WORKSHOP: Performing Calculations Using Formulas in Geometry

A. Substitution Rules

1. Write the formula out in its general form before substituting.
2. Place brackets around each numerical quantity/value that replaces a variable in a formula.
3. Promote any existing brackets in a formula to square brackets during your substitution step.
4. When the formula includes π, use the π button on your calculator and do **not** replace the π symbol with a numerical value during your substitution step (*note: if your calculator does not have a π button, then you will replace the π symbol with 3.14159 and replace the = with* ≐ at this step in addition to the final step).

Ex. 1. Substitute the appropriate values into each of the following formulas:

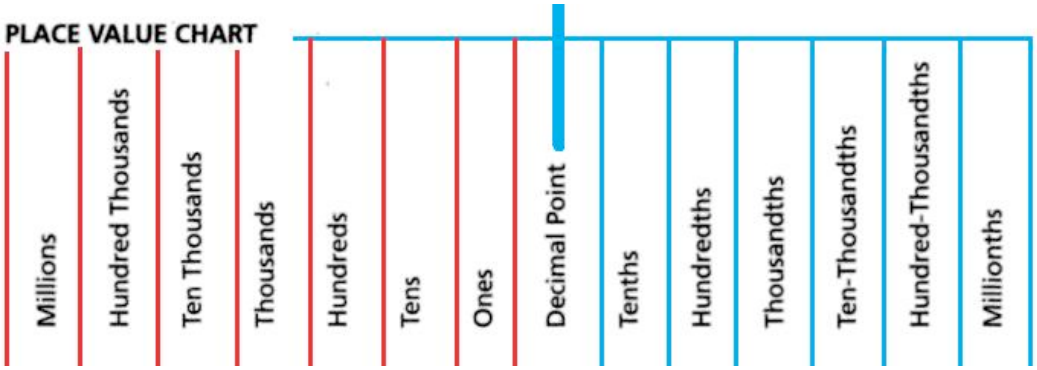
a) Given: $C = 2\pi r$, where r = 57 cm

b) Given: $A = \frac{(a+b)h}{2}$, where a = 3.4 mm, b = 2.2 mm, h = 5.0 mm

c) Given: $V = \frac{\pi r^2 h}{3}$, where r = 3.75 m, h = 6.1 m

d) Given: $A = \pi r^2 + 2\pi rh + \pi rs$, where r = 5 m, s = 8 m, h = 11 m

B. Rounding and Place Value Review



1. When rounding to a given place value, the number directly ***to the right*** determines how you will round.
2. a) If the number to the right is 5 or greater, round the required digit up one and remove all values thereafter.
   b) If the number to the right is less than 5, keep the required digit the same and remove all values thereafter.

Ex. 2. Round 725.6499 to the given place value.

a) Unit: _____ **b)** Tenth: _____ **c)** Hundredth: _____

d) Thousandth: _____ **e)** Ten: _____ **f)** Whole number: _____

C. Calculation Rules

1. Before you begin your calculations, be aware of the required rounding for your final answer.
2. If the problem is multi-step and requires an intermediate value (i.e. using Pythagorean theorem), round this value to at least one decimal place greater than the rounding required for your final answer.
3. Perform the final calculation (i.e. perimeter, area, volume) by typing the entire expression into your calculator in one step. *Note*: For formulas in fraction form, you will need to type brackets around your numerator and denominator.

Ex. 3. **a)** Go to **Ex. 1.** and complete the calculations for each equation. Round questions **a)** and **d)** to the nearest unit and round questions **b)** and **c)** to the nearest tenth.

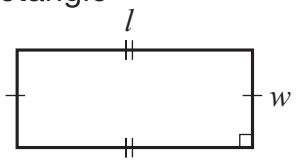
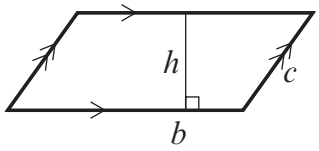
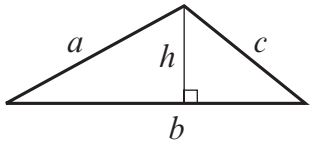
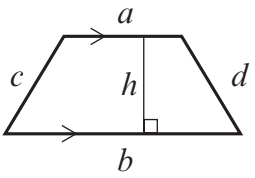
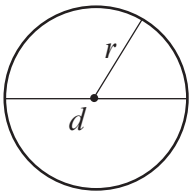
b) Given the following: $h^2 = c^2 - b^2$, $A = \frac{bh}{2}$, $c = 4.68$ m, and $b = 2.3$ m

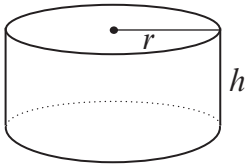
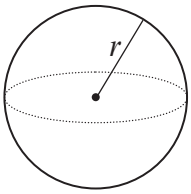
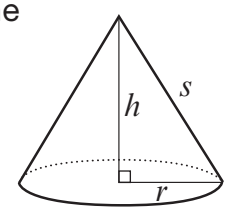
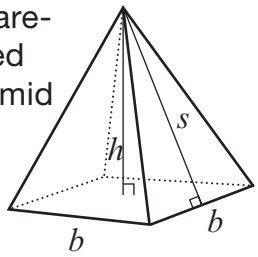
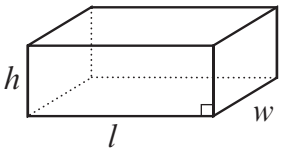
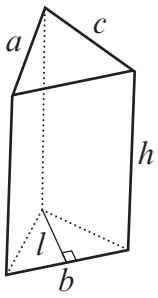
Calculate the value of *h* and use it to find the value of *A*, to one decimal place.

HW: Worksheet #7

Formula Sheet

Grade 9 Academic

Geometric Figure	Perimeter	Area
Rectangle 	$P = l + l + w + w$ or $P = 2(l + w)$	$A = lw$
Parallelogram 	$P = b + b + c + c$ or $P = 2(b + c)$	$A = bh$
Triangle 	$P = a + b + c$	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$
Trapezoid 	$P = a + b + c + d$	$A = \frac{(a + b)h}{2}$ or $A = \frac{1}{2}(a + b)h$
Circle 	$C = \pi d$ or $C = 2\pi r$	$A = \pi r^2$

Geometric Figure	Surface Area	Volume
Cylinder 	$A_{\text{base}} = \pi r^2$ $A_{\text{lateral surface}} = 2\pi rh$ $A_{\text{total}} = 2A_{\text{base}} + A_{\text{lateral surface}}$ $= 2\pi r^2 + 2\pi rh$	$V = (A_{\text{base}})(\text{height})$ $V = \pi r^2 h$
Sphere 	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$ or $V = \frac{4\pi r^3}{3}$
Cone 	$A_{\text{lateral surface}} = \pi rs$ $A_{\text{base}} = \pi r^2$ $A_{\text{total}} = A_{\text{lateral surface}} + A_{\text{base}}$ $= \pi rs + \pi r^2$	$V = \frac{(A_{\text{base}})(\text{height})}{3}$ $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$
Square-based pyramid 	$A_{\text{triangle}} = \frac{1}{2}bs$ $A_{\text{base}} = b^2$ $A_{\text{total}} = 4A_{\text{triangle}} + A_{\text{base}}$ $= 2bs + b^2$	$V = \frac{(A_{\text{base}})(\text{height})}{3}$ $V = \frac{1}{3}b^2 h$ or $V = \frac{b^2 h}{3}$
Rectangular prism 	$A = 2(wh + lw + lh)$	$V = (A_{\text{base}})(\text{height})$ $V = lwh$
Triangular prism 	$A_{\text{base}} = \frac{1}{2}bl$ $A_{\text{rectangles}} = ah + bh + ch$ $A_{\text{total}} = A_{\text{rectangles}} + 2A_{\text{base}}$ $= ah + bh + ch + bl$	$V = (A_{\text{base}})(\text{height})$ $V = \frac{1}{2}blh$ or $V = \frac{blh}{2}$

 Date: _____

Worksheet #7

1. Round each number to the given place value.

   a) 409.0196 ≐ _____ (hundredths)

   b) $\pi \doteq$ _____ (hundred thousandths)

   c) 15.9761 ≐ _____ (tens)

   d) 0.4604 ≐ _____ (hundredths)

   e) $\sqrt{37} \doteq$ _____ (tenths)

   f) $\sqrt{24.036} \doteq$ _____ (unit)

For questions #2–8, complete each required calculation.

i) Use the EQAO Formula Sheet provided.
ii) Be sure to follow the substitution and calculation rules in Lesson 5.
iii) Round each final answer as indicated.
iv) Use a ∴ statement, using the word "approximately" when necessary, and the appropriate units to complete each solution.

2. Given a parallelogram where $b = 5.26 \text{ cm}, c = 3.68 \text{ cm}, \text{and } h = 2.50 \text{ cm},$ determine the:
   a) perimeter, to the nearest unit.
   b) area, to the nearest tenth.

3. Given a trapezoid where $a = 14.3 \text{ m}, b = 17.1 \text{ m}, c = 8.7 \text{ m}, d = 7.3 \text{ m}, \text{and } h = 6.9 \text{ m},$ determine the area to one decimal.

4. Given a circle with a diameter of 4.28 mm, determine the:
   a) circumference, to the nearest hundredth.
   b) area, to two decimals.

5. Given a cylinder with a radius of 8 cm and a height of 5 cm, determine the:
   a) total surface area, to the nearest whole number.
   b) volume, to the nearest tenth.

6. Given a sphere with a radius of 9.05 m, determine the volume to two decimals.

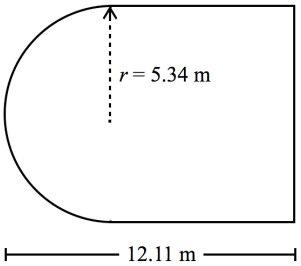
7. Given a cone with a radius of 5 ft a height of 12 ft and a slant height of 13 ft, determine the:
   a) total surface area, to the nearest tenth.
   b) volume, to the nearest unit.

8. Given a square-based pyramid where $b = 7.37 \text{ cm}$, and $h = 12.1 \text{ cm}$, you wish to determine the total surface area, to the nearest hundredth. To do so, you will first need to complete an intermediate step to find the slant height (s). Determine the:
   a) slant height (s) by using the Pythagorean Theorem. Round your answer to the appropriate number of decimal places based on the rounding requirement for total surface area [to be calculated in part b)].

   b) total surface area, to the nearest hundredth.

9. In each of the following solutions, identify the ***two*** errors and explain each error. Make the necessary corrections to complete each solution.

a) Determine the perimeter of the following figure, to the nearest hundredth.

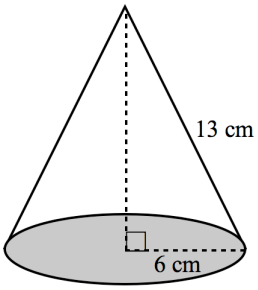


$$C = \frac{2\pi r}{2}$$
$$= \pi(5.34)$$
$$\doteq 16.8$$

$$P \doteq 6.77 + 10.68 + 6.77 + 16.8$$
$$\doteq 41.02$$

$\therefore$ *the perimeter of the figure is 41.02 m.*

b) Determine the volume of the cone, to one decimal.



$$h^2 = 13^2 + 6^2$$
$$= 205$$
$$h = \sqrt{205}$$
$$\doteq 14.32$$

$$V = \frac{\pi r^2 h}{3}$$
$$\doteq \frac{\pi(6)^2(14.32)}{3}$$
$$\doteq 539.8$$

$\therefore$ *the volume of the cone is approximately 539.8 cm^3.*

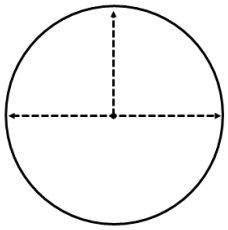
Answers:

1. a) 409.02 b) 3.14159 c) 20 d) 0.46 e) 6.1 f) 5
2. a) 18 cm b) 13.2 cm^2
3. 108.3 m^2
4. a) 13.45 mm b) 14.39 mm^2
5. a) 653 cm^2 b) 1005.3 cm^3
6. 3104.81 m^3
7. a) 282.7 ft^2 b) 314 ft^3
8. a) 12.649 cm b) 240.76 cm^2
9. a) Error #1: Round the circumference calculation to 3 decimals. Error #2: Use the word "approximately" in the "therefore" statement. When the errors are corrected, the final rounded answer will be 41.00 m.
9. b) Error #1: The Pythagorean calculation is incorrect. Instead of calculating the hypotenuse, the calculation should be for the leg of the right triangle. Error #2: The answer for the volume was rounded incorrectly. It should have been 539.9 cm^3. When the errors are corrected, the final rounded answer will be 382.6 cm^3.

 Date: _____

Perimeter

Formulas



C =

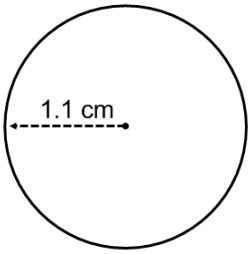
d =

r =

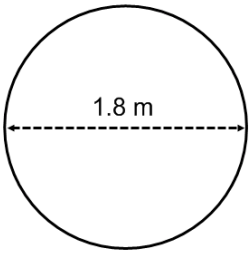
π =

Ex. 1. Find the circumference to one decimal place.

a)

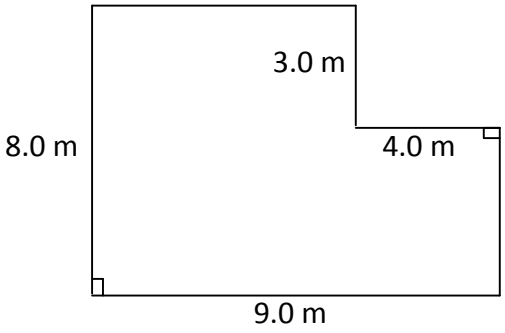


b)

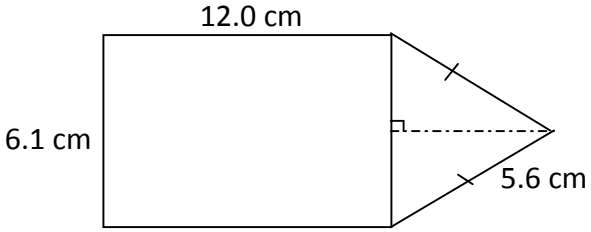


Ex. 2. Find the perimeter (the distance around the outside) of each figure, to one decimal place.

a)

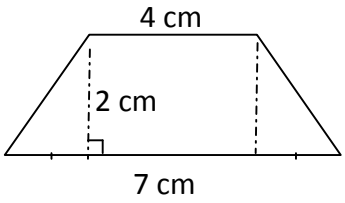


b)

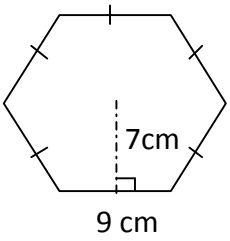


Ex. 3. Find the perimeter of each figure, to the nearest unit.

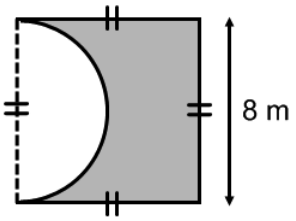
a)



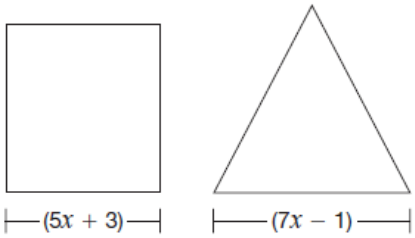
b)



Ex. 4. Find the perimeter of the shaded figure, to the nearest tenth.



Ex. 5. A square and an equilateral triangle below have the same perimeter. Find x .



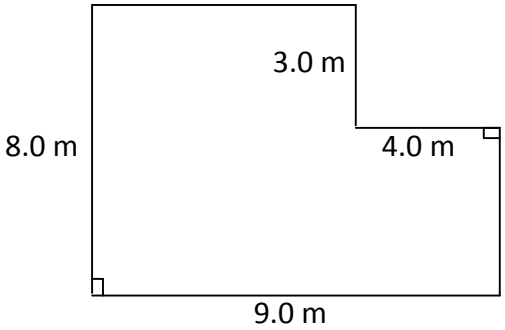
HW: p. 510 #1bd; p. 484 #11; p. 512 #1 (circumference only), 2b; p. 440 #2a (perimeter only), 4 (perimeter only), 5ac (perimeter only).

 Date: _____

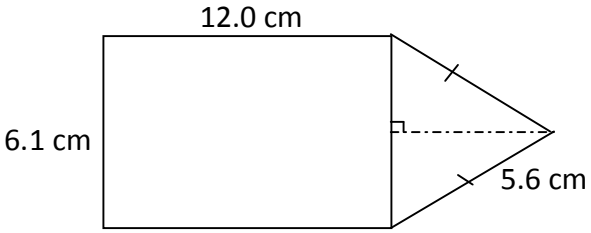
Area

Ex. 1. Find the area of each figure to one decimal place.

a)

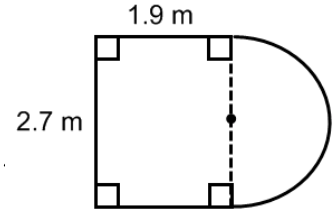


b)

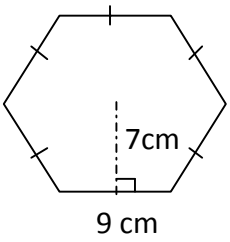


Ex. 2. Find the area of each figure to the nearest unit.

a)

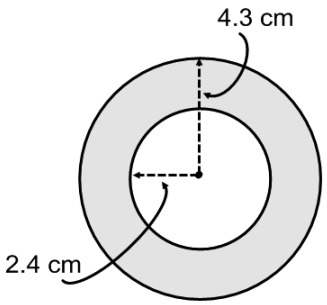


b)

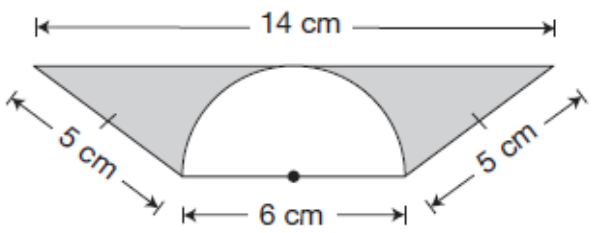


Ex. 3. Find the area of the shaded region to one decimal place.

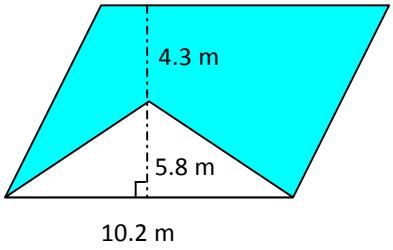
a)



b)



Ex. 4. Find the shaded area to one decimal place.



Ex. 5. Find the radius and diameter of a circle with an area of 78.5 m^2 to one decimal place.

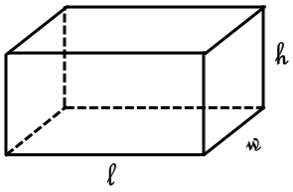
HW: p. 510 #2bcef, 3bc; p. 512 #1 (area only), 2a, 3; p. 440-441 #2a (area only), 3, 5ac (area only); p. 484 #12

Volume

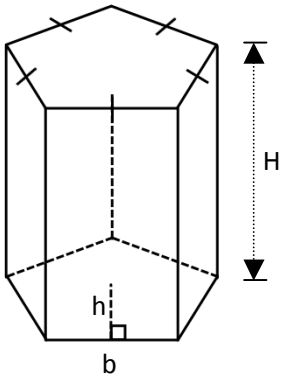
The volume of a prism or cylinder is the product of the area of the base and the height.

Prisms

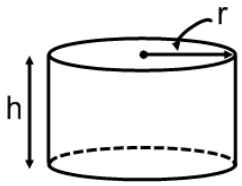
A.



B.



Cylinders

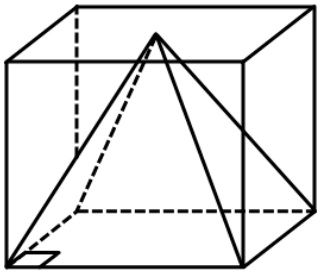


Pyramids

All pyramids have triangular sides.
The base can be any polygon.

The volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height.

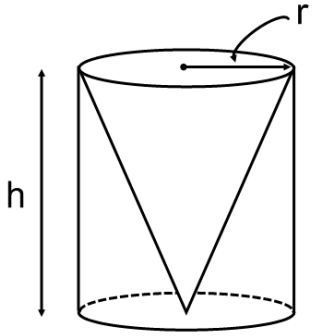
$$V_{pyramid} = \frac{1}{3} A_{base} \times height$$



Cones

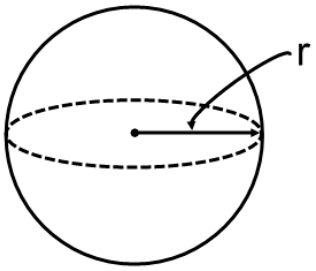
The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same radius and height.

$$V_{cone} = \frac{1}{3} A_{base} \times height \text{ or } V_{cone} = \frac{1}{3} \pi r^2 h$$



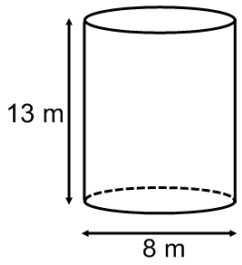
Spheres

$$V_{sphere} = \frac{4}{3} \pi r^3$$

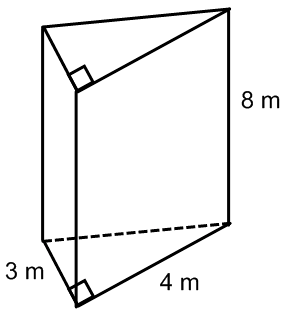


Ex. 1. Calculate each volume, to one decimal place.

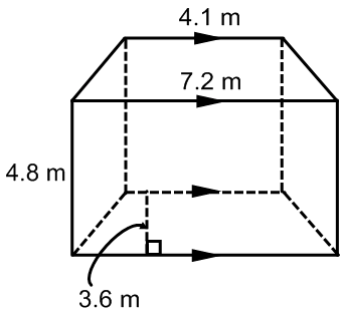
a)



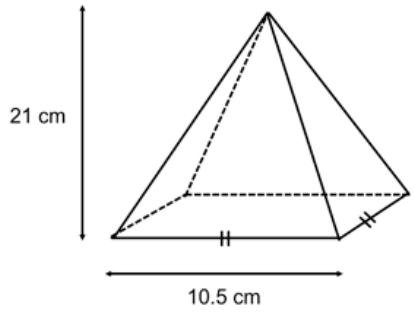
b)



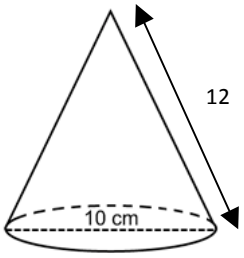
c)



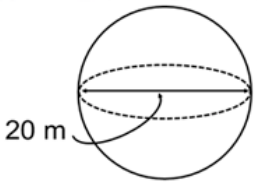
d)



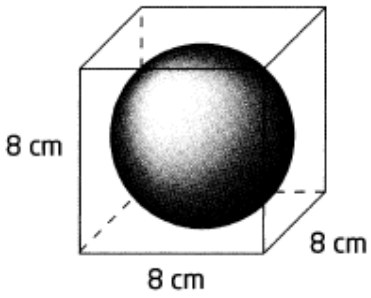
e)



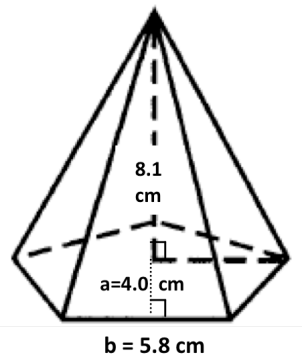
f)



Ex. 2. A sphere just fits inside an 8 cm by 8 cm by 8 cm cubic box. What percent of the box is empty to the nearest percent?



Ex. 3. An oil lamp with a reservoir in the shape of a pyramid has a regular pentagonal base as shown and a height of 8.1 cm. If the oil comes in 750 ml bottles, how many times can the lamp be completely filled with one bottle of oil? (1 ml = 1 cm^3)



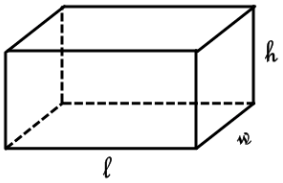
Ex. 4. The volume of a rectangular prism is represented by $12x^3$. Determine an expression for the area of the base if the height is represented by $3x$.

HW: p. 515 #1ac (volume only), 2a, 3, 4; p. 464-465 #1, 2, 6, 7, 10 (1 ml = 1 cm^3); p. 470-472 #2, 6, 9a, 14

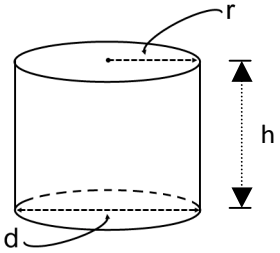
 Date: _____

Surface Area

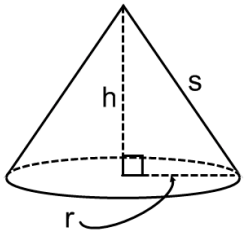
Rectangular Prism S.A. = $A_{left\ \&\ right} + A_{bottom\ \&\ top} + A_{front\ \&\ back}$



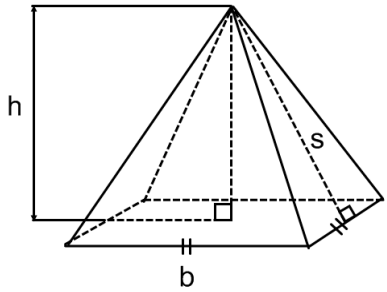
Cylinder S.A. = $A_{top} + A_{base} + A_{lateral\ surface}$



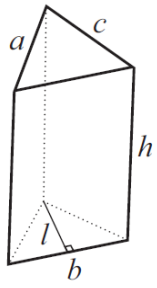
Cone S.A. = $A_{base} + A_{lateral\ surface}$



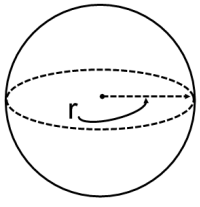
Pyramid S.A. = $A_{bottom} + 4A_{side}$



Triangular Prism S.A. = $A_{top} + A_{bottom} + A_{rectangles}$

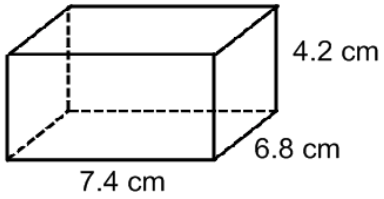


Sphere S.A. =

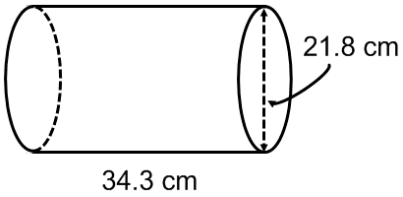


Ex. 1. Calculate the total surface area to one decimal place.

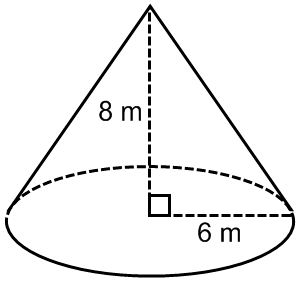
a)



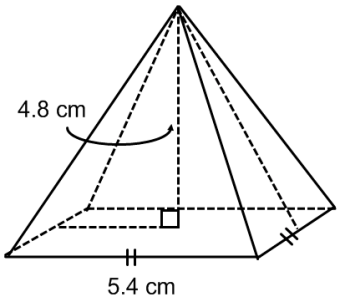
b)



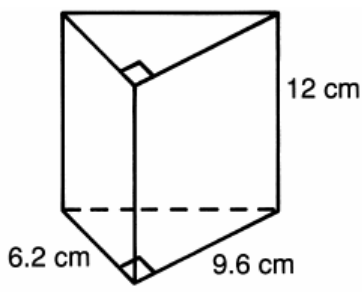
c)



d)



e)

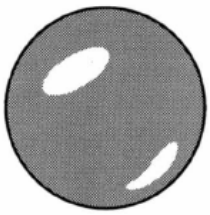


Ex. 2. A spherical weather balloon 16 m in diameter is to be painted with a special reflective coating.

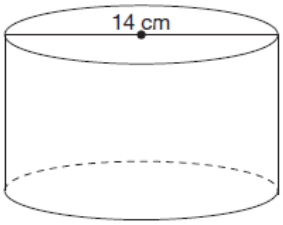
a) Determine the area, to the nearest square metre, to be painted.

b) If 1 tin of paint covers 20 m^2, how many tins of paint are required?

c) What is the total cost of painting the balloon if each tin costs $14.95?



Ex. 3. The cylinder pictured below has a surface area of 660 cm^2. Determine the height of the cylinder to one decimal place.

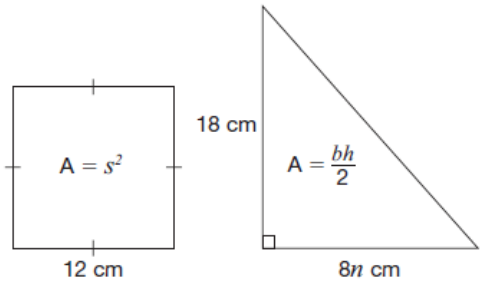


Ex. 4. If the radius of a sphere is doubled, the surface area will increase by a factor of _____ and the volume will increase by a factor of _____ .

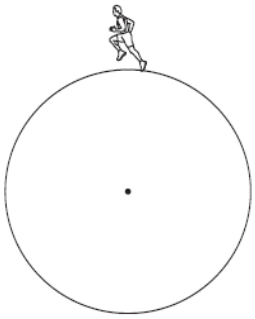
HW: p. 515 #1ac (surface area only), 5bc; p. 424 #2ac; p. 426 #6 (surface area only); p. 455-456 #3, 5, 8c, 13; p. 470 #8; p. 238 #14ab

Applications: Perimeter, Area, Surface Area and Volume

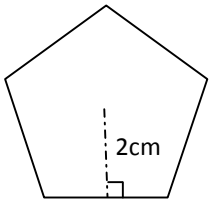
Ex. 1. A square and the triangle below have the same area. Find n .



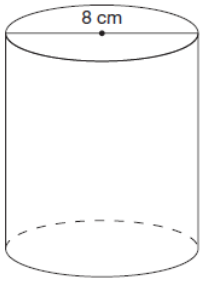
Ex. 2. The distance covered in 5 laps of a circular track is 400π metres. What is the shortest distance between any point on the track and the centre to the nearest tenth of a metre?



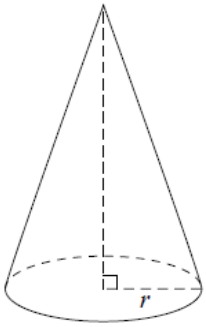
Ex. 3. Find the side length of the given regular polygon if the area is 27.5 cm^2 (to one decimal place).



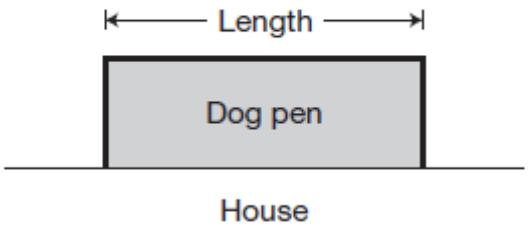
Ex. 4. The cylinder below has a volume of 150 cm^3. Find the lateral surface area to the nearest hundredth.



Ex. 5. The cone shown is 20 cm high and has a total volume of 1000 cm^3. Determine the slant height to the nearest tenth.



Ex. 6. Marcus is building a rectangular dog pen along the side of his house as shown below.



Marcus has 20 m of fencing for the 3 sides of the dog pen.
What is the length of the dog pen with the maximum area?

a 4 m

b 5 m

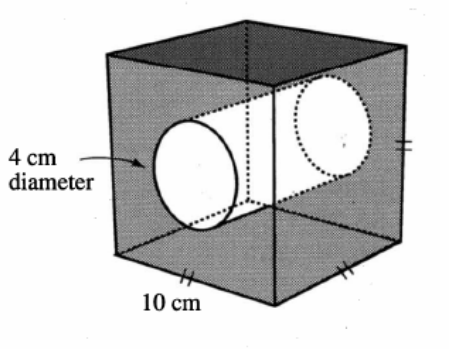
c 10 m

d 12 m

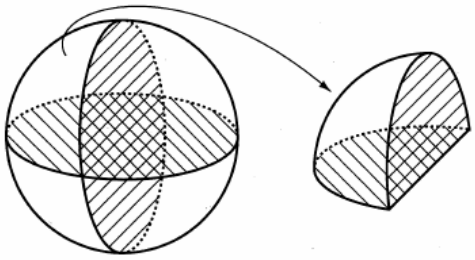
Ex. 7. a) Determine the maximum area of a rectangle with a perimeter of 32 cm.
Note: The rectangle with a maximum area for a given perimeter is a square.

b) Determine the minimum length of wood needed to build a rectangular frame for an area of 132 cm^2.
Note: The rectangle with a minimum perimeter for a given area is a square.

Ex. 8. A metal casting has dimensions as shown. If the density of the material is 6.8 g/cm^3, determine the mass of the casting, in kg.
Note: Mass = Density x Volume



Ex. 9. A window display is made from a Styrofoam ball 1.8 m in diameter. The ball is cut into 4 equal parts as shown.
Determine the cost of covering the one quarter sphere on all 3 surfaces with "glitter" paint, if one tin of paint covers 1.5 m^2, and costs $4.95. Include 13% tax in the total cost.



HW: p. 250 #8; p. 441 #7; p. 472 #16; p. 484-485 #1-5, 7, 8, 14, 16, 17, 21, 22.