

b) ① parallel to $3x - 5y - 30 = 0$ and has ② the same x-intercept as $y = -\frac{9}{5}x + 6$.

① $3x - 5y - 30 = 0$

$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{30}{-5}$

$y = \frac{3}{5}x - 6$

$\boxed{\therefore m = \frac{3}{5}}$

② $y = -\frac{9}{5}x + 6$

For x-int: Let $y = 0$

$0 = -\frac{9}{5}x + 6$

$5(0) = \frac{\cancel{5}}{1}\left(-\frac{9}{\cancel{5}}x\right) + 5(6)$

$0 = -9x + 30$

$9x = 30$

$x = \frac{30}{9}$

$\therefore x = \frac{10}{3}$ $\quad \therefore pt\left(\frac{10}{3}, 0\right)$

③ $m = \frac{3}{5}$, $b =$ ___

$x = \frac{10}{3}$, $y = 0$

Find b:

$0 = \frac{\cancel{3}}{\cancel{5}}\left(\frac{\cancel{10}}{\cancel{3}}\right) + b$

$0 = 2 + b$

$-2 = b$

$\therefore y = \frac{3}{5}x - 2$ is the equation.

Applications

Ex. 1. In the relation $C = 60 + 15n$, C represents the total cost of holding an event at a hall in $, and n represents the number of guests. Use the equation to determine the:

a) ***slope*** and explain what it means as a ***rate of change***

b) C**− *intercept*** and explain what it means

c) number of guests that can attend an event if the total cost can't exceed $1900

$C = 15n + 60$

$y = mx + b$

a) slope $= 15$

$= \frac{15 \text{ \$}}{1 \text{ \# of guest}}$

The rate of change is the price increases by $15/guest

b) The C-int is 60 and means that the flat fee is $60 $ with no guests the charge is $60.

c) Find n if C=1900

$1900 = 15n + 60$

$-15n = 60 - 1900$

$\frac{-15n}{-15} = \frac{-1840 \div 5}{-15 \div 5}$

$n = \frac{368}{3}$

$n = 122\frac{2}{3}$

$n = 122\frac{2}{3}$

$15\overline{)1840}$ = 122, −15, 34, −30, 40, 30, 10

$3\overline{)368}$ = 122, −3, 06, −6, 08, −6, 2

∴ the most guests he could invite would be 122.

Ex. 2. Hannah's total pay for a two week pay period includes a base salary and a percent of her sales. The following table shows her total pay for three different sales levels. Use the table to determine:

a) the equation that represents the relationship between Hannah's total pay and her sales

b) the meaning of the slope and y − intercept in this relation

c) Hannah's total pay when her sales are $47 000

Sales ($)	Total pay ($)
15 000	1700
17 500	1825
28 000	2350

a) Let x represent her total sales, in $

Let y represent her total pay, in $.

(15000, 1700) (17500, 1825)

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1825 - 1700}{17500 - 15000}$

$m = \frac{125}{2500}$

$m = \frac{5}{100}$

$m = \frac{1}{20}$

Find b:

$1700 = \frac{1}{\cancel{20}}(\cancel{15000}) + b$

$1700 = 750 + b$

$1700 - 750 = b$

$b = 950$

∴ the equation is

$y = \frac{1}{20}x + 950$

b) $m = \frac{1}{20} = \frac{5}{100} = 5\%$

The slope means that she earns 5% of her total sales

The y-int of 950 is the base salary she earns. ($950)

c) Find y if x=47000

$y = \frac{1}{\cancel{20}}(\cancel{47000}) + 950$

$y = 2350 + 950$

$y = 3300$

∴ her total pay is $3300 when she has sales of $47000.

Ex. 3. The Stacey family is flying home from their Aunt Kelsey's home in their Cesna. They are travelling at a constant speed. After 2 hours of travel, they are 560 km from home and after 4 hours of travel, they are 280 km from home.

linear

a) Determine an equation to represent this distance-time relationship.

b) What do the slope and y-intercept of your equation mean in this situation?

c) Determine the length of their trip home.

d) Graph the relation.

a) Let x represent the time spent flying, in hours.
Let y represent the distance from home, in km.
$(2, 560)$ $(4, 280)$ ← must state these

Find m: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{280 - 560}{4 - 2}$

$m = \frac{-280}{2}$ $\frac{\text{km}}{\text{hrs}}$

$m = -140$ km/hr

$m = -140$, $b = \_\_\_$

$x = 2$, $y = 560$

Find b: $y = mx + b$

$560 = -140(2) + b$

$560 = -280 + b$

$560 + 280 = b$

$840 = b$ $(0, 840)$

$\therefore y = -140x + 840$ is the equation.

b) The slope of -140 means they are flying home at a rate (speed) of 140 km/h.
The y-int of 840 means they were initially 840 km from home.

c) Find x if $y = 0$:

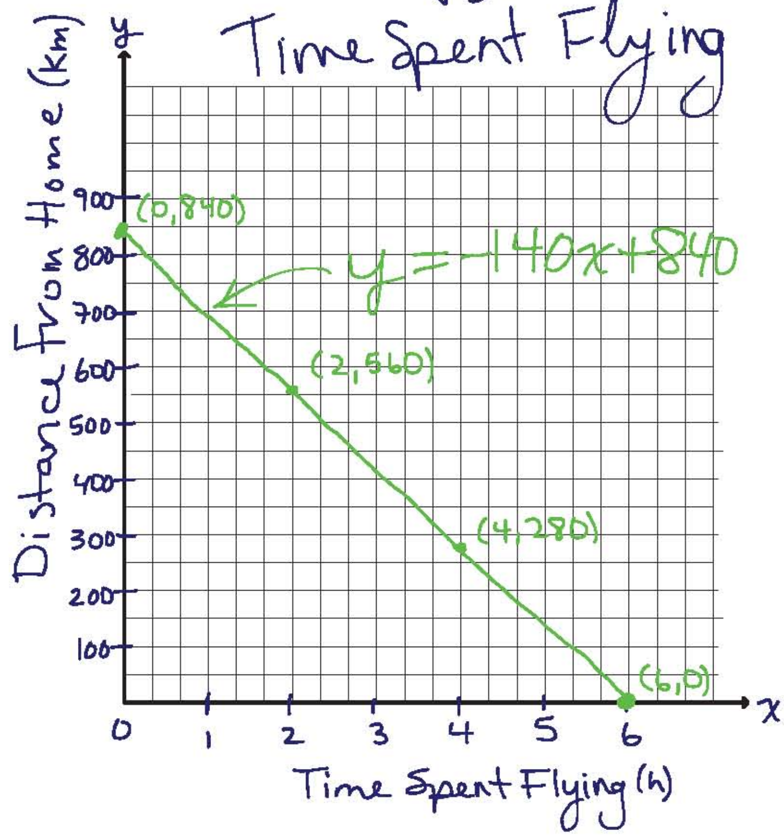
$0 = -140x + 840$

$\frac{140x}{140} = \frac{840}{140}$

$x = 6$

$\therefore$ it took 6 hours to get home.
$(6, 0)$

Distance from Home vs. Time Spent Flying

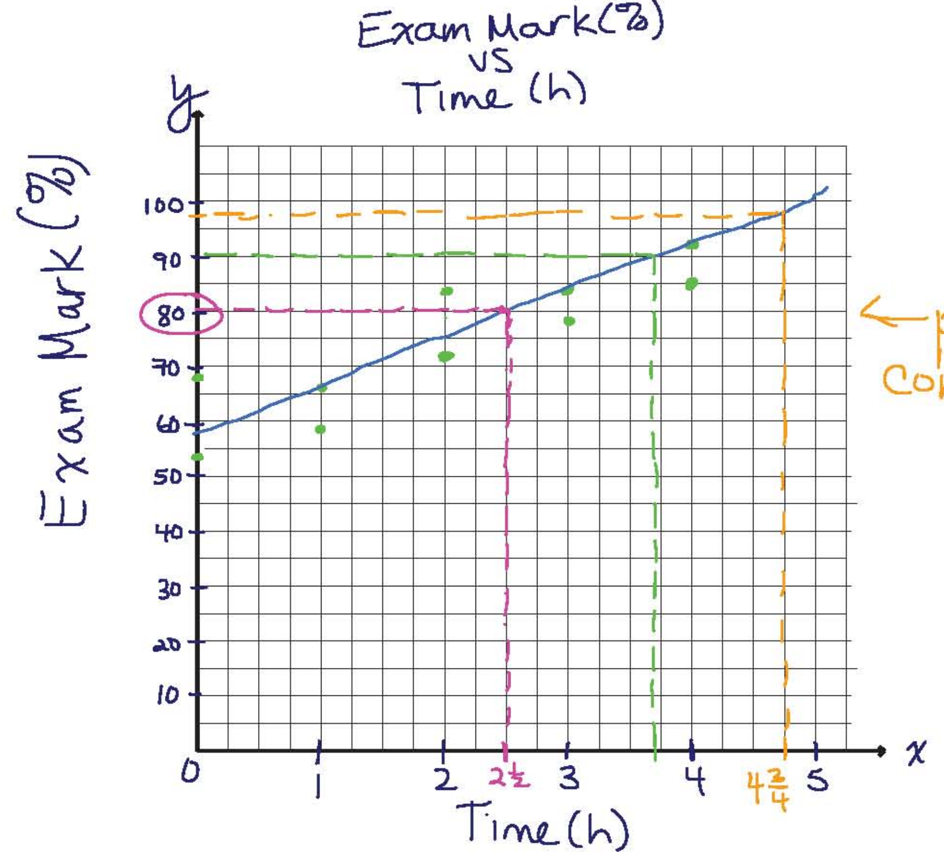


Scatter Plots and Line of Best Fit

Ex. 1. A teacher wants to find whether the length of time students study is related to their exam mark. She collects data using a survey and organized the data into a table.

	Student	A	B	C	D	E	F	G	H	I	J	
x	Time (h)	4	0	3	1	3	0	1	2	2	4	0–4
y	Exam Mark (%)	85	68	78	66	84	53	58	84	72	92	53–92

a) Draw and label the scatter plot for this data.



← positive correlation

Strategy for drawing a line of best fit:

- follows the trend of the points.
- even distribution of points above and below (number & space)
- must go through at least 2 points.

b) Describe the trend, if one exists.

As the time spent studying increases, the exam mark increases.

As (ind. variable) increases, the (dep. variable) increase/decrease

c) Draw the line of best fit.

d) Predict Drake's mark if he studied for $2\frac{1}{2}$ hours.

$\doteq 80\%$ (interpolate)

e) Predict Stacey's mark if she studied for $4\frac{3}{4}$ hours.

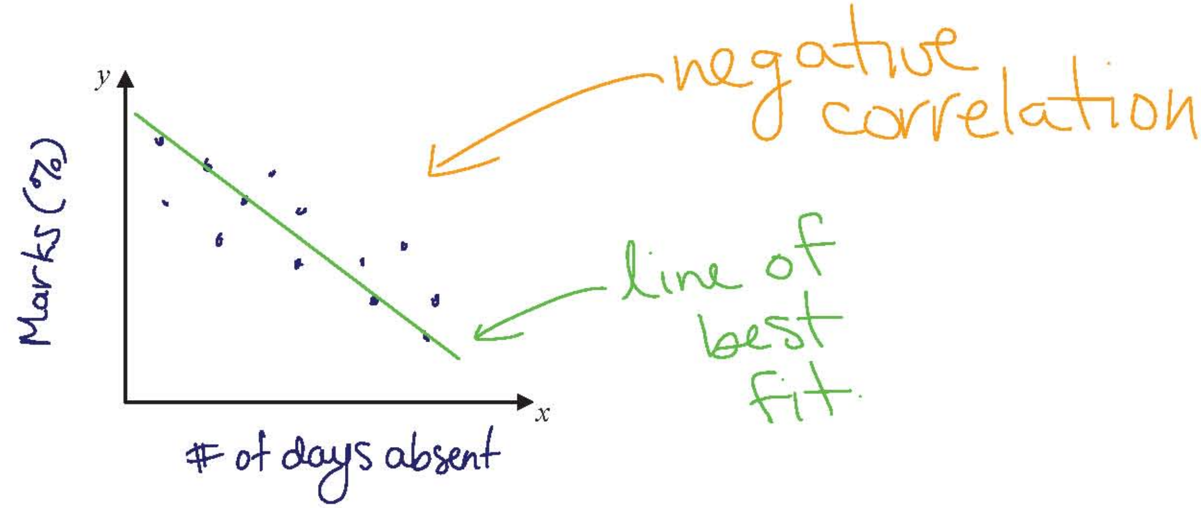
$\doteq 98\%$ (extrapolate)

f) How long should you study for if you want 90% ?

$\doteq 3\frac{3}{4}$ hour (interpolate)

Summary:

1. A **scatter plot** is a graph that shows the relationship between a set of numeric data. In the example on side 1, marks increased as the independent variable (time spent studying) increased. Below, sketch a scatter plot where marks ***decrease*** as the independent variable increases. Label the axes.



2. The points in a scatter plot may show a general pattern or trend. From the trend you can describe a ***relationship***. The relationship will have one of two **types of correlation**, either:

   Positive correlation – As the independent variable increases, the dependent variable increases so the cluster of points appear to ***rise up*** to the right .

   Negative correlation – As the independent variable increases, the dependent variable decreases so the cluster of points appear to ***fall down*** to the right.

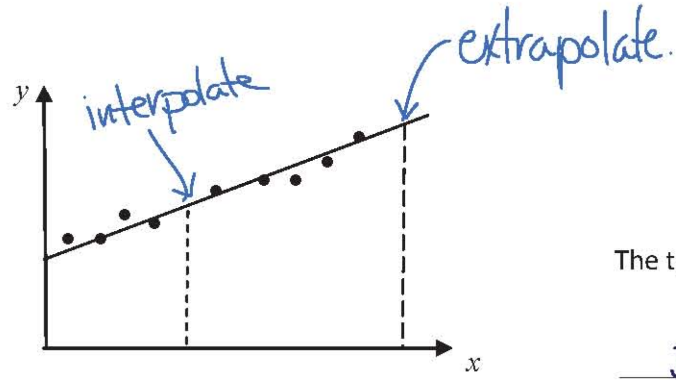
3. A line that approximates a trend is called a line of best fit.

   This line should pass through as many points as possible with an equal number of points above and below the line.

4. The line can be used to make predictions.

   Interpolate – to estimate values inside the range of data given.

   Extrapolate– to estimate values outside the range of data given.

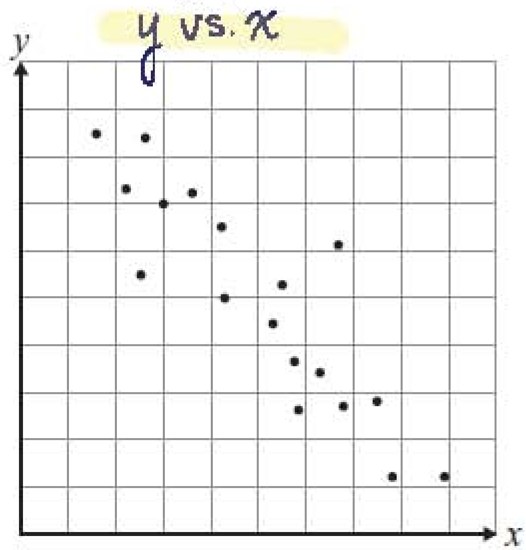


The type of correlation here is: positive correlation

Scatter Plots and *Equations* of Lines of Best Fit

Ex. 1. Consider the graph below.

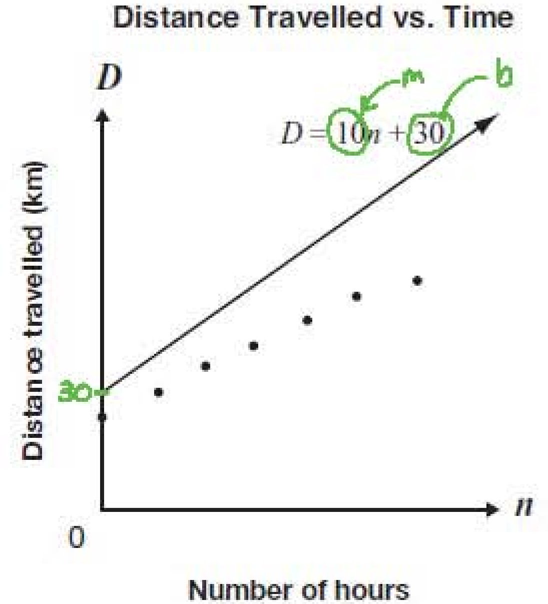
y vs. x



Which relationship is most likely to be represented by this graph?

~~A.~~ height vs. weight
~~B.~~ pay vs. number of hours worked
C. gas remaining vs. distance travelled
~~D.~~ volume of water in a bucket vs. its mass

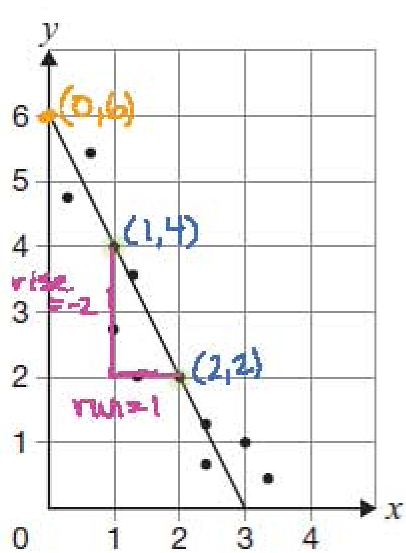
Ex. 2. Consider the graph below.



Which equation best represents the ***line of best fit*** for the data?

~~A.~~ $D = 5n + 33$
B. $D = 8n + 23$
~~C.~~ $D = 10n + 18$
~~D.~~ $D = 12n + 25$

Ex. 3. Determine an ***equation*** for the ***line of best fit*** drawn on the scatter plot below. } * only use points on the line of best fit.



$y = mx + b$

slope: $m = \frac{\text{rise}}{\text{run}}$
$= \frac{-2}{1}$
$= -2$

y-int: $b = 6$ (from graph)

$\therefore$ $y = -2x + 6$ is the equation of the line of best fit.

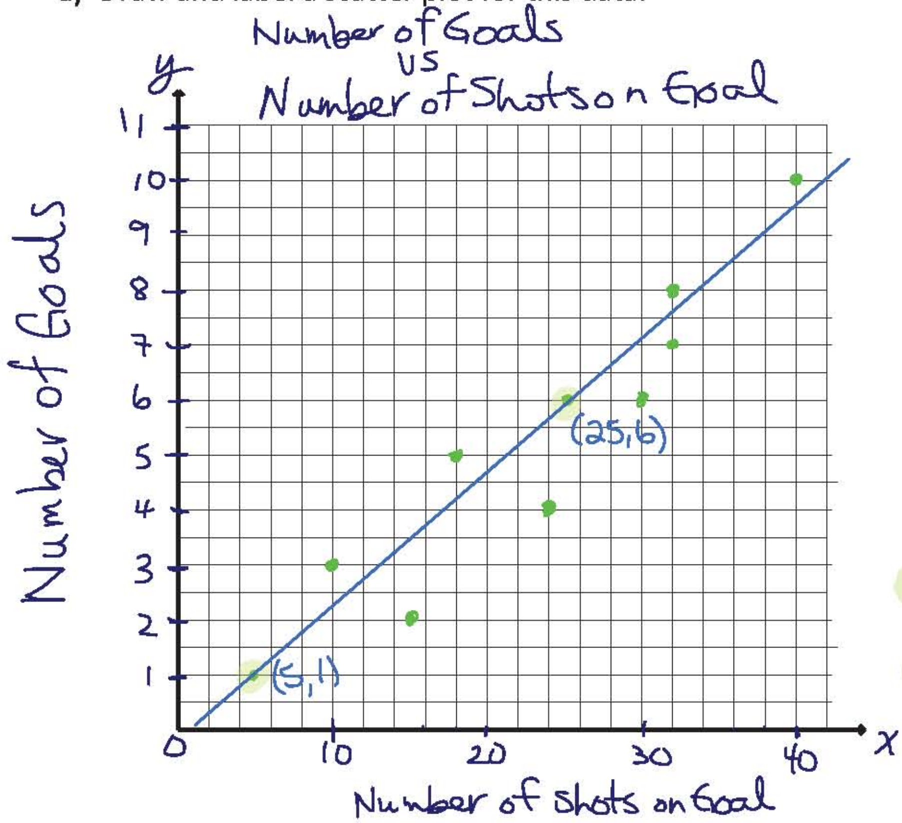
Ex. 4. A hockey team is interested in the relationship between the number of shots on goal and the number of goals scored. The following table shows the results from ten games.

x	**Number of Shots on Goal**	10	32	18	5	40	25	30	15	32	24
y	**Number of Goals Scored**	3	8	5	1	10	6	6	2	7	4
					*		*				

5–40

1–10

a) Draw and label a scatter plot for this data.



b) Describe the relationship (trend) between the variables if one exists.

As the number of shots on goal increases, the number of goal increases.

c) What type of ***correlation*** exists between the variables?

positive correlation.

d) Draw a ***line of best fit.***
*(Make sure the line passes through the two points with a *.)*

e) Determine an ***equation*** for the ***line of best fit***.

Using points (5,1) and (25,6).

Find m: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-1}{25-5}$

$= \frac{5}{20}$

$\boxed{\therefore m = \frac{1}{4}}$ ← # of goals / ← # shots on goal

Find b: if $x = 5, y = 1, m = \frac{1}{4}$

$1 = \frac{1}{4}(5) + b$

$\frac{4}{4} = \frac{5}{4} + b$

$\frac{4}{4} - \frac{5}{4} = b$

$\boxed{-\frac{1}{4} = b}$

$\therefore$ the equation of the line of best fit is $y = \frac{1}{4}x - \frac{1}{4}$

f) Use the equation to predict the number of:

i) goals scored if there were 21 shots on goal

Find y if $x = 21$

$y = \frac{1}{4}(21) - \frac{1}{4}$

$y = \frac{21}{4} - \frac{1}{4}$

$y = \frac{20}{4}$

$y = 5$

$\therefore$ they should score 5 goals with 21 shots on goal.

ii) shots on goal if 8 goals were scored

Find x if $y = 8$:

$4(8) = \cancel{4}\left(\frac{1}{\cancel{4}}x\right) - \cancel{4}\left(\frac{1}{\cancel{4}}\right)$

$32 = x - 1$

$32 + 1 = x$

$x = 33$

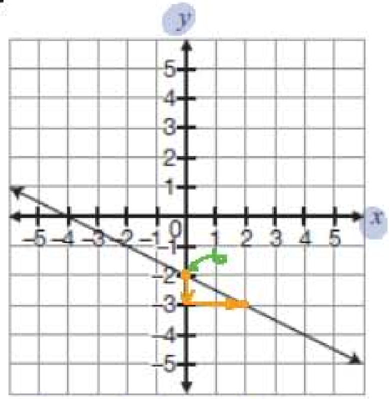
$\therefore$ they should take 33 shots on goal to score 8.

Unit 7 Review – Part I

Warm-up:

1. Determine an equation for each of the following linear relations.

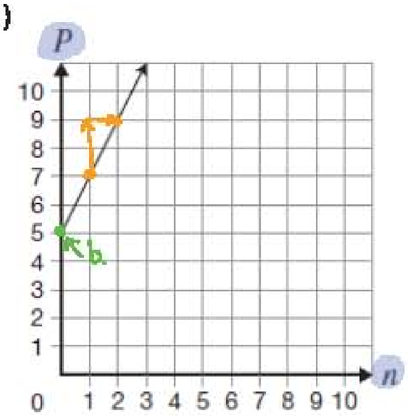
a)



$m = -\frac{1}{2}$, $b = -2$

$\therefore y = -\frac{1}{2}x - 2$ is the equation

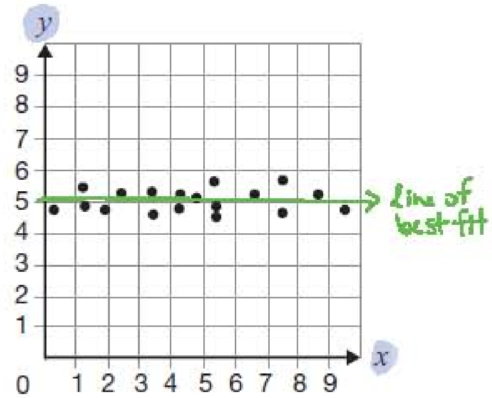
b)



$m = \frac{2}{1}$, $b = 5$

$\therefore P = 2n + 5$ is the equation

c)



$\therefore y = 5$ is the equation of the line of best fit.

2. The total cost of swimming at a community swimming pool is made up of a membership fee and a cost per swim. At this community centre, Jake pays a total of \$100 and swims 40 times. Paula pays a total of \$70 and swims 25 times. Determine:

a) an equation for the total cost of swimming at this centre.

b) the total cost of swimming three times a week for the year.

a) Let x represent the number of swims.
Let y represent the total cost, in \$

(40, 100) (25, 70)

① Find m: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{70 - 100}{25 - 40}$

$m = \frac{-30}{-15}$

$\boxed{\therefore m = 2}$

$m = 2$, $b =$ ___

$x = 25$, $y = 70$

② Find b: $y = mx + b$

$70 = 25(2) + b$

$70 = 50 + b$

$70 - 50 = b$

$\boxed{\therefore b = 20}$

$\therefore$ the equation is $y = 2x + 20$.

** Note: the membership fee is \$20; cost per swim is \$2.

b) Find y if

$x = 52 \times 3$

$x = 156$

$y = 2(156) + 20$

$y = 312 + 20$

$y = 332$

$\therefore$ it will cost \$332 for 3 swims a week for a year

3. Determine the value of k if the points $A(-3,-6)$, $B(1,k)$ and $C(4,8)$ are *collinear.*

$x_1\ y_1$ $x_2\ y_2$ $x_1\ y_1$ $x_2\ y_2$

If collinear: $m_{AB} = m_{BC}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\frac{k-(-6)}{1-(-3)} = \frac{8-k}{4-1}$$

$$\frac{k+6}{4} = \frac{8-k}{3}$$

$$\frac{12^{3}}{1}\left(\frac{k+6}{4_1}\right) = \frac{12^{4}}{1}\left(\frac{8-k}{3_1}\right)$$

$$3k+18 = 32-4k$$

$$3k+4k = 32-18$$

$$7k = 14$$

$$\boxed{\therefore k = 2}$$

4. Determine the equation of a line that is perpendicular to the line $5x-3y+15=0$ ① with the same x-intercept as the line $y=\frac{1}{2}x+3$. ②

① $5x-3y+15=0$

Rearrange for slope:

$$\frac{-3y}{-3} = \frac{-5x}{-3} \frac{-15}{-3}$$

$$y = \frac{5}{3}x + 5$$

$$m = \frac{5}{3}$$

$$\boxed{\therefore m_{perp} = -\frac{3}{5}}$$

② $y = \frac{1}{2}x+3$

For x-int; let $y=0$:

$$0 = \frac{1}{2}x + 3$$

$$2(0) = \frac{2}{1}\left(\frac{1}{2}x\right) + 2(3)$$

$$0 = x + 6$$

$$x = -6$$

$$\boxed{\therefore \text{point } (-6,0)}$$

③ $m = -\frac{3}{5}$, $b = \_\_$

$x=-6 \quad y=0$

Find b: $y = mx+b$

$$0 = -\frac{3}{5}\left(\frac{-6}{1}\right) + b$$

$$0 = \frac{18}{5} + b$$

$$-\frac{18}{5} = b$$

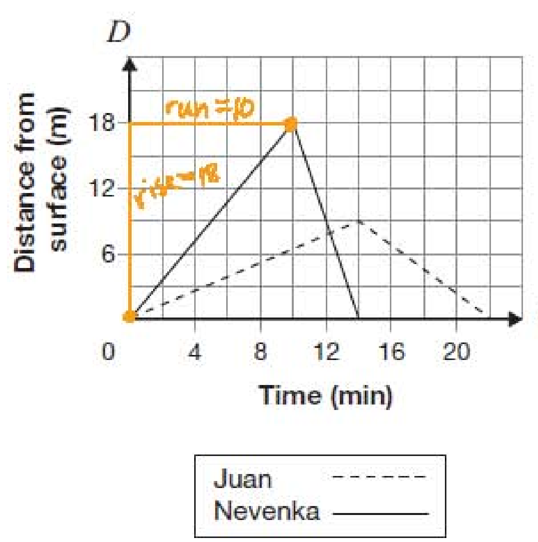
$\therefore y = -\frac{3}{5}x - \frac{18}{5}$ is the equation.

5. Nevenka and Juan scuba dive. The graph below represents the relationship between the distance from the surface, in metres, and time, in minutes, for both divers as they swim down from the surface and then swim back up.

a) Complete the description of Nevenka's dive.

Nevenka enters the water and swims down to a depth of 18 m *below the surface of the water at a rate of* $\frac{18}{10} = \frac{9}{5} = 1.8$ m/min. *Nevenka then immediately swims back up to the water's surface at a rate of* $\frac{18}{4} = \frac{9}{2} = 4.5$ m/min

Distance from Surface vs. Time



b) Complete the description of Juan's dive.

Juan enters the water and swims down to a depth of 9 m *below the surface of the water at a rate of* $\frac{9}{14}$ m/min. *Juan then immediately swims back up at a rate of* $\frac{9}{8} = 1\frac{1}{8}$ m/min.

Unit 7 Review – Part II

Warm-up:

1. Wing length is a reliable method for determining the age of young birds. Below is an example of data for a particular species.

Wing length (cm)	Age (days)
*1.5	4
3.1	8
3.2	10
*4.0	12
5.2	16

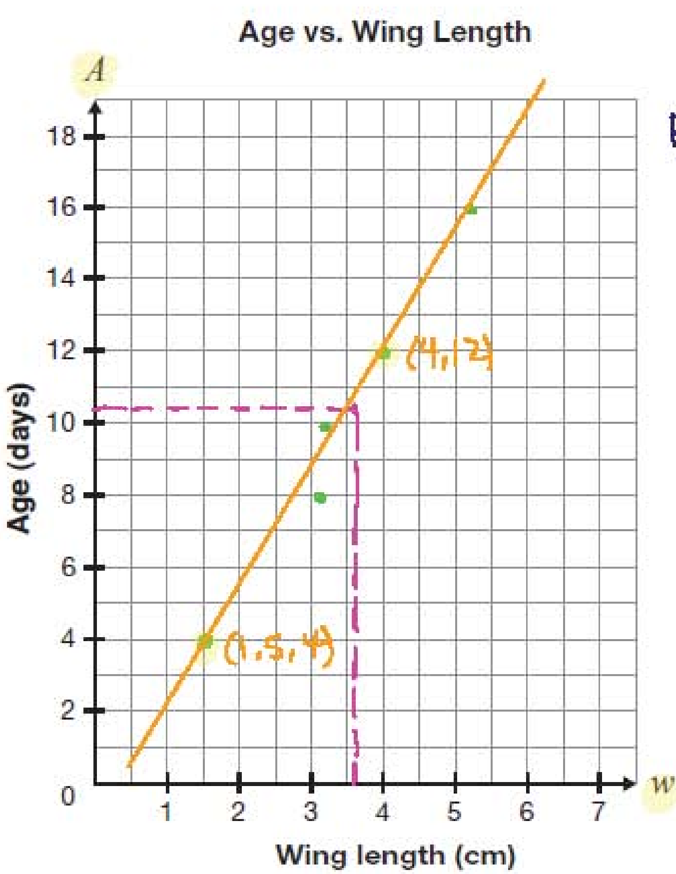
a) Construct a scatter plot on the grid provided for the data in the table.

b) Sketch a line of best fit.

c) Determine an equation for the line of best fit. *(Make sure the line passes through the two points with a *.)*

d) Use the graph to estimate the age of a bird with a wing length of 3.6 cm.

e) Use the equation to estimate the wing length of a three week old bird.



c) (1.5, 4) (4, 12)

Find m:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{12-4}{4-1.5}$

$m = \frac{8}{2.5}$ ×10 ×10

$m = \frac{80}{25}$

$m = \frac{16}{5}$

Find b:

$y = mx + b$

$12 = \frac{16}{5}(4) + b$

$\frac{12}{1} = \frac{64}{5} + b$

$\frac{60}{5} = \frac{64}{5} + b$

$-\frac{4}{5} = b$

$m = \frac{16}{5}$
$b =$ __
$x = 4$
$y = 12$

$\therefore A = \frac{16}{5}w - \frac{4}{5}$ is the equation of the line of best fit.

d) The bird is approx. 10 days old. (interpolation)

e) 3 weeks
= 3 × 7 days
= 21 days

Find w if A = 21:

$21 = \frac{16}{5}w - \frac{4}{5}$

$5(21) = \cancel{5}\left(\frac{16}{\cancel{5}}w\right) - \cancel{5}\left(\frac{4}{\cancel{5}}\right)$

$105 = 16w - 4$

$105 + 4 = 16w$

$\frac{109}{16} = \frac{16w}{16}$

$w = \frac{109}{16}$

$w \doteq 6.8$

$16\overline{)109.00}$ = 6.81
−96
130
−128
20
−16
4

∴ the wing length is approx 6.8 cm at 3 weeks.

2. Abigail buys a prepaid card for her cellphone. When she talks on her phone, a fee per minute is deducted from the value of the prepaid card. The table below shows information about the remaining value of the card.
   a) Determine the equation that represents the relationship between the remaining value and the total number of minutes used.
   b) What do the slope and V-intercept mean in this situation?
   c) How many minutes can she use before she has to reload her card?

Total number of minutes used, t	Remaining value, V ($)
10	22.00
20	19.00

a) (10, 22) (20, 19)

Find m:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{19-22}{20-10}$ $\frac{\$}{min}$

$m = \frac{-3}{10}$ \$/min

$m = -0.3$ \$/min

$m = \frac{-3}{10}$, b = ___

$x = 10$, $y = 22$

Find b: $y = mx + b$

$22 = -\frac{3}{\cancel{10}_1}(\cancel{10}^1) + b$

$22 = -3 + b$

$22 + 3 = b$

$b = 25$

$\therefore V = -0.3t + 25$

or $V = -\frac{3}{10}t + 25$ is the equation.

b) The slope of −0.3 means that \$0.30 is deducted each minute.
The V-intercept of 25 means that the card had an initial value of \$25.

c) Find t if V=0

$0 = -0.3t + 25$

$0.3t = 25$

$t = \frac{25 \times 10}{0.3 \times 10}$

$t = \frac{250}{3}$ → $t = 83\frac{1}{3}$ or $t = 83.\overline{3}$

∴ She should reload after $83\frac{1}{3}$ minutes (83 min 20 sec.)

3. Tyler walks along a line leading from a motion sensor. Use the information from the graph below to describe Tyler's walk.

Description of Tyler's Walk:

Initially Tyler is 2m *from the motion sensor.*
After $\frac{1}{2}$ *of a second he walks* away *from the motion sensor at a speed of* $1\frac{1}{3}$ m/sec *for* 1.5 *seconds until he is* 4m *away from the sensor. He remains* 4m *away from the sensor for* 1.5 *seconds before he walks back.*

Tyler walks back toward the sensor at a speed of 2m/sec *for* 1.5 *seconds until he is* 1m *away from the sensor. He remains at* 1m *away from the sensor for* 1 *second.*

Distance from Motion Sensor vs. Time