

SLOPE



MPM1D1

Unit 7: Equations of Linear Relations

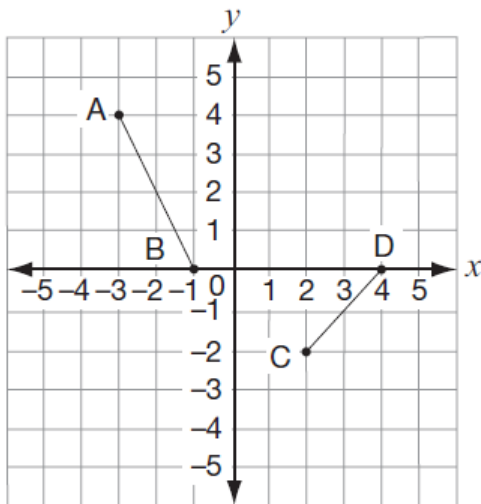
Name: _____

Slopes and Rates of Change

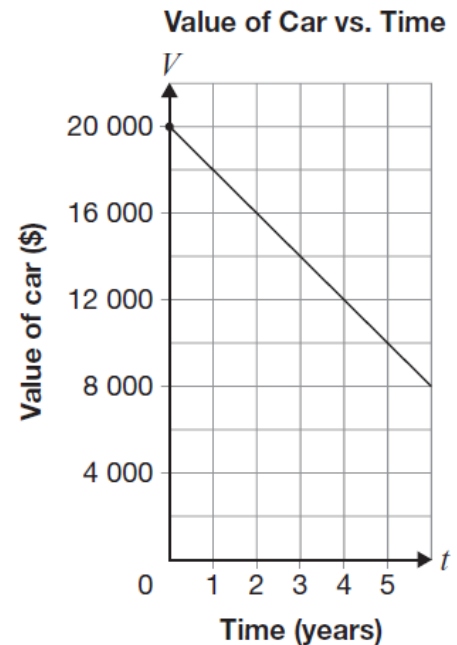
Rate of Change: The **rate of change** of a linear relation tells us how one variable changes as the other variable changes.

- i) Graphically, the **rate of change** is the **steepness** of the line where $slope = \frac{rise}{run}$.
- ii) Algebraically, the **rate of change** is the **slope, m** , in the equation $y = mx + b$.

Ex. 1. Use the graph to determine the **slope, (rate of change)** of each line segment.

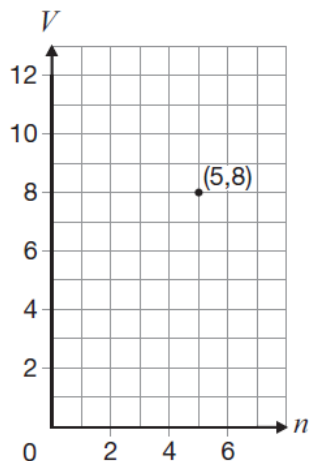


- Ex. 2.** Use the graph to determine the:
- a) **rate of change** in the value of the car
 - b) **V -intercept** and explain what it means
 - c) **equation** of the linear relation
 - d) **t -intercept** and explain what it means



Ex. 3. The point on the grid below belongs to a linear relation that has $-\frac{3}{2}$ as its *rate of change*.

Write the coordinates of two other points that would be on the line passing through this point.



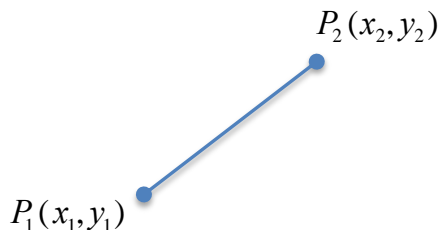
The Slope Formula:

Using the endpoints $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ for a line segment, (or any two points on a line) develop a formula for the slope.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \underline{\hspace{2cm}}$$



Ex. 4. Use the *slope formula* to calculate the slope of the line through each pair of points.

- a)** $A(3, -5)$ and $B(-1, -3)$ **b)** $C(-0.5, 5.4)$ and $D(0.7, 2.1)$ **c)** $E(-3, -2)$ and $F(1, -2)$

Ex. 5. The point $(-1, -1)$ lies on a line with slope $\frac{1}{2}$. Determine the y - coordinate of the point on the line with x - coordinate 9.

Collinear: Three or more points are collinear if they all lie on the same line.

ie. Points A , B , and C are **collinear** if $m_{AB} = m_{BC}$.

Ex. 6. Determine whether the points $P(-6,12)$, $Q(3,6)$ and $R(12,0)$ are collinear.

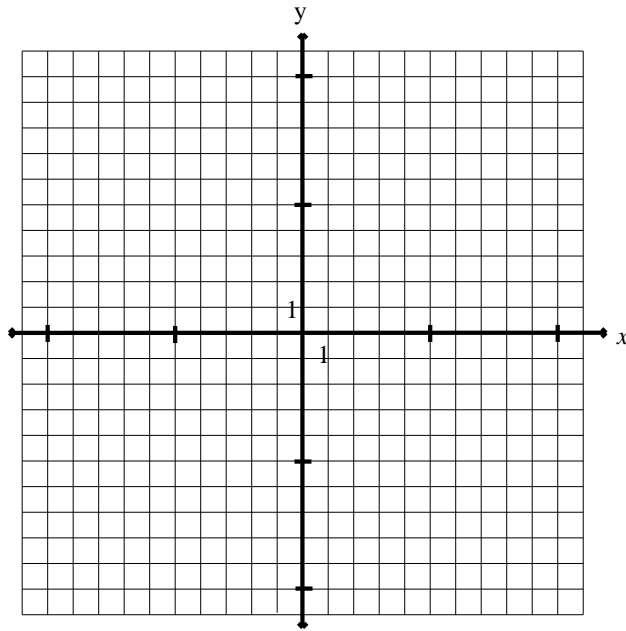
Ex. 7. The table below represents the linear relationship between cost and repair time at an appliance store. Determine the **rate of change** and explain what it means.

Repair time, t (h)	Cost, C (\$)
3	205
6	385
8	505

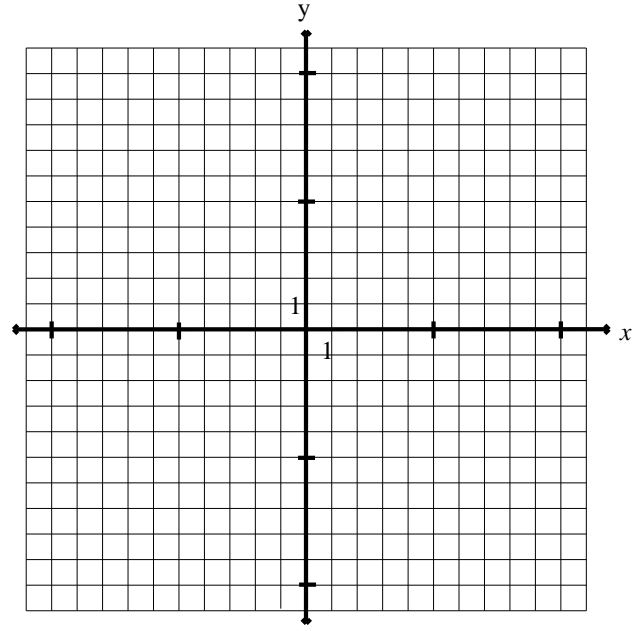
Ex. 8. A bathtub is filling with water at a constant rate. After 3 minutes the water is 7.5 cm deep, and after 8 minutes the water is 15 cm deep. At what rate is the depth of water increasing?

Grids For Homework: p. 278

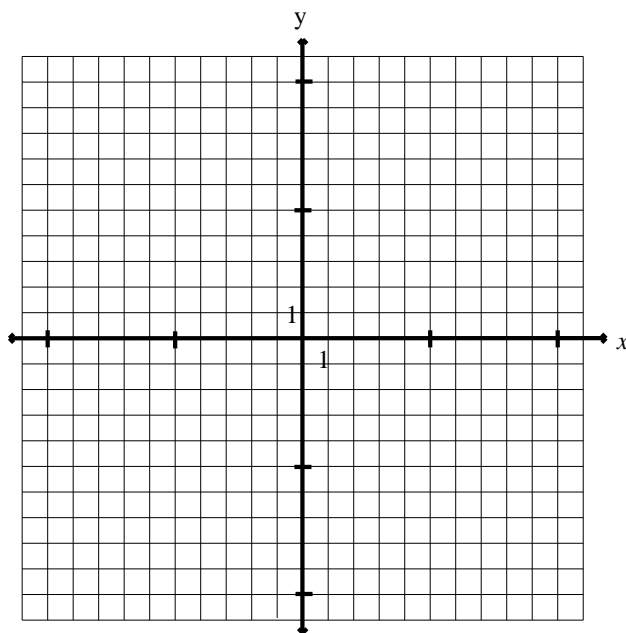
#4. a) and b)



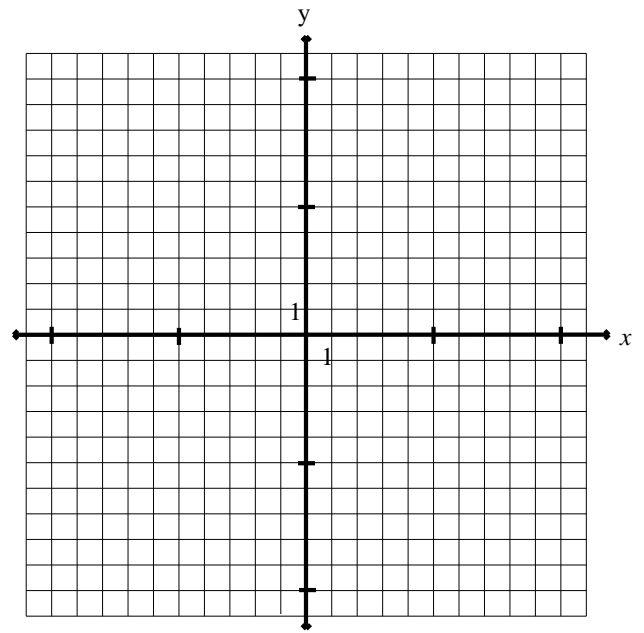
#4. c) and d)



#7.



#8.



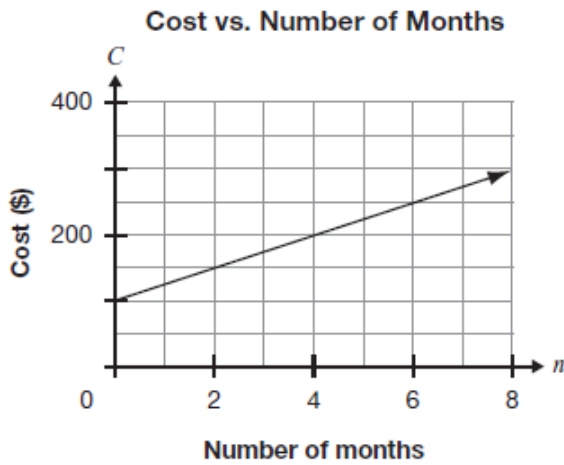
Using a Point and Slope to Determine the Equation of a Line

Warmup:

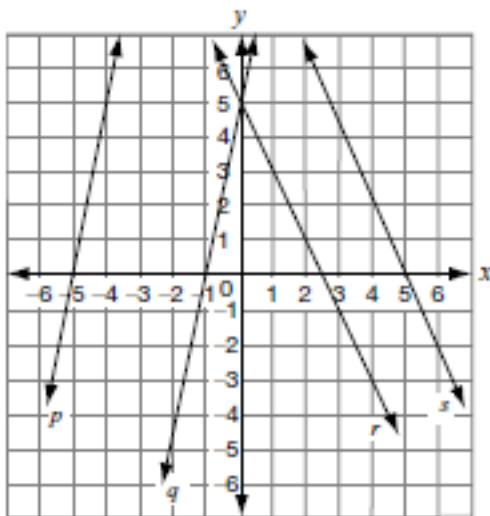
1. If the equation $y = mx + 5$ represents a line that passes through the point $(3, 7)$, find m .

2. If the equation $y = -\frac{5}{6}x + b$ represents a line that passes through the point $(-2, 1)$, find b .

Ex. 1. The graph below represents the cost to belong to a local gym. Determine the equation that represents the graph in the form $y = mx + b$.



Ex. 2. Determine the equation of each line in the form $y = mx + b$.



Ex. 3. Determine the equation of each line in the form $y = mx + b$ described below.

a) has a y -intercept of 0 and a slope of $\frac{1}{2}$

b) passing through $A(0, -3)$, with a slope of -1

c) with a slope of -5 , passing through $A(7, -3)$

d) passing through $A(-2, 1)$, with a slope of $\frac{3}{2}$

e) has an x -intercept of $1\frac{1}{4}$ and a slope of $-\frac{2}{5}$

f) with a slope of $\frac{3}{5}$, passing through $(4, 2)$

Using Two Points to Determine the Equation of a Line

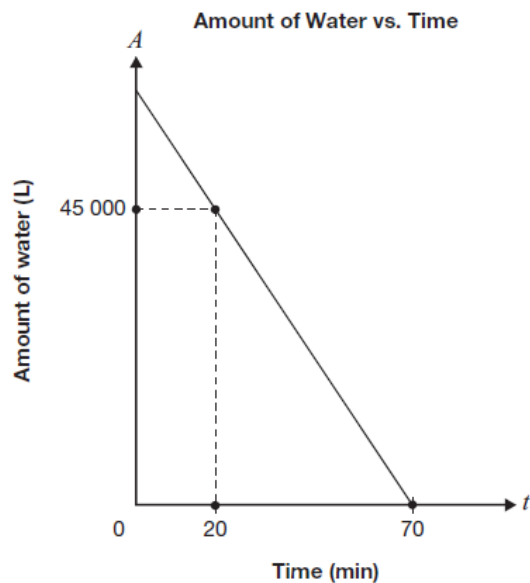
Ex. 1. Find the equation of the line that passes through the point $(-3, 2)$ and has the following slope:

a) $m = \frac{5}{6}$

b) $m = 0$

Ex. 2. Determine the value of k if the points $X(2, 3)$, $Y(8, k)$ and $Z(29, k + 7)$ are *collinear*.

Ex. 3. The graph below represents the relationship between the amount of water, A , in a pool as it drains and time, t . Determine the *equation* of this relation by first determining the *initial amount* of water in the pool and the *rate of change* of this relation.



Ex. 4. Determine the equation of the line that passes through the given two points.

a) $(-6, 10)$ and $(2, 4)$

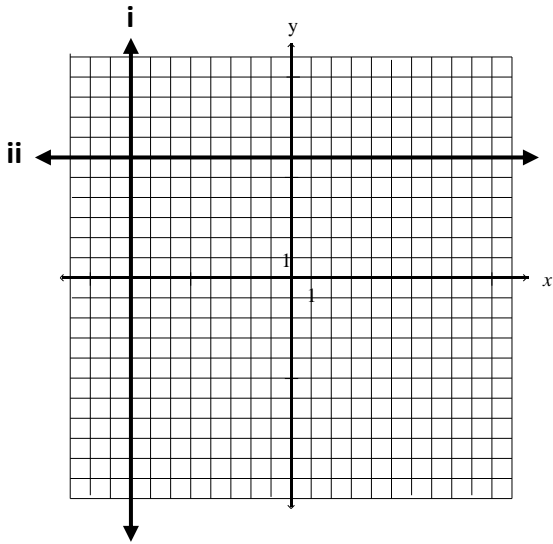
b) y -intercept is -4 and $(-5, -4)$

Ex. 5. Determine the equation of the line with the same x -intercept as the line $5x - 3y + 15 = 0$ and the same y -intercept as the line $x + 8y + 2 = 0$.

Ex. 3. Find the equation of each of the following lines given:

a) i) _____

ii) _____



b) $(-1, 9)$ and $(-1, -2)$ are points on the line

c) the line is horizontal and passes through $(3, 5)$

d) the slope is undefined and $(-3, 1)$ is on the line

e) the line is parallel to $y = -3$ and passes through $(2, 6)$

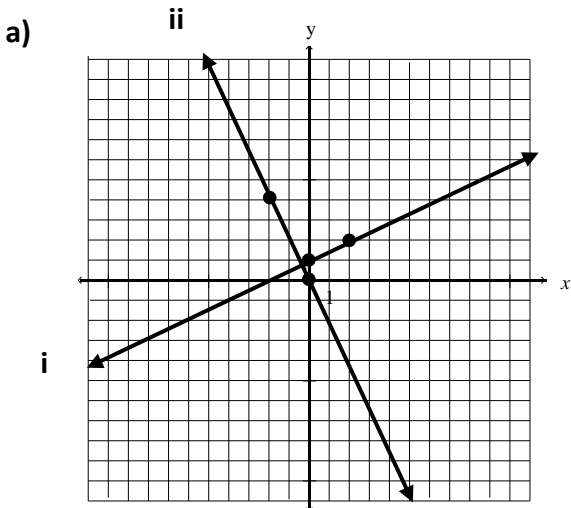
f) the line is perpendicular to the x -axis and passes through $(-3, -8)$

g) perpendicular to $x - 5 = 0$ with a y -intercept of 1

h) passes through $(5, 7)$ and $(-2, 7)$

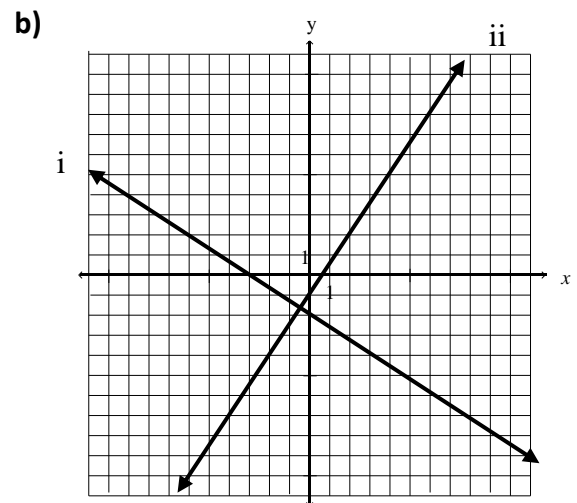
PART B: Parallel and Perpendicular Lines

Ex. 4. Find the equation of each line below in slope, y -intercept form.



i) $m =$ $b =$ \therefore

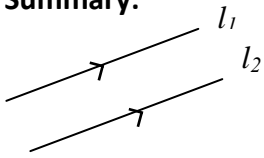
ii) $m =$ $b =$ \therefore



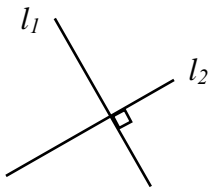
i) $m =$ $b =$ \therefore

ii) $m =$ $b =$ \therefore

Summary:



Parallel Lines have slopes that are _____ .



Perpendicular Lines have slopes that are _____
_____ of each other.

Ex. 5. State whether the following lines are **parallel**, **perpendicular** or **neither** by comparing slopes.

- a) $y = -\frac{1}{4}x + 3$ and $y = \frac{-2}{8}x - 1$ b) $y = -\frac{5}{2}x$ and $y = -0.4x + 2$ c) $y = -x - 3$ and $y = x + 5$

Ex. 6. Determine if lines k and l are **parallel**, **perpendicular** or **neither** by comparing slopes.

k : through points $(-3, -7)$, $(9, 2)$ and l : through points $(-1, 5)$, $(2, 1)$

Ex. 7. Determine the equation of the line in slope, y -intercept form that is:

- a) perpendicular to $2x - y - 4 = 0$ and has the same y -intercept as $\frac{2}{3}x + \frac{3}{4}y + 6 = 0$.

- b) parallel to $3x - 5y - 30 = 0$ and has the same x -intercept as $y = -\frac{9}{5}x + 6$.

Review and follow Ex. 3c from Lesson 2. Show all work on lined paper.

Why Did Gyro Go Into a Bakery?



For each exercise below, find the equation of the line that has the given slope and passes through the given point. Circle the letter next to the correct equation. Then write this letter in each box at the bottom of the page that contains the number of that exercise.

① $m = 2; (3, 2)$	<input type="checkbox"/> G	$y = 2x + 1$	<input type="checkbox"/> R	$y = 2x - 4$
② $m = -3; (1, 4)$	<input type="checkbox"/> O	$y = -3x + 7$	<input type="checkbox"/> P	$y = -3x + 2$
③ $m = -5; (-1, 3)$	<input type="checkbox"/> M	$y = -5x - 2$	<input type="checkbox"/> D	$y = -5x + 6$
④ $m = 3; (-4, -7)$	<input type="checkbox"/> V	$y = 3x + 1$	<input type="checkbox"/> E	$y = 3x + 5$
⑤ $m = -1; (5, -2)$	<input type="checkbox"/> U	$y = -x + 3$	<input type="checkbox"/> C	$y = -x - 1$
⑥ $m = \frac{1}{2}; (6, 1)$	<input type="checkbox"/> W	$y = \frac{1}{2}x - 5$	<input type="checkbox"/> H	$y = \frac{1}{2}x - 2$
⑦ $m = -\frac{2}{3}; (3, 4)$	<input type="checkbox"/> A	$y = -\frac{2}{3}x - 7$	<input type="checkbox"/> I	$y = -\frac{2}{3}x + 6$
⑧ $m = \frac{4}{3}; (-2, 0)$	<input type="checkbox"/> K	$y = \frac{4}{3}x + \frac{5}{2}$	<input type="checkbox"/> F	$y = \frac{4}{3}x + \frac{8}{3}$
⑨ $m = -\frac{1}{4}; (2, 1)$	<input type="checkbox"/> J	$y = -\frac{1}{4}x + \frac{3}{2}$	<input type="checkbox"/> D	$y = -\frac{1}{4}x - \frac{3}{8}$
⑩ $m = 4; (-1, \frac{1}{2})$	<input type="checkbox"/> A	$y = 4x - \frac{2}{3}$	<input type="checkbox"/> T	$y = 4x + \frac{9}{2}$
⑪ $m = -2; (0, 0)$	<input type="checkbox"/> L	$y = -2x$	<input type="checkbox"/> B	$y = -2x - 2$
⑫ $m = 0; (-5, \frac{3}{4})$	<input type="checkbox"/> S	$y = \frac{3}{4}$	<input type="checkbox"/> N	$y = -5x$



9	5	12	10	8	2	1	10	6	4	12	3	4	11	11	2	8	7	10
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Review and follow Ex. 4a from Lesson 3. Show all work on lined paper.

What Happened When Two Fruit Companies Merged ?

For each exercise below, find the equation of the line passing through the given points. Circle the two letters next to the correct equation. Then write these letters in the two boxes at the bottom of the page that contain the number of that exercise.

Answers:

① (1, 5) (2, 7)

IS $y = \frac{2}{3}x + 3$

TH $y = \frac{1}{2}x - 4$

② (0, 1) (3, -8)

AP $y = -\frac{3}{2}x + 8$

UI $y = -3x + 5$

③ (2, -3) (4, -2)

ST $y = \frac{1}{2}x - 7$

DE $y = 2x + 3$

④ (2, 5) (4, 2)

CT $y = -3x + 1$

EY $y = 4x + 7$

⑤ (-3, -5) (-1, 3)

LO $y = -\frac{3}{2}x - 4$

IL $y = 2x + 1$

Answers:

⑥ (3, -1) (-6, -4)

HA $y = \frac{1}{2}x - 1$

ER $y = -\frac{3}{4}x + 4$

⑦ (4, 1) (-4, 7)

IS $y = \frac{1}{3}x + \frac{8}{3}$

EL $y = -2x - 1$

⑧ (-1, 2) (3, 4)

PE $y = -x + 2$

EA $y = -\frac{3}{4}x + 2$

⑨ (-1, -4) (2, 0)

SO $y = \frac{4}{3}x - 2$

AR $y = \frac{1}{3}x - 2$

⑩ (3, -1) (-3, 5)

MA $y = \frac{1}{2}x + \frac{5}{2}$

FE $y = \frac{4}{3}x - \frac{8}{3}$

3	3	5	5	8	8	1	1	4	4	7	7	9	9	2	2	10	10	6	6
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Applications

Ex. 1. In the relation $C = 60 + 15n$, C represents the total cost of holding an event at a hall in \$, and n represents the number of guests. Use the equation to determine the:

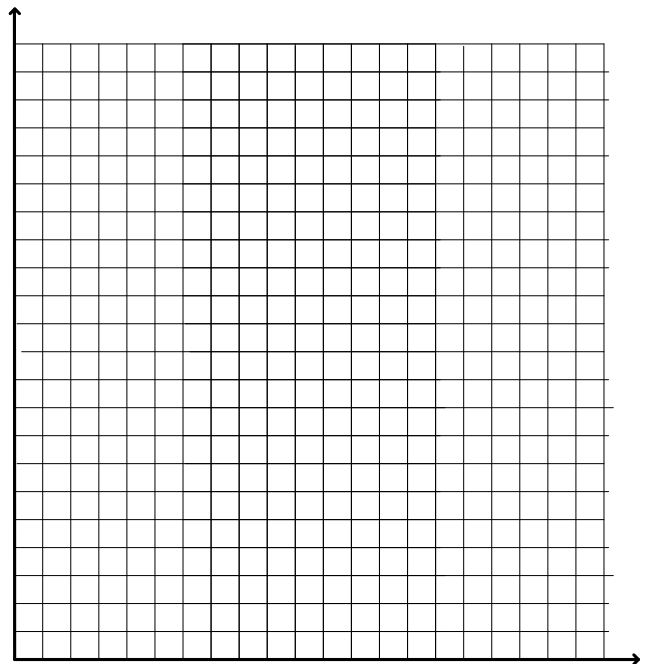
- slope** and explain what it means as a **rate of change**
- C – **intercept** and explain what it means
- number of guests that can attend an event if the total cost can't exceed \$1900

Ex. 2. Hannah's total pay for a two week pay period includes a base salary and a percent of her sales. The following table shows her total pay for three different sales levels. Use the table to determine:

- the equation that represents the relationship between Hannah's total pay and her sales
- the meaning of the slope and y – intercept in this relation
- Hannah's total pay when her sales are \$47 000

Sales (\$)	Total pay (\$)
15 000	1700
17 500	1825
28 000	2350

- Ex. 3.** The Stacey family is flying home from their Aunt Kelsey's home in their Cessna. They are travelling at a constant speed. After 2 hours of travel, they are 560 km from home and after 4 hours of travel, they are 280 km from home.
- Determine an equation to represent this distance-time relationship.
 - What do the slope and y -intercept of your equation mean in this situation?
 - Determine the length of their trip home.
 - Graph the relation.



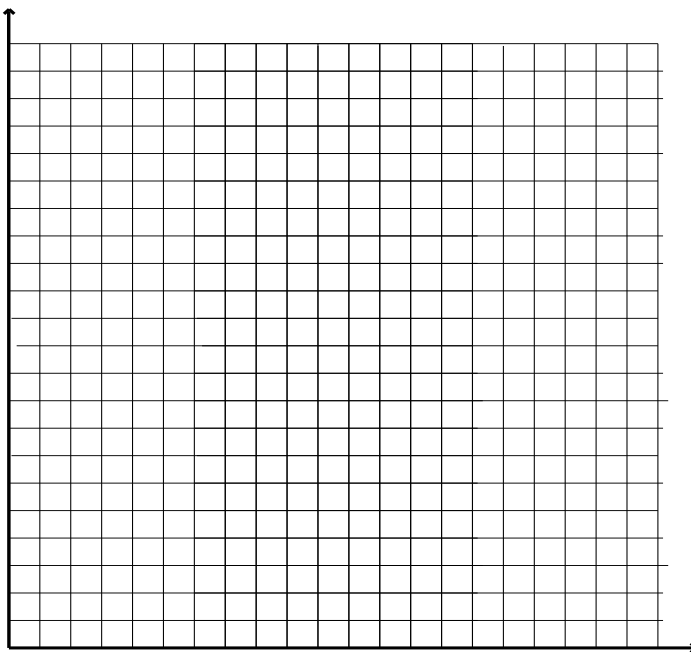
Scatter Plots and Line of Best Fit

Ex. 1. A teacher wants to find whether the length of time students study is related to their exam mark. She collects data using a survey and organized the data into a table.

Student	A	B	C	D	E	F	G	H	I	J
Time (h)	4	0	3	1	3	0	1	2	2	4
Exam Mark (%)	85	68	78	66	84	53	58	84	72	92

a) Draw and label the scatter plot for this data.

Strategy for drawing a line of best fit:



b) Describe the trend, if one exists.

c) Draw the line of best fit.

d) Predict Drake’s mark if he studied for $2\frac{1}{2}$ hours.

e) Predict Stacey’s mark if she studied for $4\frac{3}{4}$ hours.

f) How long should you study for if you want 90% ?

Summary:

1. A **scatter plot** is a graph that shows the relationship between a set of numeric data. In the example on side 1, marks increased as the independent variable (time spent studying) increased. Below, sketch a scatter plot where marks **decrease** as the independent variable increases. Label the axes.



2. The points in a scatter plot may show a general pattern or trend. From the trend you can describe a **relationship**. The relationship will have one of two **types of correlation**, either:

Positive correlation – As the independent variable increases, the dependent variable _____ so the cluster of points appear to **rise up** to the right .

Negative correlation – As the independent variable increases, the dependent variable _____ so the cluster of points appear to **fall down** to the right.

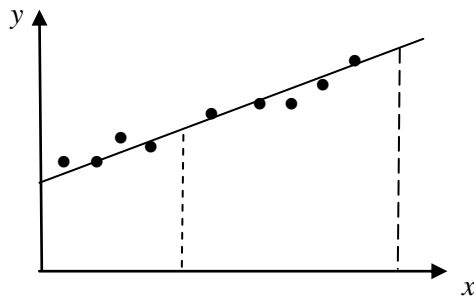
3. A line that approximates a trend is called a _____.

This line should pass through as many points as possible with an equal number of points above and below the line.

4. The line can be used to make predictions.

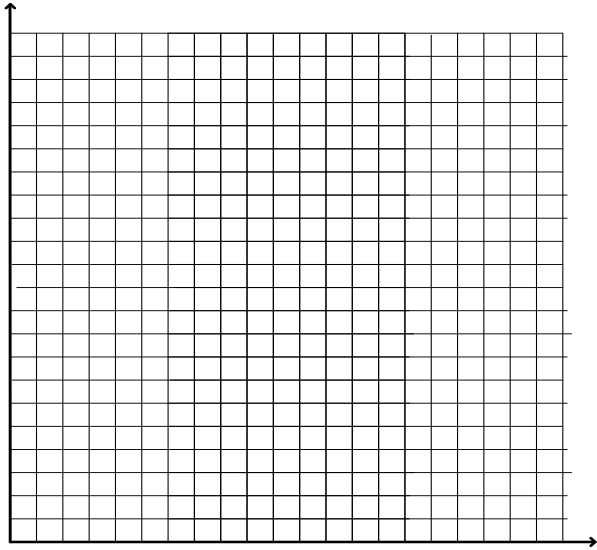
Interpolate – to estimate values _____ the range of data given.

Extrapolate– to estimate values _____ the range of data given.

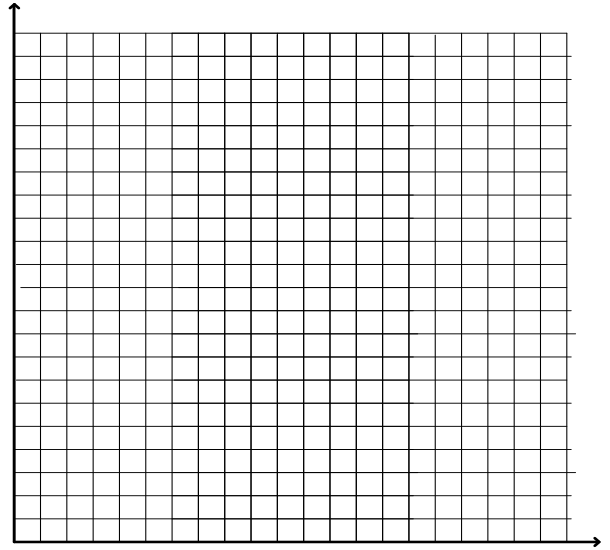


The type of correlation here is:

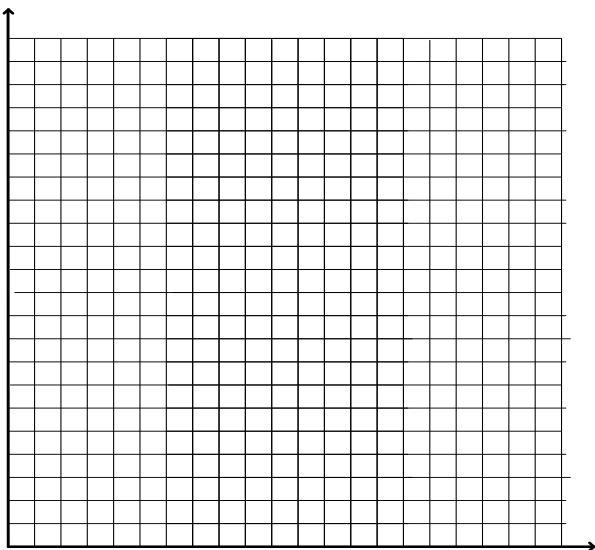
p. 337 #3.



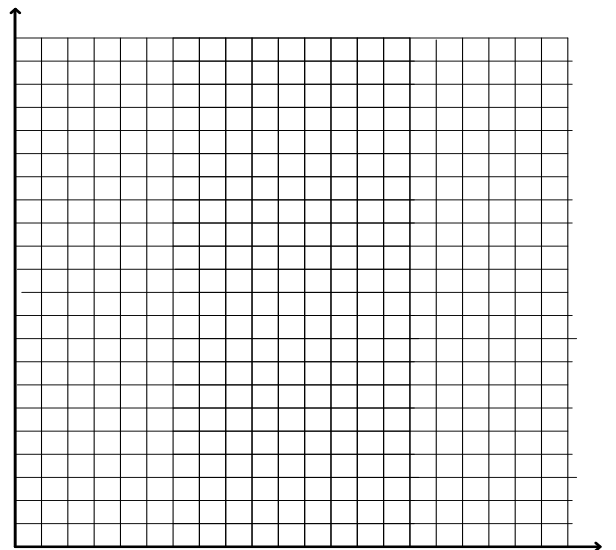
p. 338 #5.



p. 350 #3.

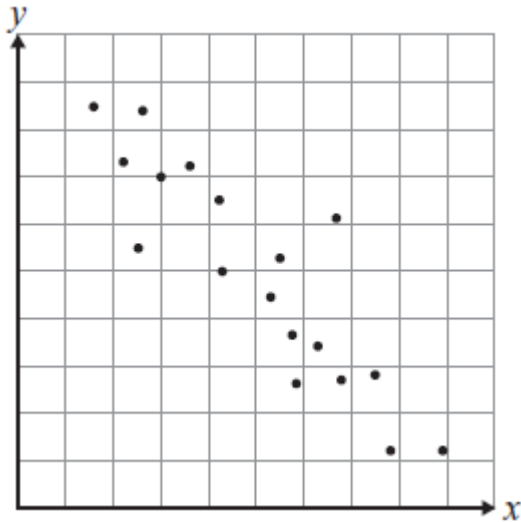


p. 351 #8.



Scatter Plots and *Equations of Lines of Best Fit*

Ex. 1. Consider the graph below.

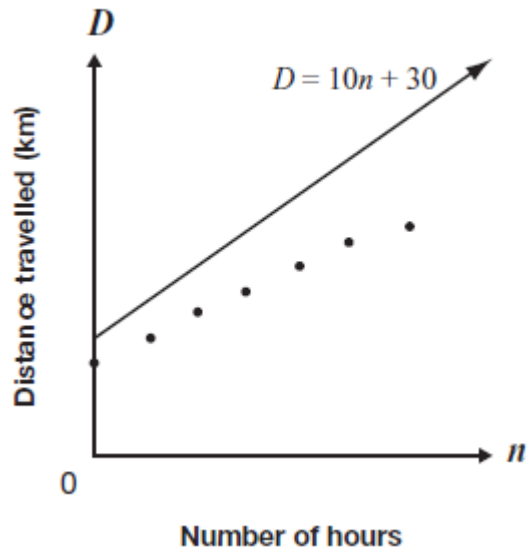


Which relationship is most likely to be represented by this graph?

- A height vs. weight
- B pay vs. number of hours worked
- C gas remaining vs. distance travelled
- D volume of water in a bucket vs. its mass

Ex. 2. Consider the graph below.

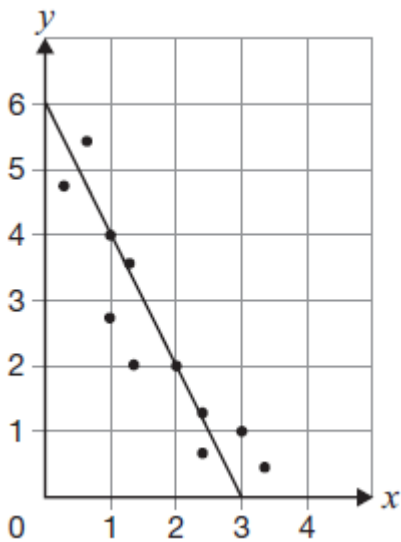
Distance Travelled vs. Time



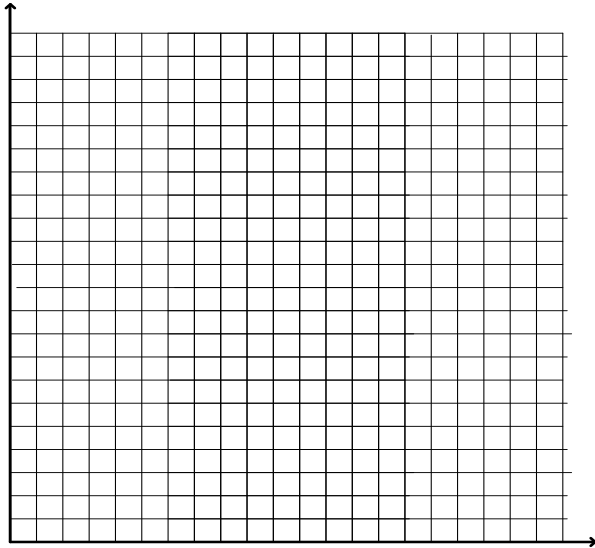
Which equation best represents the *line of best fit* for the data?

- A $D = 5n + 33$
- B $D = 8n + 23$
- C $D = 10n + 18$
- D $D = 12n + 25$

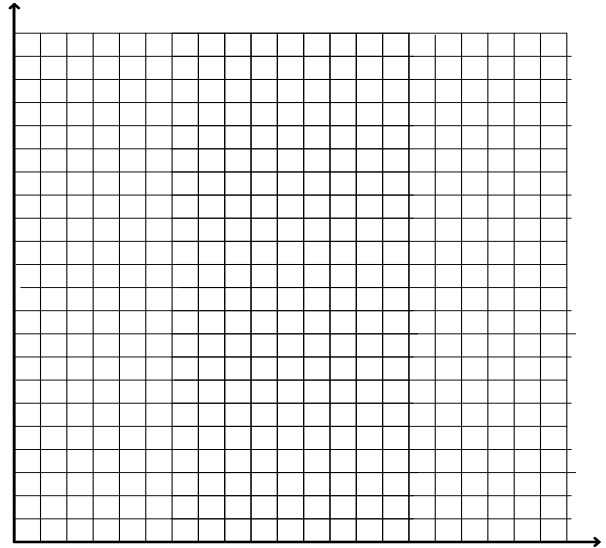
Ex. 3. Determine an *equation* for the *line of best fit* drawn on the scatter plot below.



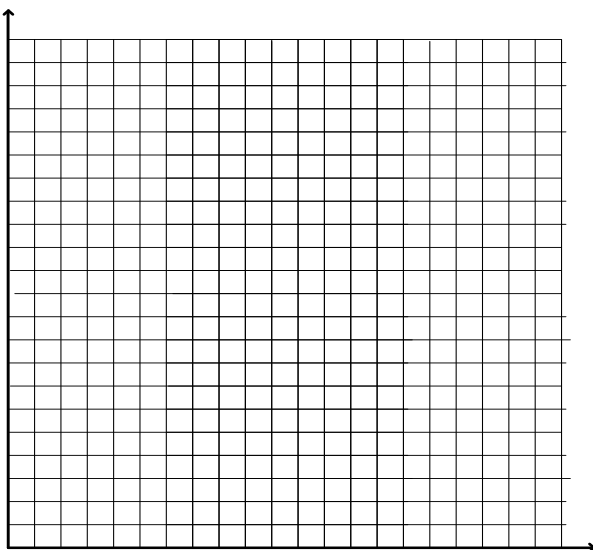
p. 337 #2.



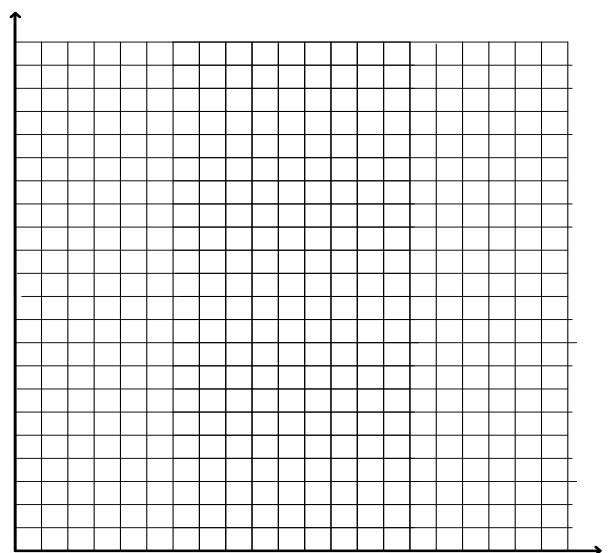
p. 338 #4.



p. 343 #4.



p. 374 #4. (*Independent variable is # of hours of TV watched*)

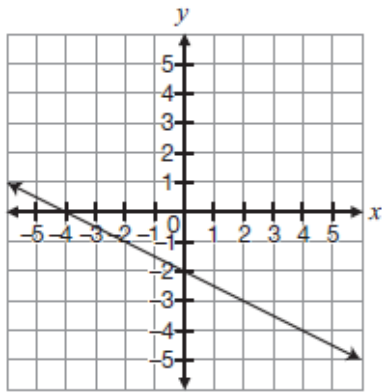


Unit 7 Review – Part I

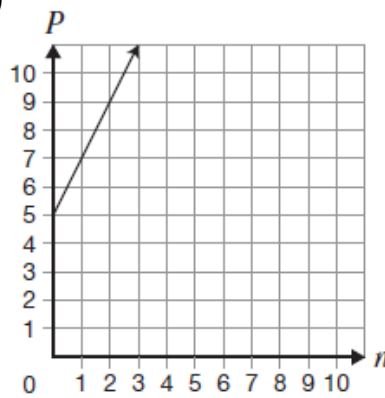
Warm-up:

1. Determine an equation for each of the following linear relations.

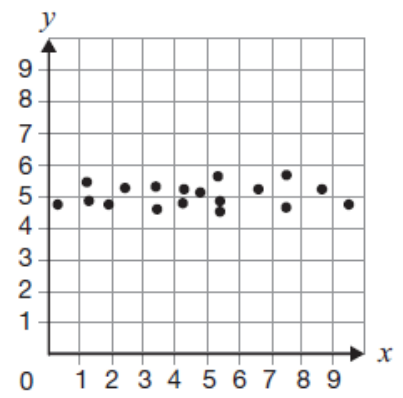
a)



b)



c)



2. The total cost of swimming at a community swimming pool is made up of a membership fee and a cost per swim. At this community centre, Jake pays a total of \$100 and swims 40 times. Paula pays a total of \$70 and swims 25 times. Determine:

- a) an equation for the total cost of swimming at this centre.
- b) the total cost of swimming three times a week for the year.

3. Determine the value of k if the points $A(-3, -6)$, $B(1, k)$ and $C(4, 8)$ are **collinear**.

4. Determine the equation of a line that is perpendicular to the line $5x - 3y + 15 = 0$ with the same x -intercept as the line $y = \frac{1}{2}x + 3$.

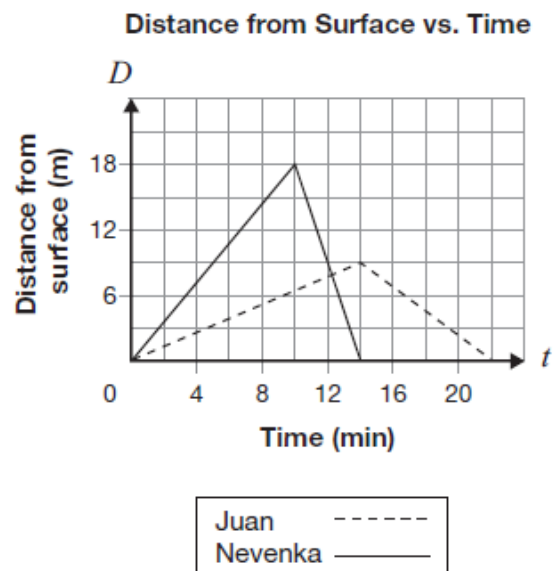
5. Nevenka and Juan scuba dive. The graph below represents the relationship between the distance from the surface, in metres, and time, in minutes, for both divers as they swim down from the surface and then swim back up.

a) Complete the description of Nevenka's dive.

Nevenka enters the water and swims down to a depth of _____ below the surface of the water at a rate of _____. Nevenka then immediately swims back up to the water's surface at a rate of _____.

b) Complete the description of Juan's dive.

Juan enters the water and swims down to a depth of _____ below the surface of the water at a rate of _____. Juan then immediately swims back up at a rate of _____.



Unit 7 Review – Part II

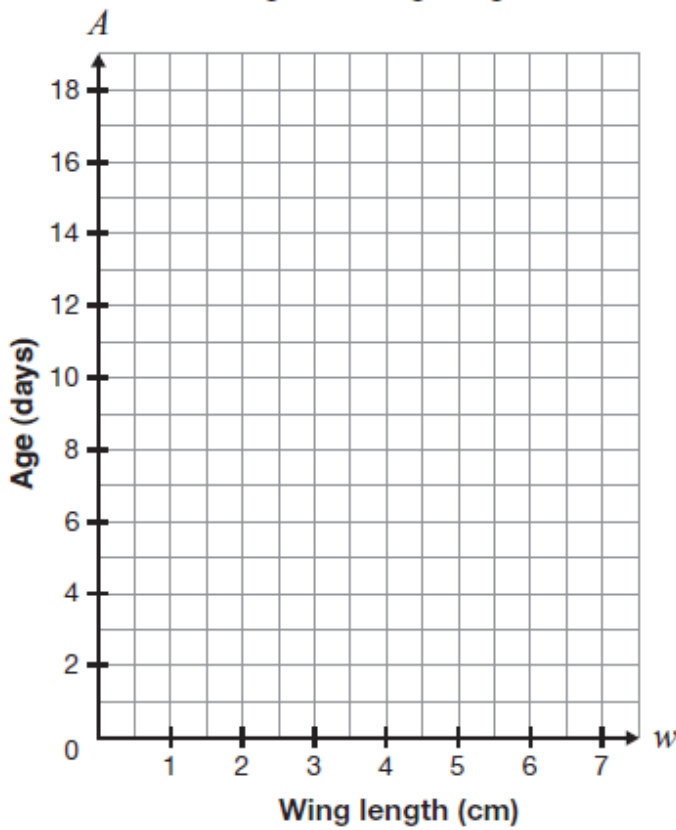
Warm-up:

1. Wing length is a reliable method for determining the age of young birds. Below is an example of data for a particular species.

Wing length (cm)	Age (days)
*1.5	4
3.1	8
3.2	10
*4.0	12
5.2	16

- a) Construct a scatter plot on the grid provided for the data in the table.
- b) Sketch a line of best fit.
- c) Determine an equation for the line of best fit. *(Make sure the line passes through the two points with a *.)*
- d) Use the graph to estimate the age of a bird with a wing length of 3.6 cm.
- e) Use the equation to estimate the wing length of a three week old bird.

Age vs. Wing Length



2. Abigail buys a prepaid card for her cellphone. When she talks on her phone, a fee per minute is deducted from the value of the prepaid card. The table below shows information about the remaining value of the card.
- Determine the equation that represents the relationship between the remaining value and the total number of minutes used.
 - What do the slope and V -intercept mean in this situation?
 - How many minutes can she use before she has to reload her card?

Total number of minutes used, t	Remaining value, V (\$)
10	22.00
20	19.00

3. Tyler walks along a line leading from a motion sensor. Use the information from the graph below to describe Tyler's walk.

Description of Tyler's Walk:

Initially Tyler is _____ from the motion sensor.
 After _____ of a second he walks _____
 from the motion sensor at a speed of _____
 for _____ seconds until he is _____
 away from the sensor. He remains _____
 away from the sensor for _____ seconds
 before he walks back.

Tyler walks back toward the sensor at a speed of _____
 for _____ seconds until he is _____
 away from the sensor. He remains at _____
 away from the sensor for _____ second.

