 Exam Date: _____ Time: _____ Room: _____

Numerical Skills

Name: _____

1. Evaluate.

a) $\frac{1}{3}-\frac{5}{6}-1\frac{1}{3}$

$=\frac{2}{6}-\frac{5}{6}-\frac{8}{6}$

$=-\frac{11}{6}$

b) $1.5-\left(\frac{3}{4}+\frac{1}{4}\right)$

$=\frac{3}{2}-1$

$=\frac{3}{2}-\frac{2}{2}$

$=\frac{1}{2}$

c) $\frac{-4}{5}\left(\frac{-3}{4}+\frac{1}{3}\right)$

$=-\frac{4}{5}\left(-\frac{9}{12}+\frac{4}{12}\right)$

$=-\frac{1}{5}\left(-\frac{5}{12}\right)$

$=\frac{1}{3}$

d) $\left(1\frac{4}{5}+0.75\right)\left(-\frac{1}{3}+\frac{-1}{4}\right)$

$=\left(\frac{9}{5}+\frac{3}{4}\right)\left(-\frac{1}{3}-\frac{1}{4}\right)$

$=\left(\frac{36}{20}+\frac{15}{20}\right)\left(-\frac{4}{12}-\frac{3}{12}\right)$

$=\left(\frac{51}{20}\right)\left(-\frac{7}{12}\right)$

$=-\frac{119}{80}$

e) $\frac{-5}{7}\times\frac{28}{5}\div\left(-\frac{8}{6}\right)$

$=\frac{5}{7}\times\frac{28}{5}\times\frac{6}{8}$

$=3$

f) $3\frac{1}{2}+\frac{8}{15}\div\frac{4}{45}$

$=\frac{7}{2}+\frac{8}{15}\times\frac{45}{4}$

$=\frac{7}{2}+\frac{12}{2}$

$=\frac{19}{2}$ [or $9\frac{1}{2}$]

g) $\sqrt{3^2}+4^3+12^2$

$=3+64+144$

$=211$

h) $\frac{5\sqrt{16}-8}{3}$

$=\frac{5(4)-8}{3}$

$=\frac{20-8}{3}$

$=\frac{12}{3}$

$=4$

2. Which of the following are rational numbers?

a) $0.7=\frac{7}{9}$ b) $3.14=3\frac{14}{100}=\frac{314}{100}$ c) $1\frac{3}{4}=\frac{7}{4}$ d) $0.6=\frac{6}{10}$ e) $\sqrt{5}$ [irrational]

3. Write as a fraction in lowest terms.

a) 0.62

$=\frac{62}{100}$

$=\frac{31}{50}$

b) 1.4

$=\frac{14}{10}$

$=\frac{7}{5}$

c) -7.25

$=-7\frac{1}{4}$

$=-\frac{29}{4}$

4. Write in decimal form.

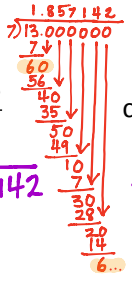
a) $-\frac{22}{3}$

$=-7\frac{1}{3}$

$=-7.\dot{3}$

b) $\frac{13}{7}$

$=1.\overline{857142}$



c) $-4\frac{7}{8}$

$=-4.875$

d) 5.0×10^5

$=500\,000$

e) 2.85×10^{-4}

$=0.000\,285$

5. Write in scientific notation.

a) $37\,300\,000$

$=3.73\times10^7$

b) $0.000\,000\,000\,015\,4$

$=1.54\times10^{-11}$

c) $200\,000\,000\,000\times0.000\,000\,000\,7$

$=2.0\times10^{11}\times7.0\times10^{-10}$

$=2.0\times7.0\times10^{11}\times10^{-10}$

$=14\times10$

$=1.4\times10^2$

6. Simplify.

a) $a \cdot a^3$

$= a^4$

b) $(2a^2)(3a)$

$= 6a^3$

c) $b^7 \div b^3$

$= b^4$

d) $10x^{10} \div 20x^5$

$= \frac{1}{2}x^5$

e) $3a^0$

$= 3(1)$

$= 3$

7. Evaluate.

a) $2^{-1} + 3^2$

$= \frac{1}{2} + 9$

$= 9\frac{1}{2}$ [or $\frac{19}{2}$]

b) $\left(\frac{-4}{3}\right)^{-1}$

$= -\frac{3}{4}$

c) $\left(\frac{2}{3}\right)^{-3}$

$= \left(\frac{3}{2}\right)^3$

$= \frac{27}{8}$

d) $\left(\frac{3}{4}\right)^0$

$= 1$

e) $2^2 \times 3^{-2}$

$= 4\left(\frac{1}{9}\right)$

$= \frac{4}{9}$

f) $\frac{5}{2^{-3}}$

$= 5(2^3)$

$= 5(8)$

$= 40$

g) $\frac{3^6 \times 3^7}{3^{10}}$

$= \frac{3^{13}}{3^{10}}$

$= 3^3$

$= 27$

8. Simplify, then evaluate for the given values.

a) $\left(x^{-5}\right)^2\left(x^7\right)^3\left(x^2\right)^{-6}$ when $x = 2$

$= (x^{-10})(x^{21})(x^{-12})$

$= x^{-1}$

$= 2^{-1}$

$= \frac{1}{2}$

b) $\frac{\left(m^5\right)^2}{m^{11}} + \frac{\left(n^2\right)^3}{n^7}$ when $m = 3$ and $n = 4$

$= \frac{m^{10}}{m^{11}} + \frac{n^6}{n^7}$

$= m^{-1} + n^{-1}$

$= 3^{-1} + 4^{-1}$

$= \frac{1}{3} + \frac{1}{4}$

$= \frac{4}{12} + \frac{3}{12}$

$= \frac{7}{12}$

c) $a^2 + 5b$ for $a = -\frac{1}{3}$ and $b = -\frac{2}{15}$

$= \left(-\frac{1}{3}\right)^2 + 5\left(-\frac{2}{15}\right)$

$= \frac{1}{9} - \frac{2}{3}$

$= \frac{1}{9} - \frac{6}{9}$

$= -\frac{5}{9}$

9. On a particular corner kick, the height of the soccer ball in metres is given by the formula $h = -2t^2 + 5t$, where t is the time in seconds. What is the height of the ball at 2 seconds?

Let $t = 2$

$h = -2(2)^2 + 5(2)$

$= -2(4) + 10$

$= -8 + 10$

$= 2$

$\therefore$ the ball is 2 metres high at 2 seconds.

Algebra Skills

1. Simplify.

a) $4xy-12y^2+7xy+9y^2$

$= 4xy + 7xy - 12y^2 + 9y^2$

$= 11xy - 3y^2$

b) $2a^2-4a-(3a^2-7a)$

$= 2a^2 - 4a - 3a^2 + 7a$

$= 2a^2 - 3a^2 - 4a + 7a$

$= -a^2 + 3a$

c) $xy+x^2y-4xy+3yx^2$

$= xy - 4xy + x^2y + 3x^2y$

$= -3xy + 4x^2y$

d) $5y^3(2y^4+3y^2+1)$

$= 10y^7 + 15y^5 + 5y^3$

e) $k(5-k)-3(2k-k^2)$

$= 5k - k^2 - 6k + 3k^2$

$= -k^2 + 3k^2 + 5k - 6k$

$= 2k^2 - k$

f) $2t(t+3)+4t(t-2)$

$= 2t^2 + 6t + 4t^2 - 8t$

$= 2t^2 + 4t^2 + 6t - 8t$

$= 6t^2 - 2t$

g) $2x(3x+5y)-2(x^2+3xy)-5y(3x^2-2x)$

$= 6x^2 + 10xy - 2x^2 - 6xy - 15x^2y + 10xy$

$= 6x^2 - 2x^2 + 10xy - 6xy + 10xy - 15x^2y$

$= 4x^2 + 14xy - 15x^2y$

h) $3x(2x+y)-2x[3-(2x+4)]$

$= 6x^2 + 3xy - 2x(3 - 2x - 4)$

$= 6x^2 + 3xy - 2x(-2x - 1)$

$= 6x^2 + 3xy + 4x^2 + 2x$

$= 6x^2 + 4x^2 + 3xy + 2x$

$= 10x^2 + 3xy + 2x$

2. Simplify.

a) $(5a^3b^2c)(2ab^3)$

$= 10a^4b^5c$

b) $(-30x^{10}y^6z)\div(-15x^2y^{-3})$

$= 2x^8y^9z$

c) $\left(\dfrac{-16x^5y^3}{4x^2y^2}\right)^3$

$= (-4x^3y)^3$

$= -64x^9y^3$

d) $\dfrac{4xy-8}{4}$

$= \dfrac{4xy}{4} - \dfrac{8}{4}$

$= xy - 2$

e) $\dfrac{15x^2y^3-20x^5y^2+25x^3y^4}{5xy^2}$

$= \dfrac{15x^2y^3}{5xy^2} - \dfrac{20x^5y^2}{5xy^2} + \dfrac{25x^3y^4}{5xy^2}$

$= 3xy - 4x^4 + 5x^2y^2$

f) $\left(\dfrac{10a^3b^2-8a^2b^3}{2a^2b^2}\right)+\left(\dfrac{15a-21b}{-3}\right)$

$= \left(\dfrac{10a^3b^2}{2a^2b^2} - \dfrac{8a^2b^3}{2a^2b^2}\right) + \left(\dfrac{15a}{-3} - \dfrac{21b}{-3}\right)$

$= 5a - 4b - 5a + 7b$

$= 3b$

3. Factor completely.

a) $3a^2-9a+6ab$

$= 3a(a - 3 + 2b)$

b) $3x^2y-6xy^3$

$= 3xy(x - 2y^2)$

c) $-x^2-10x$

$= -x(x + 10)$

d) $16x^3y^2-4x^2y^3$

$= 4x^2y^2(4x - y)$

e) $-10p^2q-5pq$

$= -5pq(2p + 1)$

f) $3a^2b^3c-10ab^2c^5+7a^3bc^2$

$= abc(3ab^2 - 10bc^4 + 7a^2c)$

4. Solve the following equations. Do a formal check for a) and b).

a) $2x-8=3x+2$

$2x - 3x = 2 + 8$

$\frac{-x}{-1} = \frac{10}{-1}$

$x = -10$

LS $= 2x - 8$ RS $= 3x + 2$

$= 2(-10) - 8$ $= 3(-10) + 2$

$= -20 - 8$ $= -30 + 2$

$= -28$ $= -28$

$\therefore$ LS = RS, $\therefore x = -10$.

b) $6(2-x)=3(x+2)$

$12 - 6x = 3x + 6$

$-6x - 3x = 6 - 12$

$\frac{-9x}{-9} = \frac{-6}{-9}$

$x = \frac{2}{3}$

LS $= 6(2-x)$ RS $= 3(x+2)$

$= 6(2 - \frac{2}{3})$ $= 3(\frac{2}{3} + 2)$

$= 12 - 4$ $= 2 + 6$

$= 8$ $= 8$

$\therefore$ LS = RS, $\therefore x = \frac{2}{3}$.

c) $\dfrac{x+3}{7}=-2$

$7\left(\frac{x+3}{7}\right) = 7(-2)$

$x + 3 = -14$

$x = -14 - 3$

$x = -17$

d) $\frac{3x}{5} = \frac{x}{2} - \frac{3}{5}$

$10^{2}\left(\frac{3x}{5_1}\right) = 10^{5}\left(\frac{x}{2_1}\right) - 10^{2}\left(\frac{3}{5_1}\right)$

$2(3x) = 5x - 6$

$6x = 5x - 6$

$6x - 5x = -6$

$x = -6$

e) $\frac{y+4}{3} = \frac{y+1}{2}$

$^{2}6\left(\frac{y+4}{3_1}\right) = {}^{3}6\left(\frac{y+1}{2_1}\right)$

$2(y+4) = 3(y+1)$

$2y + 8 = 3y + 3$

$2y - 3y = 3 - 8$

$\frac{-y}{-1} = \frac{-5}{-1}$

$y = 5$

f) $\frac{x-1}{2} + \frac{3x+2}{2} = \frac{5}{3}$

$^{3}6\left(\frac{x-1}{2_1}\right) + {}^{3}6\left(\frac{3x+2}{2_1}\right) = {}^{2}6\left(\frac{5}{3_1}\right)$

$3(x-1) + 3(3x+2) = 10$

$3x - 3 + 9x + 6 = 10$

$12x + 3 = 10$

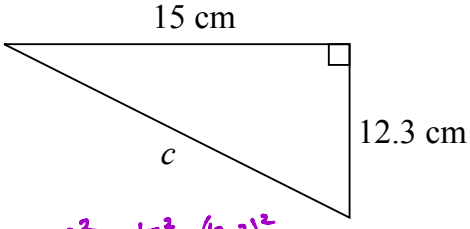
$12x = 10 - 3$

$\frac{12x}{12} = \frac{7}{12}$

$x = \frac{7}{12}$

5. Solve for the indicated side. Round answers to the nearest unit.

a)



$c^2 = 15^2 + (12.3)^2$

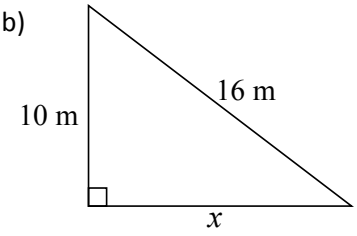
$= 225 + 151.29$

$= 376.29$

$c = \sqrt{376.29}$

$\doteq 19$ cm

b)



$x^2 = 16^2 - 10^2$

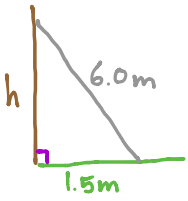
$= 256 - 100$

$= 156$

$x = \sqrt{156}$

$\doteq 12$ m

6. Marsha leans a 6.0 m ladder against a wall. The base of the ladder is 1.5 m from the wall. How far up the wall will the ladder reach?



Let h represent the height up the wall the ladder will reach, in m.

$h^2 = 6^2 - 1.5^2$

$= 36 - 2.25$

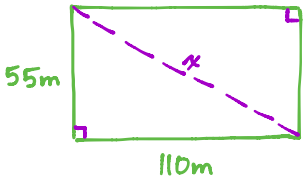
$= 33.75$

$h = \sqrt{33.75}$

$\doteq 5.8$

∴ the ladder will reach approximately 5.8 m up the wall.

7. On his way to school Abdul cuts across a vacant lot that measures 110 m by 55 m. He walks diagonally from corner to corner. One day, a fence is built around the lot and he has to walk around. How much **farther** does he have to walk?



Let x represent the length of the diagonal, in m.

$x^2 = 110^2 + 55^2$

$= 12100 + 3025$

$= 15125$

$x = \sqrt{15125}$

Extended walk $= 110 + 55 - \sqrt{15125}$

$= 165 - \sqrt{15125}$

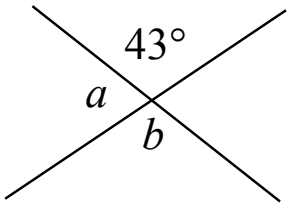
$\doteq 42$

∴ Abdul has to walk approximately 42 m farther.

Angles, Triangles and Parallel Lines

1. Determine the value of each variable. State any angle properties used for each question that has a *. (Drawings are not to scale.)

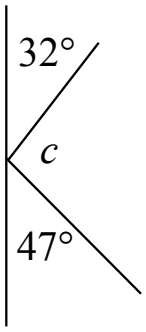
a)



$a = 137°$

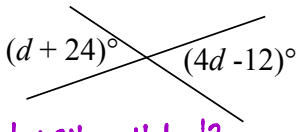
$b = 43°$

*b)



$c = 101°$ [supplementary angles]

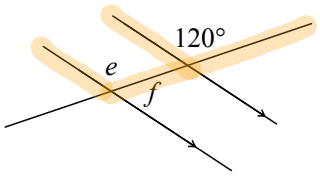
*c)



$d + 24 = 4d - 12$
$d - 4d = -12 - 24$
$\frac{-3d}{-3} = \frac{-36}{-3}$
$d = 12$

$d = 12$ [opposite angles]

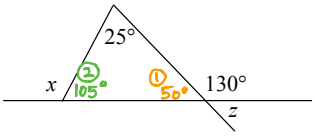
d)



$e = 120°$

$f = 60°$

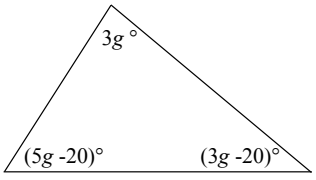
e)



$x = 75°$

$z = 50°$

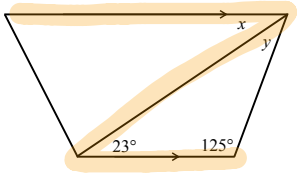
*f)



$3g + 5g - 20 + 3g - 20 = 180$
$11g - 40 = 180$
$11g = 180 + 40$
$\frac{11g}{11} = \frac{220}{11}$
$g = 20$

$g = 20$ [Sum of interior angles of a triangle]

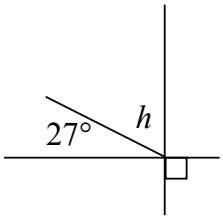
g)



$x = 23°$

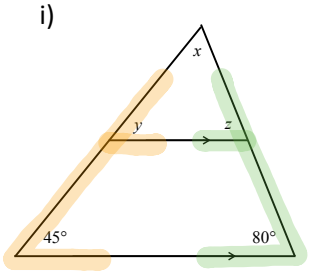
$y = 32°$

*h)



$h = 63°$ [Complementary angles]

i)

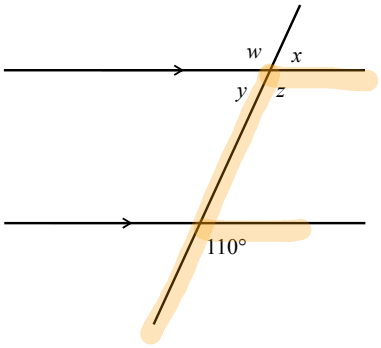


$x = 55°$

$y = 45°$

$z = 80°$

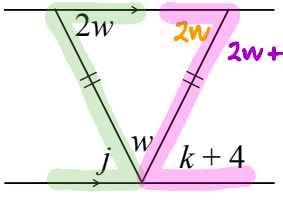
*j)



$w = 110°$ [opposite angle]
$x = 70°$

$y = 70°$ [Supplementary angle]
$z = 110°$ [corresponding angle]

k)



$2w + 2w + w = 180$
$\frac{5w}{5} = \frac{180}{5}$
$w = 36$

$w = 36°$

$j = 72°$

$k = 68°$

$k + 4 = 72$
$k = 72 - 4$
$= 68$