

Introduction to Rational Numbers

NUMBER SETS

Set	Definition	Examples	Symbol
Natural Numbers	the counting numbers	1, 2, 3, 4, ...	N
Whole Numbers	the counting numbers and zero	0, 1, 2, 3, 4, ...	W
Integers	positive and negative whole numbers	..., -3, -2, -1, 0, 1, 2, 3, ...	Z
Rational Numbers	-numbers that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$ - numbers that either <i>repeat</i> or <i>terminate</i> in their decimal form	4, $-\frac{1}{2}$, 8.2, $1.\bar{3}$, $-2\frac{1}{3}$	Q
Irrational Numbers	- numbers that do not <i>repeat</i> or <i>terminate</i> in their decimal form	π , e , $\sqrt{2}$, $\sqrt{3}$	\bar{Q}

*Final answers: no negative in denominator.

Ex. 1. Simplify each rational number to a positive or negative fraction in lowest terms.

a) $\frac{-2}{4}$
 $= -\frac{1}{2}$

b) $\frac{-9}{-12}$
 $= +\frac{3}{4}$

c) $\frac{-8}{10}$
 $= +\frac{4}{5}$

d) $\frac{-14}{-21}$
 $= -\frac{2}{3}$

Ex. 2. Rewrite the following rational numbers as *quotients of two integers* ($\frac{a}{b}$ form).

a) 4
 $= \frac{4}{1}$

b) 0
 $= \frac{0}{1}$

c) $-3\frac{7}{8}$
 $= -\frac{31}{8}$
 $= \frac{-31}{8}$

d) $-1\frac{4}{5}$
 $= -\frac{9}{5}$
 $= \frac{-9}{5}$

no mixed

Ex. 3. Rewrite the following rational numbers as *decimals* (terminating or repeating).

a) $1\frac{1}{4}$
 $= 1.25$

b) $\frac{2}{3}$
 $= 0.\bar{6}$

$3 \overline{) 2.00 \dots}$
 $\underline{-18}$
 20
 $\underline{-18}$
 $2 \dots$

c) $\frac{2}{-5}$
 $= -\frac{2}{5}$
 $= -0.4$

$5 \overline{) 2.0}$
 $\underline{-10}$
 0

d) $-1\frac{1}{6}$
 $= -1\frac{1}{6}$
 $= -1.\bar{1}\bar{6}$

$6 \overline{) 7.000 \dots}$
 $\underline{-6}$
 10
 $\underline{-6}$
 40
 $\underline{-36}$
 $40 \dots$

Ex. 4. Rewrite the following rational numbers as *quotients of two integers* ($\frac{a}{b}$ form).

a) 0.55
 $= \frac{55}{100}$
 $= \frac{11}{20}$

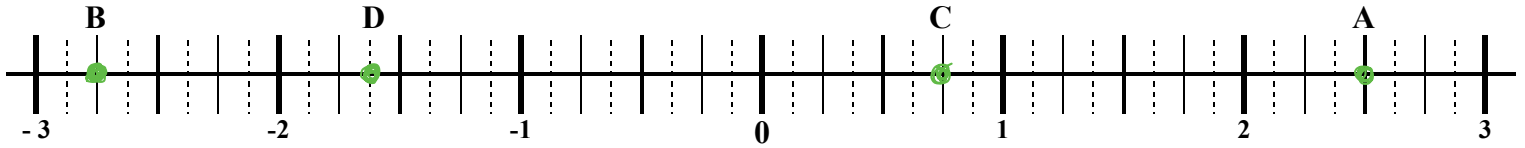
b) -2.8
 $= -2\frac{8}{10}$
 $= -2\frac{4}{5}$
 $= \frac{-14}{5}$

c) $0.\bar{3}$
 $= \frac{3}{9}$
 $= \frac{1}{3}$

d) $-4.\bar{7}$
 $= -4\frac{7}{9}$
 $= \frac{-43}{9}$

no mixed

Ex. 4. Identify the values represented by: i) **A** and **B** in decimal form $A: 2.5, B: -2.75$
 ii) **C** and **D** in simplified fraction form $C: \frac{3}{4}, D: -1\frac{5}{8}$



Ex. 5. Place $<$, $>$, or $=$ to make true statements. Explain how you know each statement is true.

a) $-2.5 < -1.25$
∵ -2.5 is further to the left from 0.

b) $\frac{3}{-5} > \frac{-8}{10}$
 $-\frac{3}{5} > -\frac{8}{10}$
 $-\frac{6}{10} > -\frac{8}{10}$

c) $-2 > \frac{-21}{10}$
 $-\frac{2}{1} > -\frac{21}{10}$
 $-\frac{20}{10} > -\frac{21}{10}$

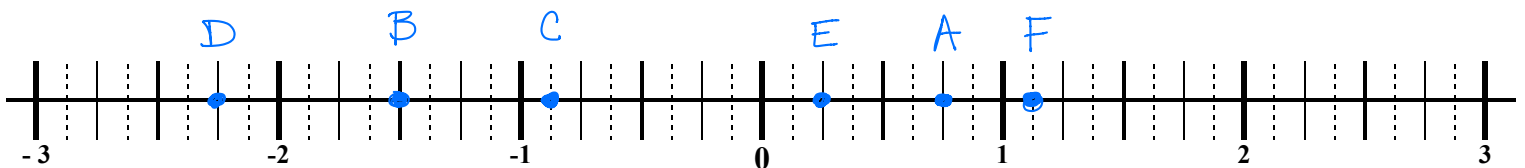
d) $-2.\bar{6} = -\frac{-8}{-3}$
 $-2\frac{6}{9} = -\frac{8}{3}$
 $-2\frac{2}{3} = -\frac{8}{3}$

Ex. 6. Write and use a mathematical expression to determine the rational number which is:

a) $1\frac{5}{6}$ more than $-3\frac{1}{4}$
 $= -3\frac{1}{4} + 1\frac{5}{6}$
 $= -\frac{13}{4} + \frac{11}{6}$
 $= -\frac{39}{12} + \frac{22}{12}$
 $= -\frac{17}{12}$ or $-1\frac{5}{12}$

b) 3.1 less than -5.5
 $= -5.5 - 3.1$
 $= -8.6$

Ex. 7. Graph each rational number on the number line: $\frac{3}{4}$, $-\frac{3}{2}$, $\frac{14}{-16}$, $-\frac{9}{4}$, $-\frac{2}{-8}$, $1\frac{1}{8}$



Multiplying and Dividing Rational Numbers

- Rules :**
- i) Simplify signs. *← only for multiply/divide*
 - ii) Convert all mixed numbers to improper fractions and give all whole numbers a denominator of 1.
 - iii) If dividing, remember "KCI Raiders".
 - iv) Reduce by dividing out common factors in the numerator and denominator.
 - v) Multiply numerators and multiply denominators.
 - vi) Reduce the final answer to lowest terms if necessary.

Ex. 1. Evaluate.

$$\begin{aligned} \text{a) } & \frac{-7}{8} \times \frac{3}{-14} \\ & = + \frac{\cancel{7}^1}{8} \times \frac{3}{\cancel{14}_2} \\ & = \frac{3}{16} \end{aligned}$$

$$\begin{aligned} \text{b) } & 2\frac{1}{4} \div \frac{-3}{8} \\ & = -\frac{9}{4} \div \frac{3}{8} \\ & = -\frac{\cancel{9}^3}{\cancel{4}_1} \times \frac{\cancel{8}^2}{\cancel{3}_1} \\ & = -\frac{6}{1} \\ & \rightarrow = -6 \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{-8}{-9} \times \frac{-3}{5} \div (-40) \\ & = + \frac{8}{9} \times \frac{3}{5} \div \frac{40}{1} \\ & = \frac{\cancel{8}^1}{\cancel{9}_3} \times \frac{\cancel{3}^1}{5} \times \frac{1}{\cancel{40}_8} \\ & = \frac{1}{75} \end{aligned}$$

$$\begin{aligned} \text{d) } & -1\frac{5}{9} \div 2\frac{1}{3} \div \frac{-4}{-9} \\ & = -\frac{14}{9} \div \frac{7}{3} \div \frac{4}{9} \\ & = -\frac{\cancel{14}^2}{\cancel{9}_3} \times \frac{3}{\cancel{7}_1} \times \frac{\cancel{9}^1}{\cancel{4}_2} \\ & = -\frac{3}{2} \text{ or } -\frac{1}{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \text{e) } & -1\frac{1}{4} \div \frac{7}{8} \div (-1.25) \\ & = -\frac{5}{4} \div \frac{7}{8} \div \frac{1}{4} \\ & = -\frac{5}{4} \div \frac{7}{8} \div \frac{5}{4} \\ & = -\frac{\cancel{5}^1}{\cancel{4}_1} \times \frac{8}{7} \times \frac{\cancel{4}^1}{\cancel{5}_1} \\ & = -\frac{8}{7} \end{aligned}$$

Ex. 2. Evaluate.

$$\begin{aligned} \text{a) } & -3.5(-0.6) \\ & = + 2.10 \\ & = 2.1 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{8.6}{-0.2} \\ & = -\frac{8.6 \times 10}{0.2 \times 10} \\ & = -\frac{86}{2} \\ & = -43 \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{-0.45}{-0.009} \\ & = -\frac{0.45 \times 1000}{0.009 \times 1000} \\ & = -\frac{450}{9} \\ & = -50 \end{aligned}$$

Ex. 3. Evaluate.

$$\begin{aligned} \text{a) } & \left(-\frac{1}{4}\right)^2 \\ & = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right) \\ & = +\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) \\ & = +\frac{1}{16} \end{aligned}$$

$$\begin{aligned} \text{b) } & -\left(-\frac{2}{5}\right)^4 \\ & = -\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) \\ & = -\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) \\ & = -\frac{16}{625} \leftarrow \frac{25}{25} \end{aligned}$$

$$\begin{aligned} \text{c) } & (-1\bar{3})^3 \\ & = \left(-1\frac{3}{9}\right)^3 \\ & = \left(-1\frac{1}{3}\right)^3 \\ & = \left(-\frac{4}{3}\right)^3 \\ & = \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right) \\ & = -\left(\frac{4}{3}\right)\left(\frac{4}{3}\right)\left(\frac{4}{3}\right) \\ & = -\frac{64}{27} \text{ or } -2\frac{10}{27} \end{aligned}$$

Adding and Subtracting Rational Numbers

Rules for adding or subtracting:

- i) Eliminate brackets and double signs, making sure all denominators are positive.
- ii) Convert all mixed fractions to improper fractions and give all whole numbers a denominator of 1.
- iii) Find the lowest common denominator, LCD.
- iv) Add or subtract the numerators only. Keep the denominator the same.
- v) Reduce the final answer to lowest terms if possible. (Simplify.)

Ex. 1. Evaluate.

$$\begin{aligned} \text{a) } & \frac{3}{8} - \frac{7}{8} \\ &= -\frac{10}{8} \\ &= -\frac{5}{4} \\ &= -1\frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{b) } & -\frac{5}{6} + 1\frac{1}{2} \\ &= -\frac{5}{6} + \frac{3}{2} \\ &= -\frac{5}{6} + \frac{9}{6} \\ &= +\frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{-2}{3} - \left(-1\frac{1}{5}\right) \\ &= -\frac{2}{3} + \frac{6}{5} \\ &= -\frac{10}{15} + \frac{18}{15} \\ &= +\frac{8}{15} \end{aligned}$$

$$\begin{aligned} \text{d) } & \sqrt{\frac{9}{100}} - \sqrt{\frac{16}{25}} \\ &= \frac{3}{10} - \frac{4}{5} \\ &= +\frac{3}{10} - \frac{8}{10} \\ &= -\frac{5}{10} \\ &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{2}{-3} + \left(-\frac{3}{5}\right) + \frac{-7}{-10} \\ &= -\frac{2 \times 10}{3 \times 10} - \frac{3}{5} + \frac{7}{10} \\ &= -\frac{20}{30} - \frac{18}{30} + \frac{21}{30} \\ &= -\frac{38}{30} + \frac{21}{30} \\ &= -\frac{17}{30} \end{aligned}$$

$$\begin{aligned} \text{e) } & -3.25 - \left(-1\frac{1}{3}\right) + \frac{5}{6} - 2 \\ &= -3\frac{1}{4} + 1\frac{1}{3} + \frac{5}{6} - \frac{2}{1} \\ &= -\frac{13}{4} + \frac{4}{3} + \frac{5}{6} - \frac{2}{1} \\ &= -\frac{39}{12} + \frac{16}{12} + \frac{10}{12} - \frac{24}{12} \\ &= -\frac{63}{12} + \frac{26}{12} \\ &= -\frac{37}{12} \text{ or } -3\frac{1}{12} \end{aligned}$$

Ex. 2. Evaluate.

$$\begin{aligned} \text{a) } & 5.6 - (-3.2) \\ &= 5.6 + 3.2 \\ &= 8.8 \end{aligned}$$

$$\begin{aligned} \text{b) } & -4.5 + 7.8 \\ &= +3.3 \end{aligned}$$

$$\begin{aligned} \text{c) } & -8.9 - (-4) \\ &= -8.9 + 4 \\ &= -4.9 \end{aligned}$$

Ex. 3. The daily changes in selling price for a particular stock during a week were $-\$2.78$, $-\$5.45$, $\$0.38$, $\$1.38$, and $\$2.12$. If the selling price was $\$58.22$ at the start of the week determine:

i) the weekly change in the selling price

$$\begin{aligned} & -2.78 - 5.45 + 0.38 + 1.38 + 2.12 \\ &= -8.23 + 3.88 \\ &= -4.35 \end{aligned}$$

\therefore the weekly change is $-\$4.35$

ii) the selling price at the end of the week

$$\begin{aligned} & 58.22 + (-4.35) \\ &= 58.22 - 4.35 \\ &= 53.87 \end{aligned}$$

\therefore the selling price is $\$53.87$ at the end of the week.

$$\begin{array}{r} 2.78 \\ +5.45 \\ \hline 8.23 \\ 0.38 \\ 1.38 \\ +2.12 \\ \hline 3.88 \\ \hline 8.23 \\ -3.88 \\ \hline 4.35 \end{array}$$

Order of Operations with Rational Numbers

Ex. 1. Evaluate by following the order of operations.

$$-\frac{7}{6} \quad \text{a) } -1\frac{5}{6} - \frac{3}{4} \left(\frac{-8}{9} \right)$$

$$= -\frac{11}{6} + \frac{2}{4} \times \frac{8}{9}$$

$$= -\frac{11}{6} + \frac{2}{3}$$

$$= -\frac{11}{6} + \frac{4}{6}$$

$$= -\frac{7}{6}$$

$$= -1\frac{1}{6}$$

$$-\frac{9}{4}$$

$$\text{c) } \frac{\left[\frac{-5}{8} + \left(\frac{3}{4} \right)^2 \right]}{\left[\left(\frac{1}{2} - \frac{2}{3} \right)^2 \right]} \div$$

$$= \frac{-\frac{5}{8} + \frac{9}{16}}{\left(\frac{3}{6} - \frac{4}{6} \right)^2}$$

$$= \frac{-\frac{10}{16} + \frac{9}{16}}{\left(-\frac{1}{6} \right)^2}$$

$$= \frac{-\frac{1}{16}}{\frac{1}{36}}$$

$$= -\frac{1}{16} \div \frac{1}{36}$$

$$= -\frac{1}{16} \times \frac{36}{1}$$

$$= -\frac{9}{4}$$

$$= -2\frac{1}{4}$$

$$\frac{47}{14} \quad \text{b) } -\frac{4}{7} \times \left(-6\frac{1}{4} - \frac{-3}{8} \right)$$

$$= -\frac{4}{7} \times \left(-\frac{25}{4} + \frac{3}{8} \right)$$

$$= -\frac{4}{7} \times \left(-\frac{50}{8} + \frac{3}{8} \right)$$

$$= -\frac{4}{7} \times \left(-\frac{47}{8} \right)$$

$$= +\frac{4}{7} \times \frac{47}{8}$$

$$= \frac{47}{14}$$

$$= 3\frac{5}{14}$$

Ex. 2. Evaluate by substitution. *must substitute in brackets

$$-\frac{35}{12} \quad \text{a) } ab + 2c \quad \text{if } a = -\frac{2}{3}, b = -\frac{1}{8}, c = -1\frac{1}{2}$$

$$= \left(-\frac{2}{3} \right) \left(-\frac{1}{8} \right) + 2 \left(-1\frac{1}{2} \right)$$

$$= +\frac{2}{3} \times \frac{1}{8} - \frac{2}{1} \times \frac{3}{2}$$

$$= \frac{1}{12} - \frac{3}{1}$$

$$= \frac{1}{12} - \frac{36}{12}$$

$$= -\frac{35}{12}$$

$$= -2\frac{11}{12}$$

$$\text{b) } 4c(a-b) \quad \text{if } a = -1.3, b = 2.6, c = -0.5$$

$$= 4(-0.5) \left[(-1.3) - (2.6) \right]$$

$$= 4(-0.5) \left(-1.3 - 2.6 \right)$$

$$= 4(-0.5)(-3.9)$$

$$= +4(0.5)(3.9)$$

$$= +2(3.9)$$

$$= +7.8$$

Order of Operations with Rational Numbers Continued

Ex. 1. Evaluate.

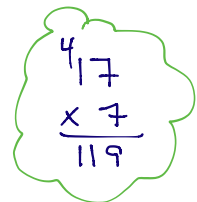
$$\begin{aligned}
 \text{a) } & -1\frac{1}{6} - \frac{-1}{-14} \left(-\frac{4}{5} + \frac{1}{3} \right) \\
 & = -\frac{7}{6} - \frac{1}{14} \left(-\frac{12}{15} + \frac{5}{15} \right) \\
 & = -\frac{7}{6} - \frac{1}{14} \left(-\frac{7}{15} \right) \\
 & = -\frac{7}{6} + \frac{1}{14} \times \frac{7}{15} \\
 & = -\frac{7}{6} + \frac{1}{30} \\
 & = -\frac{35}{30} + \frac{1}{30} \\
 & = -\frac{34}{30} \qquad \qquad \qquad = -\frac{17}{15} \\
 & \qquad \qquad \qquad \qquad \qquad \qquad = -1\frac{2}{15}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \left(\frac{2}{3} - \frac{-1}{4} \right)^2 - \left(\frac{3}{-4} + \frac{2}{3} \right)^2 \\
 & = \left(\frac{2}{3} + \frac{1}{4} \right)^2 - \left(-\frac{3}{4} + \frac{2}{3} \right)^2 \\
 & = \left(\frac{8}{12} + \frac{3}{12} \right)^2 - \left(-\frac{9}{12} + \frac{8}{12} \right)^2 \\
 & = \left(\frac{11}{12} \right)^2 - \left(-\frac{1}{12} \right)^2 \\
 & = \frac{11}{12} \times \frac{11}{12} - \left(-\frac{1}{12} \right) \left(-\frac{1}{12} \right) \\
 & = \frac{121}{144} - \frac{1}{144} \\
 & = \frac{120}{144} \quad \text{or} \quad = \frac{10}{12} \\
 & = \frac{60}{72} \quad \text{or} \quad = \frac{5}{6} \\
 & = \frac{5}{6}
 \end{aligned}$$

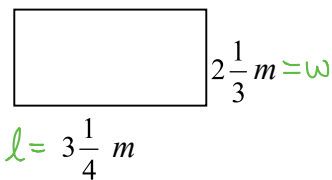
Ex. 2. Evaluate.

$$\begin{aligned}
 \text{a) } & \frac{a+c}{b} \text{ if } a = -\frac{2}{3}, b = \frac{1}{8} \text{ and } c = -1\frac{1}{2} \\
 & = \frac{\left(-\frac{2}{3}\right) + \left(-1\frac{1}{2}\right)}{\left(\frac{1}{8}\right)} \\
 & = \frac{-\frac{2}{3} - \frac{3}{2}}{\frac{1}{8}} \\
 & = \frac{-\frac{4}{6} - \frac{9}{6}}{\frac{1}{8}} \\
 & = -\frac{13}{6} \div \frac{1}{8} \\
 & = -\frac{13}{6} \times \frac{8}{1} \qquad \qquad \qquad = -17\frac{1}{3} \\
 & = -\frac{52}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & (x+y)(x-y) \text{ if } x=1.2 \text{ and } y=-0.5 \\
 & = [(1.2) + (-0.5)][(1.2) - (-0.5)] \\
 & = (1.2 - 0.5)(1.2 + 0.5) \\
 & = (0.7)(1.7) \\
 & = 1.19
 \end{aligned}$$



Ex. 3. Determine the perimeter and area of the rectangle shown



$$\begin{aligned}
 P &= 2l + 2w \\
 P &= 2\left(3\frac{1}{4}\right) + 2\left(2\frac{1}{3}\right) \\
 P &= \frac{2}{1}\left(\frac{13}{4}\right) + \frac{2}{1}\left(\frac{7}{3}\right) \\
 P &= \frac{13}{2} + \frac{14}{3} \\
 P &= \frac{39}{6} + \frac{28}{6} \\
 P &= \frac{67}{6} \\
 \therefore P &= 11\frac{1}{6} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 A &= l \times w \\
 A &= \left(3\frac{1}{4}\right)\left(2\frac{1}{3}\right) \\
 A &= \left(\frac{13}{4}\right)\left(\frac{7}{3}\right) \\
 A &= \frac{91}{12} \\
 \therefore A &= 7\frac{7}{12} \text{ m}^2
 \end{aligned}$$